Near threshold heavy quarkonium photoproduction at large momentum transfer

Xuan-Bo Tong

The Chinese University of HongKong(ShenZhen)

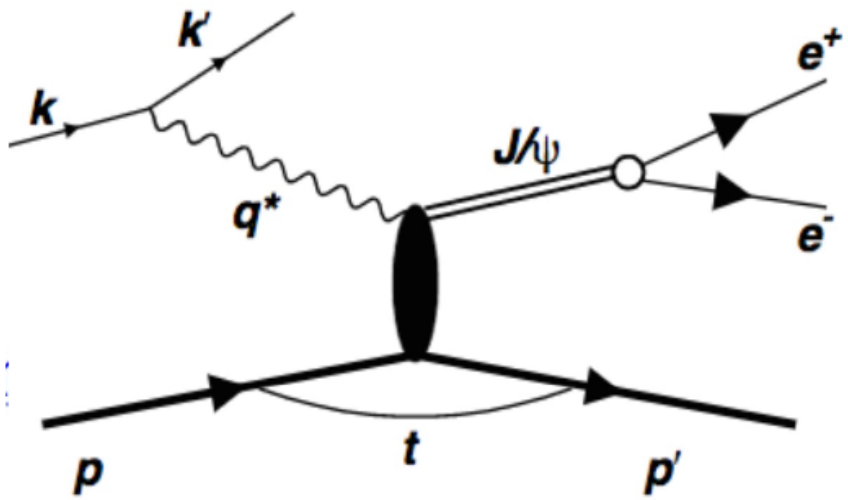
Sep 28, 2022

Refs: P.Sun, X.B. Tong, F.Yuan
Phys.Lett.B 822 (2021) 136655
Phys.Rev.D 105 (2022) 5, 054032

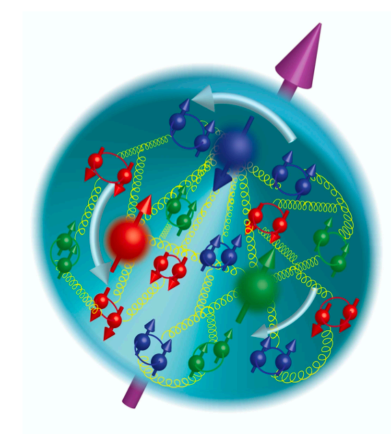
Introduction

Heavy quarkonium production:

$$\gamma^* + N \to V + N'$$



$W_{\gamma p}^2, Q^2 \gg |t|$



nucleon spin partition

In the high energy scattering, the process is well-understood theoretically:

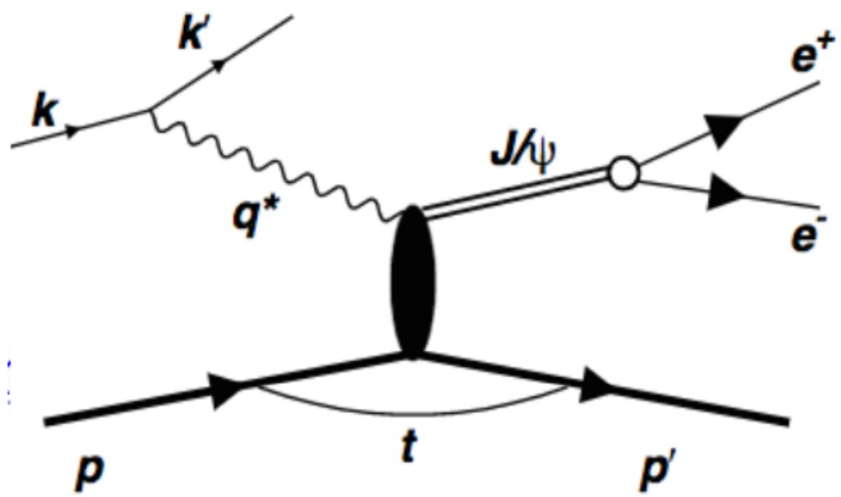
- dominated by the two gluon exchange
- described by the Generalized Parton Distribution(GPD) formalism.
- Moments of GPD E,H at |t|=0 to test Ji's spin sum-rule

e.g. Ji-1997, Collins-Frankfurt-Strikman 1997

Introduction

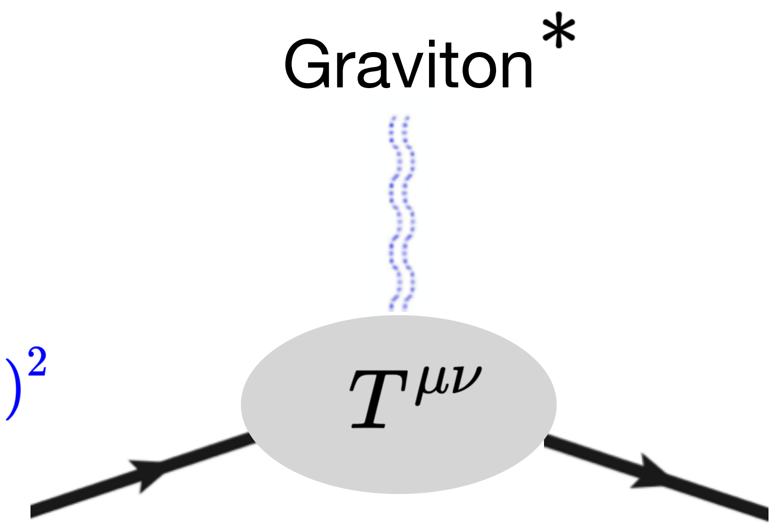
Heavy quarkonium production:

$$\gamma + N \to V + N'$$



$$Q^2 = 0$$

$$W_{\gamma p}^2 \sim (M_p + M_V)^2$$



It is suggested that Photoproductions near the threshold have direct probes to

- Gluon gravitational form factors(GFFs)
- Proton trace anomaly/mass distribution/mass radius (Scalar, Mass form factors)

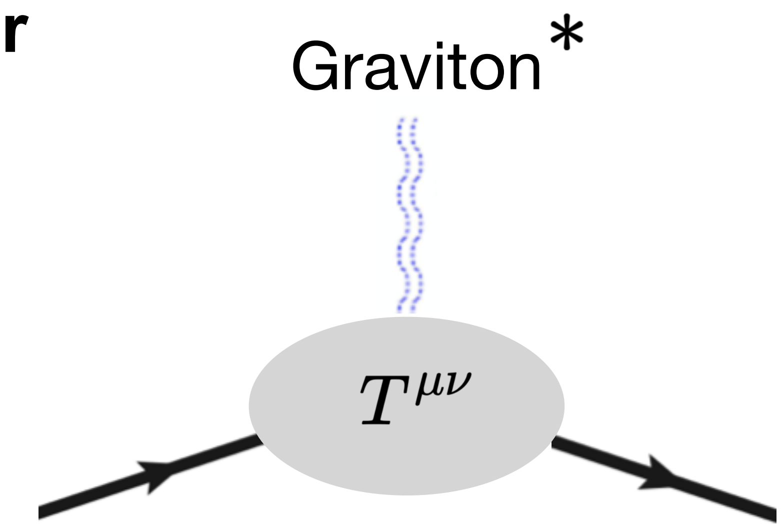
e.g. Kharzeev Phys.Rev.D 104 (2021) 5, 054015
Kharzeev-Satz-Syamtomov-Zinovjev, Eur. Phys. J. C 9, 459 (1999)
Hatta-Yang, *Phys.Rev.D* 98 (2018) 7, 074003
Hatta-Rajan-Yang, Phys.Rev.D 100 (2019) 1, 014032

Boussarie-Hatta Phys.Rev.D 101 (2020) 11
Hatta-Strikman Phys.Lett.B 817 (2021) 136295
Mamo-Zahed, Phys.Rev.D 101 (2020) 8, 086003
Phys.Rev.D 103 (2021) 9, 094010
arXiv:2204.08857

Introduction

What can we learn from GFFs?

