JLU
NEUE WEGE. SEIT 1607.

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

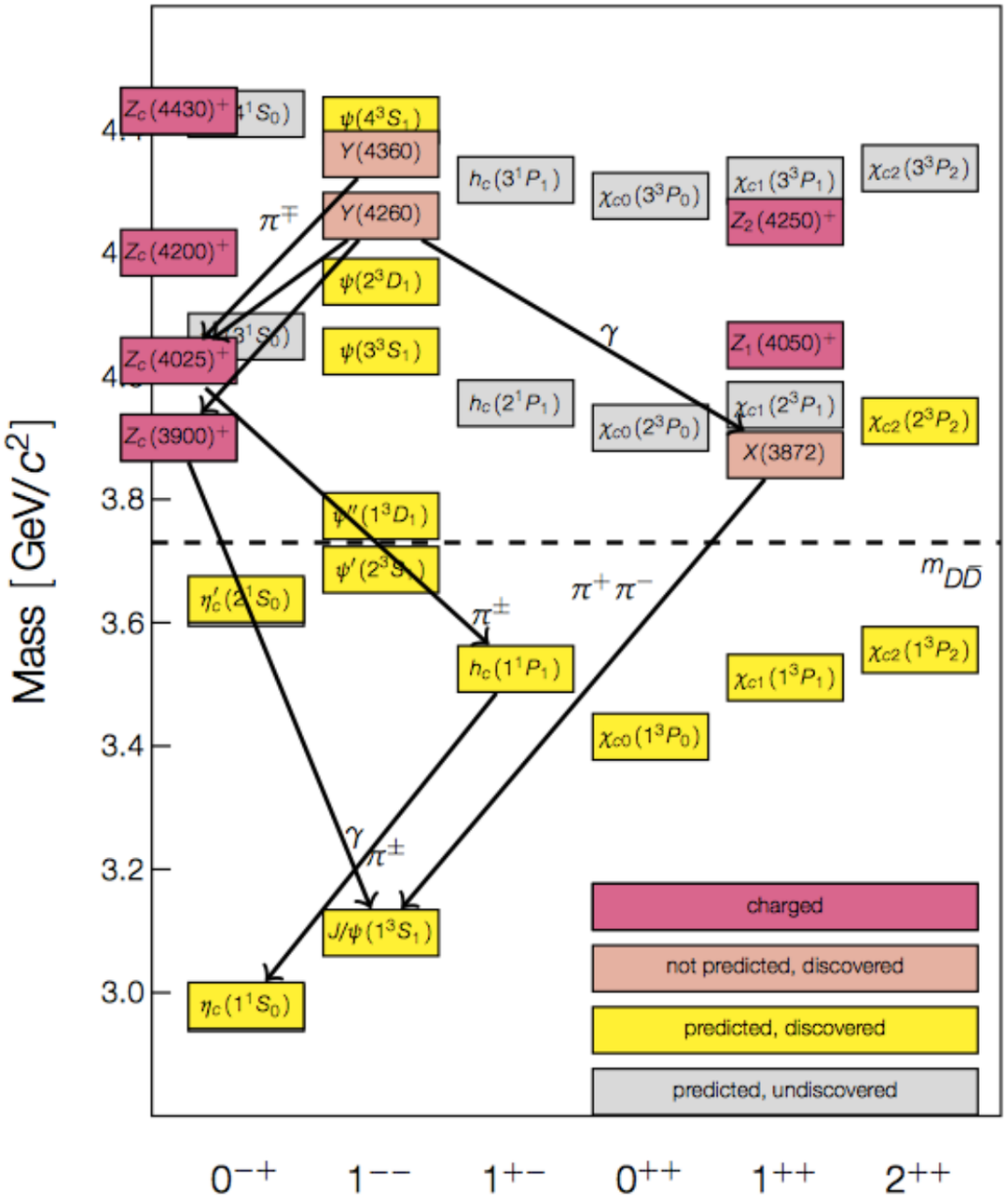
HFHF
Helmholtz Forschungsakademie Hessen für FAIR

QWG
Darmstadt 2022

Four-quark states with charm quarks from functional methods

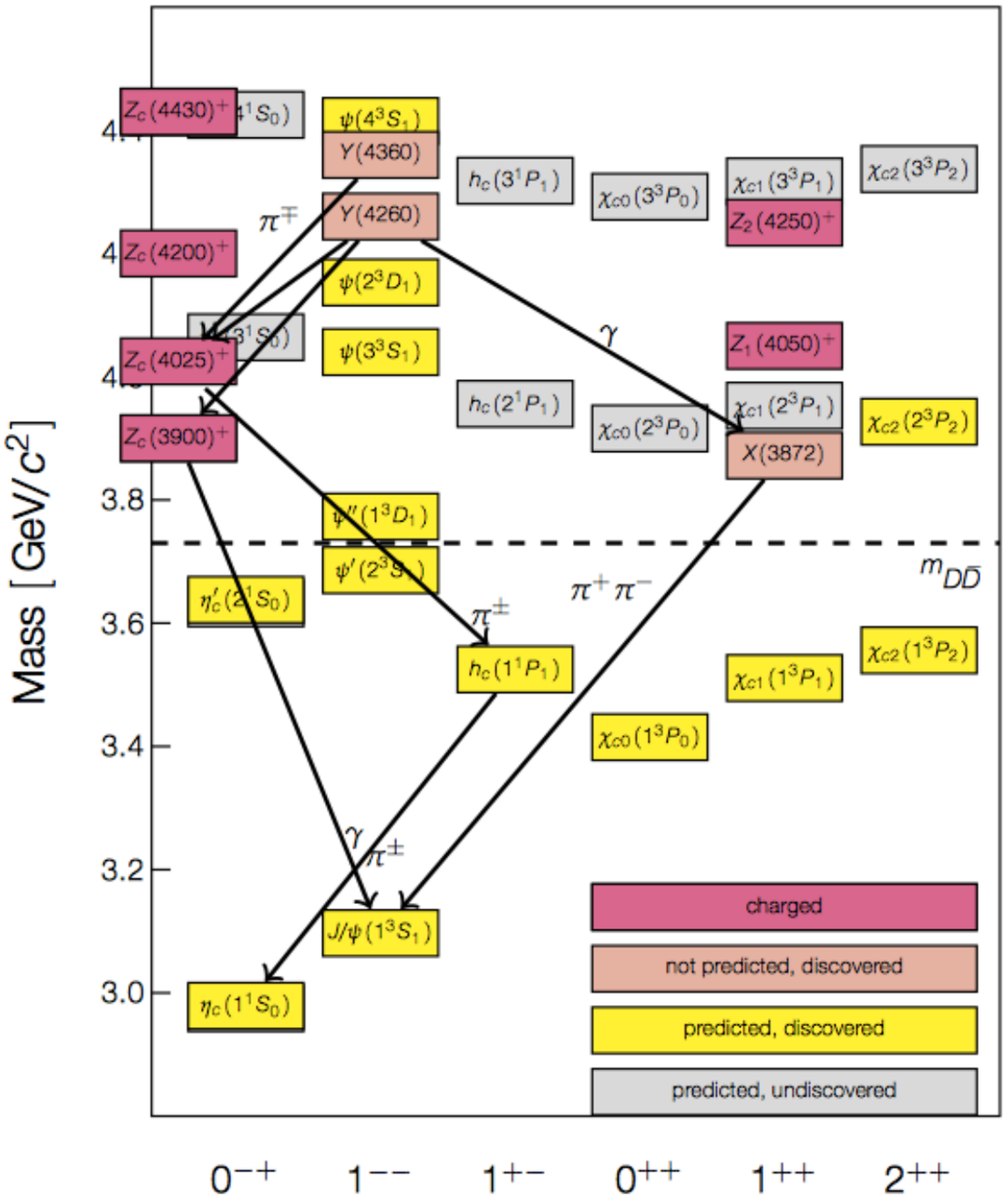
Santowsky and CF, EPJC 82 (2022) 4, 313 [2111.15310]

Tetraquark candidates with $cq\bar{q}\bar{c}$-content



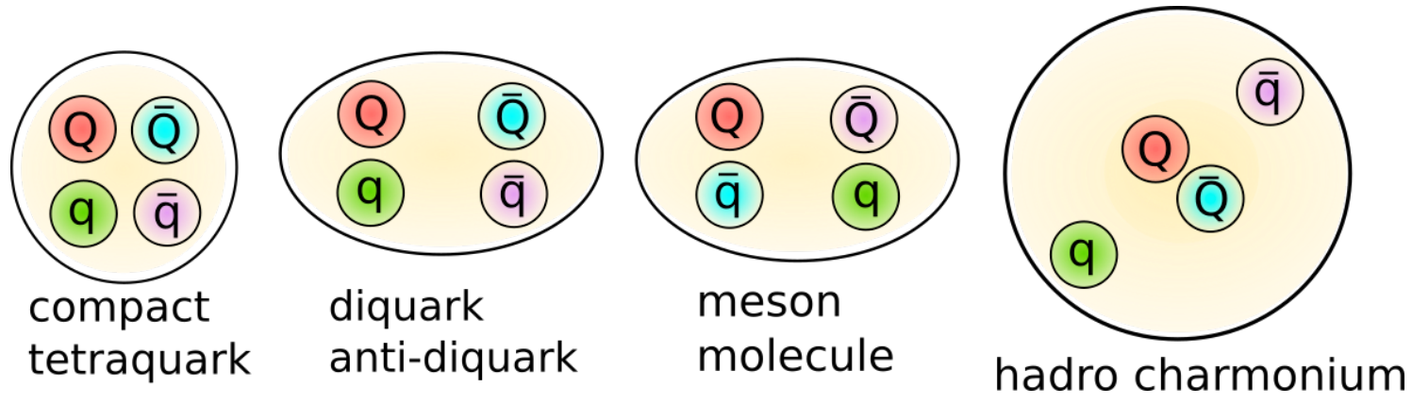
Many new unexpected states found: Belle, BABAR, BES, LHCb …

Tetraquark candidates with $cq\bar{q}\bar{c}$-content



Many new unexpected states found: Belle, BABAR, BES, LHCb …

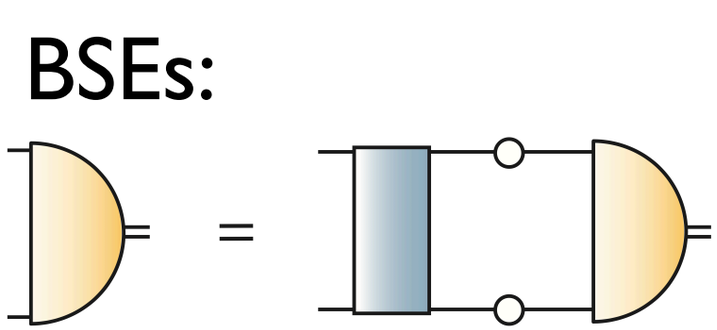
Internal structure ??

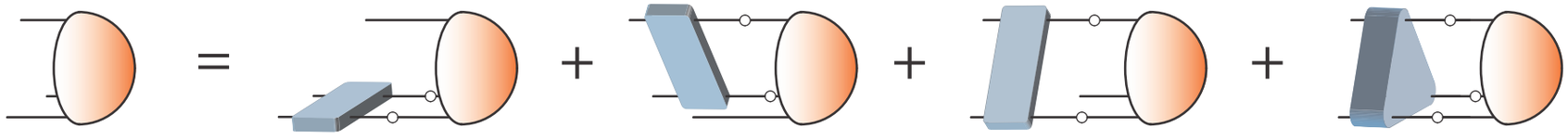


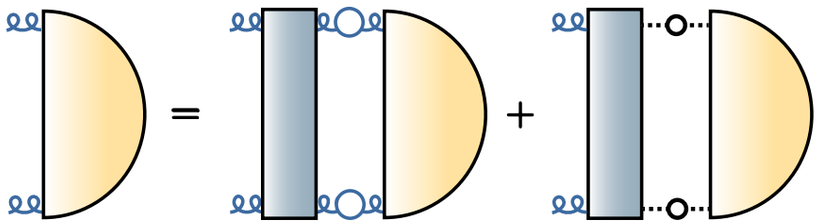
Related to details of underlying QCD forces between quarks and gluons

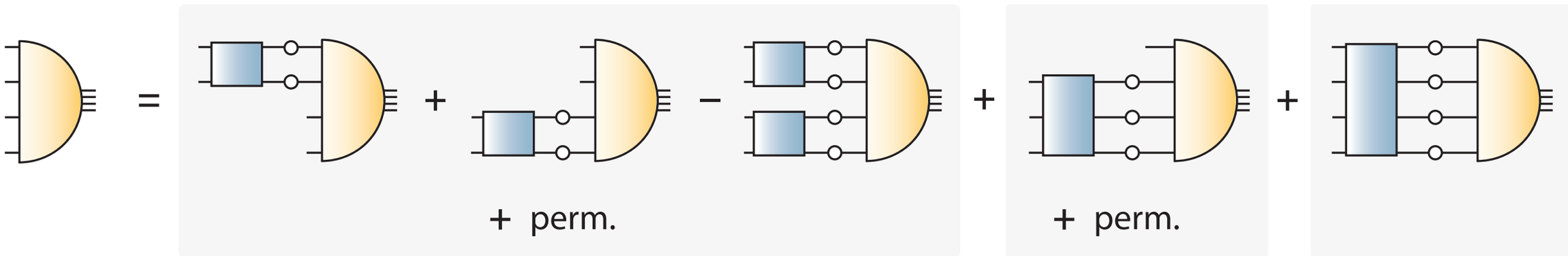
Bound states and Bethe-Salpeter equations

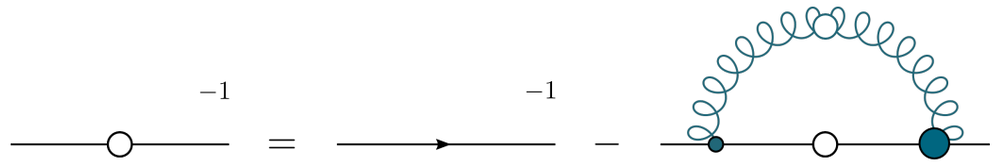
BSEs:









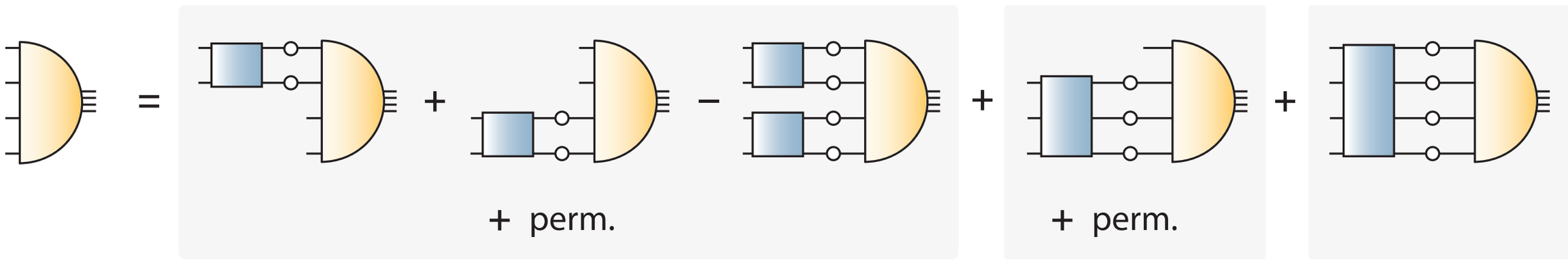


Eigenvalue equations: masses and wave functions

Four-quark states from the four-body equation

Exact equation:

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)
Heupel, Eichmann, CF, PLB 718 (2012) 545-549
Eichmann, CF, Heupel, PLB 753 (2016) 282-287



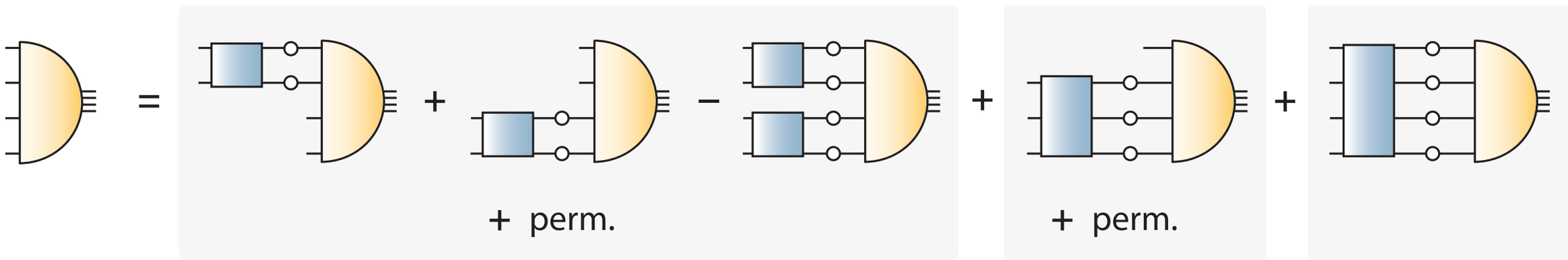
Two-body interactions

Three- and four-body interactions

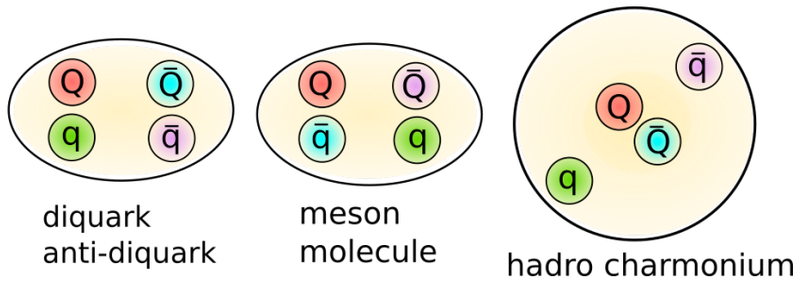
Four-quark states from the four-body equation

