Inclusive Production of Heavy Quarkonia in Potential NRQCD

HEE SOK CHUNG
KOREA UNIVERSITY

Based on
Nora Brambilla, HSC, Antonio Vairo, Phys.Rev.Lett. 126, 082003 (2021)
Nora Brambilla, HSC, Antonio Vairo, JHEP 09 (2021) 032
Nora Brambilla, HSC, Antonio Vairo, Xiang-Peng Wang, Phys.Rev.D 105, L111503 (2022)

QWG 2022 - The 15th International Workshop on Heavy Quarkonium
26-30 September 2022 GSI Darmstadt

OUTLINE

- Quarkonium production in NRQCD
- NRQCD matrix elements in pNRQCD
- Phenomenological results for J/ψ, $\psi(2S)$, and Υ

Quarkonium Production in NRQCD

- Nonrelativistic QCD provides a factorization formalism for inclusive production of a heavy quarkonium $\mathcal{Q}$:

Short-distance cross sections → *Long-distance matrix elements*

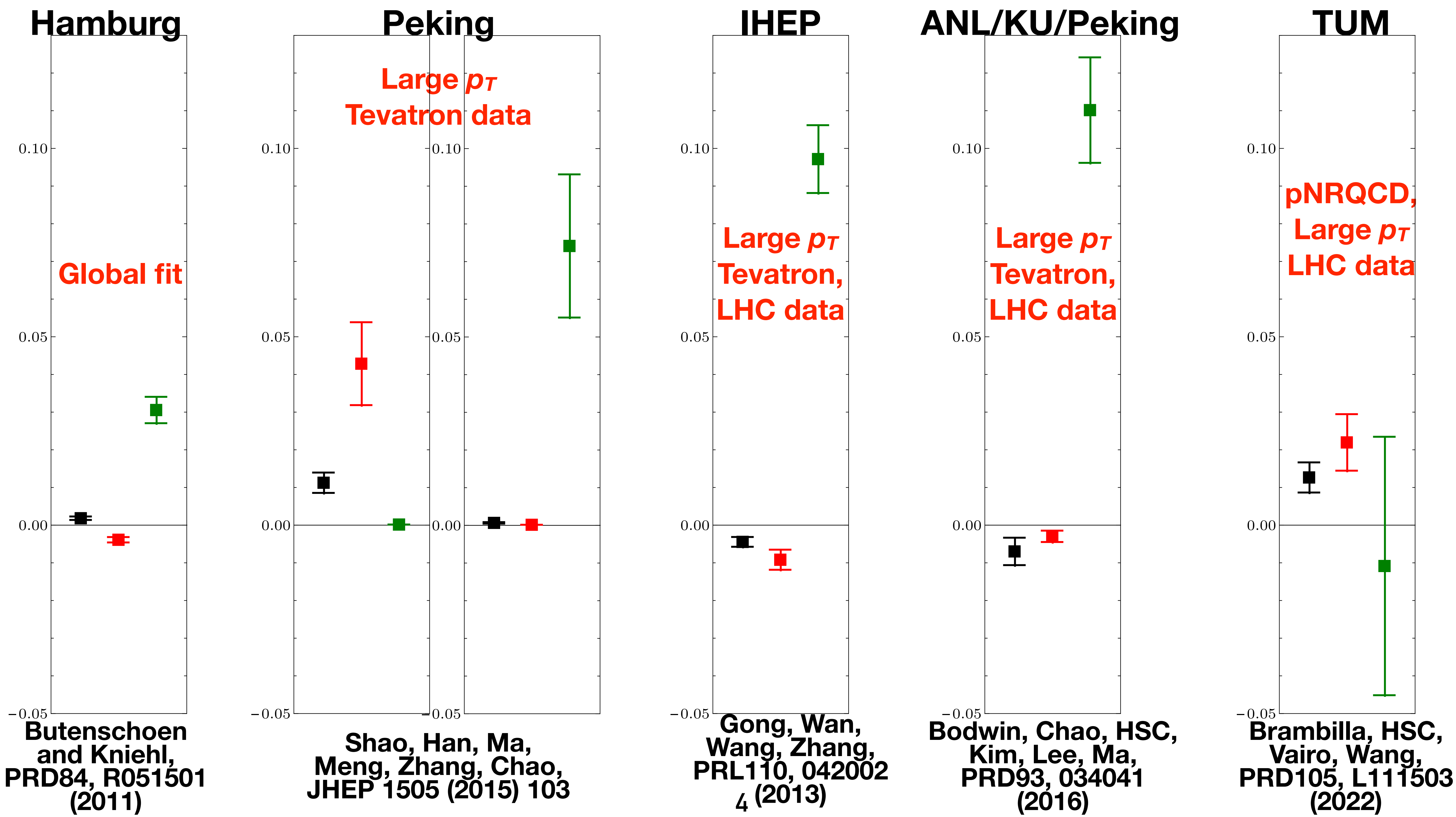
$$\sigma_{\mathcal{Q}+X} = \sum_n \hat{\sigma}_{Q\bar{Q}(n)+X} \langle \mathcal{O}^{\mathcal{Q}}(n) \rangle$$

Bodwin, Braaten, Lepage, PRD51, 1125 (1995)

- Perturbative calculation of short-distance coefficients and nonperturbative determination of matrix elements are needed to compute cross sections.
- In general it is not known how to compute matrix elements from first principles, so they are usually determined from cross section measurements. So far this approach has not lead to a comprehensive description of measurements.

J/ψ Matrix Element Determinations

- J/ψ matrix elements $\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\rangle$, $\langle \mathcal{O}^{J/\psi}({}^3P_0^{[8]})\rangle/m^2$, $\langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle$ (GeV^3)



NRQCD Matrix Elements

- NRQCD matrix elements have the form

  color singlet $\langle\Omega|\chi^\dagger\mathcal{K}_N\psi\mathcal{P}_{\mathcal{Q}(\boldsymbol{P}=\mathbf{0})}\psi^\dagger\mathcal{K}'_N\chi|\Omega\rangle$

  color octet $\langle\Omega|\chi^\dagger\mathcal{K}_N T^a\psi\Phi_\ell^{\dagger ab}\mathcal{P}_{\mathcal{Q}(\boldsymbol{P}=\mathbf{0})}\Phi_\ell^{bc}\psi^\dagger\mathcal{K}'_N T^c\chi|\Omega\rangle$

  and correspond to the probabilities for nonperturbative evolution of $Q\bar{Q}$ into $\mathcal{Q}$+anything. This happens through emission of order mv gluons.

- We aim to compute the matrix elements in the ***potential NRQCD*** effective field theory.
  In ***pNRQCD***, effects of order mv gluons can be integrated out by making use of the separation of scales mv and mv^2.

POTENTIAL NRQCD

- We work in the strong coupling regime, which is valid for charmonia and excited bottomonia. The degree of freedom is the singlet field $S(x_1, x_2)$, which describe $Q\bar{Q}$ in a color-singlet state.

$$\mathcal{L}_{\mathrm{pNRQCD}} = \mathrm{Tr}\{S^\dagger(i\partial_0 - h)S\}$$

Pineda, Soto, NPB Proc. Suppl. 64, 428 (1998)
Brambilla, Pineda, Soto, Vairo, NPB566, 275 (2000)
Brambilla, Pineda, Soto, Vairo, Rev. Mod. Phys. 77, 1423 (2005)

- In pNRQCD a quarkonium state is a color-singlet QQ bound state, which is an eigenstate of h.

- Matching to NRQCD is done nonperturbatively.

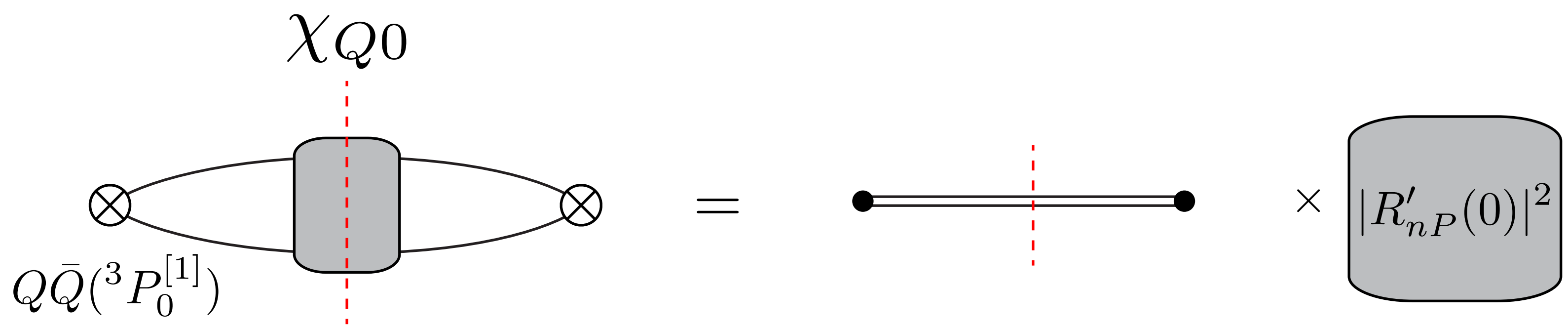
- pNRQCD has been applied to decay matrix elements to compute them in terms of wavefunctions and gluonic correlators. We have extended this formalism to production matrix elements.

Brambilla, HSC, Vairo, PRL126, 082003 (2021)
Brambilla, HSC, Vairo, JHEP 09 (2021) 032

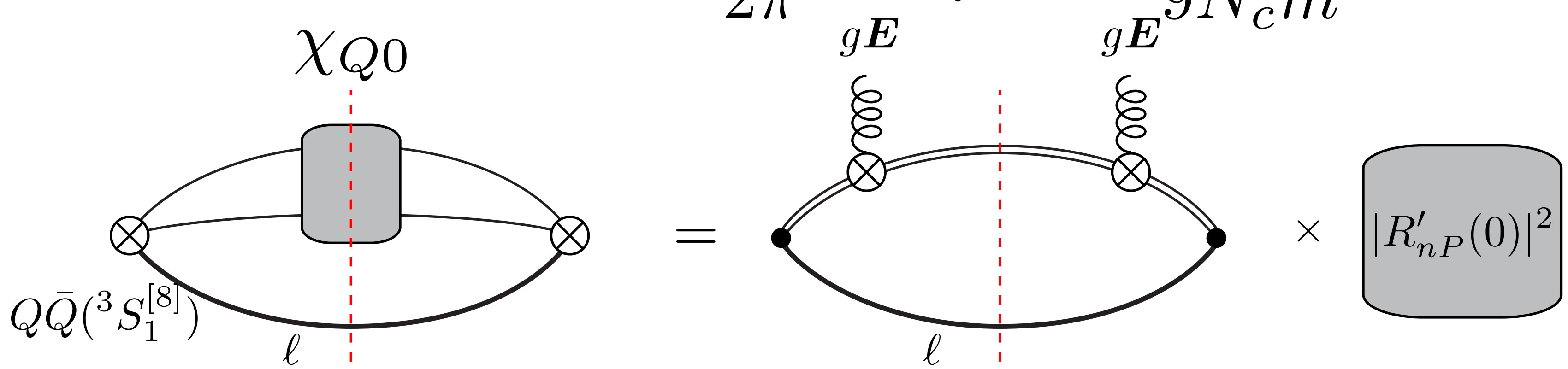
P-wave Matrix Elements in pNRQCD

- Production of χ_{QJ} :
$$\sigma_{\chi_{QJ}+X} = (2J+1)\sigma_{Q\bar{Q}(^3P_J^{[1]})}\langle \mathcal{O}^{\chi_{Q0}}(^3P_0^{[1]})\rangle + (2J+1)\sigma_{Q\bar{Q}(^3S_1^{[8]})}\langle \mathcal{O}^{\chi_{Q0}}(^3S_1^{[8]})\rangle$$

- Color singlet :
$$\langle \mathcal{O}^{\chi_{Q0}}(^3P_0^{[1]})\rangle = \frac{3N_c}{2\pi}|R^{(0)\prime}_{\chi_{Q0}}(0)|^2$$



- Color octet :
$$\langle \mathcal{O}^{\chi_{Q0}}(^3S_1^{[8]})\rangle = \frac{3N_c}{2\pi}|R^{(0)\prime}_{\chi_{Q0}}(0)|^2 \frac{\mathcal{E}}{9N_c m^2}$$