Transition matrix of QCD energy momentum tensor



$$S_M \sim \int d^4x h_{\mu\nu}(x) T^{\mu\nu}_{QCD}(x)$$

- A,B-form factors

  Spin-sum rule e.g. Ji (1997):

- C-form factors

  Presseure, shear force: Rev: Polyakov-Schweitzer 2018

  Momentum-current gravitational multipoles: Ji-Liu 2021

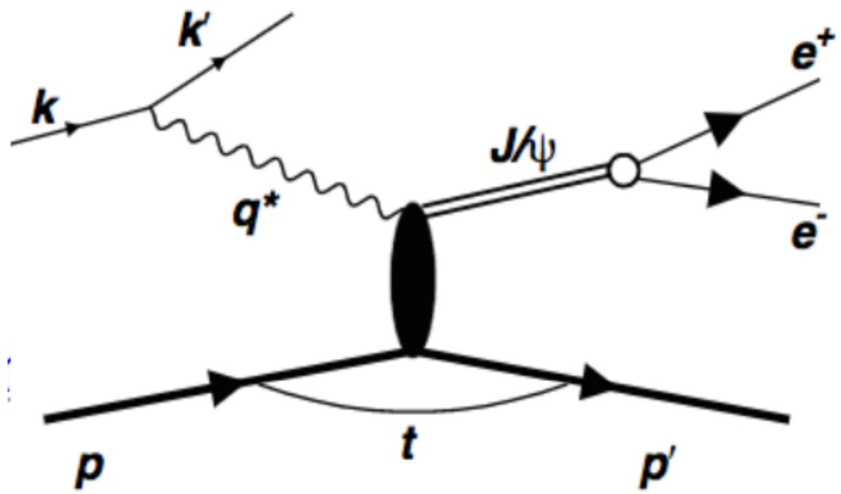
- Reconstruct the proton mass(mostly $\overline{C}, A$)

  - Ji 1996; Ji 2021; Ji-Liu 2021
  - Hatta-Rajan-Tanaka 2018;
  - Metz-Pasquini-Rodini 2020

$$\langle P', s'|T^{\mu\nu}_a(0)|P, s\rangle = \bar{u}_s(P')\Big[A_a(t)\gamma^{(\mu}\bar{P}^{\nu)} + B_a(t)\frac{i\bar{P}^{(\mu}\sigma^{\nu)\rho}\Delta_\rho}{2\Lambda} + C_a(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{\Lambda} + \bar{C}_a(t)\Lambda g^{\mu\nu}\Big]u_s(P),$$

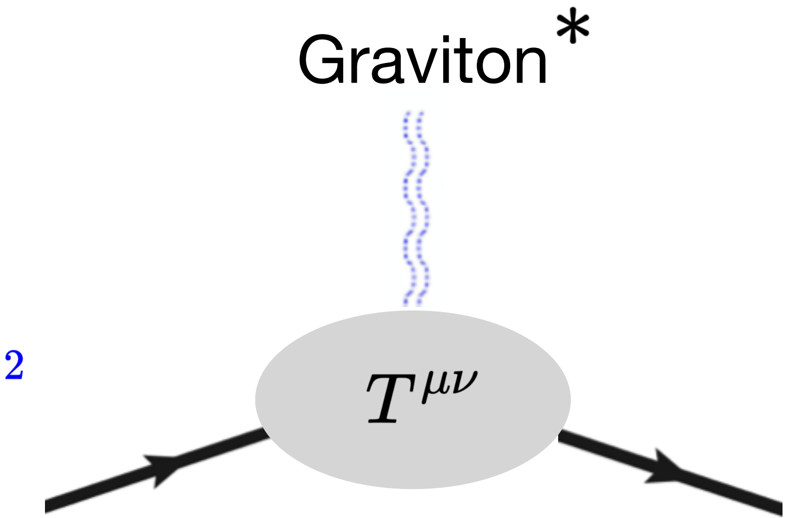
Confusing claims in the literature

$$\gamma + N \to V + N'$$



$Q^2 = 0$

$W_{\gamma p}^2 \sim (M_p + M_V)^2$



$$T_g^{\mu\nu} = F^{a,\mu\lambda} F_{\lambda}^{a,\nu} + \frac{1}{4} g^{\mu\nu} F^{a,\sigma\rho} F_{\sigma\rho}^{a}$$

Which gluon GFFs should contribute?

- Vector-meson dominance: Kharzeev Phys.Rev.D 104 (2021) 5, 054015; Kharzeev-Satz-Syamtomov -Zinovjev, Eur. Phys. J. C 9, 459 (1999) → Scalar GFF
- Holographical approach: Hatta-Yang, *Phys.Rev.D* 98 (2018) 7, 074003; Hatta-Rajan-Yang, Phys.Rev.D 100 (2019) 1, 014032; Mamo-Zahed, Phys.Rev.D 101 (2020) 8, 086003; Phys.Rev.D 103 (2021) 9, 094010; arXiv:2204.08857
- pQCD analysis: Boussarie-Hatta Phys.Rev.D 101 (2020) 11; Hatta-Strikman Phys.Lett.B 817 (2021) 136295

→ All gluon GFFs can contribute. May be dominated by A,C GFF

Guo-Ji-Liu Phys.Rev.D 103 (2021) 9, 096010 → H_g, E_g GPDs at $\xi \approx 1$

Clarify this issue in the kinematics: large momentum transfer

- Near threshold, the momentum transfer can be relatively large, e.g.