Exact equation:

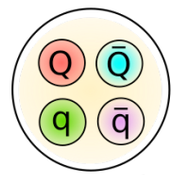
Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)
Heupel, Eichmann, CF, PLB 718 (2012) 545-549
Eichmann, CF, Heupel, PLB 753 (2016) 282-287



Two-body interactions



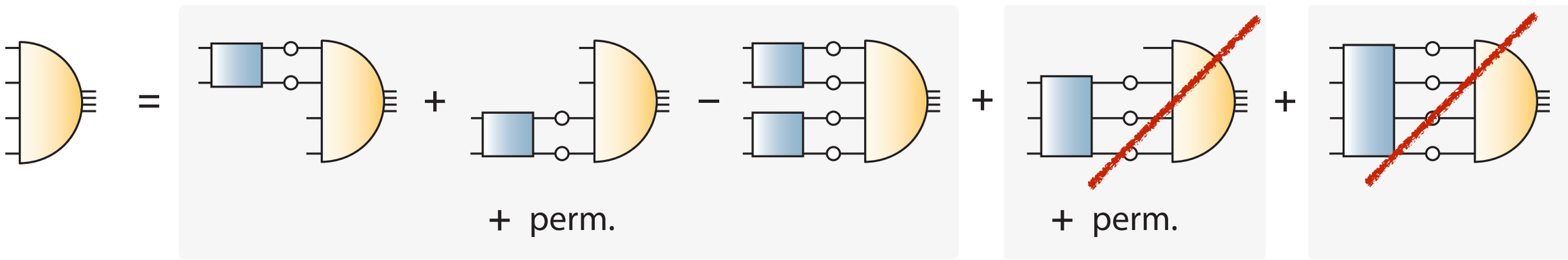
Three- and four-body interactions



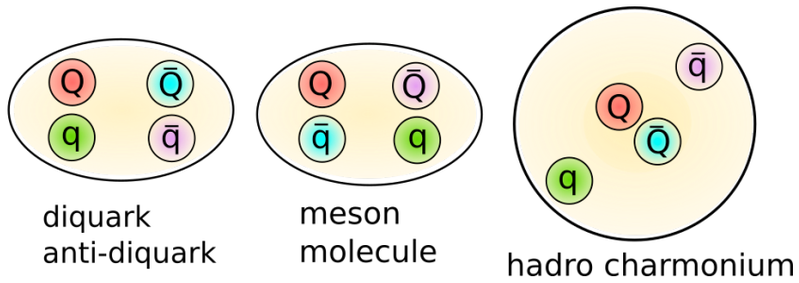
Four-quark states from the four-body equation

Exact equation:

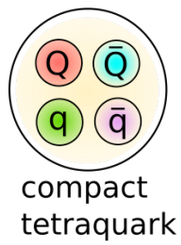
Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)
Heupel, Eichmann, CF, PLB 718 (2012) 545-549
Eichmann, CF, Heupel, PLB 753 (2016) 282-287



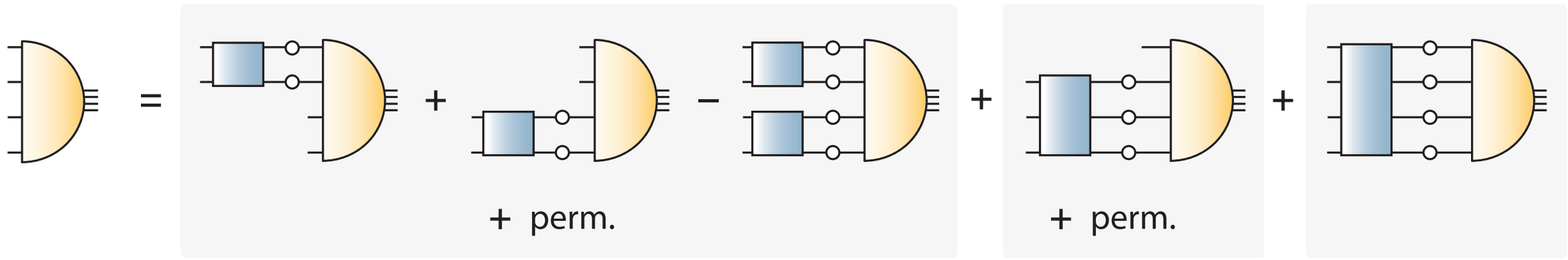
Two-body interactions



Three- and four-body interactions

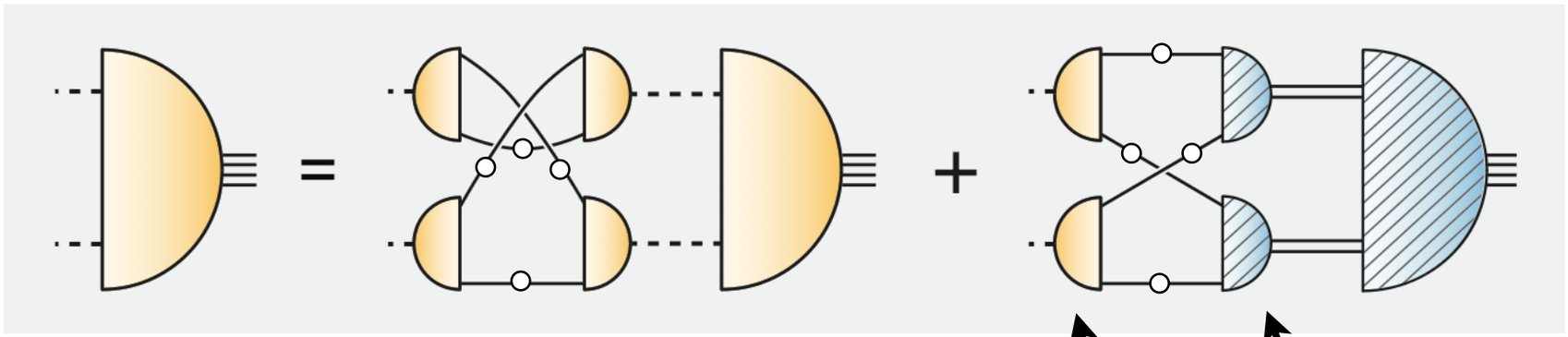


Four-quark states from the two-body equation



approximation: separable ansatz for interaction kernel

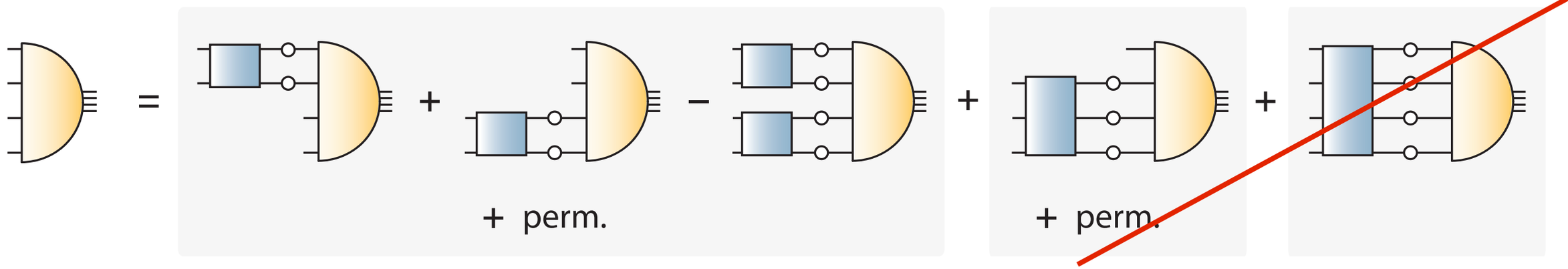
Heupel, Eichman, CF, PLB 718 (2012) 545-549



Diquark Qq-$\bar{Q}\bar{q}$

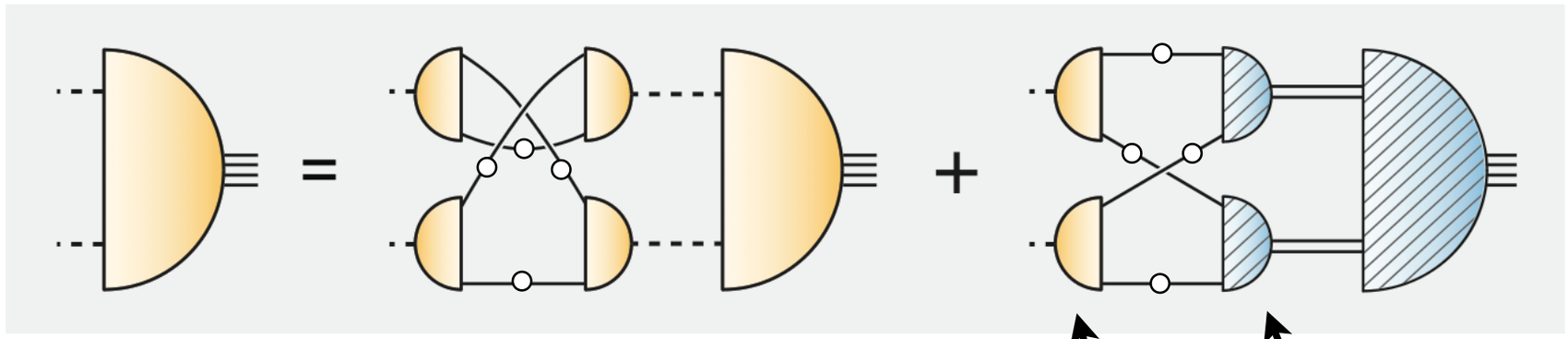
Meson $Q\bar{q}$-$\bar{Q}q$ and $Q\bar{Q}$-$q\bar{q}$

Four-quark states from the two-body equation



approximation: separable ansatz for interaction kernel

Heupel, Eichman, CF, PLB 718 (2012) 545-549



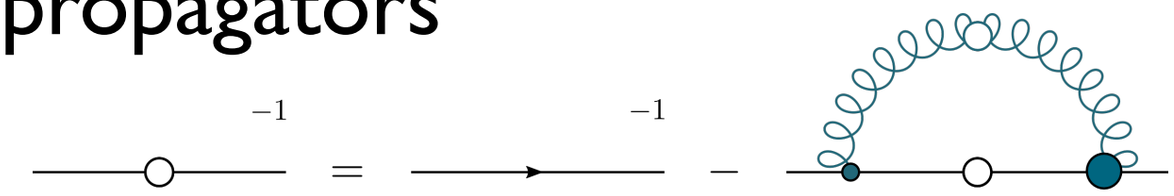
Diquark Qq-$\bar{Q}\bar{q}$

Meson $Q\bar{q}$-$\bar{Q}q$ and $Q\bar{Q}$-$q\bar{q}$

Dyson-Schwinger equations - "3PI vs RL"

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{ - \int d^4x \left(\bar{\Psi} \left(i \not{D} - m \right) \Psi - \frac{1}{4} \left(F^a_{\mu\nu} \right)^2 \right) \right\}$$

propagators

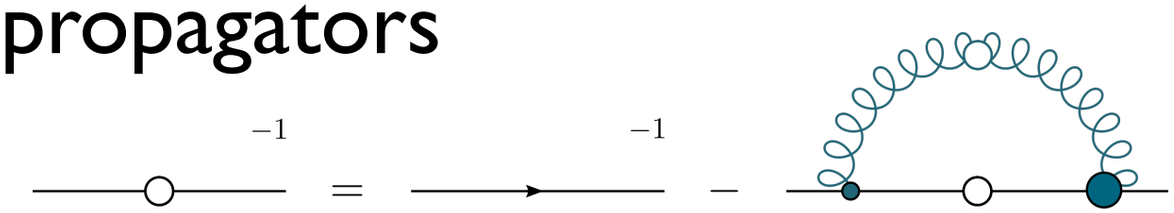


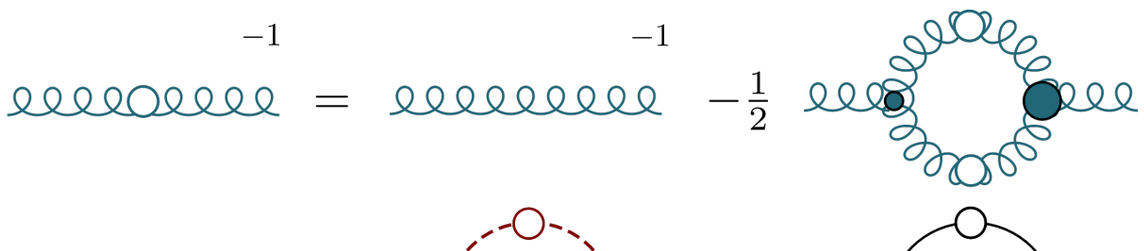
CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

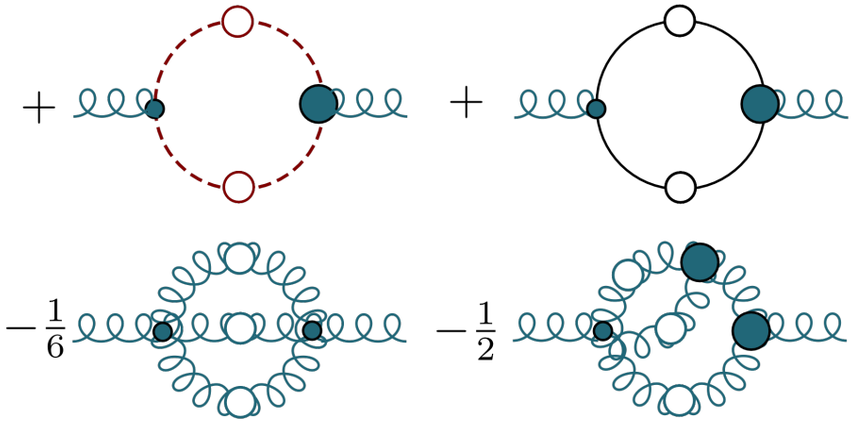
Dyson-Schwinger equations - "3PI vs RL"

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{ - \int d^4x \left(\bar{\Psi} \left(i \not{D} - m \right) \Psi - \frac{1}{4} \left(F^a_{\mu\nu} \right)^2 \right) \right\}$$