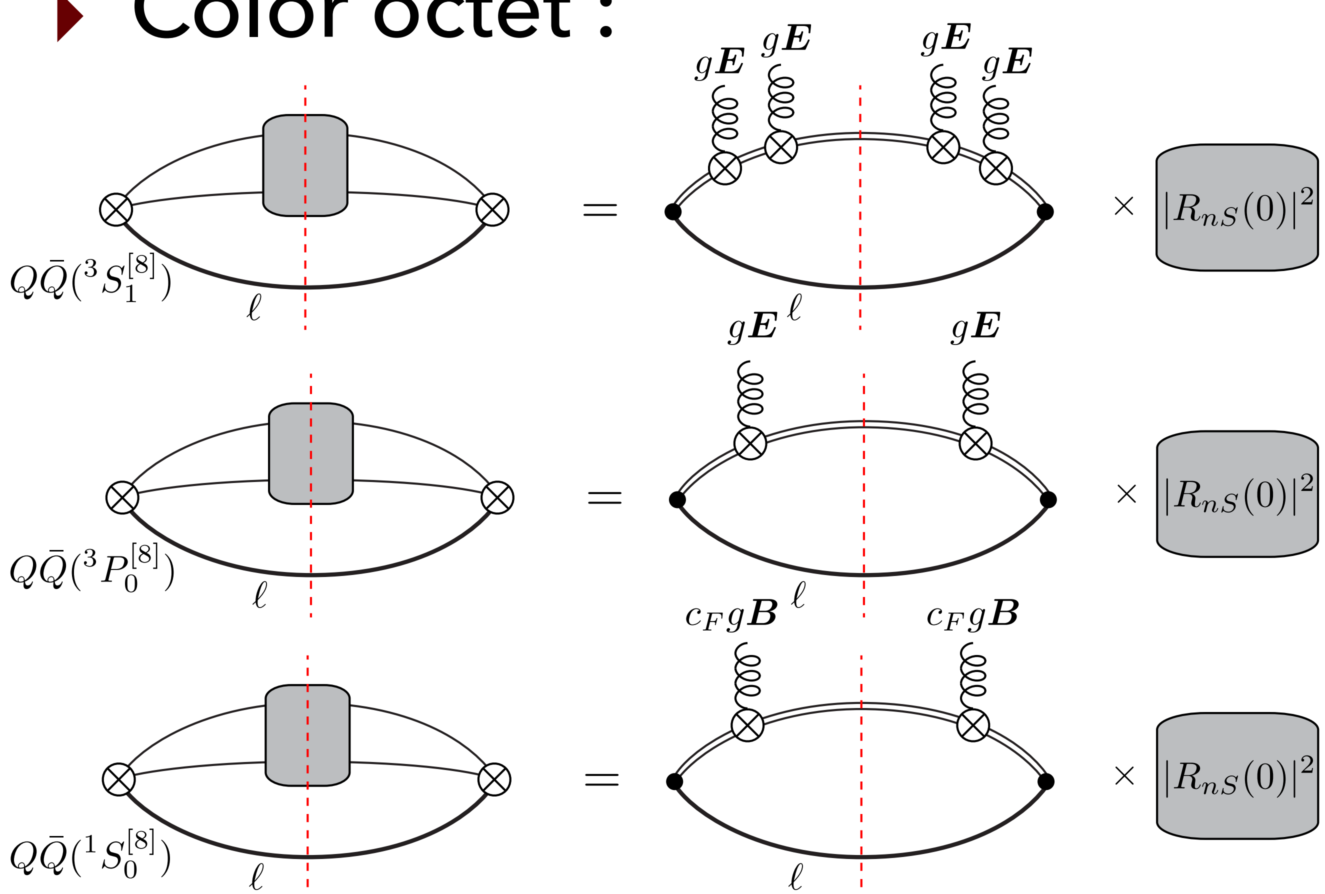
- ***One correlator $\mathcal{E}$ to rule all P-wave cross sections.***

S-wave Matrix Elements in pNRQCD

- $V = J/\psi,\ \psi(2S),\ \Upsilon(nS)$. Color singlet : $\langle \mathcal{O}^V({}^3S_1^{[1]})\rangle = \dfrac{3|R(0)|^2}{4\pi}$

$$\sigma_{V+X} = \hat{\sigma}_{Q\bar{Q}({}^3S_1^{[1]})}\langle \mathcal{O}^V({}^3S_1^{[1]})\rangle + \hat{\sigma}_{Q\bar{Q}({}^3S_1^{[8]})}\langle \mathcal{O}^V({}^3S_1^{[8]})\rangle + \hat{\sigma}_{Q\bar{Q}({}^1S_0^{[8]})}\langle \mathcal{O}^V({}^1S_0^{[8]})\rangle + \sum_{J=0,1,2} \hat{\sigma}_{Q\bar{Q}({}^3P_J^{[8]})}(2J+1)\langle \mathcal{O}^V({}^3P_0^{[8]})\rangle$$

- Color octet :



$$\langle \mathcal{O}^V({}^3S_1^{[8]})\rangle = \frac{1}{2N_c m^2}\frac{3|R(0)|^2}{4\pi}\mathcal{E}_{10;10}$$

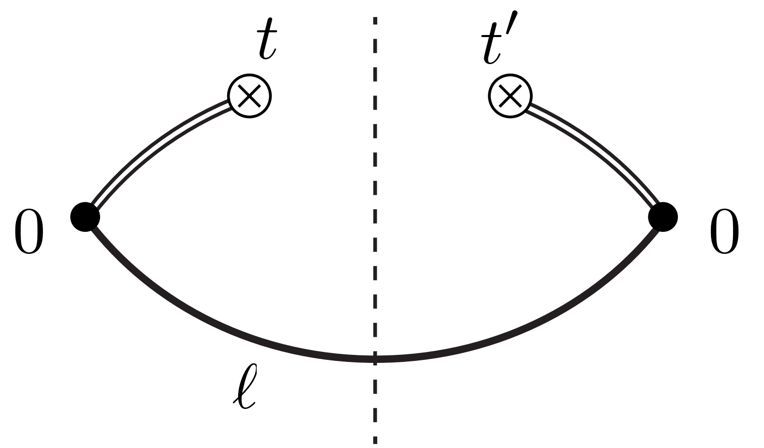
$$\langle \mathcal{O}^V({}^3P_0^{[8]})\rangle = \frac{1}{18N_c}\frac{3|R(0)|^2}{4\pi}\mathcal{E}_{00}$$

$$\langle \mathcal{O}^V({}^1S_0^{[8]})\rangle = \frac{1}{6N_c m^2}\frac{3|R(0)|^2}{4\pi}c_F^2\mathcal{B}_{00}$$

- ***Three correlators $\mathcal{E}_{10;10}$, $\mathcal{E}_{00}$, $\mathcal{B}_{00}$ to rule all S-wave production***

GLUONIC CORRELATORS

- Operator definitions of gluonic correlators are given by



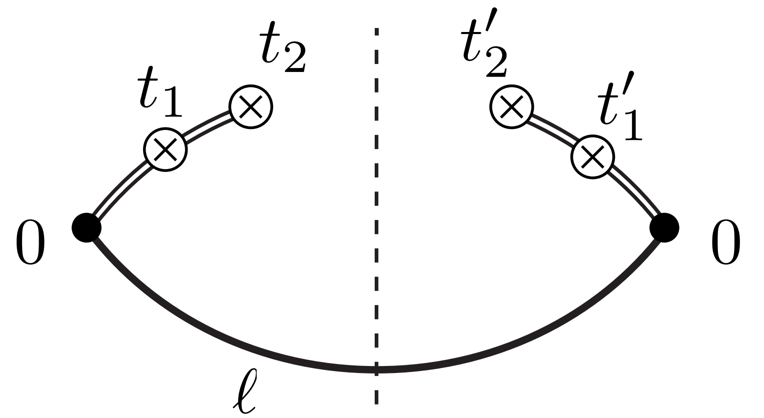
$$\mathcal{E} = \frac{3}{N_c} \left\| \int_0^\infty dt\, t\, gE^{e,i}(t) \Phi_0^{ec}(0;t) \Phi_\ell^{bc} |\Omega\rangle \right\|^2$$

P-wave production

$$\mathcal{E}_{00} = \left\| \int_0^\infty dt\, gE^{a,i}(t) \Phi_0^{ac}(0;t) \Phi_\ell^{bc} |\Omega\rangle \right\|^2$$

$$\mathcal{B}_{00} = \left\| \int_0^\infty dt\, gB^{a,i}(t) \Phi_0^{ac}(0;t) \Phi_\ell^{bc} |\Omega\rangle \right\|^2$$

S-wave production

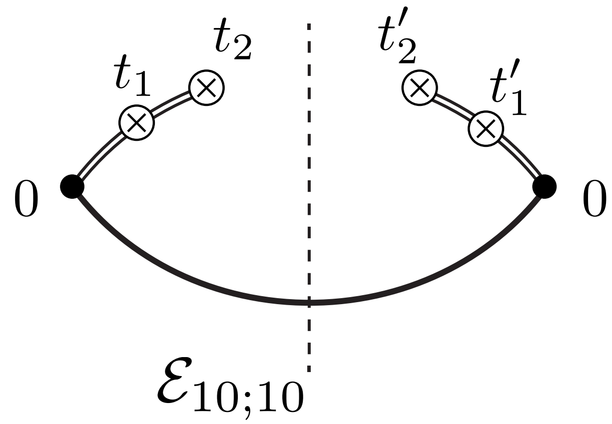


$$\mathcal{E}_{10;10} = \left\| d^{dac} \int_0^\infty dt_1\, t_1 \int_{t_1}^\infty dt_2\, gE^{b,i}(t_2) \times \Phi_0^{bc}(t_1;t_2) gE^{a,i}(t_1) \Phi_0^{df}(0;t_1) \Phi_\ell^{ef} |\Omega\rangle \right\|^2$$

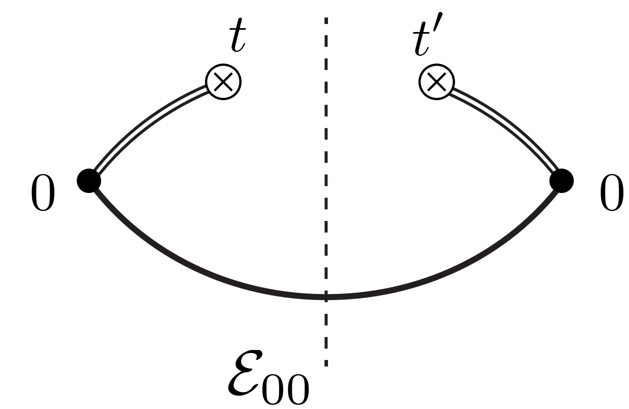
- Although they are expressed as norms, these are ultraviolet divergent and require renormalization, so they are not necessarily positive definite in dimensional regularization.

EVOLUTION EQUATIONS

- The gluonic correlators mix under scale variations:



$$\frac{d}{d\log\Lambda}\mathcal{E}_{10;10} = \frac{2\alpha_s}{3\pi}\frac{N_c^2-4}{N_c}\mathcal{E}_{00}$$



- This reproduces the known evolution equation for NRQCD matrix elements :

$$\frac{d}{d\log\Lambda}\langle\mathcal{O}^V({}^3S_1^{[8]})\rangle = \frac{6(N_c^2-4)}{N_c m^2}\frac{\alpha_s}{\pi}\langle\mathcal{O}^V({}^3P_0^{[8]})\rangle$$

- If $\mathcal{E}_{00}$ is positive, $\mathcal{E}_{10;10}(\mathbf{\Lambda})$ grows with increasing $\mathbf{\Lambda}$: in such case, $\mathcal{E}_{10;10}(\mathbf{\Lambda}{=}m_b)$ is larger than $\mathcal{E}_{10;10}(\mathbf{\Lambda}{=}m_c)$.

CROSS SECTION RATIOS

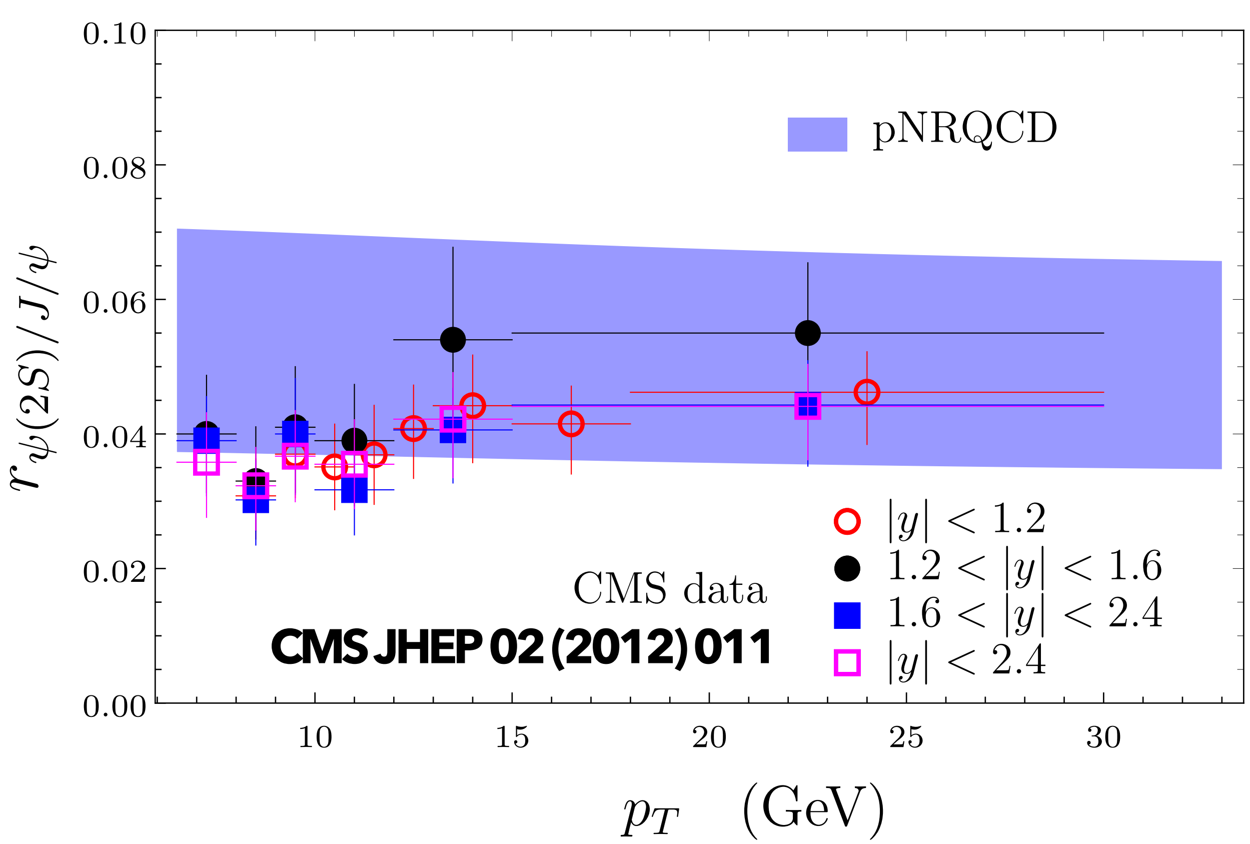
- Universality of the gluonic correlators leads to predictions for cross section ratios, independently of the correlators