$$|t|_{\rm th} = m_p M_V + \cdots$$

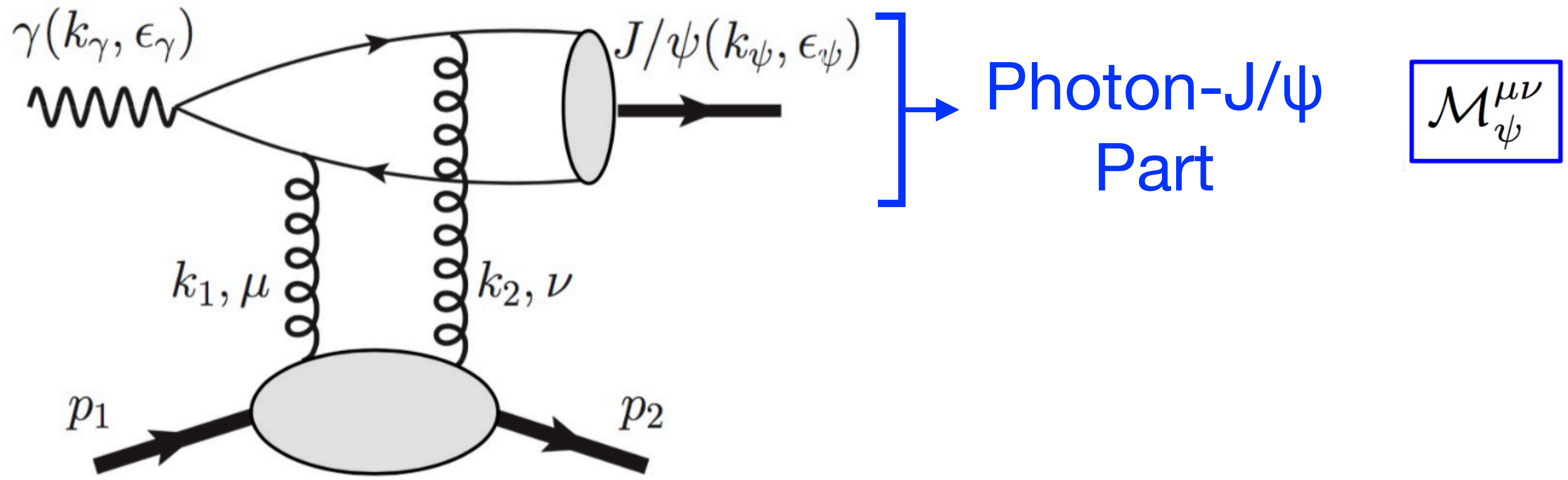
$$|t_{J/\psi}| \sim 2~\mathrm{GeV}^2$$

$$|t_\Upsilon| \sim 10\mathrm{GeV}^2$$

- We can compute both the cross section and the form factors separately in perturbative QCD
- We can check that if there is a direct connection between the near threshold production and the gluonic gravitational form factors (and how)
- Derive the power behavior of t.

Refs: P.Sun, X.B. Tong, F.Yuan
Phys.Lett.B 822 (2021) 136655
Phys.Rev.D 105 (2022) 5, 054032

Near threshold quarkonium production: kinematics



- Two limits
  - Threshold: $W_{\gamma p}^2 \sim (M_p + M_V)^2$

$$\chi = \frac{M_V^2 + 2M_p M_V}{W_{\gamma p}^2 - M_p^2} \rightarrow 1, \quad (1-\chi) \text{ is a small parameter}$$

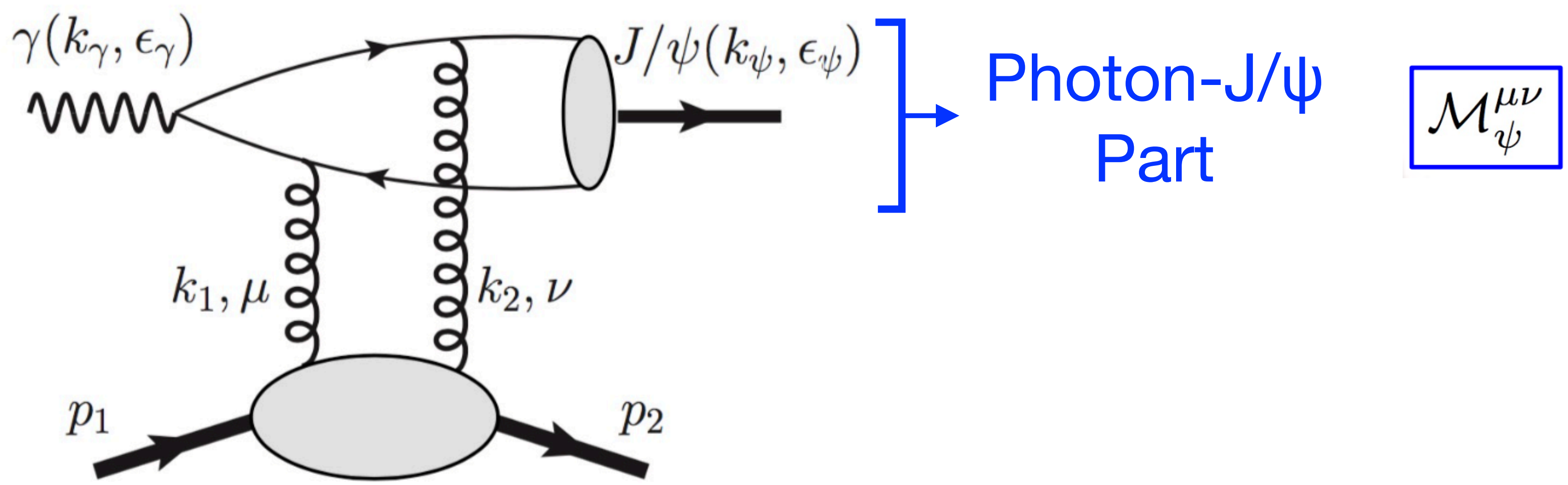
Brodsky et al PLB 498(2001)

  - Heavy quark limits:

$$W_{\gamma p}^2 \sim M_V^2 \gg (-t) \gg \Lambda_{QCD}^2$$

$$p_1 \cdot k_\gamma \sim p_1 \cdot k_\psi \sim M_V^2$$
$$p_2 \cdot k_\gamma \sim p_2 \cdot k_\psi \ll M_V^2$$

Photon-J/ψ part



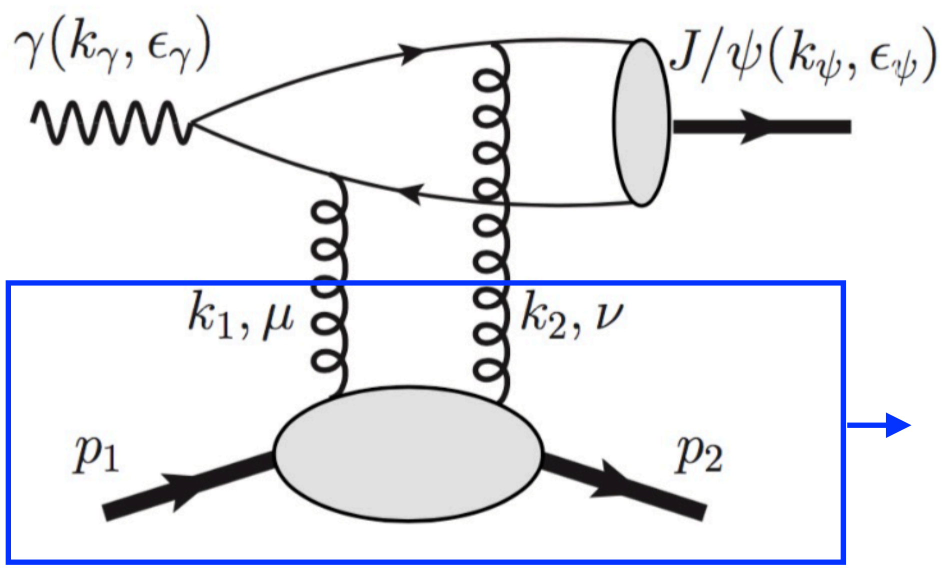
- NRQCD for heavy quadkonium production
- Propagators are of the heavy quark mass. $\sim 1/M_V$
- Take the transverse polarization for the incoming photon.

$$\mathcal{M}^{\mu\nu}_{\psi,ab} = \frac{\delta^{ab} N_\psi \left[\epsilon^*_\psi \cdot \epsilon_\gamma \mathcal{W}^{\mu\nu}_T + \epsilon^*_\psi \cdot k \mathcal{W}^{\mu\nu}_L + \mathcal{W}^{\mu\nu}_S\right]}{k_1 \cdot k_\gamma k_2 \cdot k_\gamma}$$