propagators





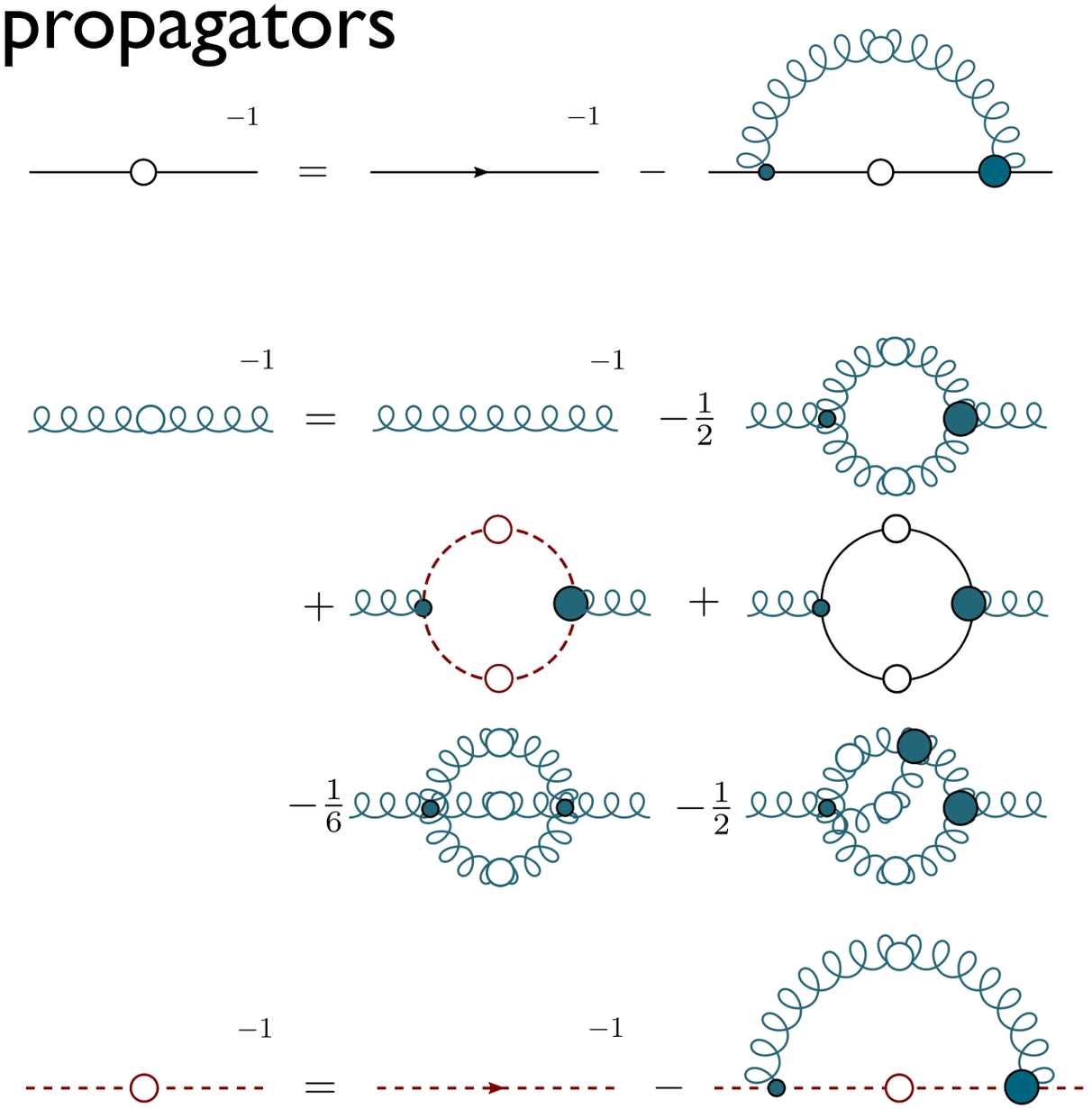


CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

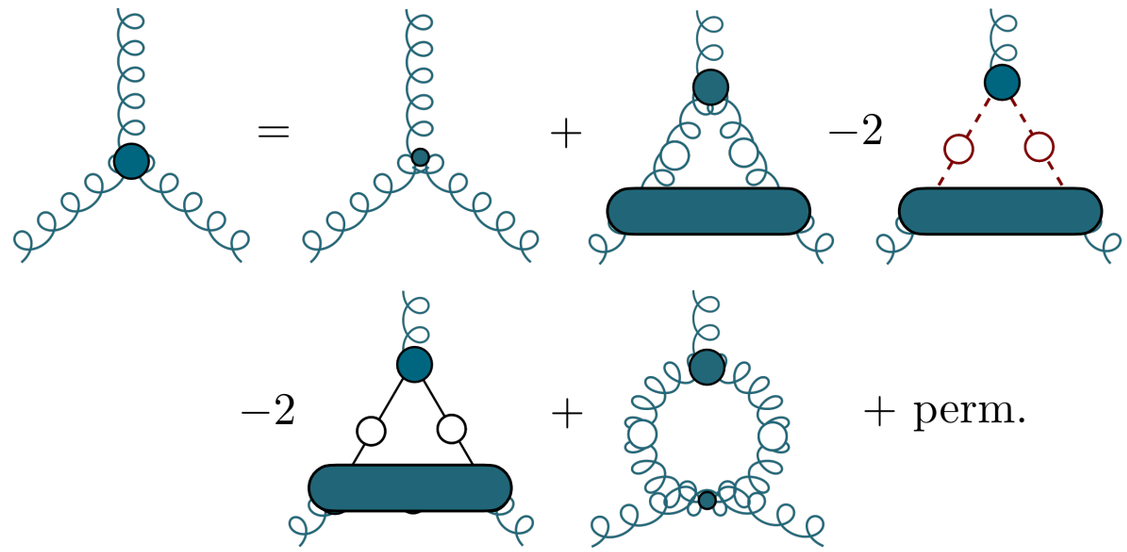
Dyson-Schwinger equations - "3PI vs RL"

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{ - \int d^4x \left(\bar{\Psi} \left(i\not{D} - m \right) \Psi - \frac{1}{4} \left(F^a_{\mu\nu} \right)^2 \right) \right\}$$

propagators



vertices

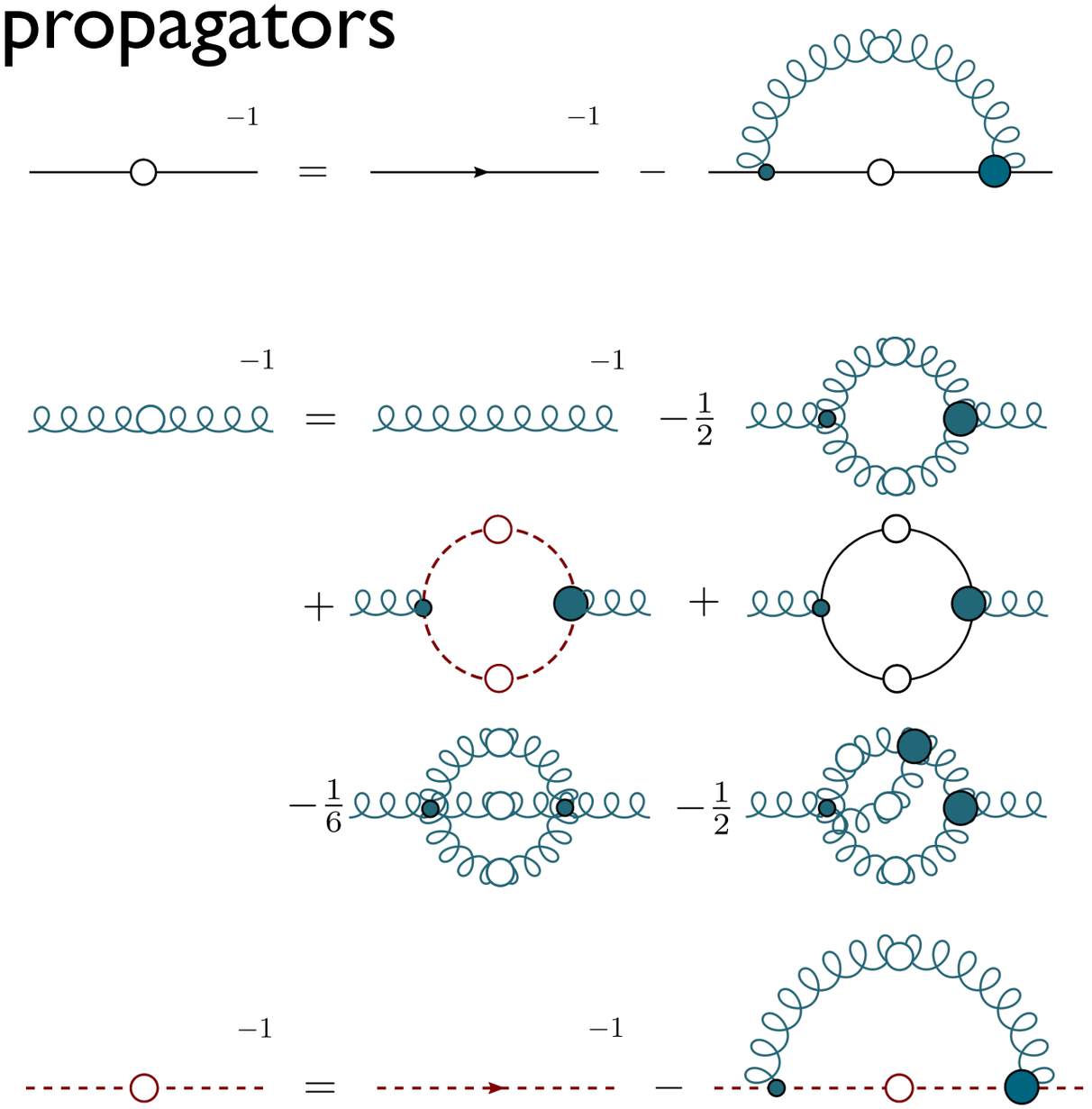


CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

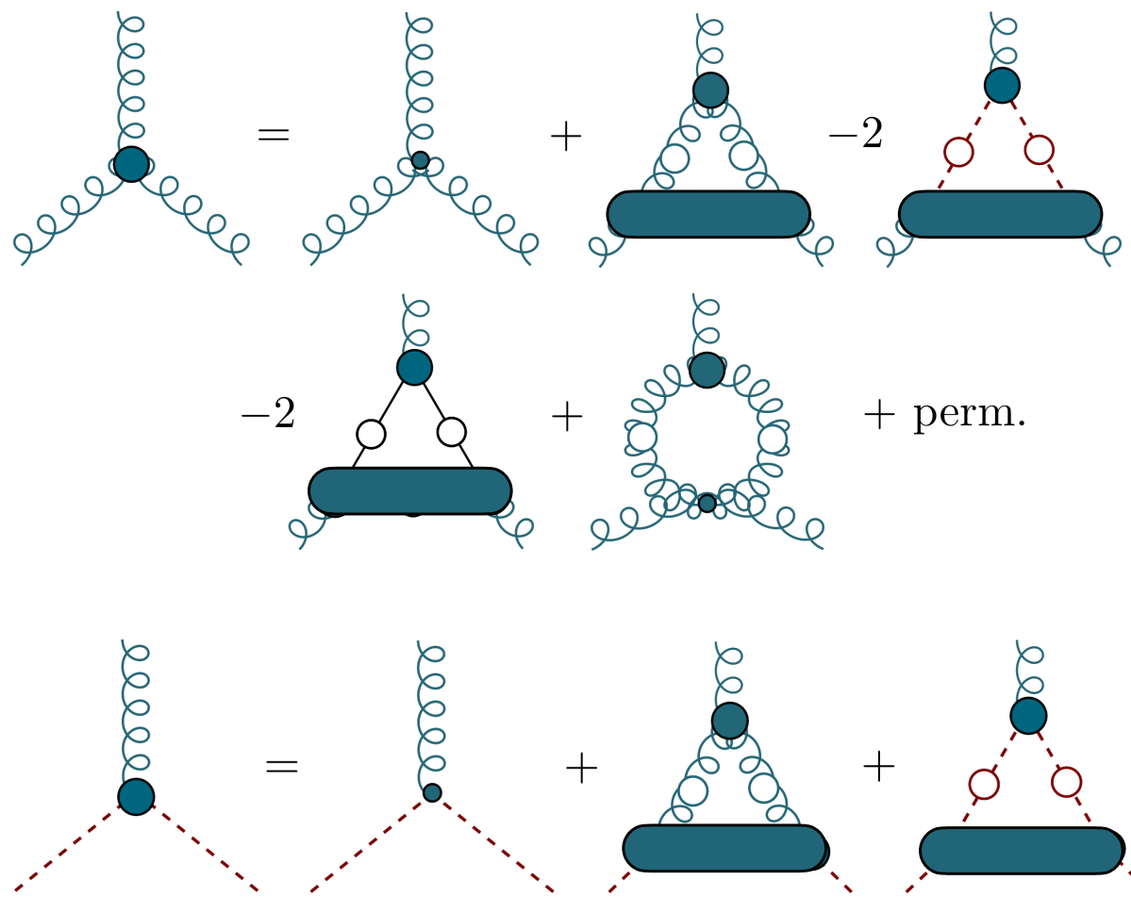
Dyson-Schwinger equations - "3PI vs RL"

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{ - \int d^4x \left(\bar{\Psi} \left(i\not{D} - m \right) \Psi - \frac{1}{4} \left(F^a_{\mu\nu} \right)^2 \right) \right\}$$

propagators



vertices

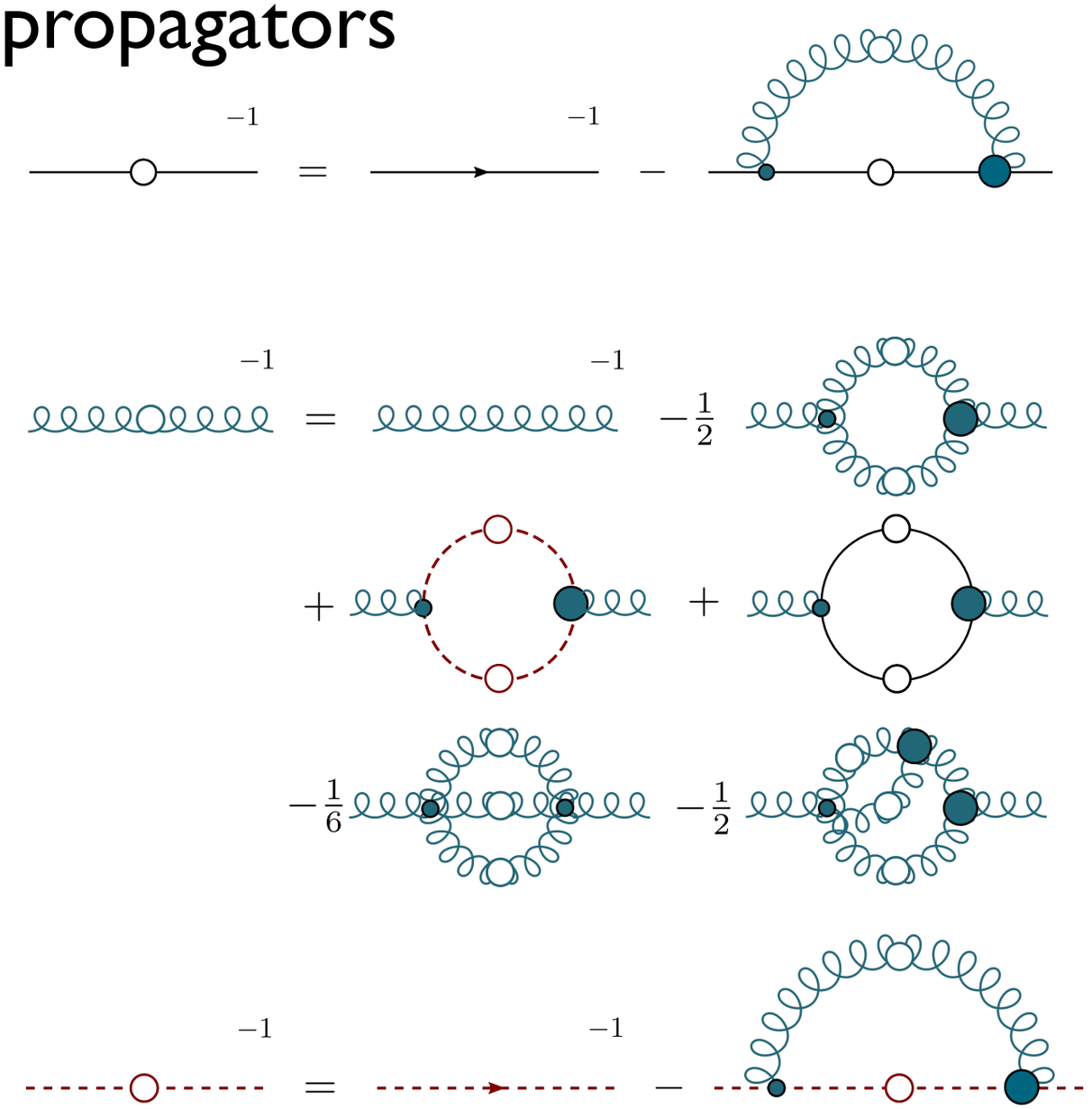


CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

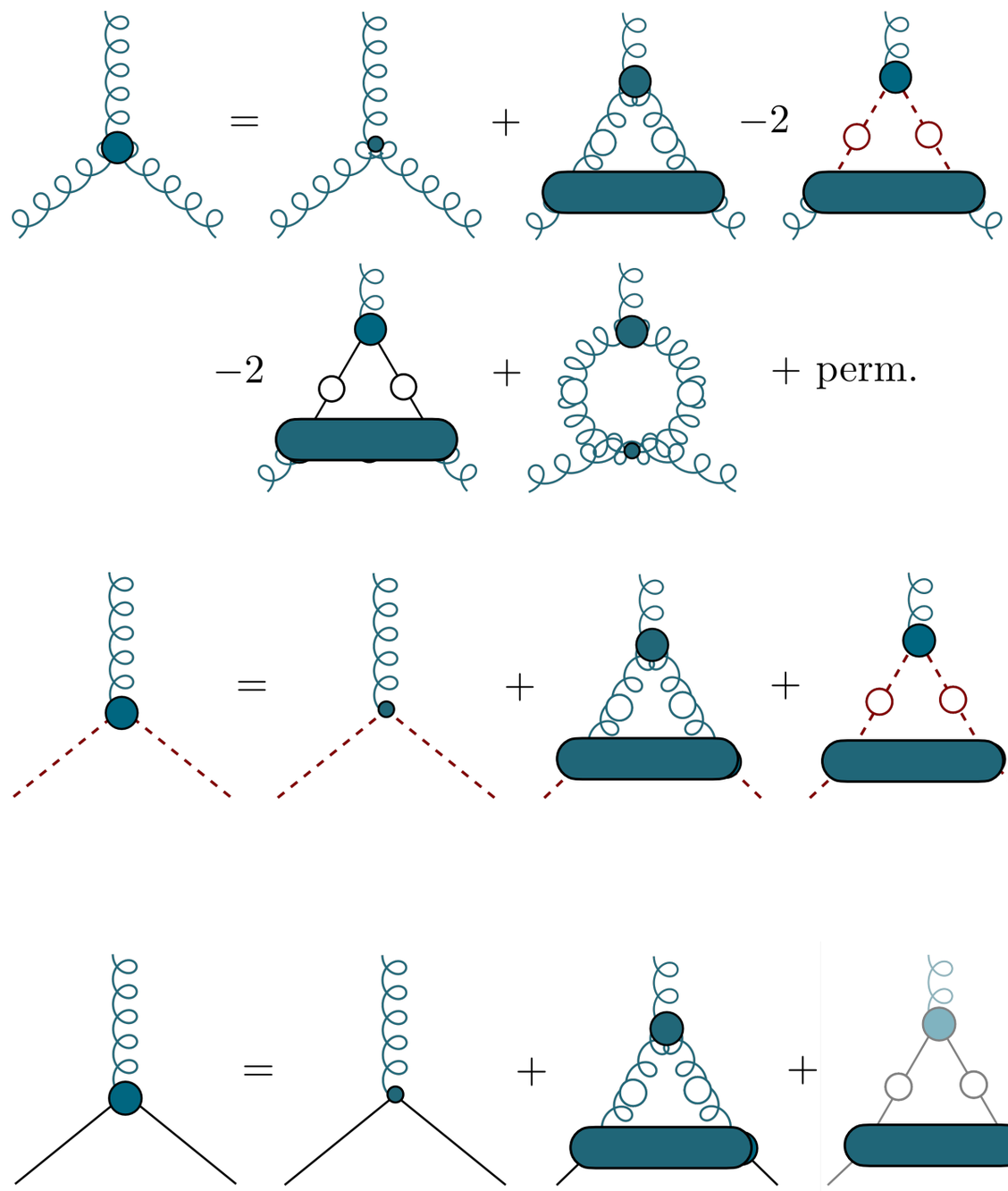
Dyson-Schwinger equations - "3PI vs RL"