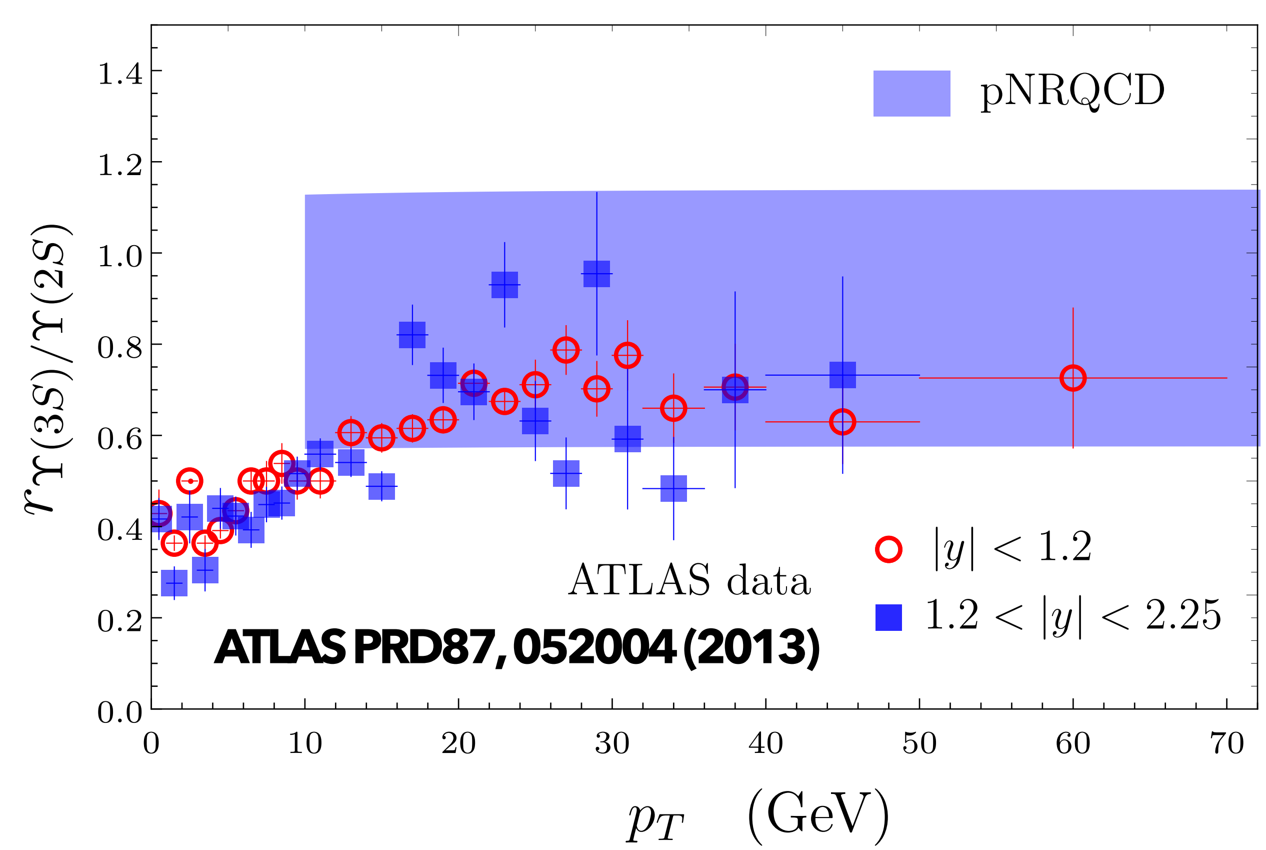
$$\frac{\sigma^{\rm direct}_{\psi(2S)}}{\sigma^{\rm direct}_{J/\psi}} = \frac{|R^{(0)}_{\psi(2S)}(0)|^2}{|R^{(0)}_{J/\psi}(0)|^2} \qquad \frac{\sigma^{\rm direct}_{\Upsilon(3S)}}{\sigma^{\rm direct}_{\Upsilon(2S)}} = \frac{|R^{(0)}_{\Upsilon(3S)}(0)|^2}{|R^{(0)}_{\Upsilon(2S)}(0)|^2}$$

Brambilla, HSC, Vairo, Wang, PRD105, L111503 (2022)

- Compared to experiment, including feeddown effects:

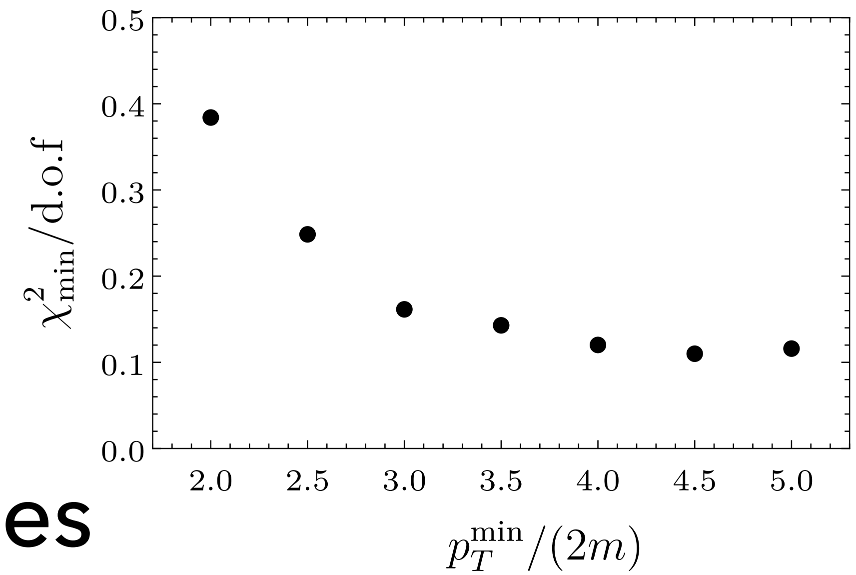
$r_{A/B} = ({\rm Br}_{A\to\mu^+\mu^-}\sigma_A)/({\rm Br}_{B\to\mu^+\mu^-}\sigma_B)$

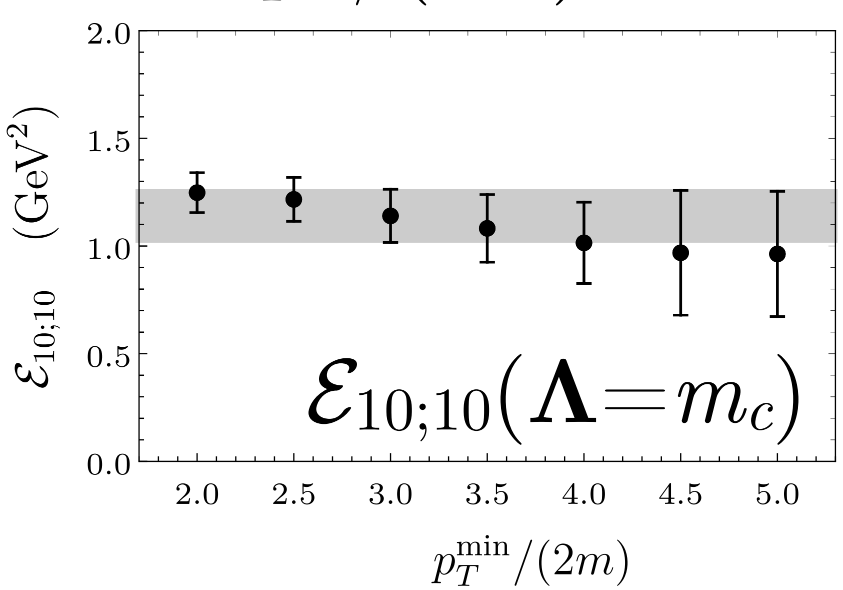


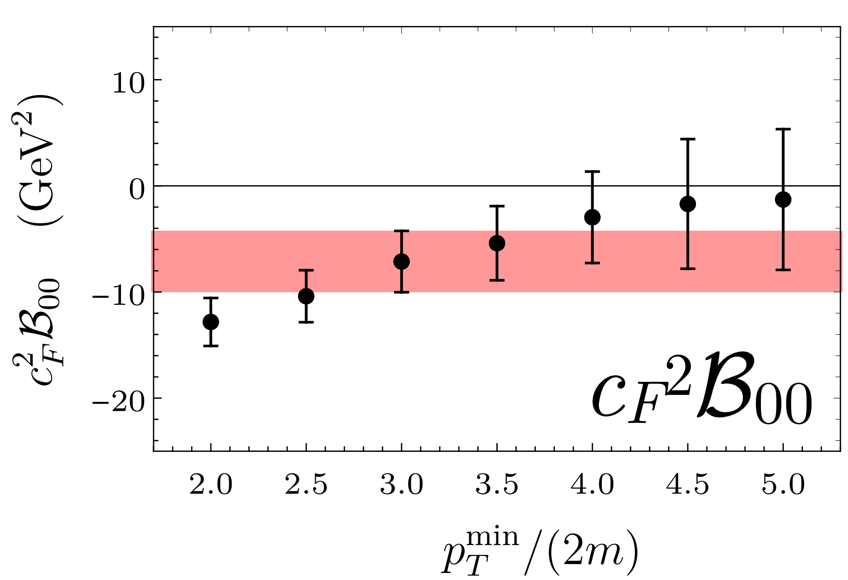


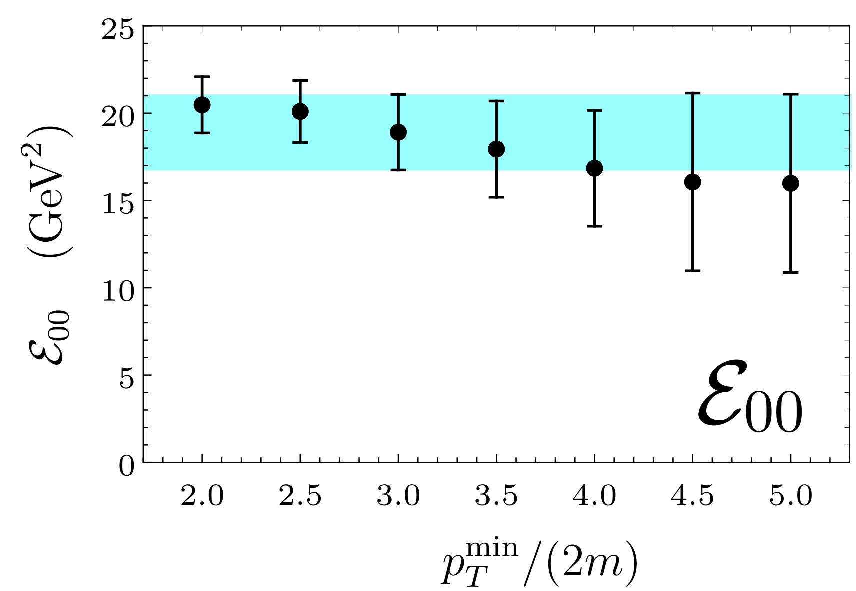
DETERMINATIONS OF GLUONIC CORRELATORS

- We determine values of gluonic correlators by comparing LHC measurements of J/ψ, $\psi(2S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ cross sections at large p_T.
- Quality of fits are good, and improve with increasing $p_T^{\rm min}$.
- Results are consistent within uncertainties for $p_T/(2m)>3$.