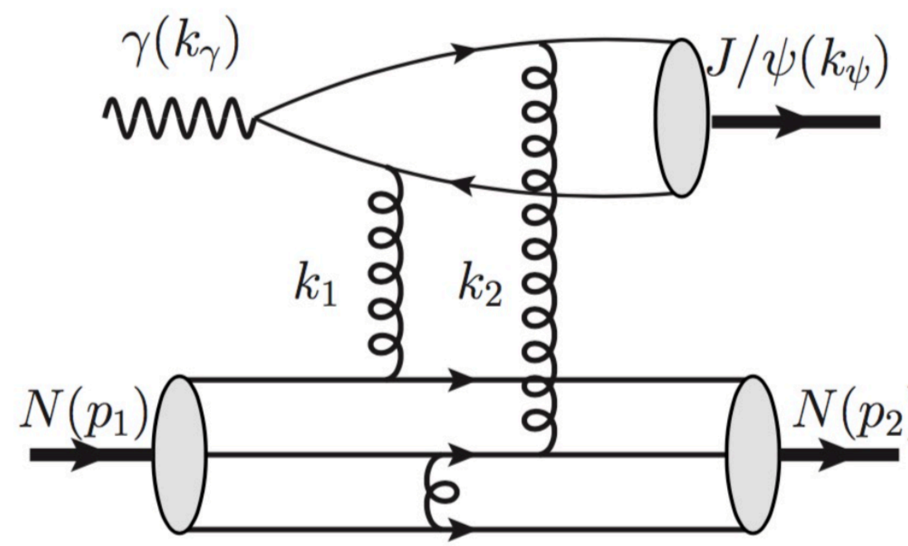
$$\mathcal{W}^{\mu\nu}_T = -k_1 \cdot k_\gamma k_2 \cdot k_\gamma g^{\mu\nu} - k_1 \cdot k_2 k^\mu_\gamma k^\nu_\gamma + k_1 \cdot k_\gamma k^\mu_2 k^\nu_\gamma + k_2 \cdot k_\gamma k^\nu_1 k^\mu_\gamma$$

→ Leading terms

Nucleon part



$$(-t) \gg \Lambda_{QCD}^2$$



- Perturbation theory can apply at the large momentum transfer,

  Similar to the nucleon EM form factors: Lepage-Brodsky 1980, Efremov-Radyushkin 1980, Belitsky-Ji-Yuan 2002

- Nucleon wave function

$$|P\rangle = \sum_{n,\lambda_i} \int \Pi_i \frac{dx_i d^2 k_{\perp i}}{\sqrt{x_i} 16\pi^3} \phi_n(x_i, k_{\perp i}, \lambda_i) |n : x_i, k_{\perp i}, \lambda_i\rangle$$

Power-counting rule → three-valence quark contribution dominates.

Brodsky et al,PRL 31, 1153(1973)
Matveev et al Lett. Nuovo Cim. 7, 719 (1973).
Ji-Ma-Yuan PRL 90,241601(2003)

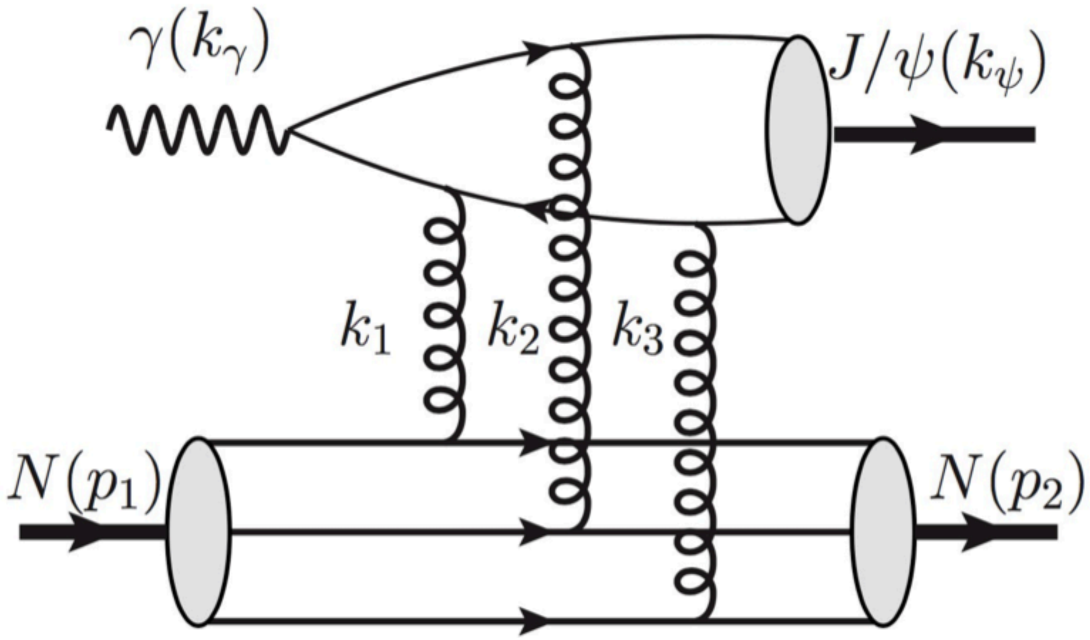
Ji-Ma-Yuan, NPB 652, 383 (2003)

- Partonic scattering

  Additional gluon exchange to generate large (-t)

Three-gluon exchange?

- Suggested by Brodsky et al PLB 498(2001) and claimed that
  - Two-gluon exchange suppressed by a threshold factor $(1-\chi)^2$;
  - Three-gluon exchange dominates at threshold though with additional $1/M_V^2$



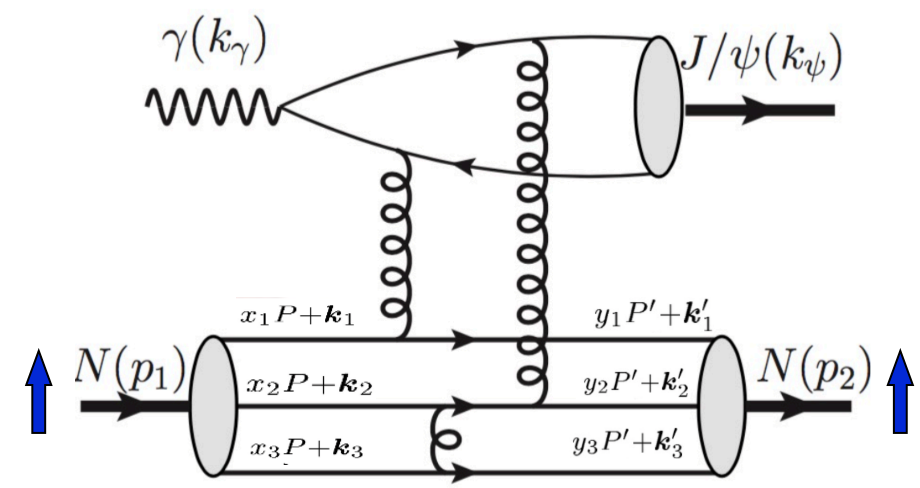
- Due to C-parity conservation, there is no contribution from three-gluon exchange

Photon-J/ψ part: $\propto f^{abc}$

Nucleon part: $\epsilon^{ijk}\epsilon^{lmn}T^a_{il}T^b_{jm}T^c_{kn} \propto d^{abc}$