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{ - \int d^4x \left(\bar{\Psi} \left(i\not{D} - m \right) \Psi - \frac{1}{4} \left(F^a_{\mu\nu} \right)^2 \right) \right\}$$

propagators



vertices

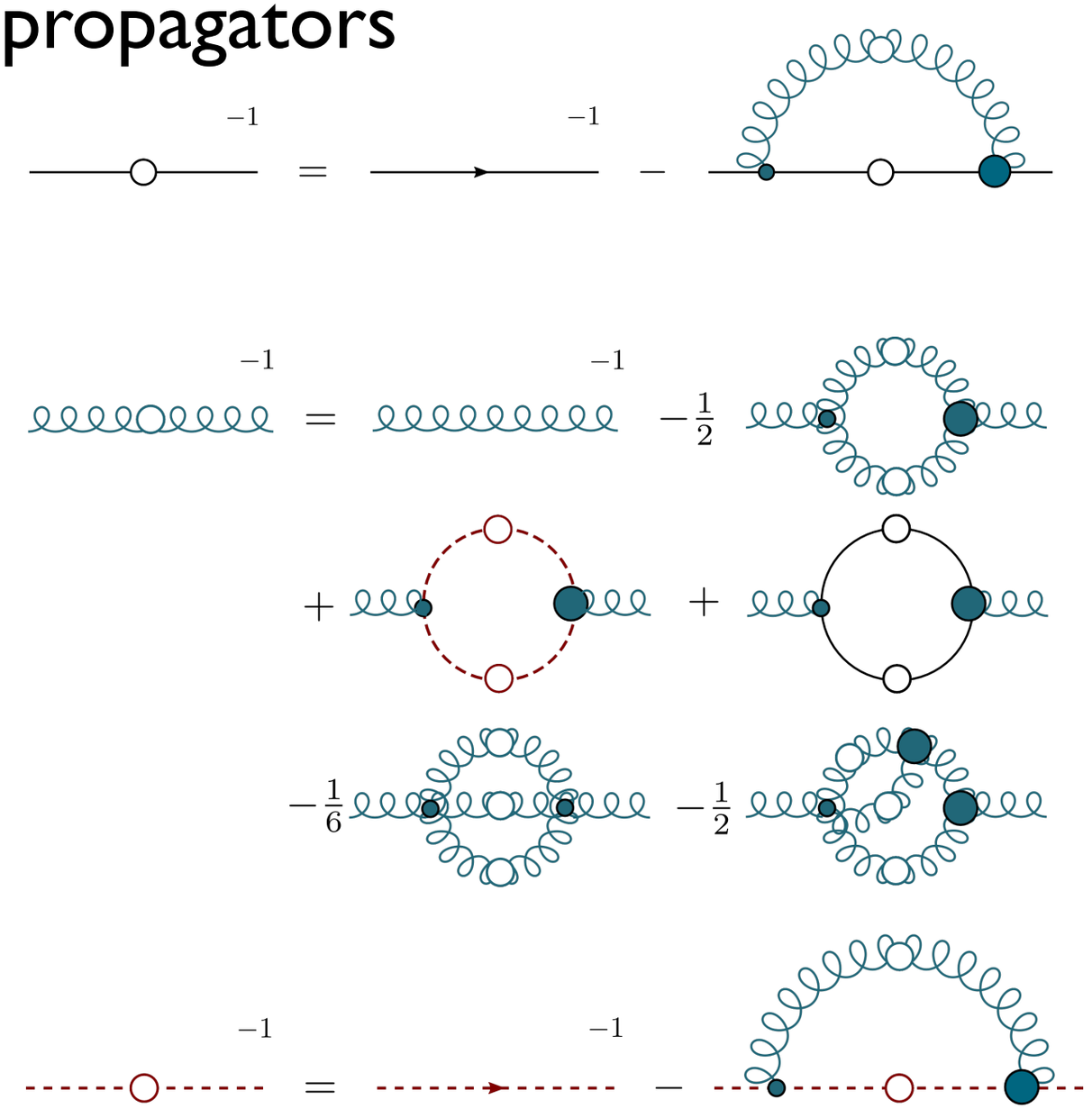


CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

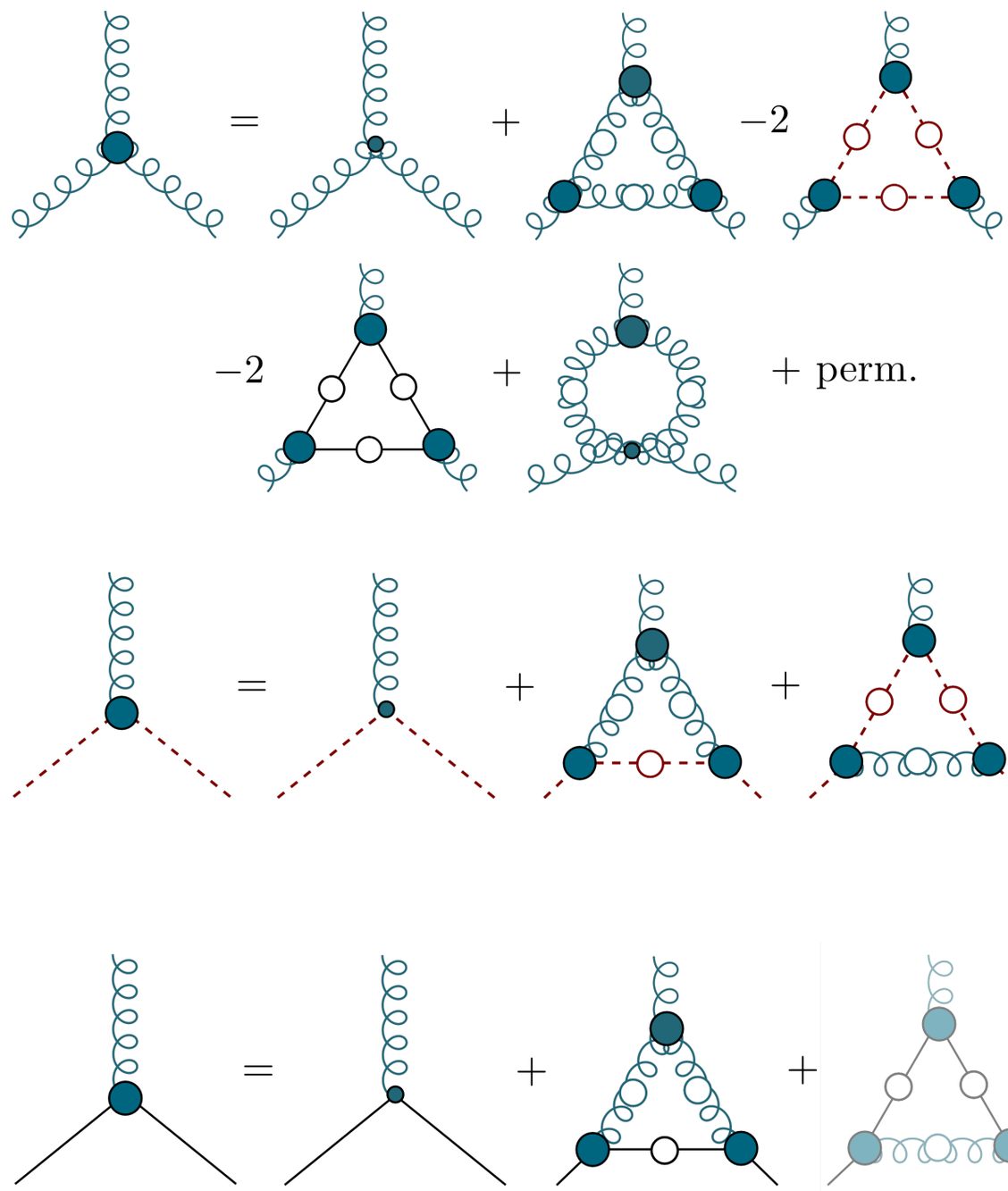
Dyson-Schwinger equations - "3PI vs RL"

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{ - \int d^4x \left(\bar{\Psi} \left(i\not{D} - m \right) \Psi - \frac{1}{4} \left(F^a_{\mu\nu} \right)^2 \right) \right\}$$

propagators



vertices

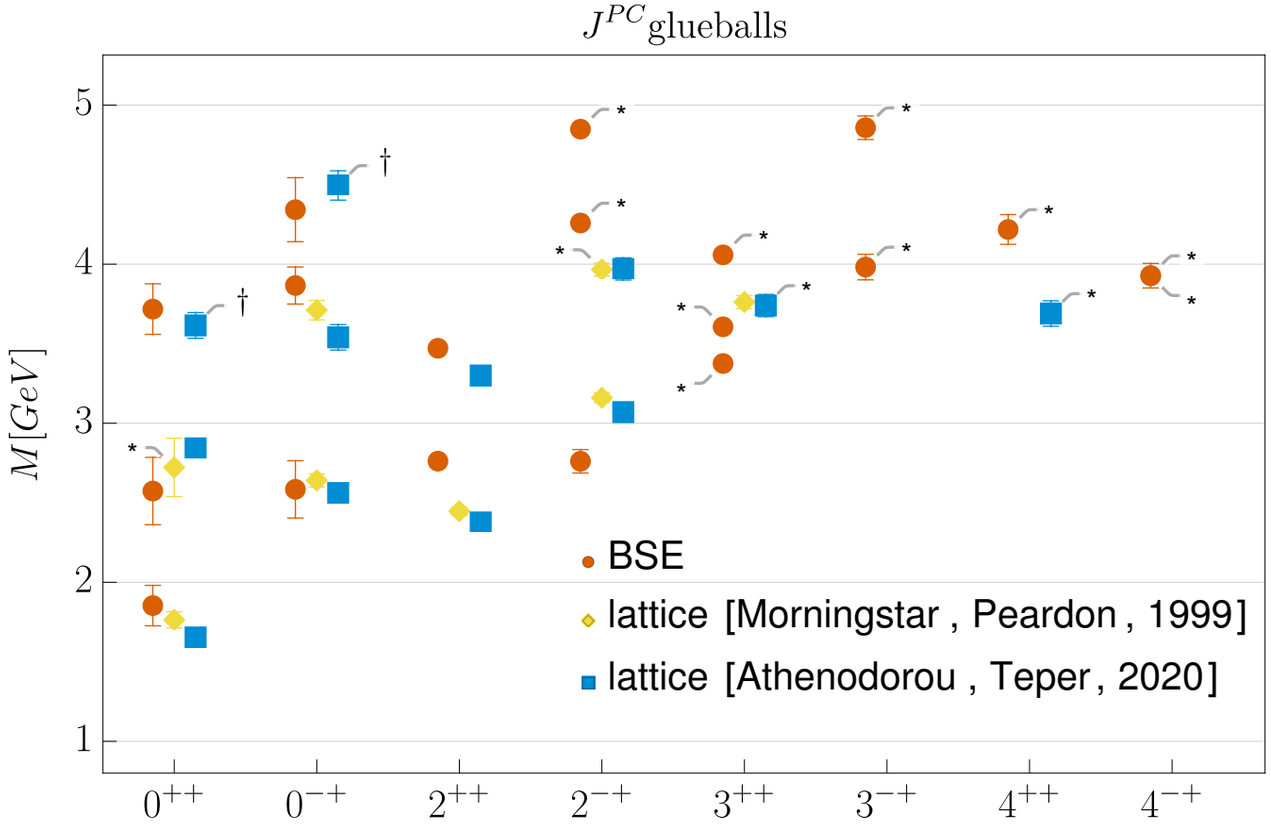


CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

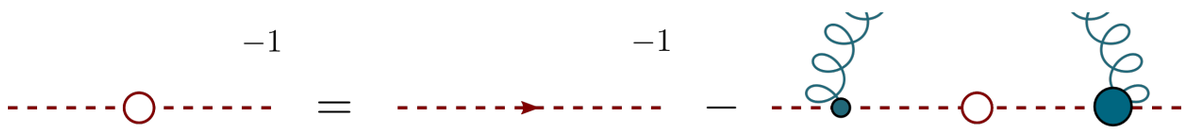
Dyson-Schwinger equations - "3PI vs RL"

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{ - \int d^4x \left(\bar{\Psi} \left(i\not{D} - m \right) \Psi - \frac{1}{4} \left(F^a_{\mu\nu} \right)^2 \right) \right\}$$

propagators

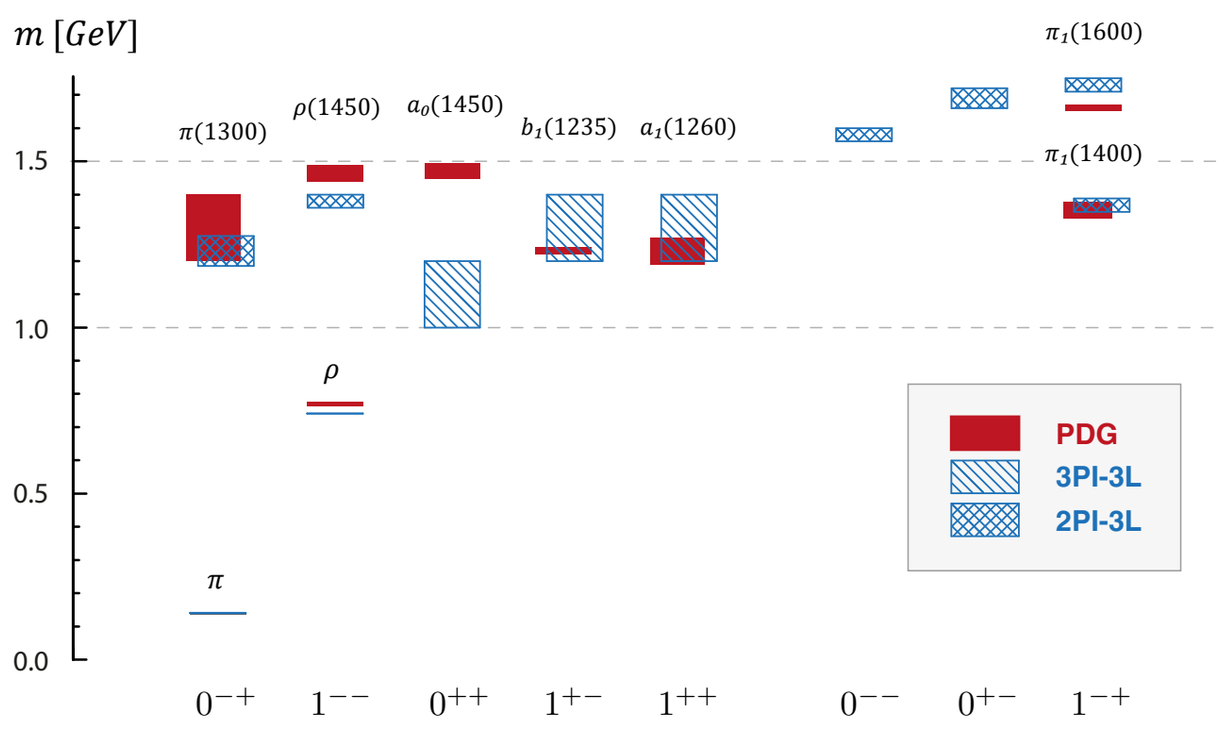


CF, Huber, Sanchis-Alepuz, EPJC 80 (2020) [arXiv:2004.00415]
Huber, CF, Sanchis-Alepuz, EPJC 81 (2021) [arXiv:2110.09180]