Determinations of Gluonic Correlators

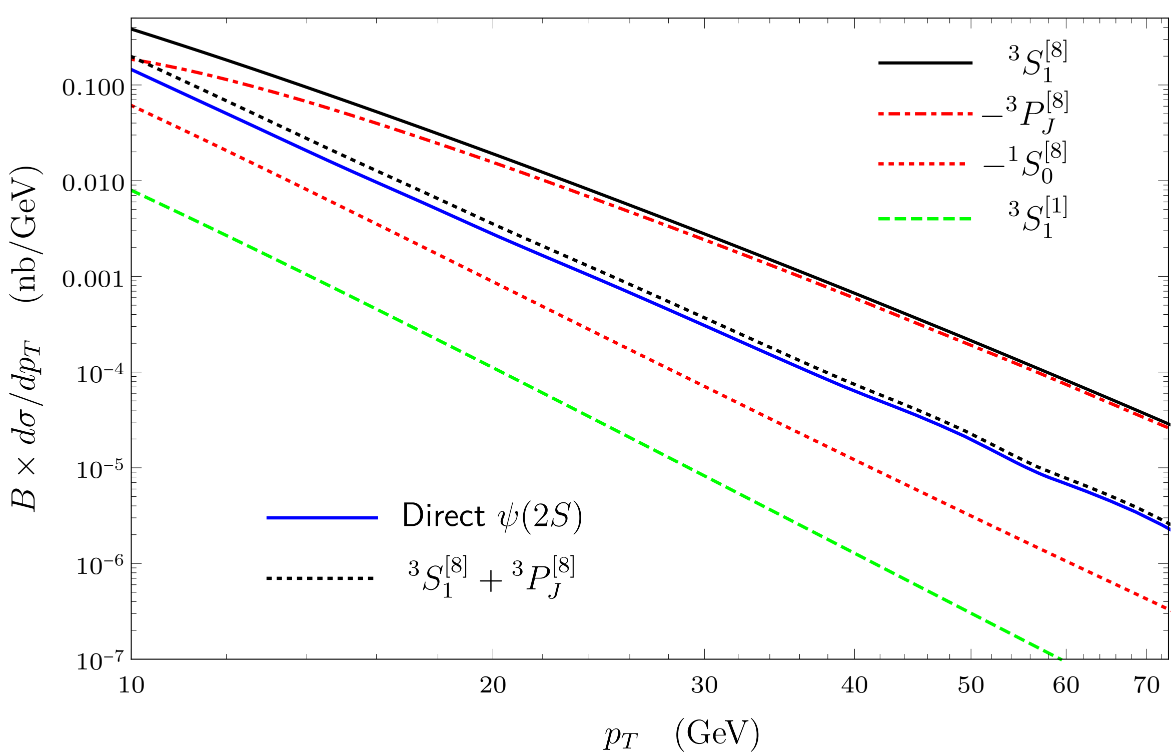
- The fits constrain $\mathcal{E}_{10;10}$ and $\mathcal{E}_{00}$ to be positive, and $\mathcal{B}_{00}$ is small.

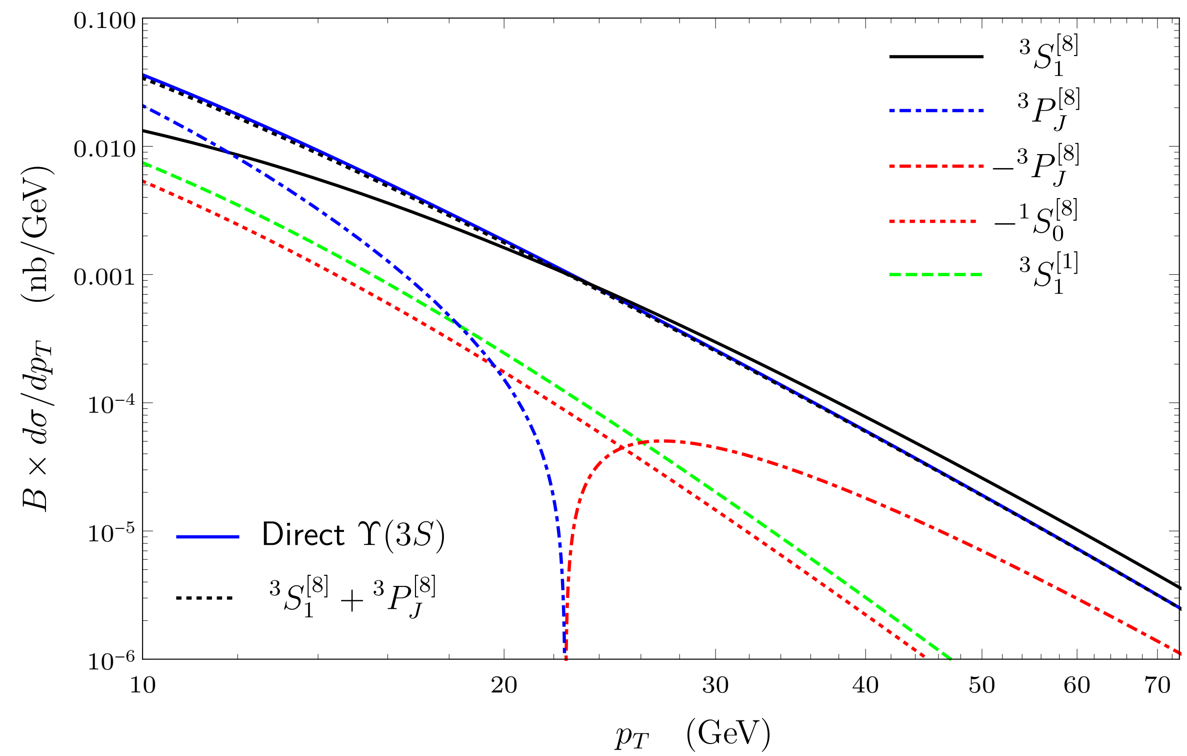
$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle$ (GeV^3)	$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle$ (GeV^3)	$\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]})\rangle/m^2$ (GeV^3)
$(1.40 \pm 0.42) \times 10^{-2}$	$(-0.63 \pm 3.22) \times 10^{-2}$	$(2.59 \pm 0.83) \times 10^{-2}$

$\frac{p_T}{2m} > 5$

- These also determine $\psi(2S)$ and Υ matrix elements.
- S-wave production is dominated by the $^3S_1{}^{[8]} + {}^3P_J{}^{[8]}$.

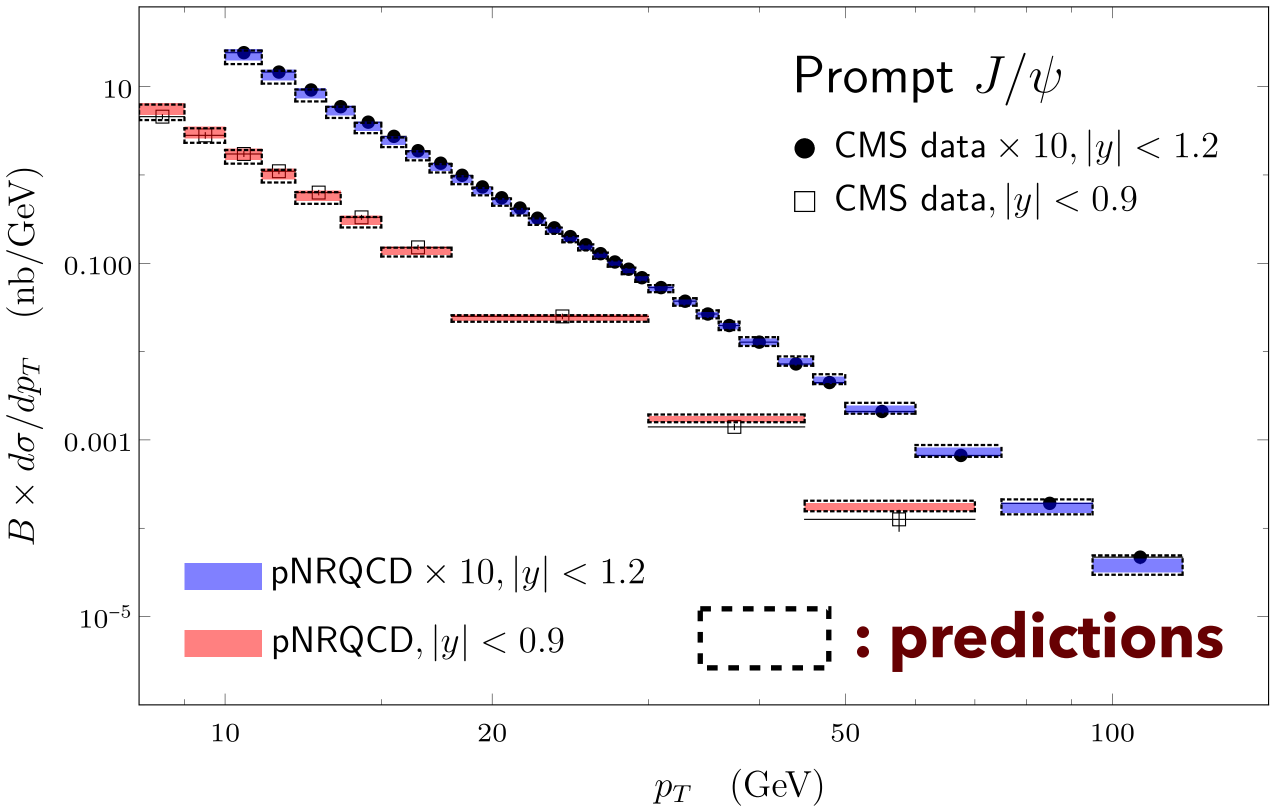
  Large cancellation occur between $^3S_1{}^{[8]}$ and $^3P_J{}^{[8]}$ channels.