Nucleon helicity configuration

- Nucleon helicity conserved ➡ Leading twist



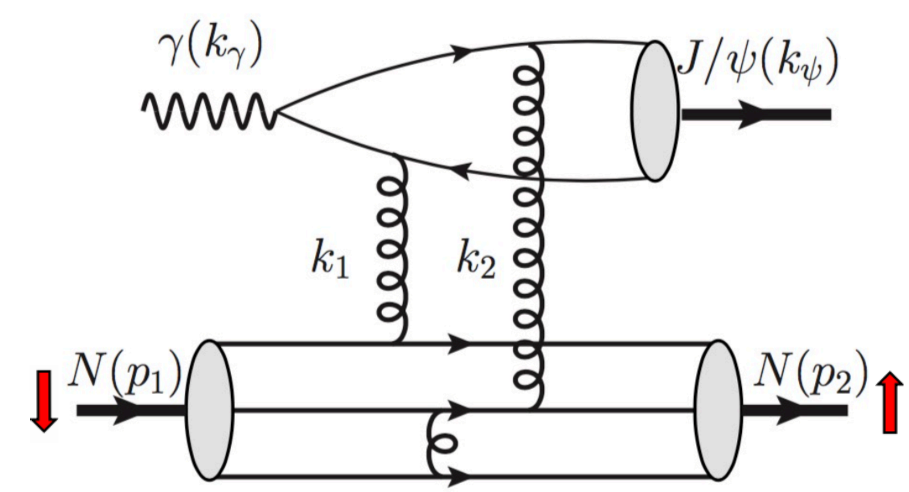
$$|P\uparrow\rangle_{1/2} = \int d[1]d[2]d[3]\left(\tilde{\psi}^{(1)}(1,2,3)\right)$$
$$\times\frac{\epsilon^{abc}}{\sqrt{6}}u^\dagger_{a\uparrow}(1)\left(u^\dagger_{b\downarrow}(2)d^\dagger_{c\uparrow}(3)-d^\dagger_{b\downarrow}(2)u^\dagger_{c\uparrow}(3)\right)|0\rangle$$

➡Twist-3 distribution amplitude:

$$\Phi_3(y_i) = -2\sqrt{6}\int\frac{d^2\vec{k}'_{1\perp}d^2\vec{k}'_{2\perp}d^2\vec{k}'_{3\perp}}{(2\pi)^6}\delta^{(2)}(\vec{k}'_{1\perp}+\vec{k}'_{2\perp}+\vec{k}'_{3\perp})\tilde{\psi}^{(1)}(1,2,3)$$

Braun-Derkachov-Korchemsky-Manashov NPB 553, 355 (1999)

- Nucleon helicity flip ➡ Sub-leading twist



The heilicties of the quarks are conserved in the high energy scattering

Need one unit orbital angular momentum!

$$|P\downarrow\rangle_{1/2} = \int d[1]d[2]d[3]\left((k_1^x-ik_1^y)\tilde{\psi}^{(3)}(1,2,3)+(k_2^x-ik_2^y)\tilde{\psi}^{(4)}(1,2,3)\right)$$
$$\times\frac{\epsilon^{abc}}{\sqrt{6}}\left(u^\dagger_{a\downarrow}(1)u^\dagger_{b\uparrow}(2)d^\dagger_{c\uparrow}(3)-d^\dagger_{a\downarrow}(1)u^\dagger_{b\uparrow}(2)u^\dagger_{c\uparrow}(3)\right)|0\rangle\ ,$$

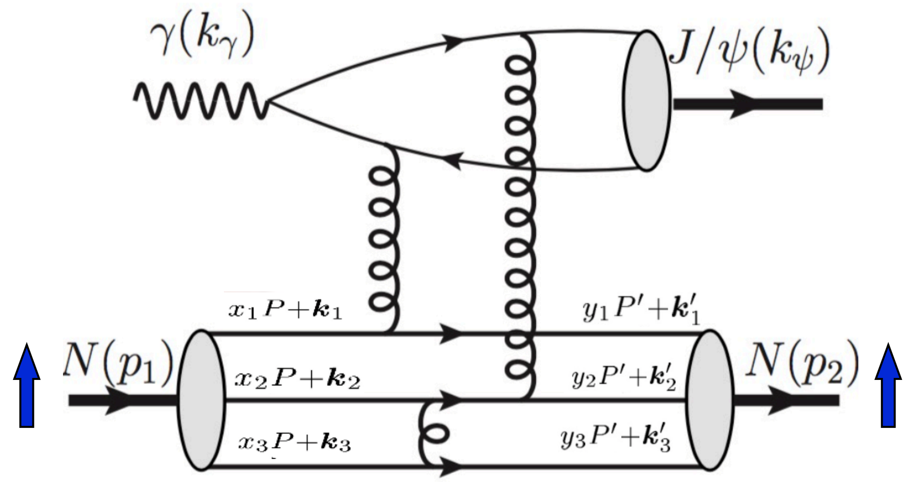
➡Twist-4 distribution amplitude: Braun-Fries-Mahnke-Stein NPB 589, 381 (2000)

$$\Psi_4(x_1,x_2,x_3) = -\frac{2\sqrt{6}}{x_2M}\int\frac{d^2\vec{k}_{1\perp}d^2\vec{k}_{2\perp}d^2\vec{k}_{3\perp}}{(2\pi)^6}\delta^{(2)}(\vec{k}_{1\perp}+\vec{k}_{2\perp}+\vec{k}_{3\perp})$$
$$\times\vec{k}_{2\perp}\cdot\left[\vec{k}_{1\perp}\tilde{\psi}^{(3)}(1,2,3)+\vec{k}_{2\perp}\tilde{\psi}^{(4)}(1,2,3)\right]\ .$$

That induces an additional 1/t suppression.

Twist-3 contribution

- Final amplitude



$$\mathcal{A}_3 = \langle J/\psi(\epsilon_\psi), N'_\uparrow | \gamma(\epsilon_\gamma), N_\uparrow \rangle$$
$$= \int [dx][dy]\Phi(x_1,x_2,x_3)\Phi^*(y_1,y_2,y_3)\frac{1}{(-t)^2}$$
$$\times \underline{\bar{U}_\uparrow(p_2)\not{k}_\gamma U_\uparrow(p_1)}\mathcal{M}_\psi^{(3)}(\epsilon_\gamma,\epsilon_\psi,\{x_i\},\{y_i\}) \;,$$

$$\mathcal{M}^{(3)} = \epsilon_\psi^* \cdot \epsilon_\gamma \frac{8e_c e g_s^6}{27\sqrt{3M_\psi^7}}\psi_J(0)\,(2\mathcal{H}_3 + \mathcal{H}'_3)$$

$$\mathcal{H}_3 = I_{13} + I_{31} + I_{12} + I_{32}, \quad I_{ij} = \frac{1}{x_i x_j y_i y_j \bar{x}_i^2 \bar{y}_i}$$

- Amplitude square

$$|\overline{\mathcal{A}_3}|^2 = \underline{(1-\chi)}G_\psi G_{p3}(t)G_{p3}^*(t) \qquad G_\psi = |N_\psi|^2 = \frac{384\pi^2 e_c^2 \alpha (4\pi\alpha_s)^2}{N_c^2 M_\psi^3}\langle 0|\mathcal{O}^\psi(^3S_1^{(1)})|0\rangle$$

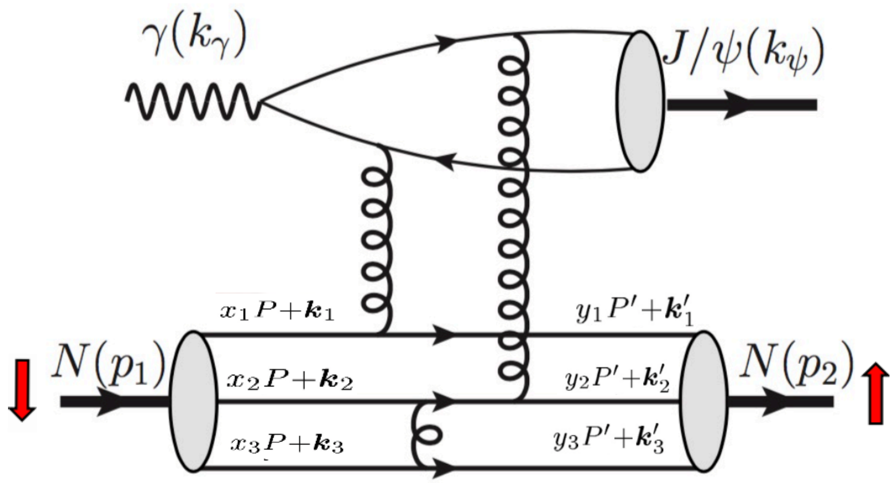
$$G_{p3}(t) = \frac{8\pi^2\alpha_s^2 C_B^2}{\underline{3t^2}}\int [dx][dy]\Phi_3(\{x\})\Phi_3^*(\{y\})\,[2\mathcal{H}_3 + \mathcal{H}'_3]$$