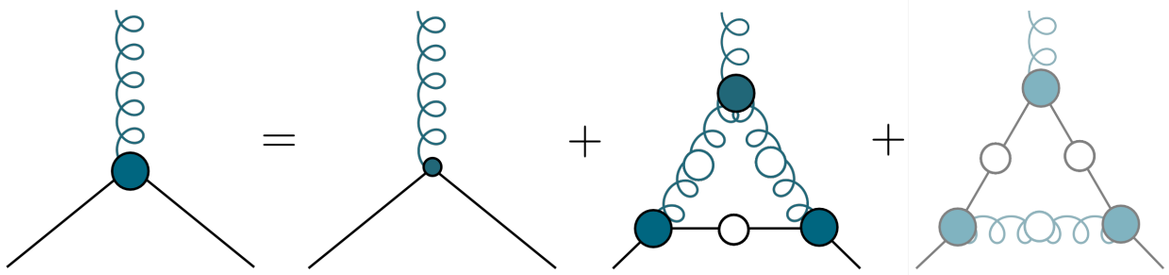


CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

vertices



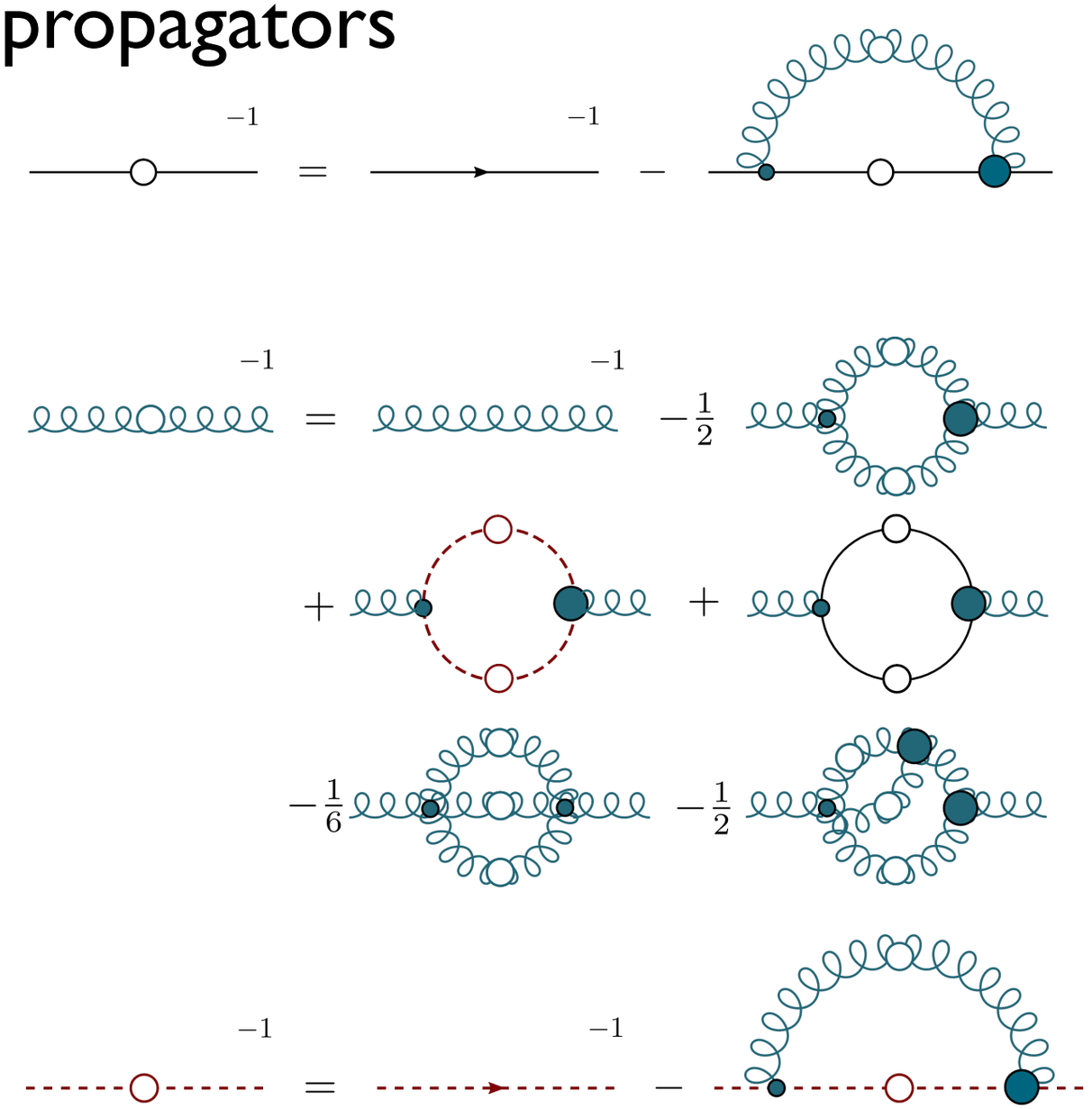
Williams, CF, Heupel, PRD93 (2016) 034026



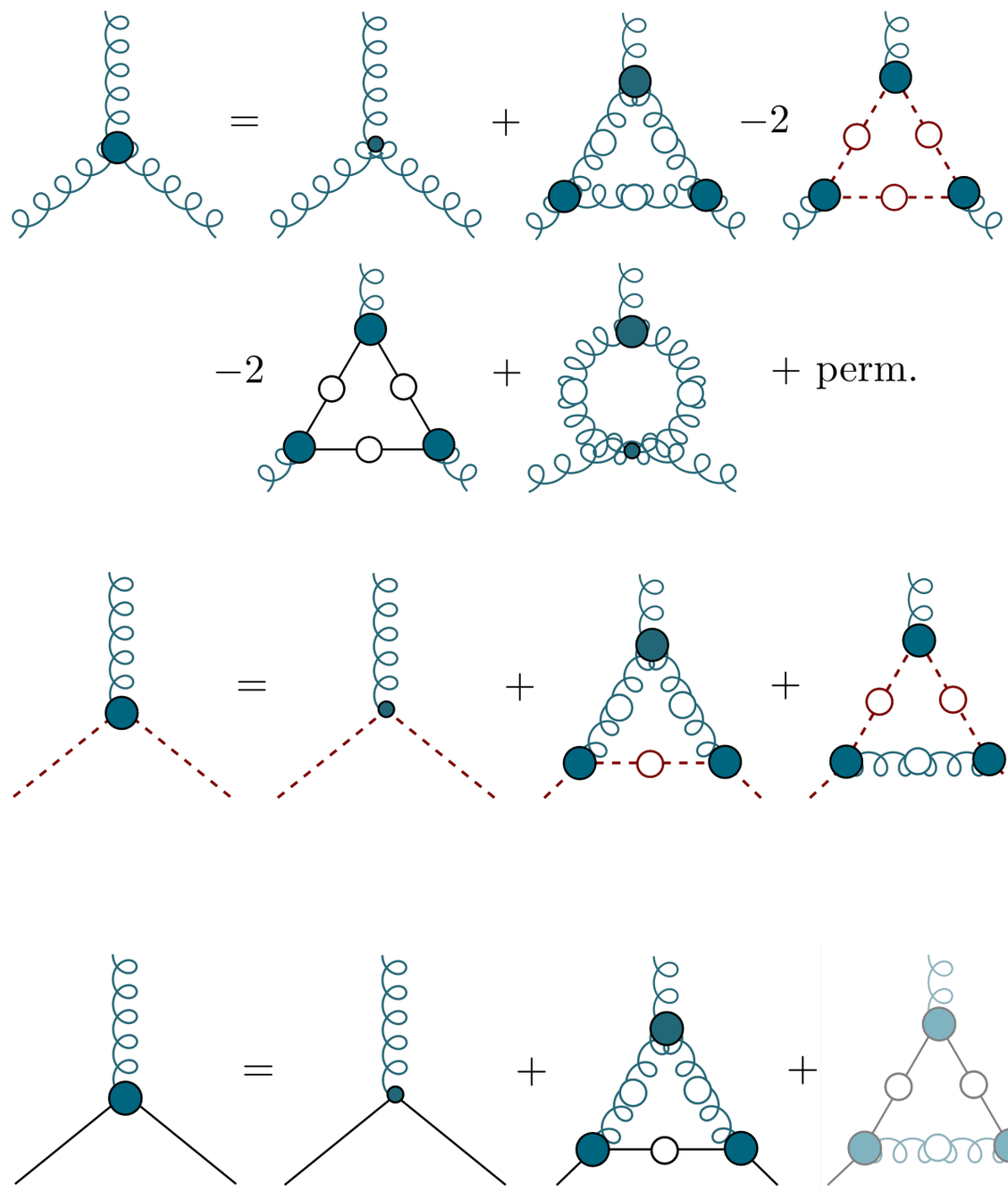
Dyson-Schwinger equations - "3PI vs RL"

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{ - \int d^4x \left(\bar{\Psi} \left(i\not{D} - m \right) \Psi - \frac{1}{4} \left(F^a_{\mu\nu} \right)^2 \right) \right\}$$

propagators



vertices

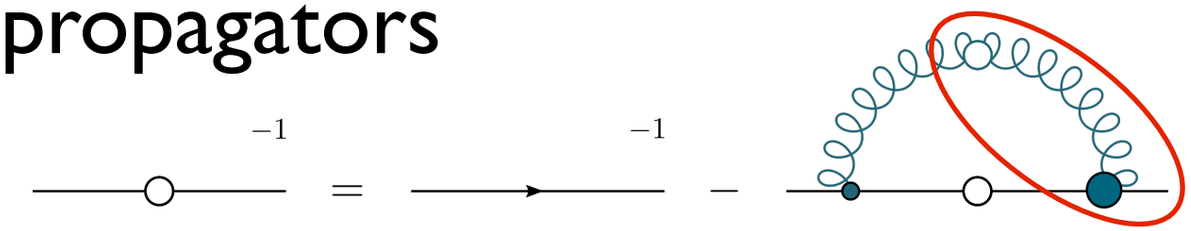


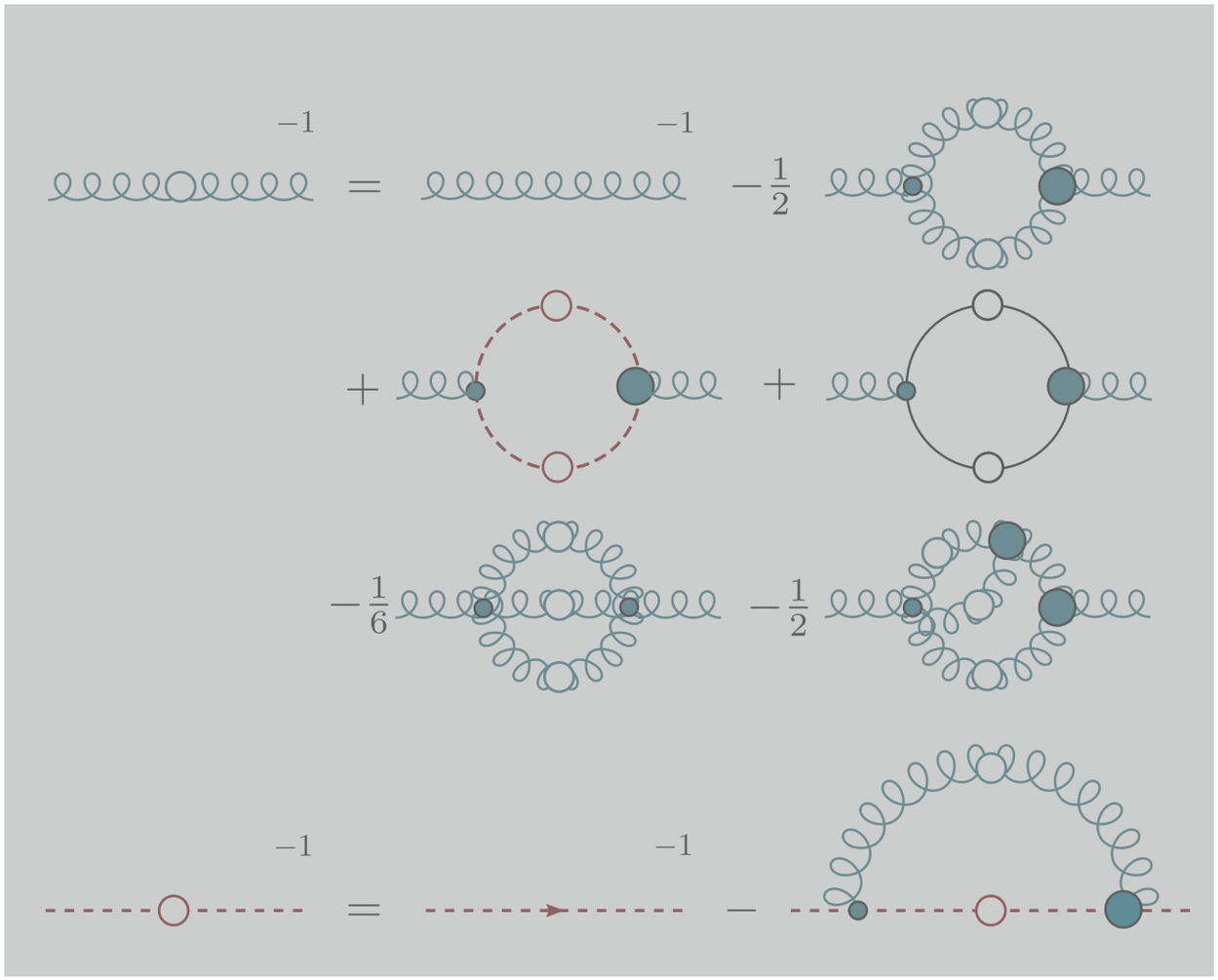
CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

Dyson-Schwinger equations - "3PI vs RL"

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{ - \int d^4x \left(\bar{\Psi} \left(i \not{D} - m \right) \Psi - \frac{1}{4} \left(F^a_{\mu\nu} \right)^2 \right) \right\}$$

propagators

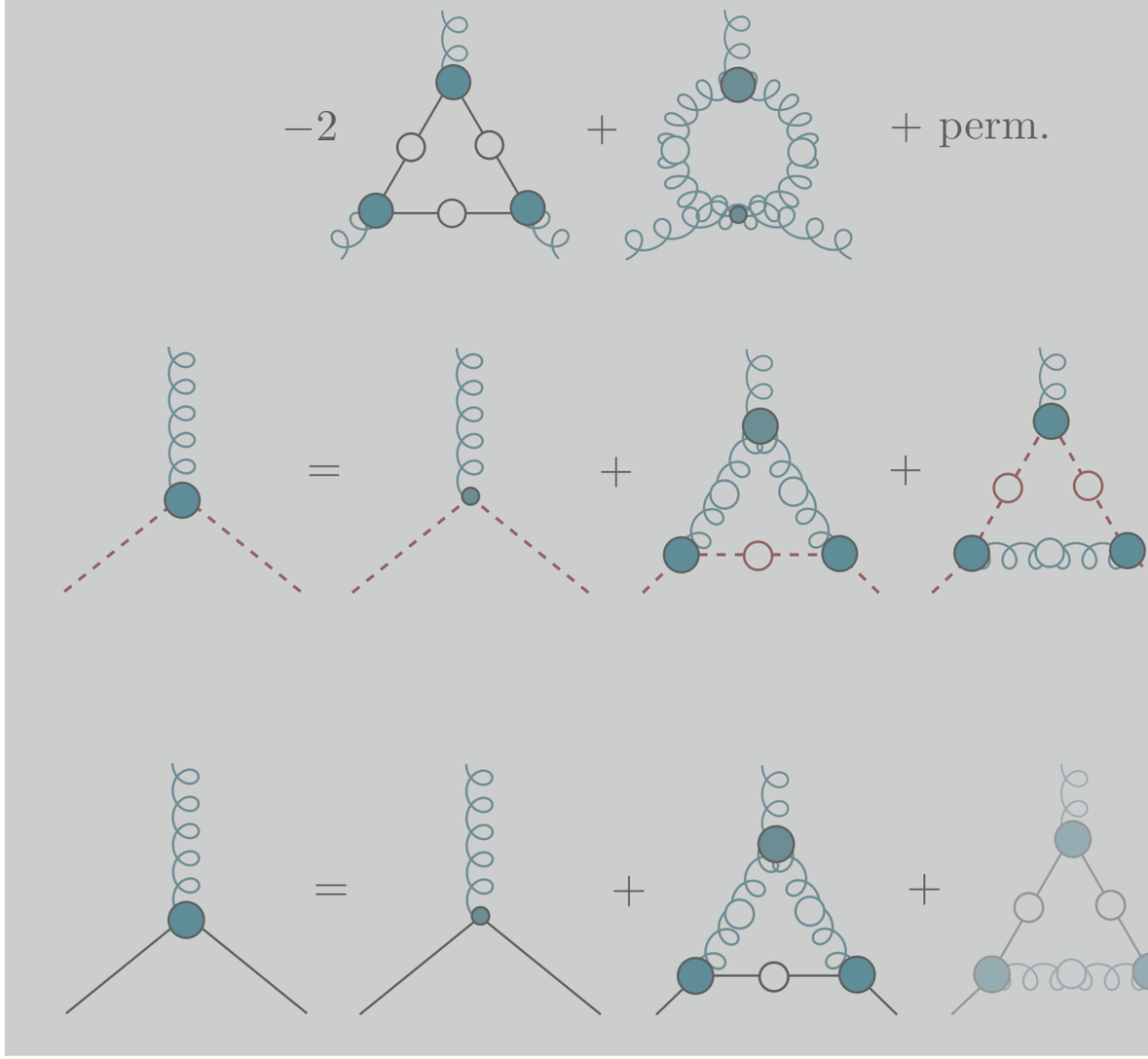




CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

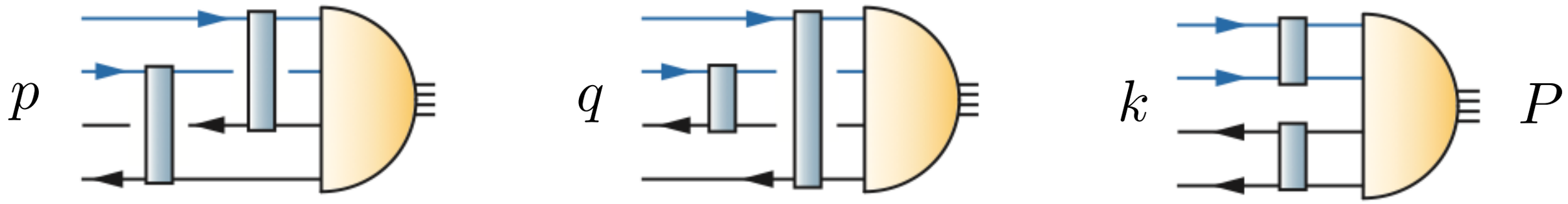
vertices

"rainbow-ladder" (RL) : model for gluon+vertex



Structure of the amplitude: X(3872)

Axialvector tetraquark:



$$\Gamma(P,p,q,k) = \sum_i f_i(s_1,\cdots,s_9) \times \tau_i(P,p,q,k) \times color \times flavor$$

768 tensor structures !!