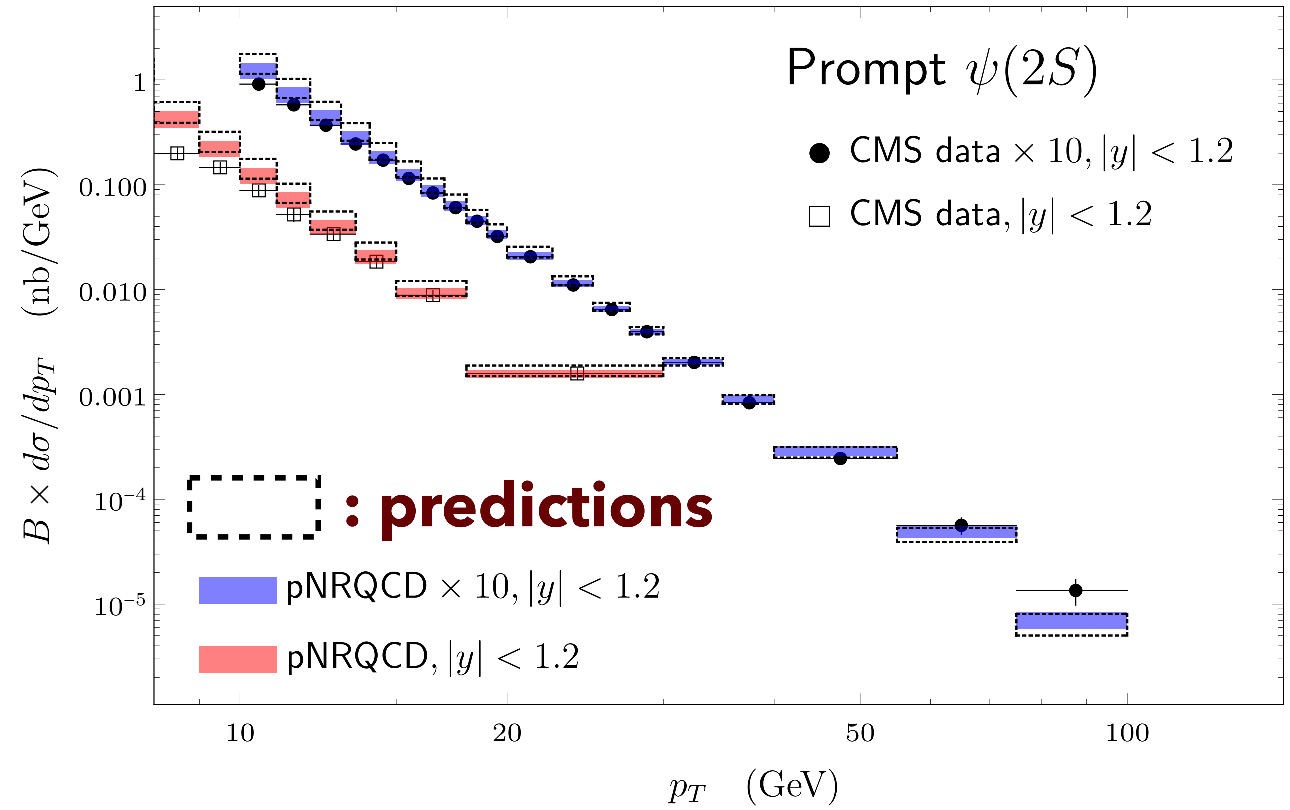




PRODUCTION RATES AT THE LHC

- J/ψ and $\psi(2S)$ production rates at the LHC



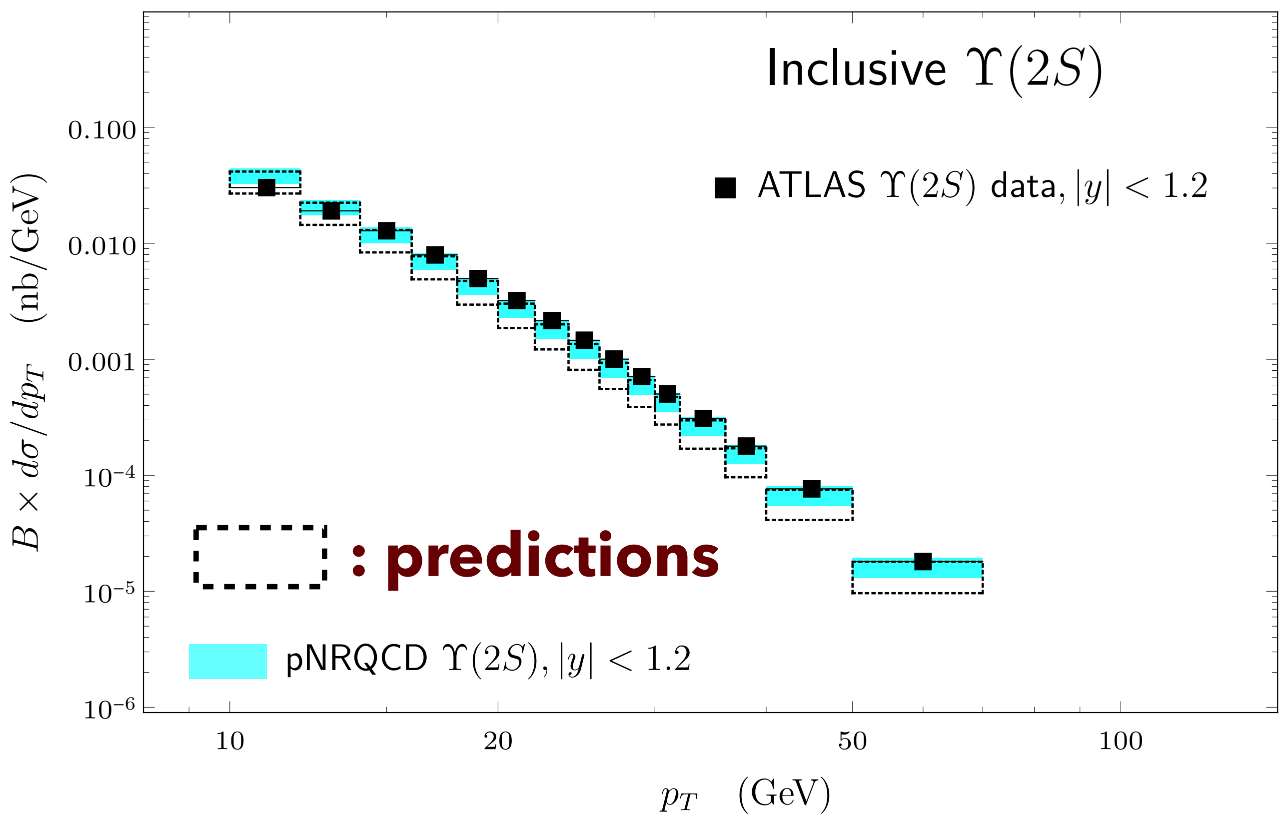


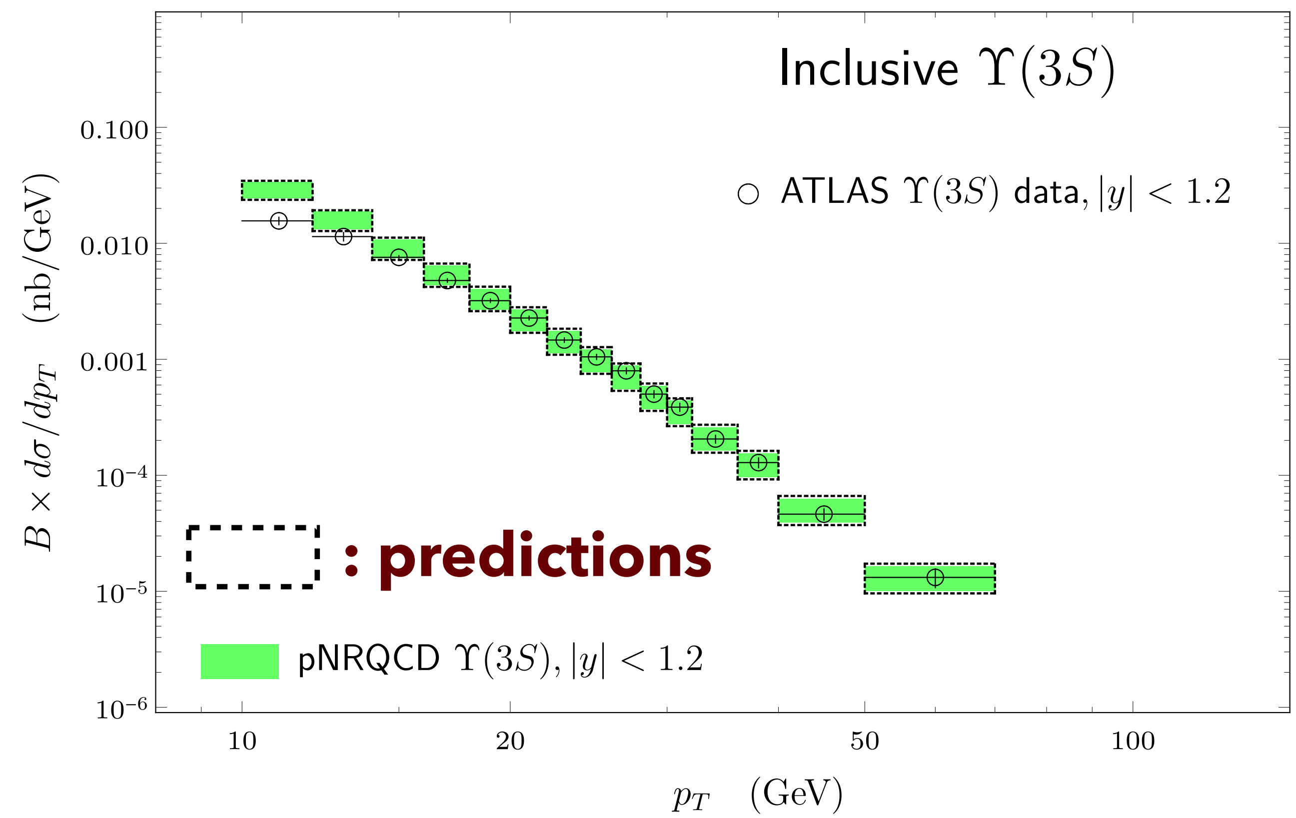
- Good agreements with LHC measurements. **CMS JHEP 02 (2012) 011, PRL114, 191802 (2015)**

- Predictions can be made by excluding cross section data from fit, results agree well with full fit.

PRODUCTION RATES AT THE LHC

- $\Upsilon(2S)$ and $\Upsilon(3S)$ production rates at the LHC



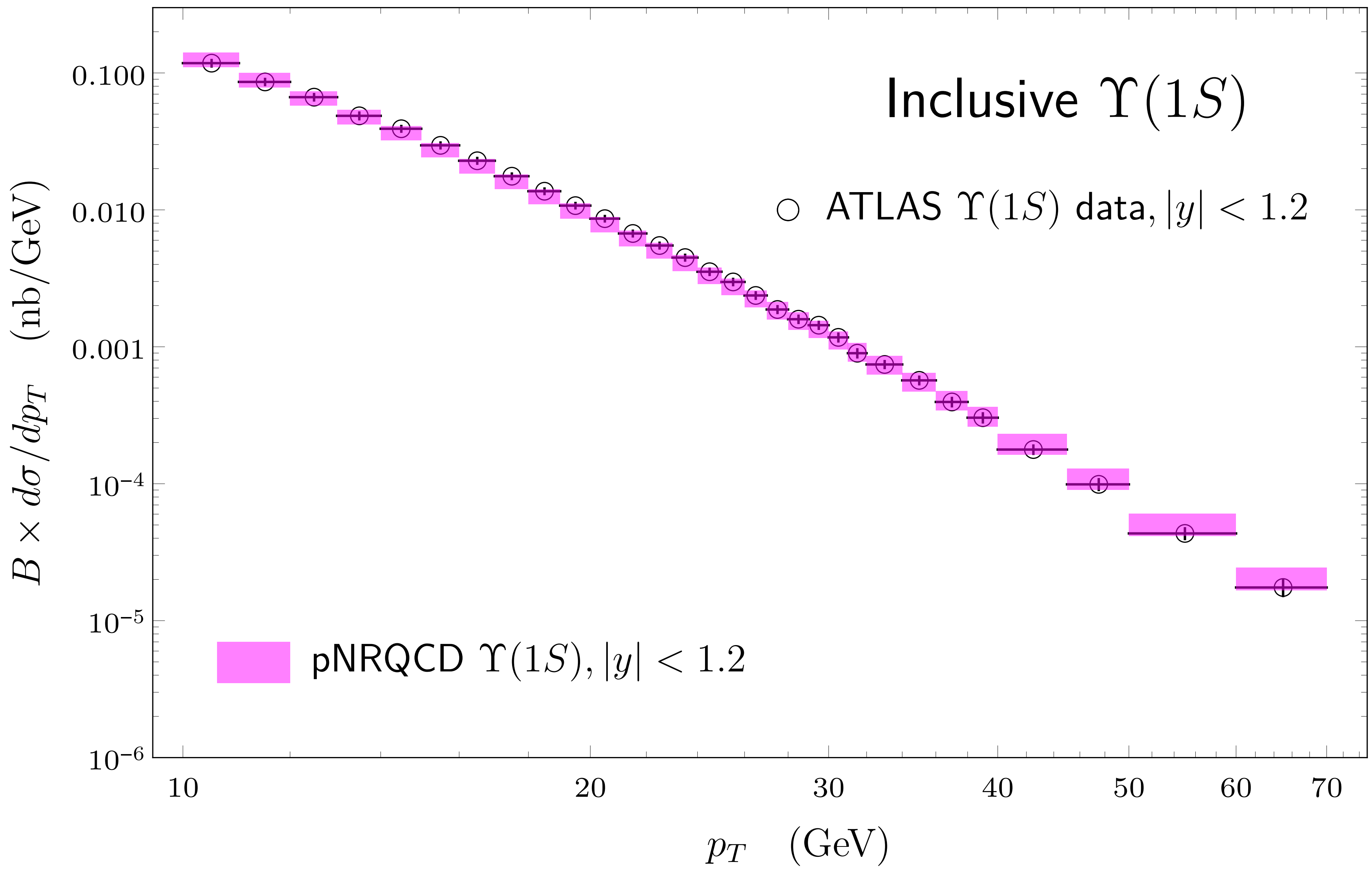


- Good agreements with LHC measurements. **ATLAS PRD87, 052004 (2013)**

- Predictions can be made by excluding cross section data from fit, results agree well with full fit.

