Triangle Singularituy in the Production of T_{cc}^+ and a soft pion

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Exoctic Heavy Hadrons

Since 2003, heavy hadrons containing four or more quarks have avoided theoretical consensus

Such states do not fit in the converntional quark or quarkonium model, therefore called exotic

First of these exotics was the $X(3872)$ found by Belle[1]
It was unique in that it was the only candidate loosely bound charm-meson molecule until 2021

The discovery of the $T_{cc}^{+}(3875)$ by LHCb provided the second example[2]

[1] Belle, PRL **91**, 262001 (2003)
[2] LHCb, Nature Physics **18**, 751–754 (2022)

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Aspects in the production of X and T_{cc}^{+} at high energy colliders could shed light onto their nature
For this talk we will focus on the T_{cc}^{+}

1 Belle, PRL **91**, 262001 (2003)
2 LHCb, Nature Physics **18**, 751–754 (2022)

Review of the T_{cc}^+

Fit parameters $\varepsilon_T = M_{T_{cc}^+} - M_{D^{*+}D^0}$

	ε_T [keV]	Γ [keV]
BW	-273 ± 63	410 ± 173
pole	-360 ± 173	$48 \pm 2^{+0}_{-14}$

Quantum numbers

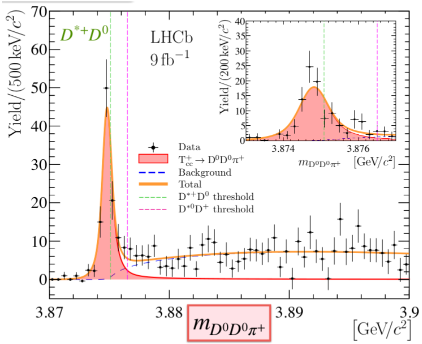
$$J^P = 1^+$$

Decay modes: $D^+D^0\pi^0$, $D^0D^0\pi^+$, $D^+D^0\gamma$

Small binding energy ($|\varepsilon_T| \ll m_\pi^2/M_D$) and quantum numbers imply T_{cc}^+ is a loosely bound charm-meson molecule

$$\left|T_{cc}^+\right\rangle = \left|D^{*+}D^0\right\rangle$$

with a small admixture of $|D^{*0}D^+\rangle$



LHCb, Nature Physics 18, 751-754 (2022),
LHCb, Nature Commun. 13, 3351 (2022)

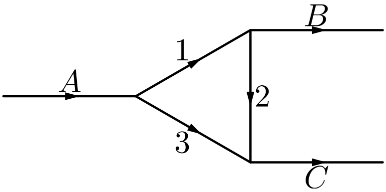
Triangle Singularities

Arises in diagrams where 3 connected internal lines that form a loop can be on-shell simultaneously

Gives $\log^2(E - E_\triangle)$ divergence in reaction rate

Location of peak $E_\triangle$ determined by masses

If internal have a decay width, divergence become narrow peak



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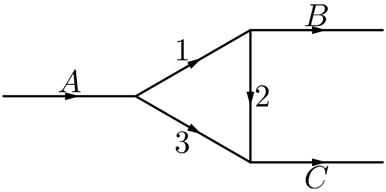
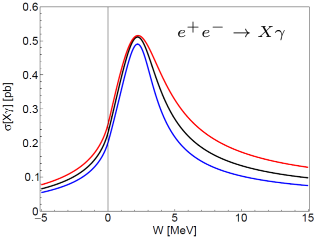


Production of T_{cc}^+ and a soft pion has triangle singularity

Previous studies[1,2,3] have shown that $e^+e^- \to X\gamma$, $B \to KX\pi$ and prompt production of $X\pi$ in pp collisions all have triangle singularities



[1] PRD **100**, no.3, 031501 (2019), [2] PRD **100**, no.7, 074028 (2019), [3] PRD **100**, no.9, 094006 (2019)

Loosely bound states

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S-wave bound states near scattering thresholds have universal properties determined by its binding energy ε

(a) Binding momentum: $\gamma = \sqrt{2\mu|\varepsilon|}$

(b) Scattering length: $a = 1/\gamma$

(c) Wavefunction:
$$\Psi(r) = \exp(-\gamma r)/r$$
$$\hat{\Psi}(k) = \frac{\sqrt{8\pi\gamma}}{k^2 + \gamma^2}$$