- Suppressed at the threshold, $\chi \to 1$
  - This behavior is similar to H_g contribution to J/ψ production in GPD formalism with $(1-\xi)$ suprresion
- Power behavior: $\frac{1}{t^4}$

Twist-4 contribution

- Final amplitude

$$\mathcal{A}_4 = \langle J/\psi(\epsilon_\psi), N'_\uparrow | \gamma(\epsilon_\gamma), N_\downarrow \rangle$$
$$= \int [dx][dy] \Psi_4(\{x\}) \Phi_3^*(\{y\}) \mathcal{M}_\psi^{(4)}(\{x\},\{y\})$$
$$\times \bar{U}_\uparrow(p_2) U_\downarrow(p_1) \frac{M_p}{(-t)^3} ,$$



- Amplitude square

$$|\overline{\mathcal{A}_4}|^2 = \widetilde{m}_t^2 G_\psi G_{p4}(t) G_{p4}^*(t) \quad \widetilde{m}_t^2 = M_p^2/(-t)$$

$$G_{p4}(t) = \frac{C_B^2 (4\pi\alpha_s)^2}{12t^2} \int [dx][dy] \Phi_3(y_1, y_2, y_3)$$
$$\times \{x_3 \Phi_4(x_1, x_2, x_3) T_{4\Phi}(\{x\},\{y\})$$
$$+ x_1 \Psi_4(x_2, x_1, x_3) T_{4\Psi}(\{x\},\{y\})\} ,$$

- No $(1-\chi)$ suppression
- Power behavior: $\frac{1}{t^5}$

Connection to the gluon GFFs?

- Scattering amplitude

$$G_{p3}(t) = \int [dx][dy]\Phi_3(x_1,x_2,x_3)\Phi_3^*(y_1,y_2,y_3) \times [2\mathcal{H}_3 + \mathcal{H}_3'] \ ,$$

$$\mathcal{H}_3 = I_{13} + I_{31} + I_{12} + I_{32},$$

$$I_{ij} = \frac{1}{x_i x_j y_i y_j \bar{x}_i^2 \bar{y}_i}$$

- Gluonic Form factors

Tong-Ma-Yuan arXiv: 2203.13493
Phys.Lett.B 823 (2021) 136751

$$A_g(t) = \int [dx][dy]\Phi_3(x_1,x_2,x_3)\Phi_3^*(y_1,y_2,y_3) \times [2\mathcal{A}_3 + \mathcal{A}_3'] \ ,$$

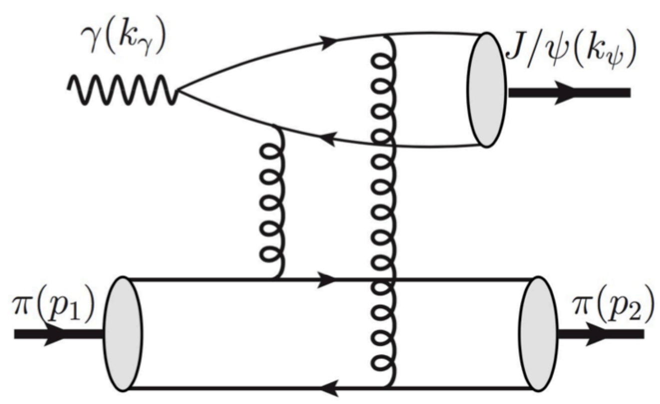
$$\mathcal{A} = \frac{4\pi^2\alpha_s^2 C_B^2}{3t^2}\Big(I_{13} + I_{12} + I_{31} + I_{32}\Big),$$

$$I_{ij} = \frac{x_i + y_i}{\bar{x}_i \bar{y}_i x_i x_j y_i y_j}$$

There is no direct connection to the gluonic gravitation form factors.

Connection to the gluon GFFs? Pion case

- Scattering amplitude

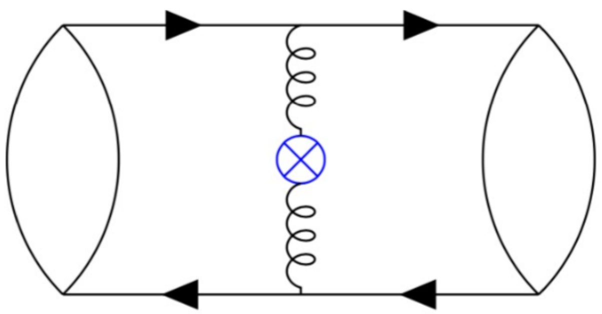


$$|\overline{\mathcal{A}^\pi}|^2 = G_\psi G_\pi(t) G_\pi^*(t)$$

$$G_\psi = |N_\psi|^2 = \frac{384\pi^2 e_c^2 \alpha (4\pi\alpha_s)^2}{N_c^2 M_\psi^3} \langle 0|\mathcal{O}^\psi(^3S_1^{(1)})|0\rangle$$

$$G_\pi(t) = \frac{8\pi\alpha_s C_F}{t} \int dx_1 dy_1 \phi^*(y_1)\phi(x_1) \frac{1}{x_1\bar{x}_1 y_1 \bar{y}_1}$$

- Gluonic Form factors



Tong-Ma-Yuan Phys.Lett.B 823 (2021) 136751

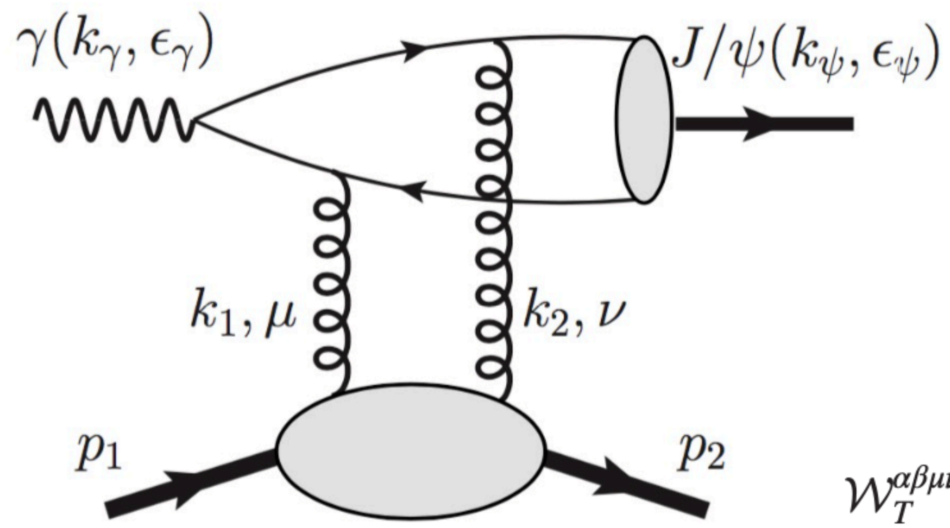
$$\langle P'|T_g^{\mu\nu}|P\rangle = 2\bar{P}^\mu \bar{P}^\nu A_g^\pi(t) + \frac{1}{2}(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2) C_g^\pi(t) + 2m^2 g^{\mu\nu} \overline{C}_g^\pi(t)$$

$$A_g^\pi(t) = C_g^\pi(t) = \frac{4m^2}{t}\overline{C}_g^\pi(t) = \frac{4\pi\alpha_s C_F}{-t} \int dx_1 dy_1 \phi^*(y_1)\phi(x_1) \left(\frac{1}{x_1\bar{x}_1} + \frac{1}{y_1\bar{y}_1}\right)$$

There is no direct connection to the gluonic gravitational form factors.