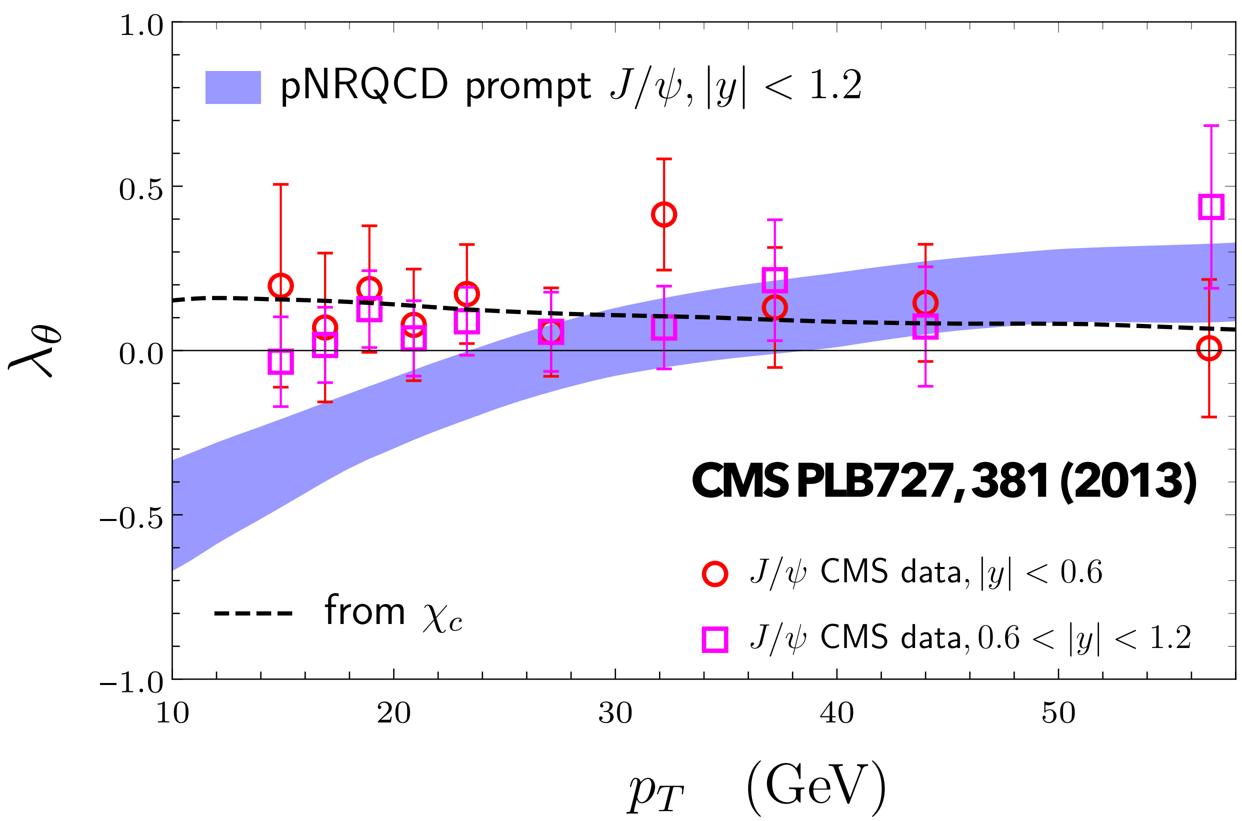
PRODUCTION RATES AT THE LHC

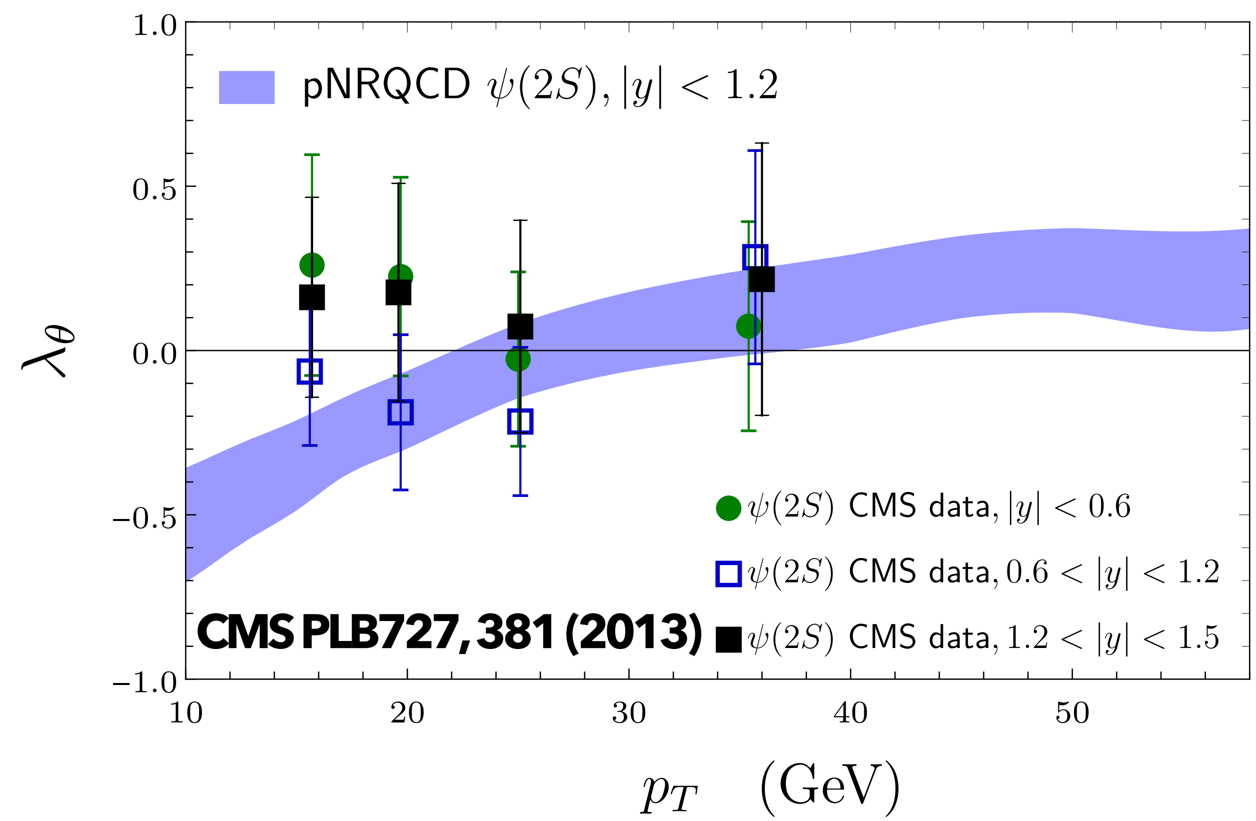
- pNRQCD prediction for $\Upsilon(1S)$ production rate at the LHC, based on J/ψ, $\psi(2S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$ data. Good agreements with measurements. **ATLAS PRD87, 052004 (2013)**



POLARIZATION AT THE LHC

- J/ψ and $\psi(2S)$ polarization λ_θ (helicity frame) at the LHC

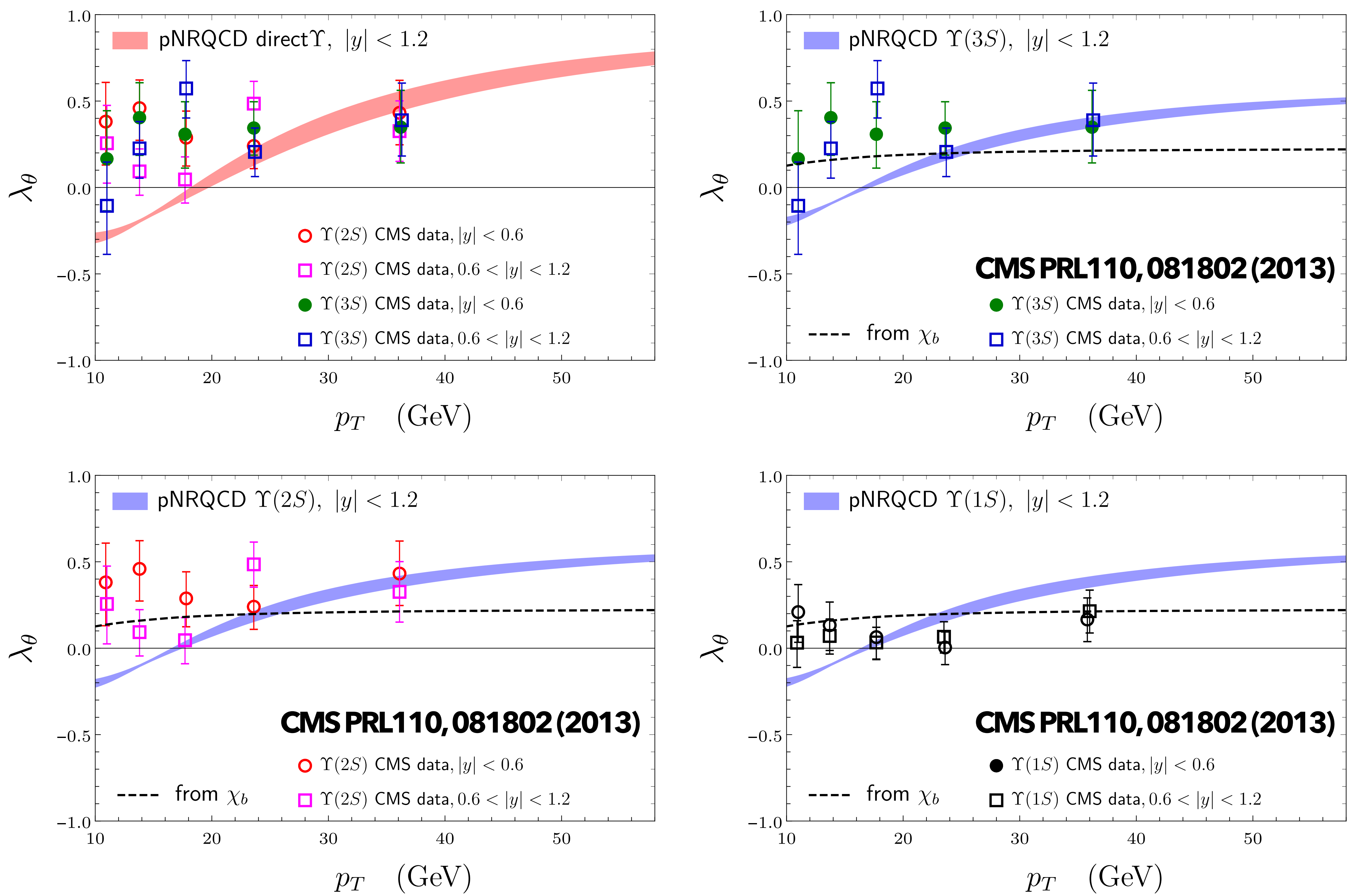




- Small values of λ_θ achieved from large cancellation between ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$, while ${}^1S_0^{[8]}$ is small.
- Polarization of direct J/ψ and direct $\psi(2S)$ are same, prompt J/ψ polarization is affected by P-wave feeddowns.

Polarization at the LHC

- Υ polarization λ_θ (helicity frame) at the LHC

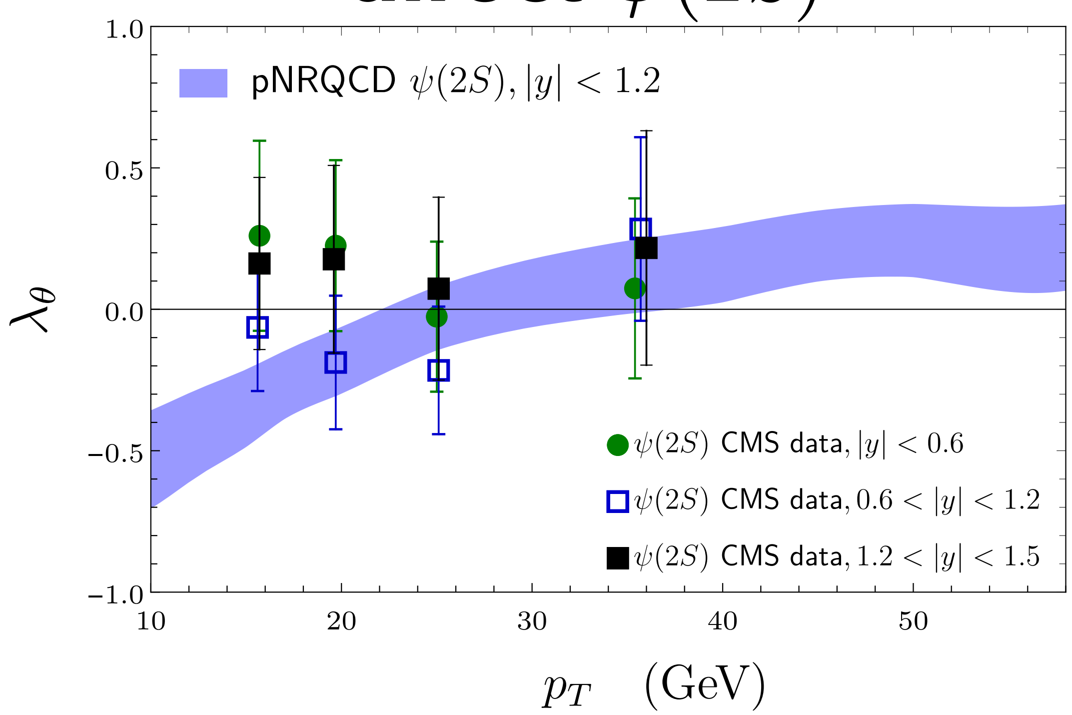


Polarization at the LHC

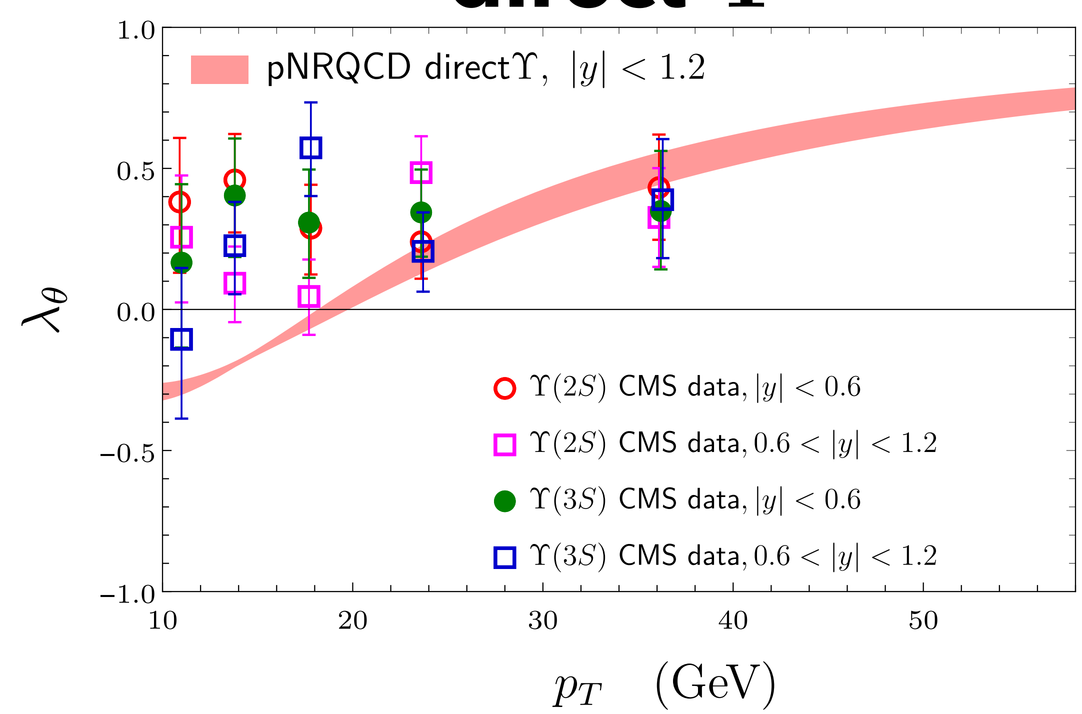
- The pNRQCD fits constrain $\mathcal{E}_{00}$ to be positive.
  In this case, $\mathcal{E}_{10;10}(\mathbf{\Lambda}=m_b)$ is larger than $\mathcal{E}_{10;10}(\mathbf{\Lambda}=m_c)$.