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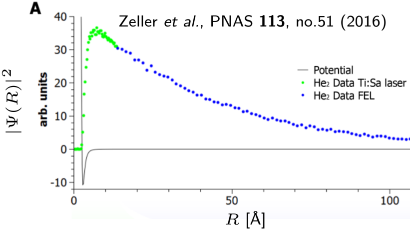
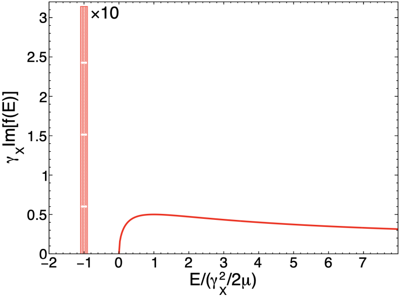
Candidates in hadron physics

- $X(3872)$:
  $|\varepsilon_X| < 0.22$ MeV $\implies \langle r \rangle > 4.8$ fm
- $T_{cc}^{+}(3875)$:
  $|\varepsilon_T| = 0.27 \pm 0.06$ MeV $\implies \langle r \rangle = 4.3 \pm 0.6$ fm



Example from atomic physics:
diamotic ^{4}He with $\varepsilon_{\mathrm{He}_2} = 150$ neV

Loosely bound states: cross sections



Scattering amplitude for complex energy E: $f(E) = \dfrac{1}{-\gamma + \sqrt{-2\mu E}}$

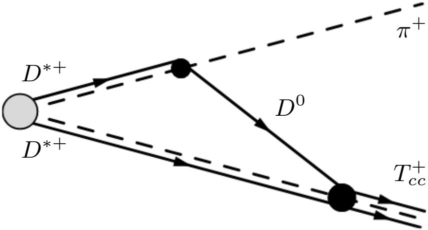
Scattering cross section given by optical theorem (now real E):

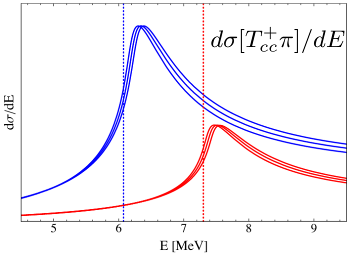
$$\mathrm{Im}[f(E+i\epsilon) = \underbrace{\frac{\pi\gamma}{\mu}\delta(E - \gamma^2/2\mu)}_{\text{bound state}} + \underbrace{\frac{\sqrt{2\mu E}}{\gamma^2 + 2\mu E}\theta(E)}_{\text{production}}$$

Large E behavior is $E^{-1/2}$

$T_{cc}^+\pi$ Cross Section

Production of T_{cc}^+ with momentum and a soft π^+, i.e. non-relativistic pions





$d\sigma[T_{cc}^+\pi^+]/dE$ has peak near 6.1 MeV above $D^{*+}D^{*0}$ threshold

$d\sigma[T_{cc}^+\pi^0]/dE$ has peak near 7.2 MeV above $D^{*+}D^{*0}$ threshold

The asymptotic behavior of the $d\sigma[T_{cc}^+\pi]/dE \propto E^{1/2}$, but expected behavior is $E^{-1/2}$

We can remedy this by looking at the loop integral

$T_{cc}^+\pi$ Cross Section

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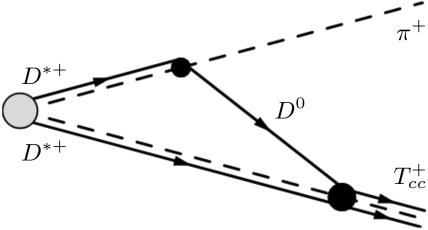
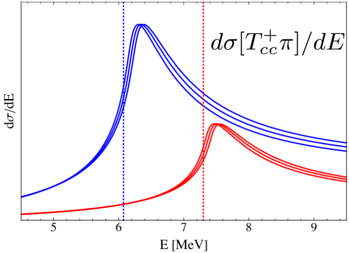
Production of T_{cc}^+ with momentum and a soft π^+, i.e. non-relativistic pions