What is the difference?

- Construct the gluonic operator



Photon-J/ψ Part → $\boxed{\mathcal{M}_\psi^{\mu\nu}}$

$$\mathcal{M}_\psi^{\mu\nu} = N_\psi \epsilon_\psi^* \cdot \epsilon_\gamma \frac{k_{\gamma,\alpha} k_{\gamma,\beta}}{k_1 \cdot k_\gamma k_2 \cdot k_\gamma} \mathcal{W}_T^{\alpha\beta\mu\nu}.$$

$$\mathcal{W}_T^{\alpha\beta\mu\nu} = -k_1^\alpha k_2^\beta g^{\mu\nu} - k_1 \cdot k_2 g^{\alpha\mu} g^{\beta\nu} + k_1^\nu k_2^\beta g^{\alpha\mu} + k_2^\mu k_1^\alpha g^{\beta\nu},$$

$$\mathcal{A} = N_\psi \epsilon_\psi^* \cdot \epsilon_\gamma \int d^4k_1 d^4k_2 \frac{k_{\gamma,\alpha} k_{\gamma,\beta}}{(k_1 \cdot k_\gamma - i\varepsilon)(k_2 \cdot k_\gamma - i\varepsilon)}$$

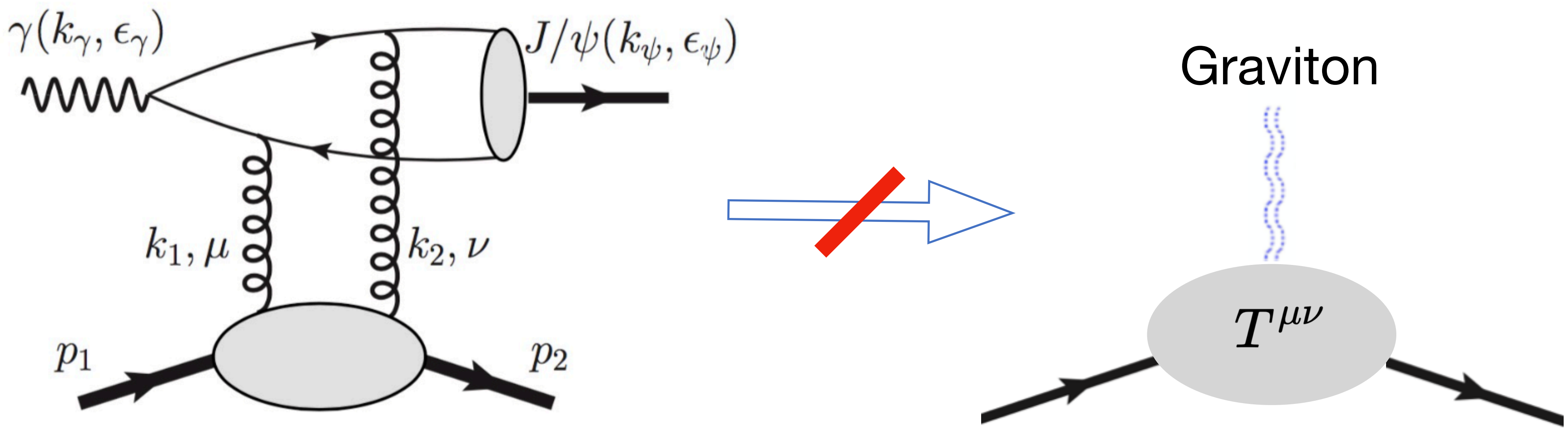
$$\times \int d^4\eta_1 d^4\eta_2 e^{ik_1\cdot\eta_1 + ik_2\cdot\eta_2} \langle N'|F^{a,\alpha}{}_\rho(\eta_1) F^{a,\beta\rho}(\eta_2)|N\rangle$$

Gluon EMT: $T_g^{\mu\nu} = F^{a,\mu\lambda} F_\lambda^{a,\nu} + \frac{1}{4} g^{\mu\nu} F^{a,\sigma\rho} F_{\sigma\rho}^a$

The prefacers that come from the quark propagator prevent the gluonic operator to be local

What is the difference?

- It is a long stretch to make this connection.



The QCD dynamics involved in the on-shell photon (massless) transition to a massive heavy quarkonium does not allow a simple interpretation.

Connection to GPD formalism

- Guo-Ji-Liu argue that the GPD formalism can can also apply in the threshold with $\xi \to 1$ Guo-Ji-Liu Phys.Rev.D 103 (2021) 9, 096010

$$\mathcal{M}(\varepsilon_V, \varepsilon) = \frac{8\sqrt{2}\pi\alpha_S(M_V)}{M_V^2}\phi^*(0)G(t,\xi)(\varepsilon_V^* \cdot \varepsilon).$$

$$G(t,\xi) = \frac{1}{2\xi}\int_{-1}^{1} \mathrm{d}x\mathcal{A}(x,\xi)\underbrace{F_g(x,\xi,t).}_{\text{GPD}} \qquad \mathcal{A}(x,\xi) \equiv \frac{1}{x+\xi-i0} - \frac{1}{x-\xi+i0}$$

- Large t gluon GPD can also be calculated in perturbative QCD
  - Ji-Hoodbhoy-Yuan, 2004, quark GPD
  - gluon GPD-> consistent check!

- Expansions around x=0?
  - Leads to A, B, C GFFs, however may have singularity; Higher-moments ~20%
  - See discussion in Hatta-Strikman Phys.Lett.B 817 (2021) 136295
    Guo-Ji-Liu Phys.Rev.D 103 (2021) 9, 096010

Summary&Outlook

- We study the near threshold quarkonium production at large nomentum, trying to understand its connection to the gluonic graviton form factors

- Find no direct connection to the gluon GFFs.

- Power behavior at large (-t) is derived, leading contributions come from the helicity-flip contributions, $1/(-t)^5$, different from the conventional power counting.

  See applications in GlueX data

  Sun-Tong-Yuan   Phys.Lett.B 822 (2021) 136655
  Phys.Rev.D 105 (2022) 5, 054032

- Our study is consistent with the GPD formalism

- Further studies for NLO-corrections/ NRQCD corrections

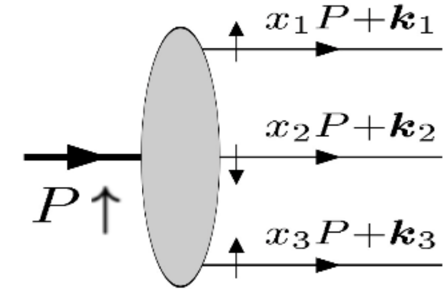
- Looking forward to Gluon GPD phenomenology.