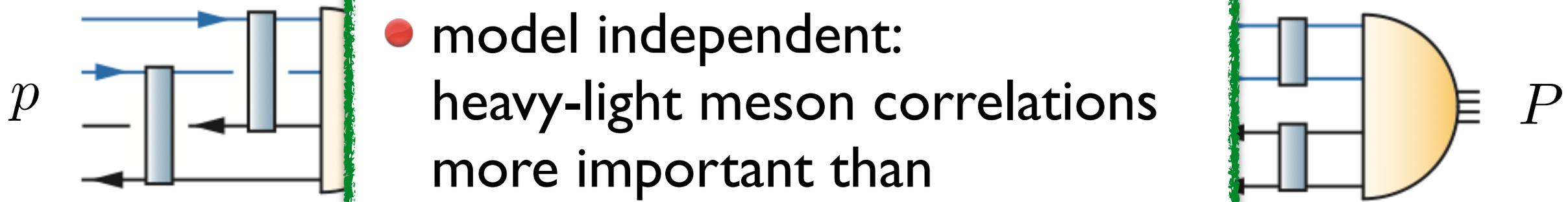
↓

- physics-guided approximation: 8 s-wave tensors are important

$D^0\,\bar{D}^{*0} + \bar{D}^0\,D^{*0} + D^{*+}\,D^-$	heavy-light meson
$J/\Psi\;\omega$	hadro-charmonium
$S\;A$	diquarks

Structure of the amplitude: X(3872)

Axialvector tetraquark:



- model independent:
  heavy-light meson correlations
  more important than
  diquark diquark correlations
  (color factor !)

$$\Gamma(P,p,q,k) = \qquad color \times flavor$$

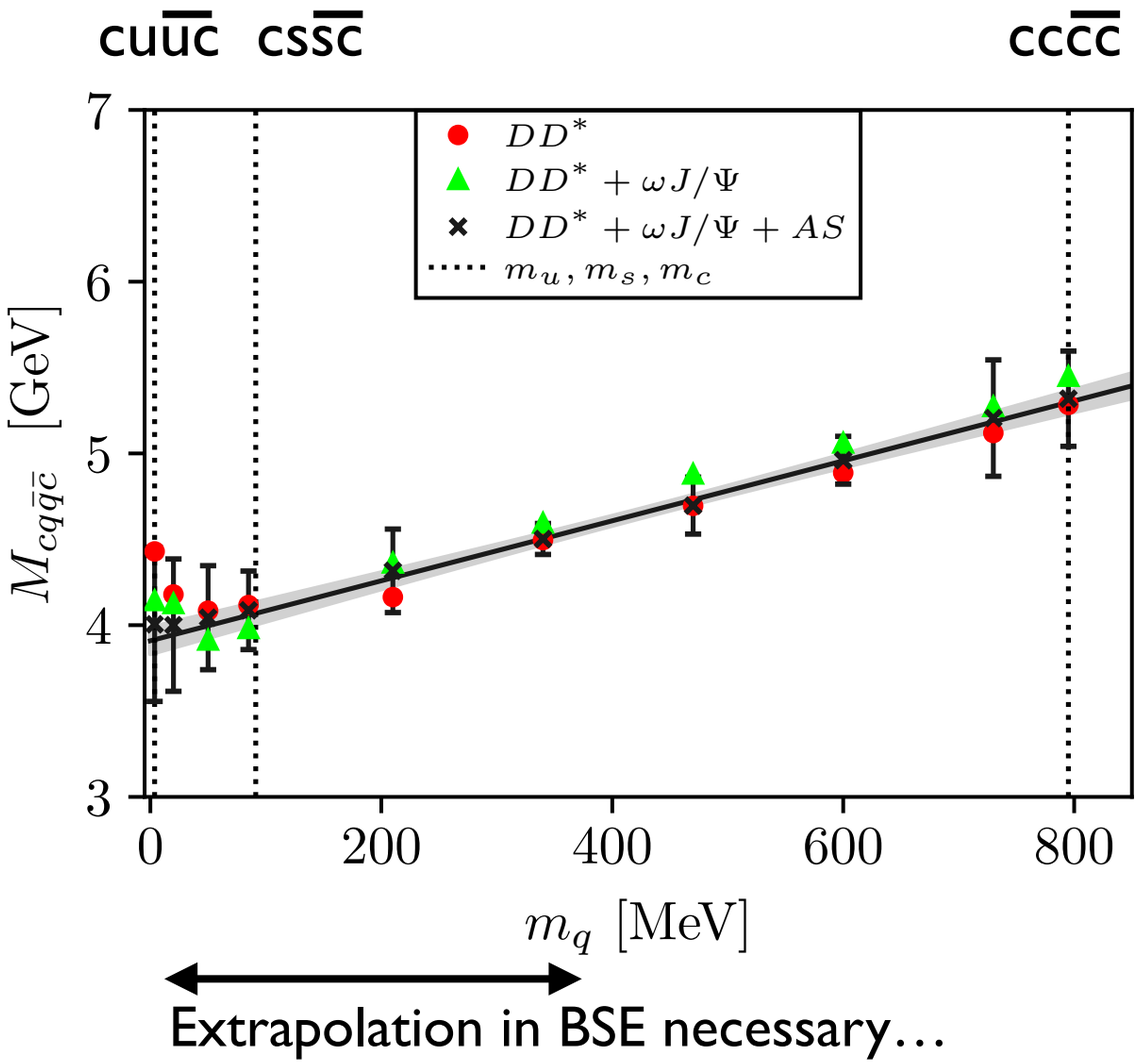
768 tensor structures !!

↓

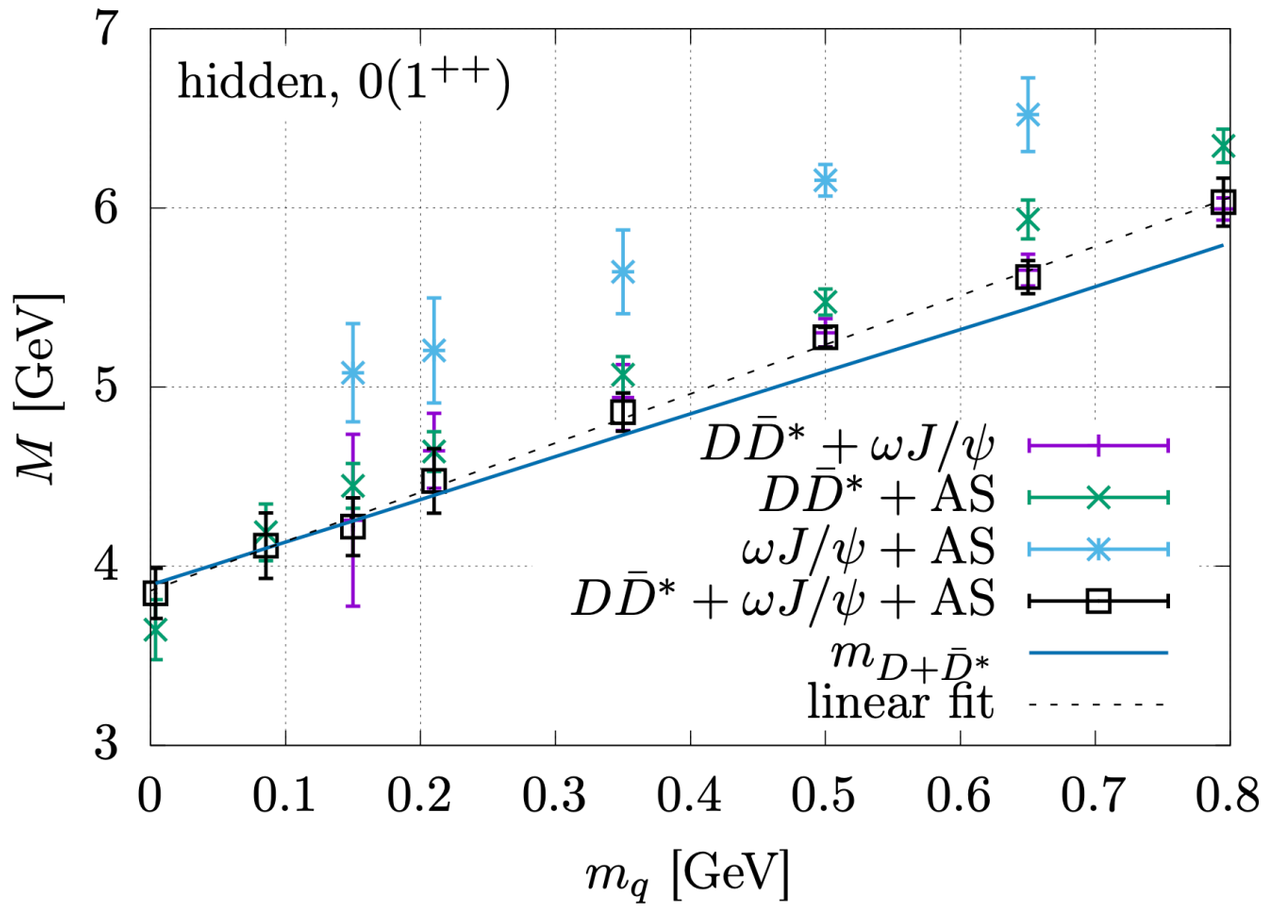
- physics-guided approximation: 8 s-wave tensors are important

$D^0\,\bar{D}^{*0} + \bar{D}^0\,D^{*0} + D^{*+}\,D^-$	heavy-light meson
$J/\Psi\;\omega$	hadro-charmonium
$S\;A$	diquarks

Four-body vs. two-body: $J^{PC} = 1^{++}$



Wallbott, Eichmann and CF, PRD100 (2019) 014033, [1905.02615]



Santowsky and CF, EPJC 82 (2022) 4, 313 [2111.15310]

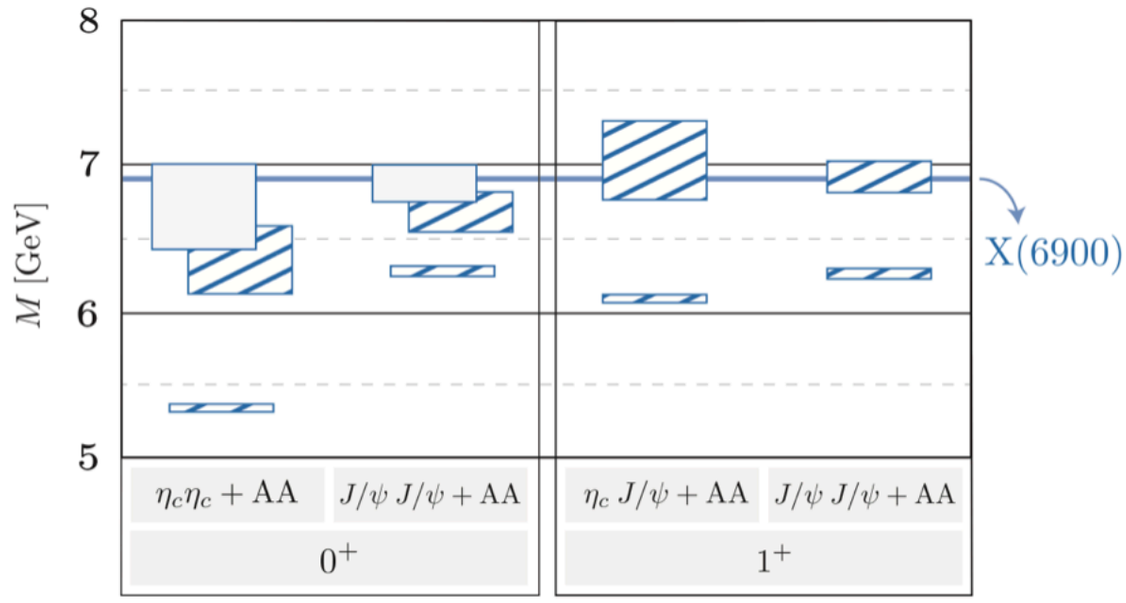
$$M_{1^{++}}^{cq\bar{q}\bar{c}} = 3916(74)\,\mathrm{MeV} \longrightarrow X(3872)$$

- DD* components dominate !

Heavy four-quark states from DSE/BSEs

	$I(J^{PC})$	four-quark	effective two-body	Exp.
hidden charm	$0(0^{++})$	3.20 (11)	3.49 (25)	
($\bar{c}c\bar{q}q$)	$0(1^{++})$	3.92 (7)	3.85 (18)	X(3872)
	$1(1^{+-})$	3.74 (9)	3.79 (31)	$Z_c(3900)$
	$1(0^{++})$		3.20 (31)	
open charm	$1(0^{+})$	3.80 (10)	3.21 (2)	
($cc\bar{q}\bar{q}$)	$0(1^{+})$	3.90 (8)	3.49 (48)	$T_{cc}(3875)$
	$1(1^{+})$	4.22 (44)	3.47 (24)	

all charm
($cc\bar{c}\bar{c}$)



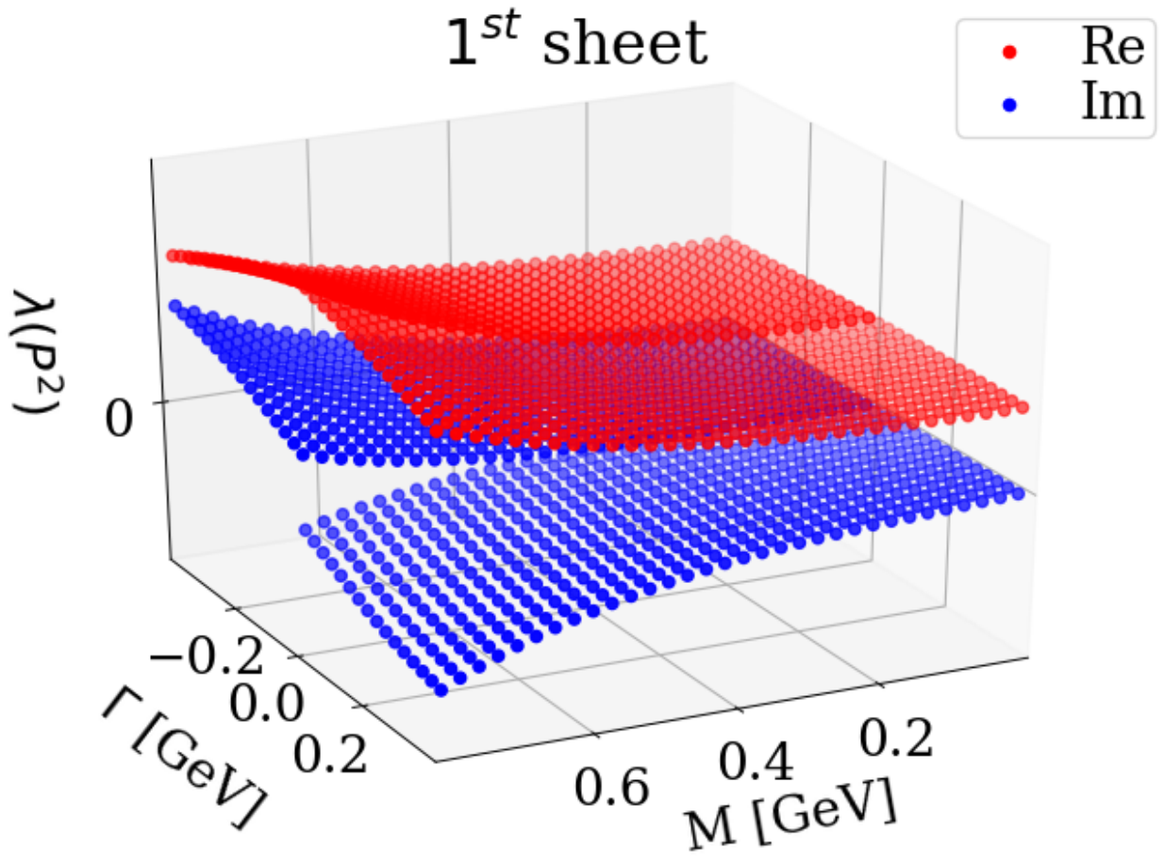
Wallbott, Eichmann and CF, PRD100 (2019) 014033, [1905.02615]
Wallbott, Eichmann and CF, PRD102 (2020), 051501, [2003.12407]
Santowsky, CF, EPJC 82 (2022) 4, 313 [2111.15310]

Work to do…

- improve two-body interactions
- further study mixing with $q\bar{q}$ in I=0 sector

Santowsky, Eichmann, CF, Wallbott and Williams, PRD 102 (2020) no.5, 056014, arXiv:2007.06495.