The loop integral has the form

$$T(q^2,\gamma^2) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{[\boldsymbol{k} + (\mu/M)\boldsymbol{q}]^2 + \gamma^2} \frac{1}{\boldsymbol{k}^2 - (\mu/\mu_\pi)\boldsymbol{q}^2 + M_* E_+}$$

$$= \frac{1}{\sqrt{8\pi\gamma}} \int \frac{d^3k}{(2\pi)^3} \hat{\Psi}(k) \frac{1}{[\boldsymbol{k} - (\mu/M)\boldsymbol{q}]^2 - (\mu/\mu_\pi)\boldsymbol{q}^2 + M_* E_+}$$

To fix, we develop coupled channel model

Ψ_{0+} coupled channel wavefunction, normalized such that if gives the same waveunction at the origin

$$\Psi(0) = \int \frac{d^3k}{(2\pi)^3} \hat{\Psi}(k) = \Psi_{0+}(0)$$

Regularize wavefunctions to have smooth UV cutoffs $\Psi \to \Psi^{(\Lambda)}$

$D^{*+}D^0$: $\gamma_T = \sqrt{2\mu|\varepsilon_T|}$

$$\hat{\Psi}^{(\Lambda)}(k) = \frac{\sqrt{8\pi\gamma_T}}{\sqrt{1+Z_{0+}}} \frac{\sqrt{(\Lambda+\gamma_T)\Lambda}}{\Lambda-\gamma_T} \left(\frac{1}{k^2+\gamma_T^2} - \frac{1}{k^2+\Lambda^2} \right)$$

$D^{*0}D^+$: $\gamma_T = \sqrt{2\mu(\delta+|\varepsilon_T|)}$, $\delta = M_{D^{*0}D^+} - M_{D^{*+}D^0}$

$$\hat{\Psi}_{0+}^{(\Lambda)}(k) = \frac{\sqrt{8\pi\gamma_T}}{\sqrt{1+Z_{0+}}} \frac{\sqrt{(\Lambda+\gamma_T)\Lambda}}{\Lambda-\gamma_{0+}} \left(\frac{1}{k^2+\gamma_{0+}^2} - \frac{1}{k^2+\Lambda^2} \right)$$

Relative probability is defined as

$$Z_{0+} = \frac{(\Lambda+\gamma_T)\gamma_T}{(\Lambda+\gamma_{0+})\gamma_{0+}}$$

Coupled Channel Model

Allows us to make replacement

$$T(q^2,\gamma^2) \longrightarrow T^{(\Lambda)}(q^2,\gamma^2)$$

Differential cross sections now have proper asymptotic behavior

Λ is scale at which model breaks down, at distance smaller than $1/\Lambda$ other components of T_{cc}^{+} wavefunction can begin to dominate

We take the cutoff to be $\Lambda \sim m_\pi$, i.e. effective range is determine by pion exchange

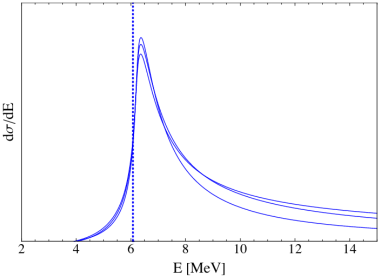
Measured T_{cc}^{+} events

LHCb has observed 117 ± 16 T_{cc}^{+} events[1]

To make prediction using our model, we can use $d\sigma[T_{cc}^{+}\pi^{-}]/dE$ which has no triangle singularity to subtract background events

Estimate events in peak due to the triangle singularity

$$\sigma[T_{cc}^{+}\pi^{+}] - \sigma[T_{cc}^{+}\pi^{-}] \approx \left(1.3^{+1.5}_{-0.8}\right) \times 10^{-2} \sigma^{(\Lambda)}[T_{cc}^{+}]$$



We predict that 1.3% come from peak of width ~ 1 MeV

More statistics needed to observe peak

[1] LHCb, Nature Physics 18, 751-754 (2022), LHCb, Nature Commun. 13, 3351 (2022)

Summary

1. We have studied the production of $T_{cc}^{+}(3875)$ and a soft pion in pp collisions
2. The production rates for $T_{cc}^{+}\pi^{+}$ and $T_{cc}^{+}\pi^{0}$ have narrow peaks about 6.1 MeV and 7.2 MeV above the $D^{*}D^{*}$ scatter threshold, respectively
3. The observation of the these peaks would be a smoking gun for the identification of the the T_{cc}^{+} as a *loosely bound charm-meson molecule*
4. We have developed a model that allows us to extend our calculation to relativistic charm mesons