- Because the ${}^3S_1^{[8]}$ channel is mostly transverse,
  Υ is more transverse than J/ψ or $\psi(2S)$ at comparable values of p_T/m, although diluted by *P*-wave feeddowns

direct $\psi(2S)$



direct Υ

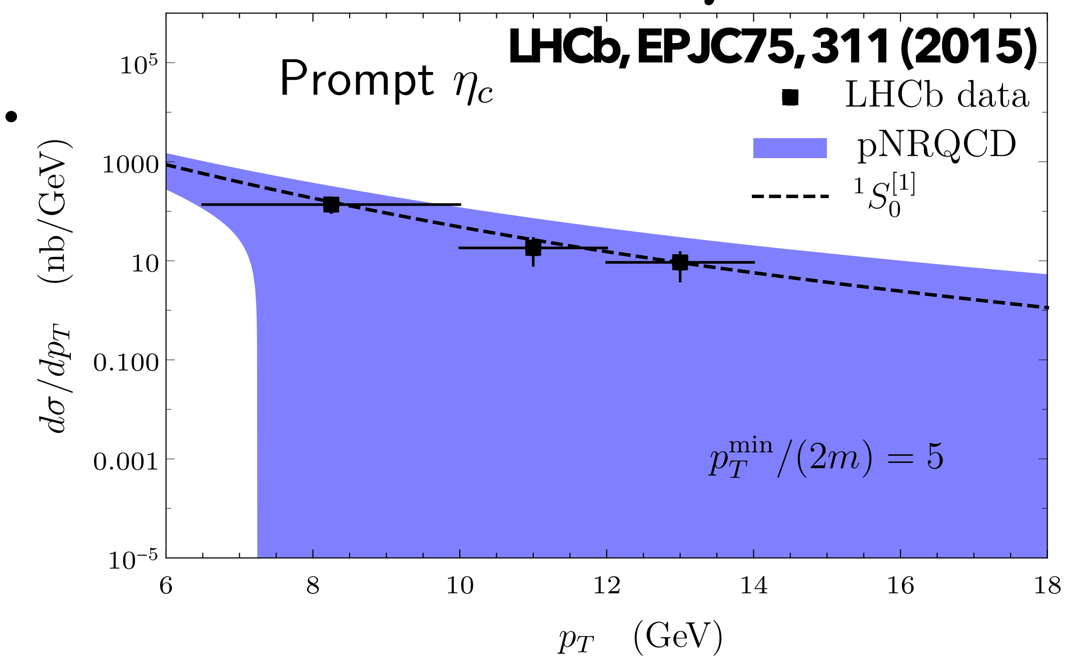


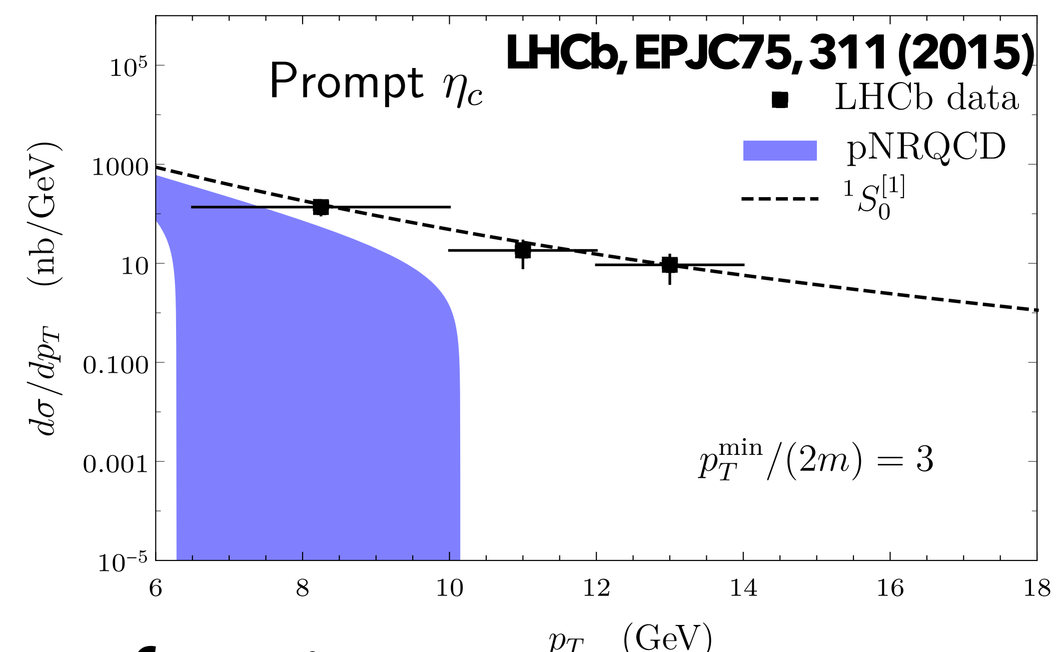
PRODUCTION OF η_c

- Heavy-quark spin symmetry allows determination of η_c matrix elements from J/ψ matrix elements.
- LHCb measurements imply near-zero $\langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle$, consistently with pNRQCD results at large $p_T{}^{\rm min}$.
- Agreement worsens with decreasing p_T cut.
- pNRQCD predicts $\eta_c(2S)/\eta_c(1S)$ ratio

$$\frac{{\rm B}_{\eta_c(2S)\to p\bar p}\times\sigma^{\rm direct}_{\eta_c(2S)}}{{\rm B}_{\eta_c(1S)\to p\bar p}\times\sigma^{\rm direct}_{\eta_c(1S)}} = (2-5)\times 10^{-2}$$

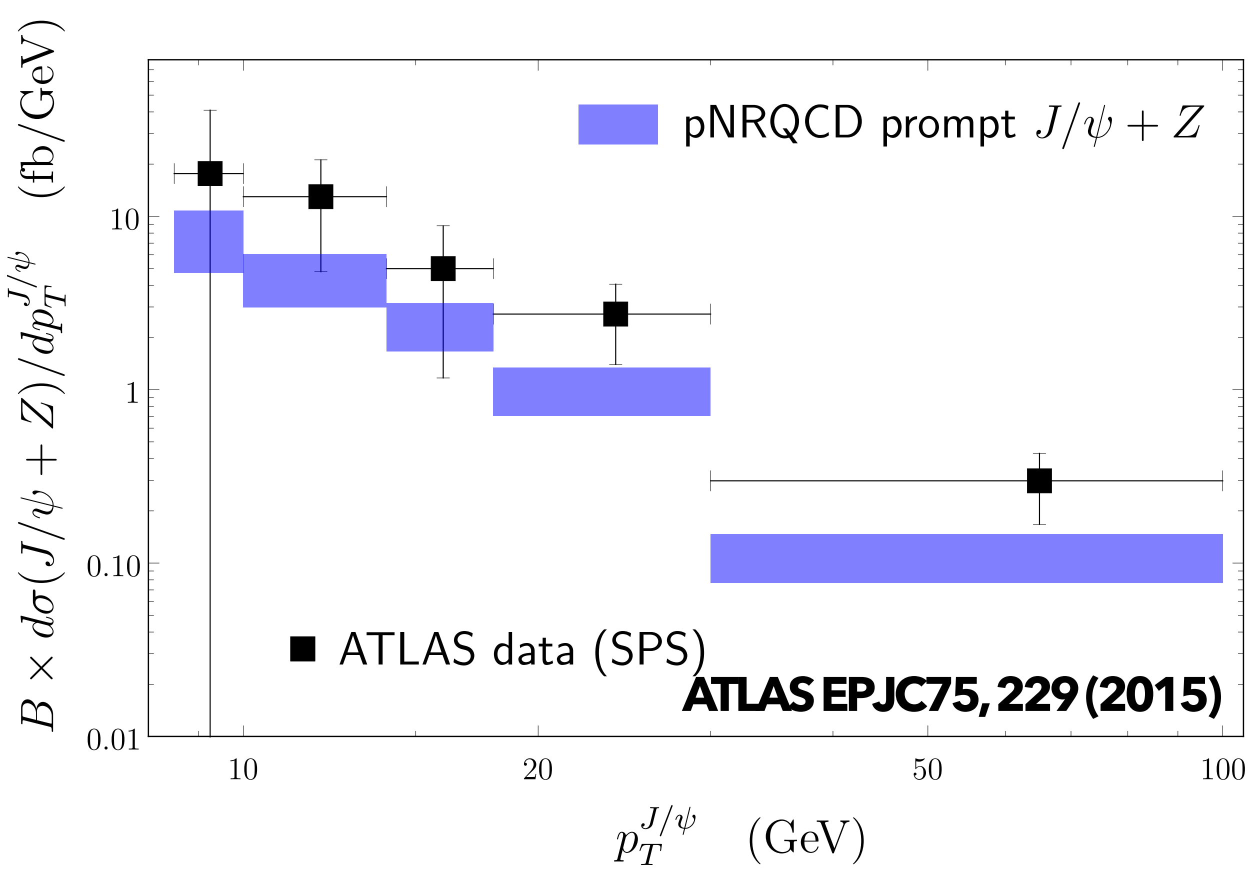
at large p_T, based on recent branching fraction measurements

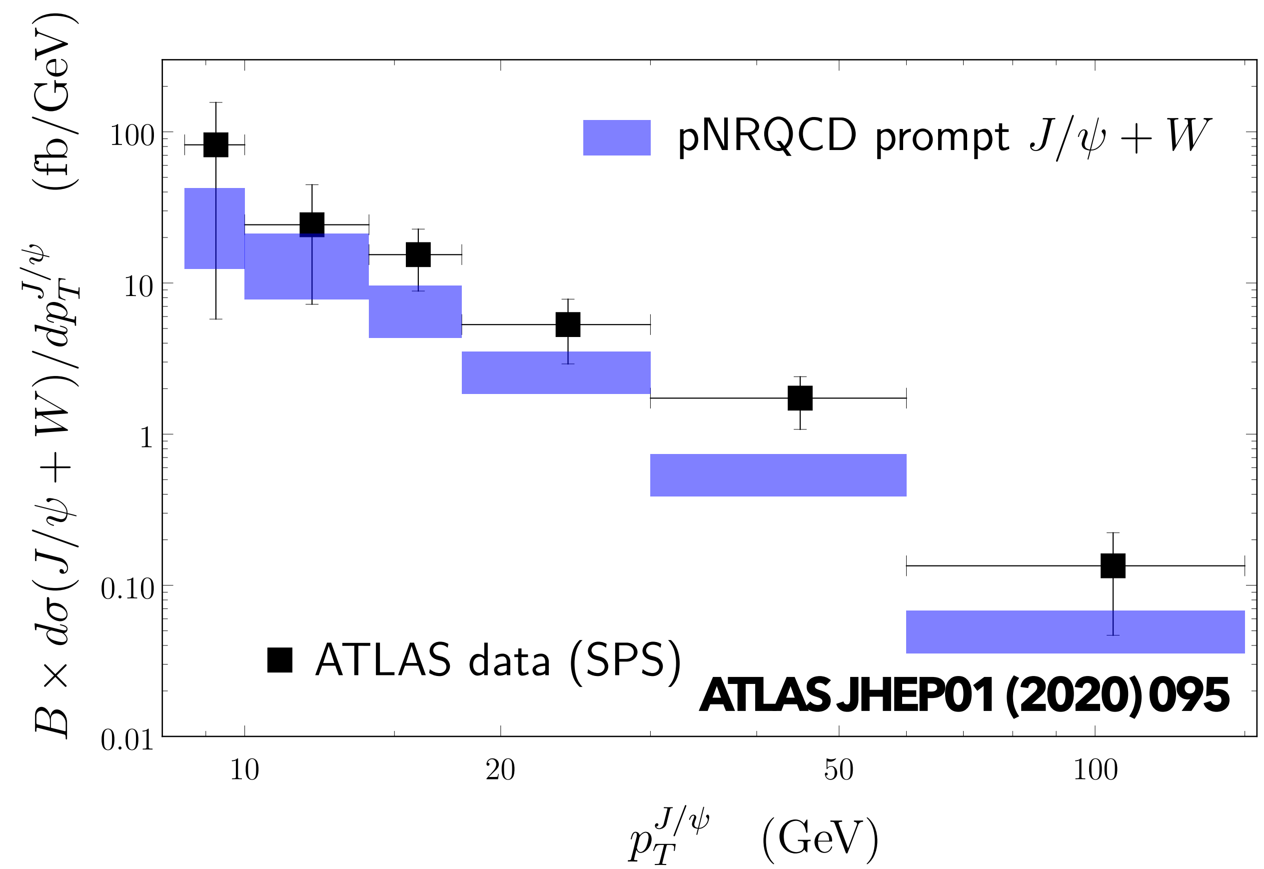




ASSOCIATED PRODUCTION OF $J/\psi + W/Z$

- Recent NLO calculation of associated production $J/\psi + W/Z$ using pNRQCD matrix elements, **Butenschoen and Kniehl, 2207.09366** compared to ATLAS measurements (DPS subtracted)





- Agree with data within uncertainties for most p_T bins.