- solve four-body BSE in the complex momentum plane



successful for ρ-meson:

Williams, PLB 798 (2019) 134943, [arXiv:1804.11161]

Summary

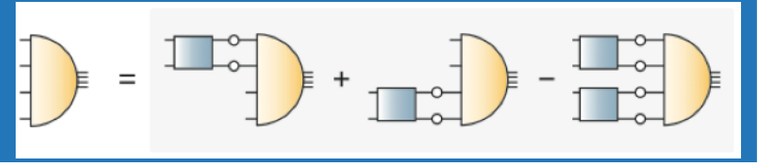
Internal dynamics very important !!

- Dynamical description of σ: π-π resonance — Eichmann, CF, Heupel, PLB 753 (2016) 282-287
- Dynamical description of X(3872) and Z(3900): DD* dominated
- First results in open charm channels — Wallbott, Eichmann and CF, PRD100 (2019) 014033, [1905.02615]; Wallbott, Eichmann and CF, PRD102 (2020) 051501, [2003.12407]
- Mixing with $q\bar{q}$ studied for light mesons — Santowsky, Eichmann, CF, Wallbott and Williams, PRD 102 (2020) no.5, 056014, [2007.06495].
- two-body vs four-body: agree
- first results in all-charm channels — Santowsky and CF, EPJC 82 (2022) 4, 313 [2111.15310]

Mini-Review: Eichmann, CF, Heupel, Santowsky, Wallbott, FBS 61 (2020) 4 38, [2008.10240]

Backup Slides

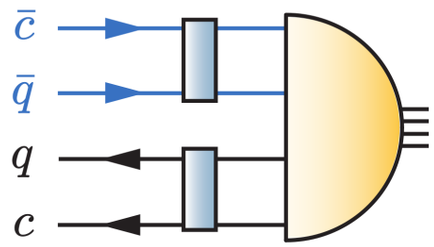
Four-body equation: permutations



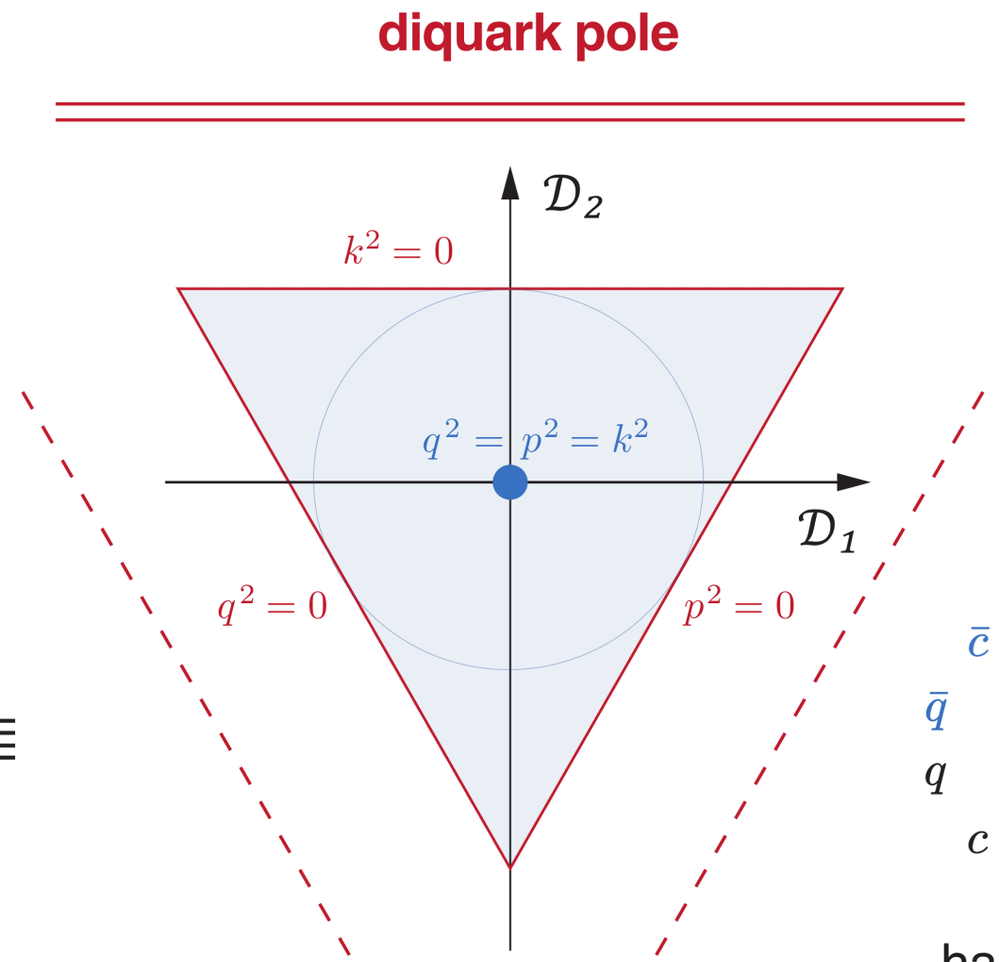
- **Singlet:** $S_0 = (p^2 + q^2 + k^2)/4$
- **Doublet:** $\mathcal{D}_1 \sim p^2 + q^2 - 2k^2$

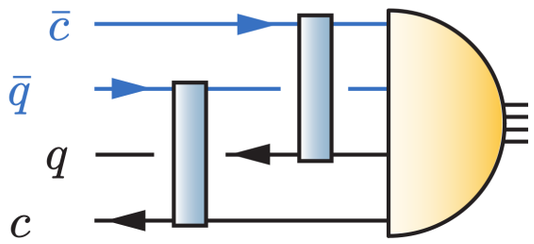
  $\mathcal{D}_2 \sim q^2 - p^2$

p, q, k : relative momenta

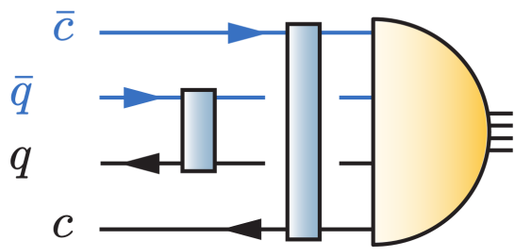


diquark-antidiquark



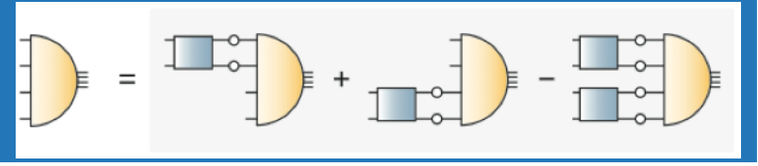


heavy-light mesons



hadrocharmonium

Four-body equation: permutations

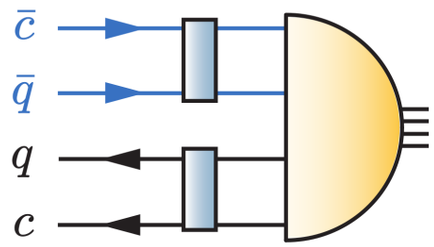


- Singlet: $S_0 = (p^2 + q^2 + k^2)/4$
- Doublet: $\mathcal{D}_1 \sim p^2 + q^2 - 2k^2$

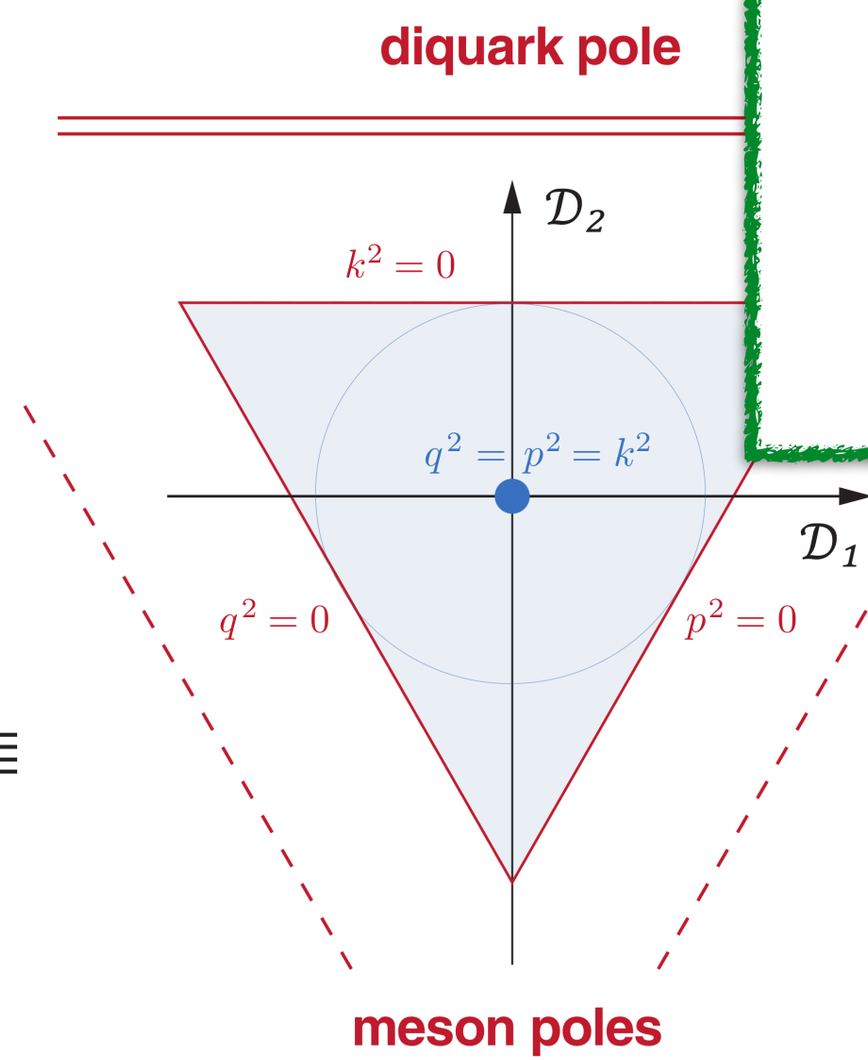
  $\mathcal{D}_2 \sim q^2 - p^2$

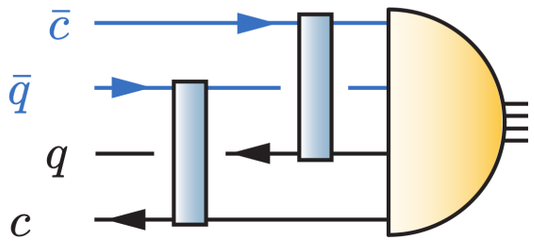
p, q, k : relative momenta

- model independent: heavy-light meson poles more important than diquark poles (color factor !)

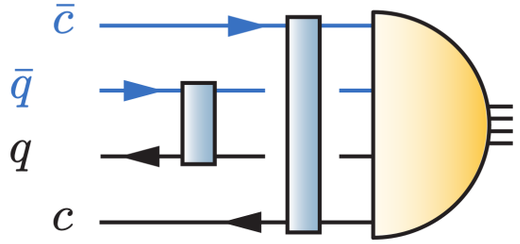


diquark-antidiquark



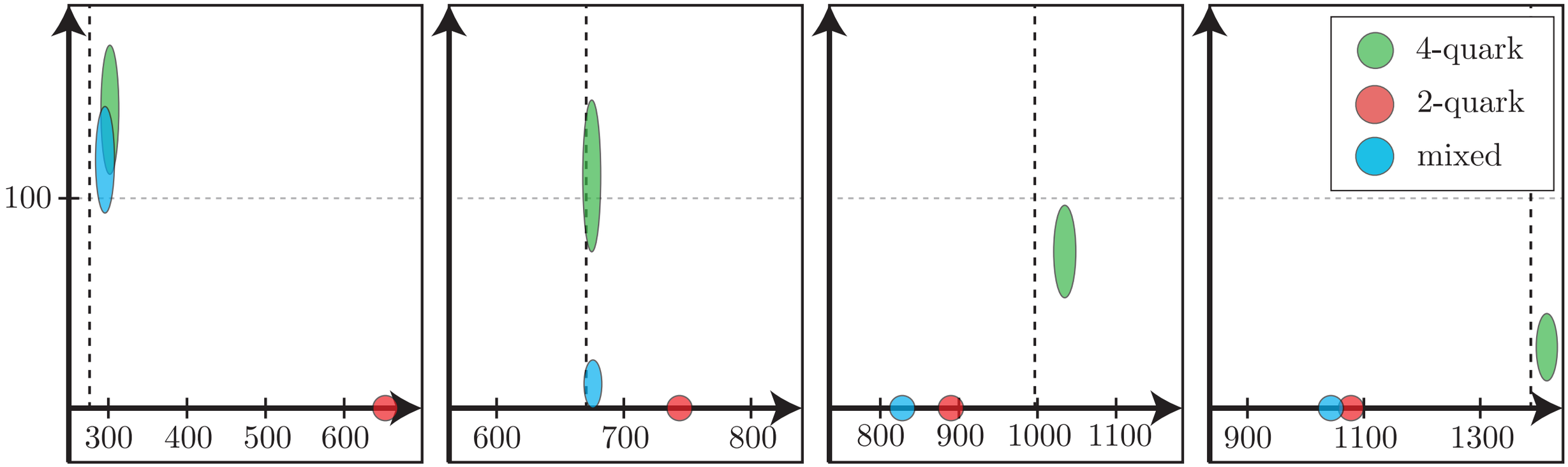


heavy-light mesons



hadrocharmonium

Mass evolution of four-quark state

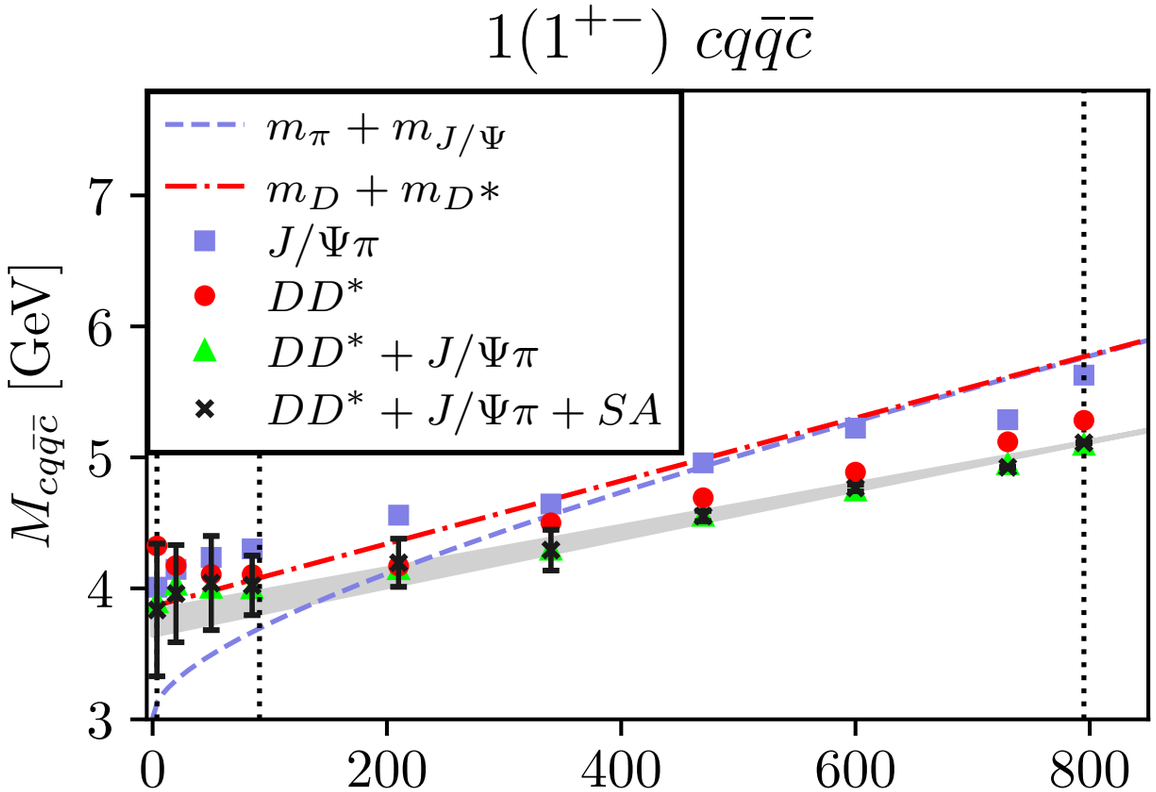


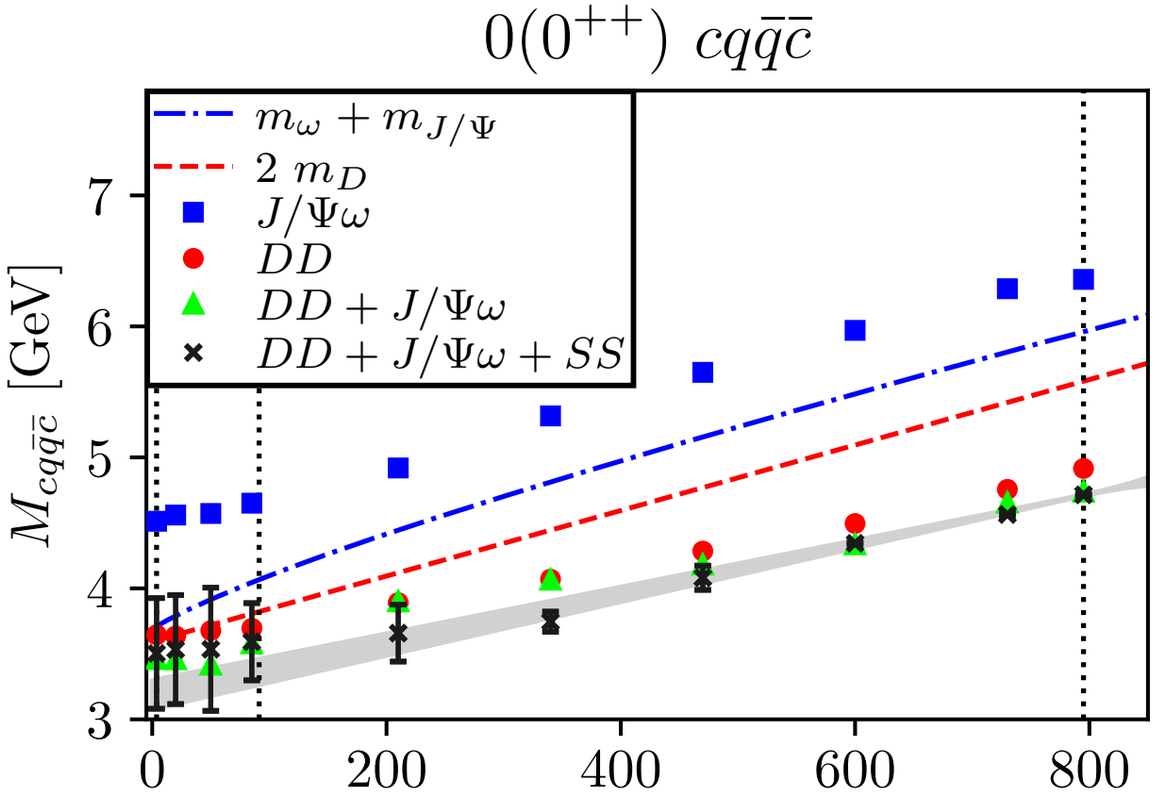
$m_q = 3.8\,\mathrm{MeV}$ $m_q = 20.0\,\mathrm{MeV}$ $m_q = 50.0\,\mathrm{MeV}$ $m_q = 85.5\,\mathrm{MeV}$