Nucleon amplitude

- Nucleon's 3-quarks WF Ji-Ma-Yuan, NPB 652, 383 (2003)

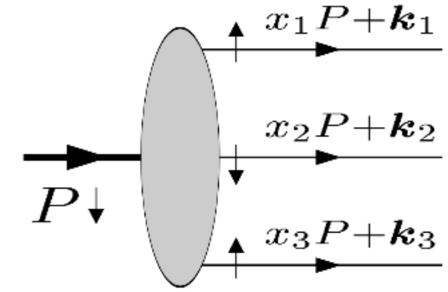
$$|P\uparrow\rangle_{1/2} = \int d[1]d[2]d[3]\left(\tilde{\psi}^{(1)}(1,2,3)\right)$$
$$\times\frac{\epsilon^{abc}}{\sqrt{6}}u^{\dagger}_{a\uparrow}(1)\left(u^{\dagger}_{b\downarrow}(2)d^{\dagger}_{c\uparrow}(3) - d^{\dagger}_{b\downarrow}(2)u^{\dagger}_{c\uparrow}(3)\right)|0\rangle$$

$L_z = 0$



$$|P\downarrow\rangle_{1/2} = \int d[1]d[2]d[3]\left((k_1^x - ik_1^y)\tilde{\psi}^{(3)}(1,2,3) + (k_2^x - ik_2^y)\tilde{\psi}^{(4)}(1,2,3)\right)$$
$$\times\frac{\epsilon^{abc}}{\sqrt{6}}\left(u^{\dagger}_{a\downarrow}(1)u^{\dagger}_{b\uparrow}(2)d^{\dagger}_{c\uparrow}(3) - d^{\dagger}_{a\downarrow}(1)u^{\dagger}_{b\uparrow}(2)u^{\dagger}_{c\uparrow}(3)\right)|0\rangle\ ,$$

$L_z = -1$



- Distribution amplitude: Integrate our the transverse momentum

Twist-3 (leading twist): Braun-Derkachov-Korchemsky-Manashov NPB 553, 355 (1999)
Braun-Fries-Mahnke-Stein NPB 589, 381 (2000)

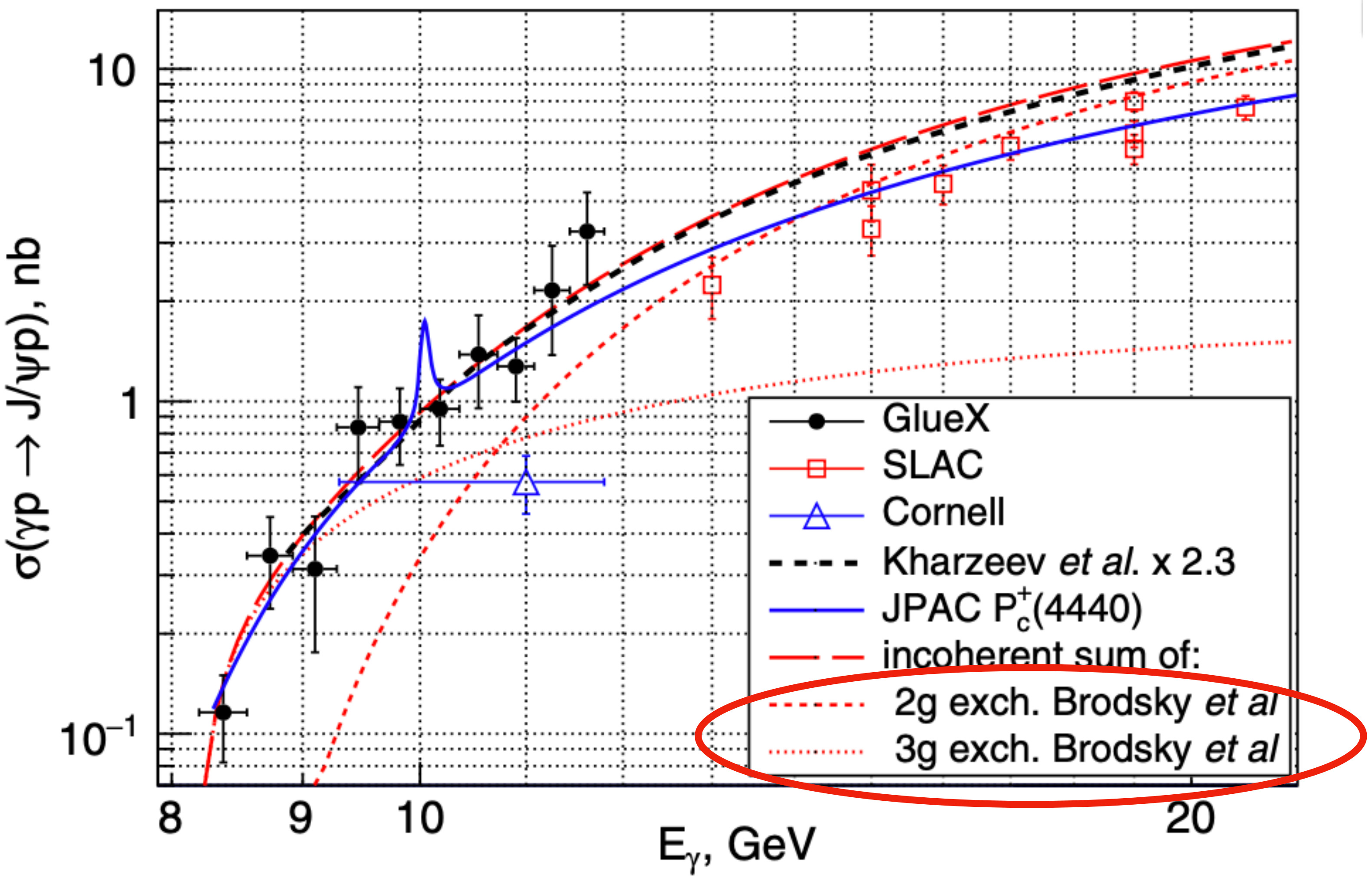
$$\Phi_3(y_i) = -2\sqrt{6}\int\frac{d^2\vec{k}'_{1\perp}d^2\vec{k}'_{2\perp}d^2\vec{k}'_{3\perp}}{(2\pi)^6}\delta^{(2)}(\vec{k}'_{1\perp}+\vec{k}'_{2\perp}+\vec{k}'_{3\perp})\tilde{\psi}^{(1)}(1,2,3)$$

Twist-4:

$$\Psi_4(x_1,x_2,x_3) = -\frac{2\sqrt{6}}{x_2M}\int\frac{d^2\vec{k}_{1\perp}d^2\vec{k}_{2\perp}d^2\vec{k}_{3\perp}}{(2\pi)^6}\delta^{(2)}(\vec{k}_{1\perp}+\vec{k}_{2\perp}+\vec{k}_{3\perp})$$
$$\times\vec{k}_{2\perp}\cdot\left[\vec{k}_{1\perp}\tilde{\psi}^{(3)}(1,2,3)+\vec{k}_{2\perp}\tilde{\psi}^{(4)}(1,2,3)\right]\ .$$

$$\Phi_4(x_2,x_1,x_3) = -\frac{2\sqrt{6}}{x_3M}\int\frac{d^2\vec{k}_{1\perp}d^2\vec{k}_{2\perp}d^2\vec{k}_{3\perp}}{(2\pi)^6}\delta^{(2)}(\vec{k}_{1\perp}+\vec{k}_{2\perp}+\vec{k}_{3\perp})$$
$$\times\vec{k}_{3\perp}\cdot\left[\vec{k}_{1\perp}\tilde{\psi}^{(3)}(1,2,3)+\vec{k}_{2\perp}\tilde{\psi}^{(4)}(1,2,3)\right]\ .$$

Confusing claims in the literature



Two gluon or three gluon exchange?