SUMMARY AND OUTLOOK

- We developed a formalism for ***computing*** quarkonium NRQCD ***production matrix elements*** using ***pNRQCD***.
- ***All S-wave quarkonium*** *cross sections* are determined by *three* ***universal gluonic correlators***.
  Same patterns *of color-octet matrix elements for* ***all S-wave quarkonia***
- pNRQCD gives predictions for ***cross section ratios*** at large p_T ***independently*** *of the color-octet matrix elements*.
  Polarization of directly produced S-wave quarkonia are *independent of radial excitation*.
- Phenomenological determination of gluonic correlators lead to $^3S_1^{[8]} + {}^3P_J^{[8]}$ **dominance**, based on ***evolution equations***.
  Good agreements with many LHC measurements.
  Caveat: *large cancellations in* $^3S_1^{[8]} + {}^3P_J^{[8]}$ *prone to radiative corrections*.

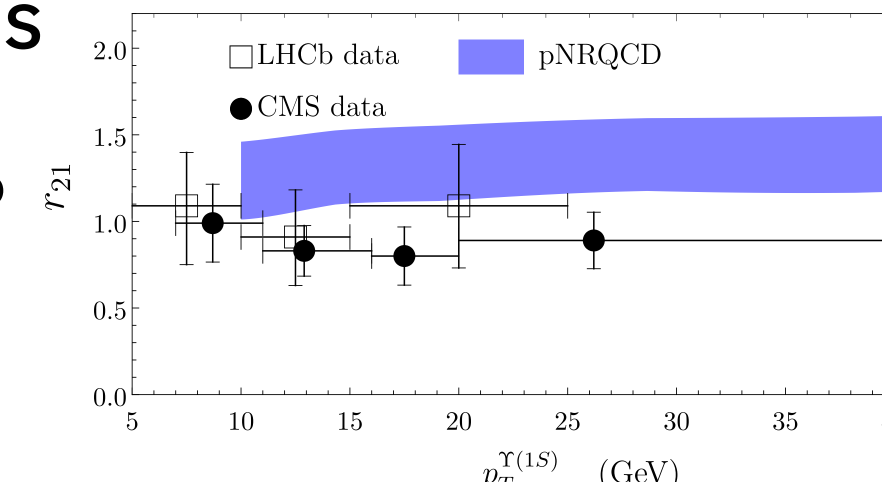
Backup

P-WAVE QUARKONIUM PRODUCTION IN pNRQCD

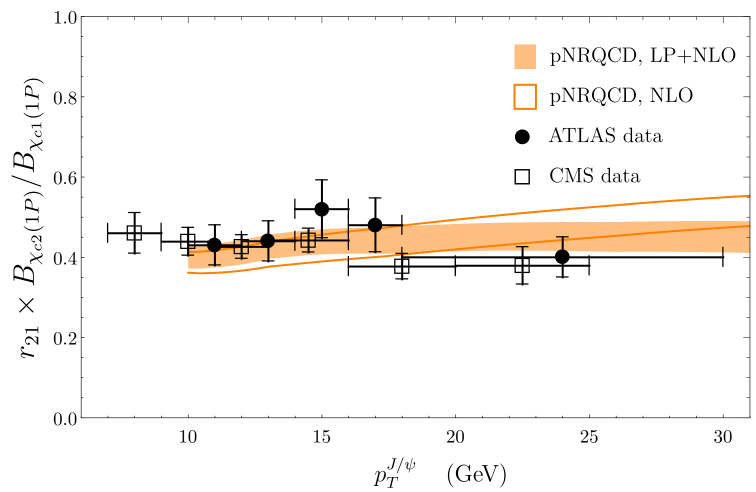
- A single nonperturbative parameter $\mathcal{E}(m_c) = 2.8 \pm 1.7$ determines all χ_{cJ} and χ_{bJ} cross sections

Brambilla, HSC, Vairo, PRL126, 082003 (2021)
JHEP 09 (2021) 032

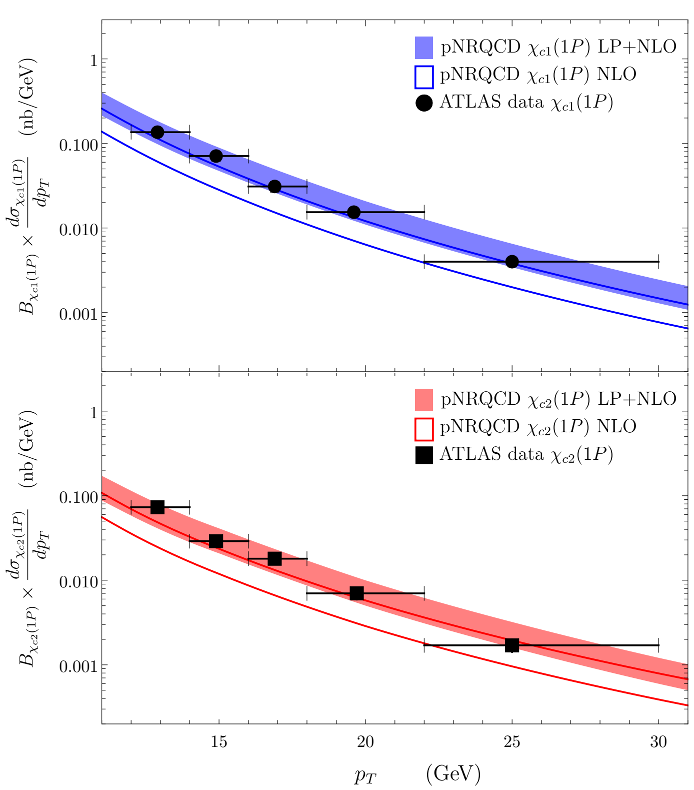
χ_{b2}/χ_{b1} ratio



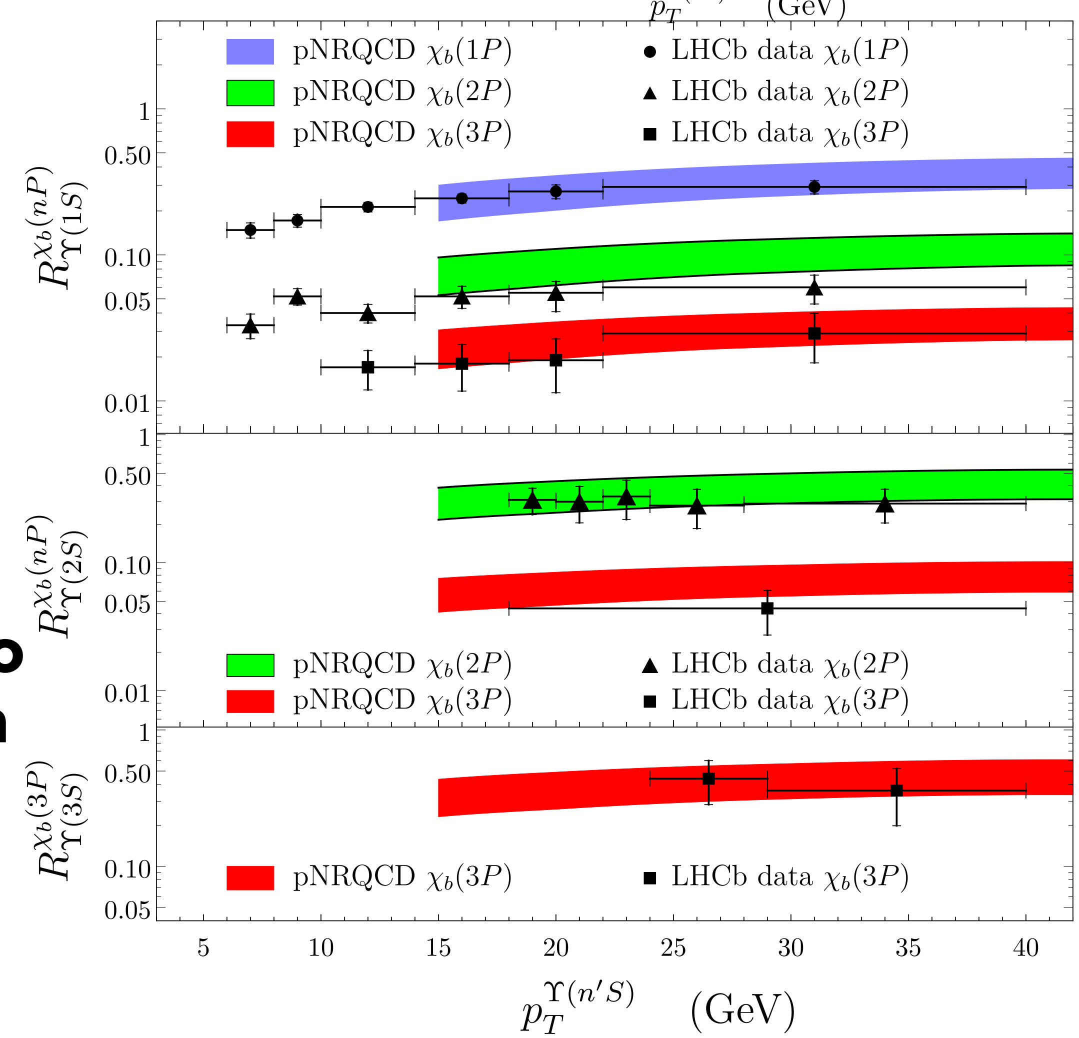
χ_{c2}/χ_{c1} ratio



χ_{c2} and χ_{c1} cross section

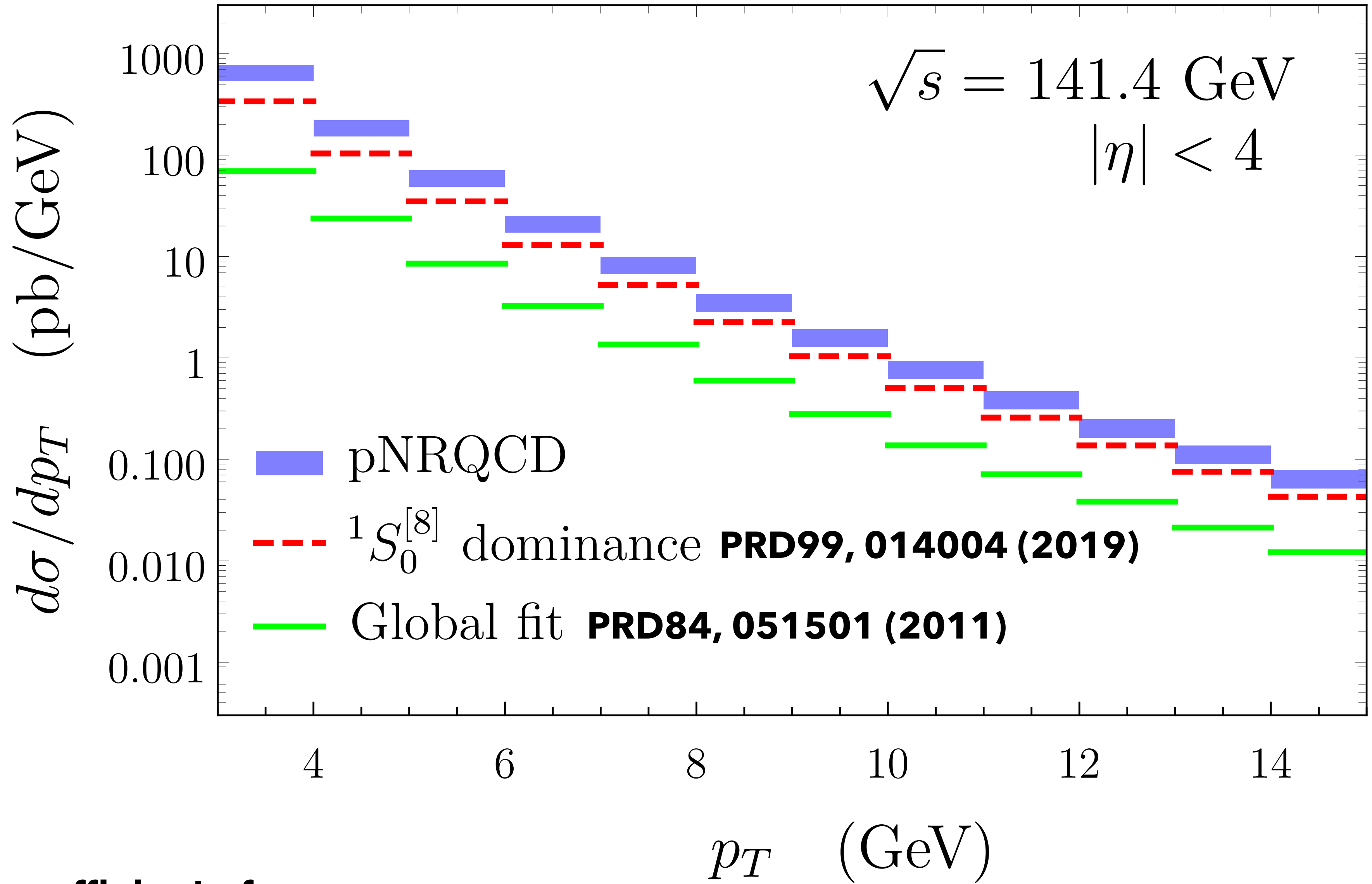


$\chi_b(nP)$ cross section normalized to Υ production



PRODUCTION AT THE EIC

- J/psi production can be measured at the EIC through $ep \to J/\psi + X$. pNRQCD prediction :



Short-distance coefficients from
Qiu, Wang, Xing, Chin. Phys. Lett. 38 (2021) 041201