- mixed state becomes qq-dominated for large m_q
- dynamical decision !

Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

$J^{PC} = 1^{+-}$ and 0^{++}

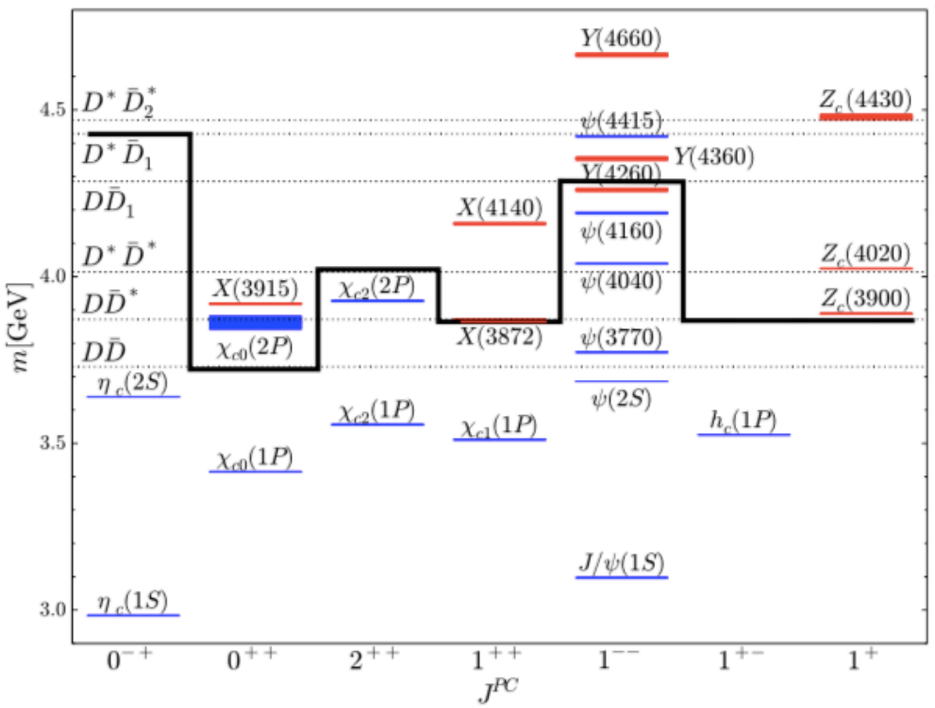




Wallbott, Eichmann and CF, PRD102 (2020)no.5, 051501, arXiv:2003.12407

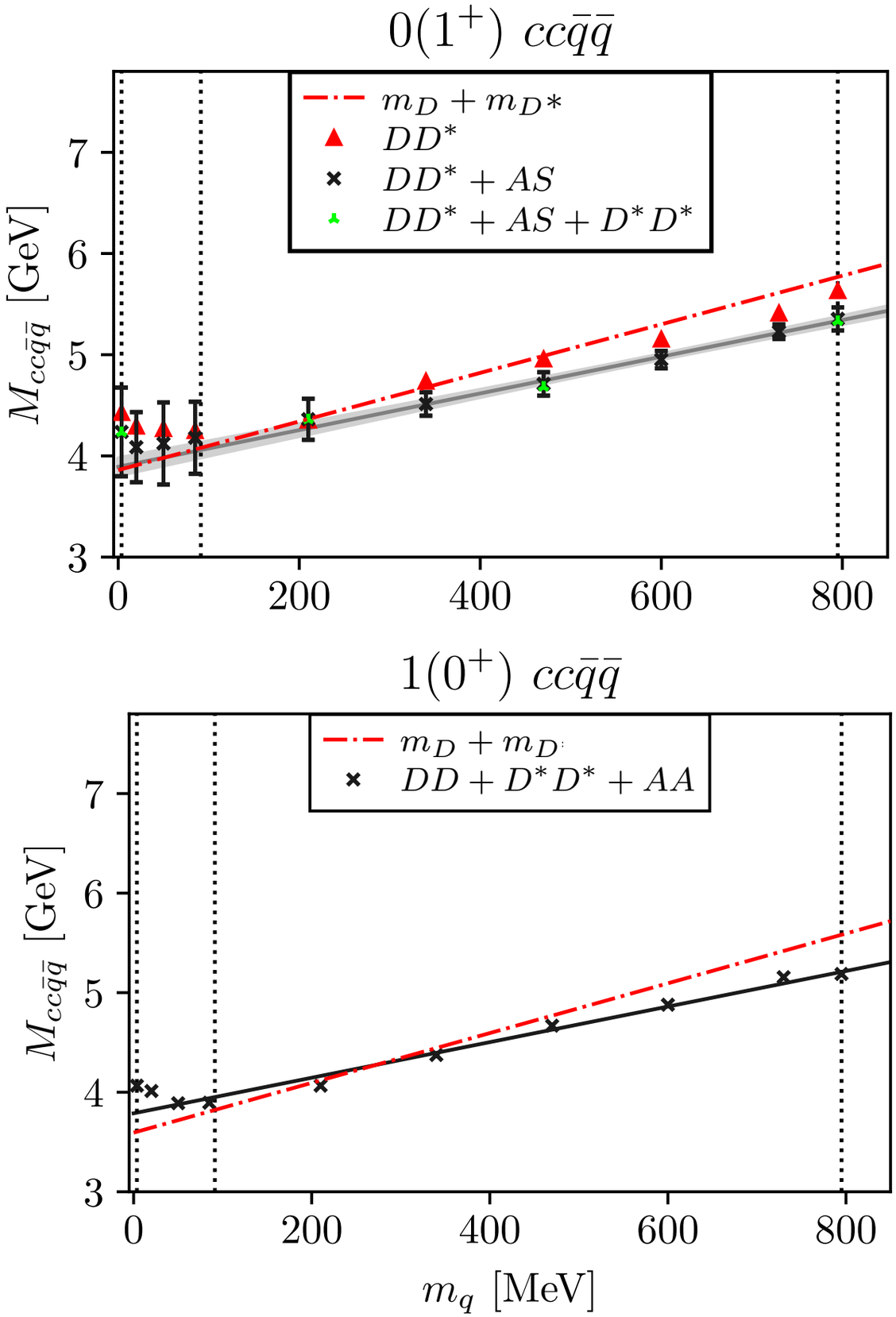
$$M^{cq\bar{q}\bar{c}}_{1^{+-}} = 3741(91) \ \rightarrow \ Z(3900)$$

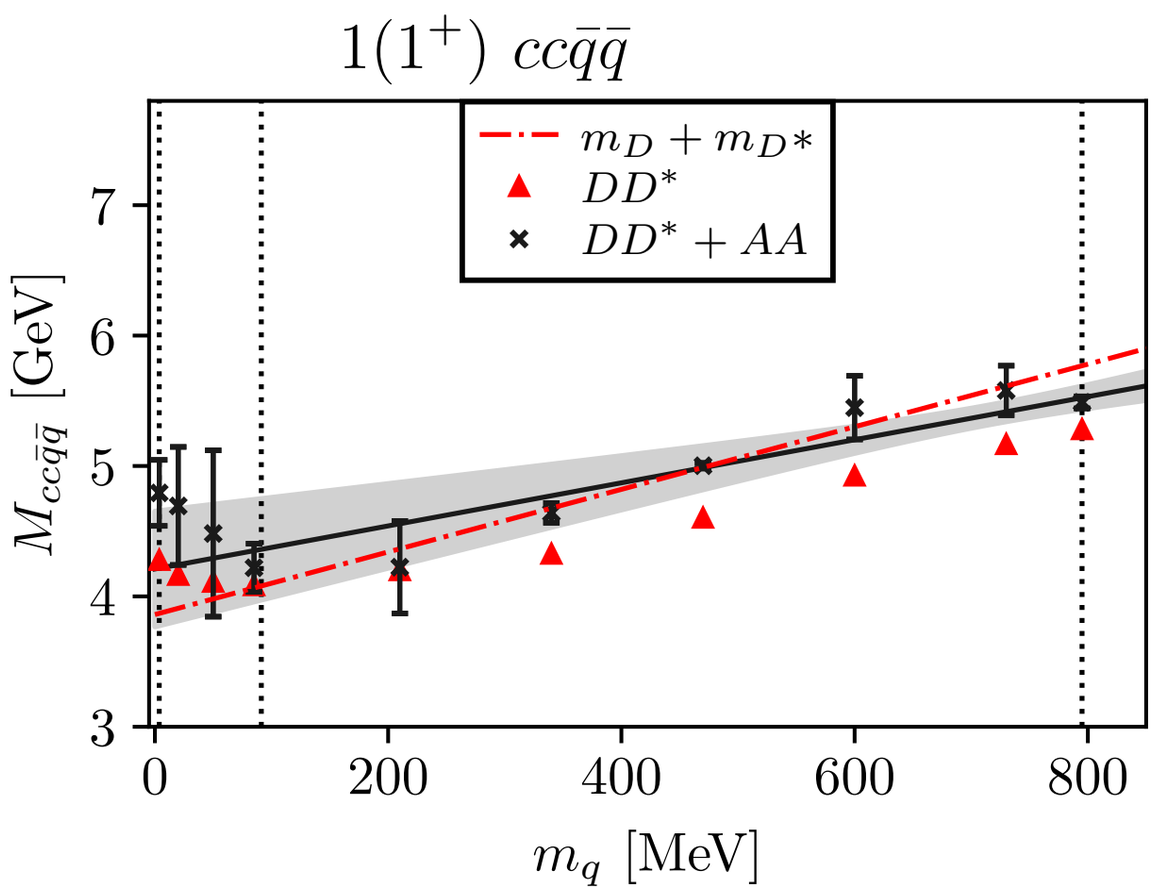
$$M^{cq\bar{q}\bar{c}}_{0^{++}} = 3195(107) \ \rightarrow \ ?$$



mass pattern matches molecule picture of Cleven et al. PRD 92 (2015) 014005:

Open charm four-quark states

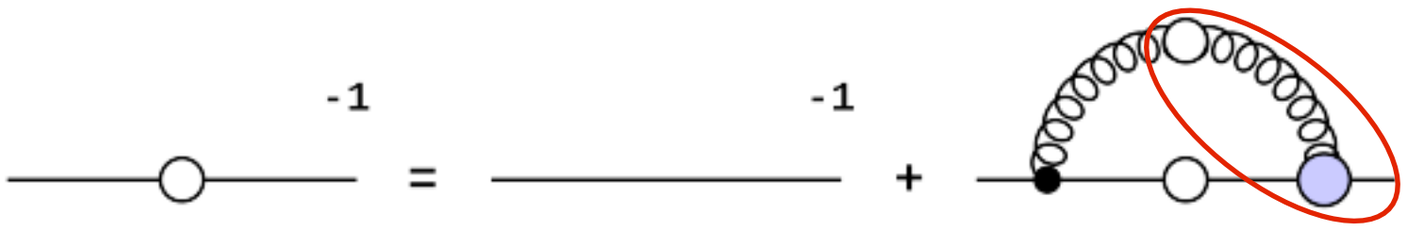




- DD(*) and diquarks important!

Wallbott, Eichmann and CF, PRD102 (2020)no.5, 051501, arXiv:2003.12407

Rainbow-ladder model for quark-gluon interaction



Combine gluon with quark-gluon vertex:

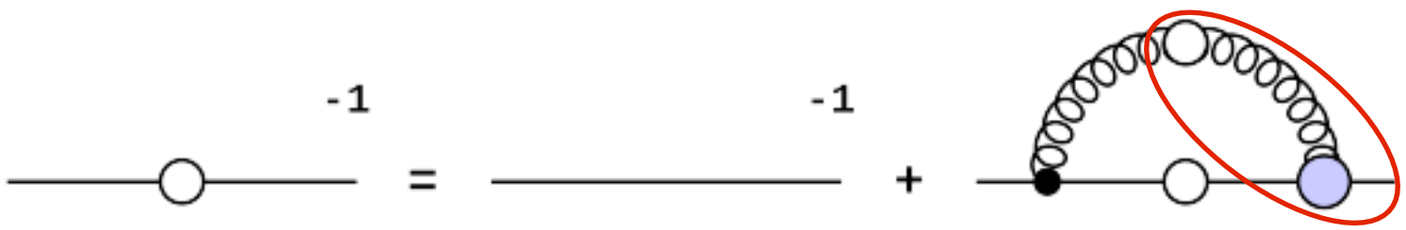
$$\Gamma^\mu(p,k) = \sum_{i=1,12} \tau_i(p,k) T_i^\mu$$

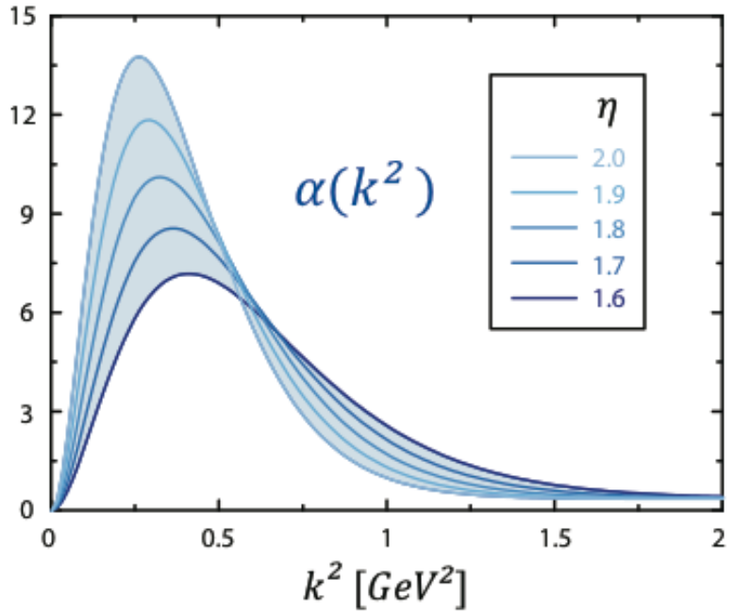
$$\sim \gamma^\mu \tau(k^2)$$ "approximation" !

$$D^{\mu\nu}(k) = \left(\delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) \frac{Z(k^2)}{k^2}$$

$$\frac{g^2}{4\pi} \tau(k^2) Z(k^2) \sim \alpha(k^2)$$

Rainbow-ladder model for quark-gluon interaction





Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

Combine gluon with quark-gluon vertex:

effective coupling

$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

- scale Λ from f_π, masses m_u=m_d, m_s from m_π, m_K
- α_{UV} from perturbation theory
- parameter η : results almost independent
- qualitatively similar to explicit calc.

Williams, EPJA 51 (2015) 5, 57.
Sanchis-Alepuz, Williams, PLB 749 (2015) 592;
Mitter, Pawlowski and Strodthoff, PRD 91 (2015) 054035
Williams, CF, Heupel, PRD93 (2016) 034026, and refs. therein