15th International Workshop on Heavy Quarkonium

NLO Results With Operator Mixing For Fully Heavy Tetraquarks In QCD Sum Rules

Ren-Hua Wu

Peking University, Beijing, China

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Based on this paper：

R.H. Wu, Y.S. Zuo, C.Y. Wang, Y.Q. Ma, C. Meng, K.T. Chao;

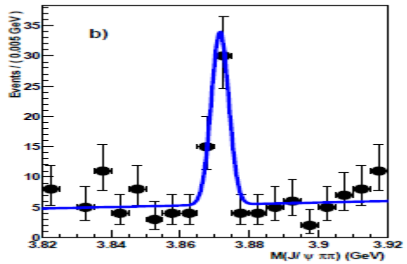
arXiv:2201.11714[hep-ph]

Outline

- Background
- Fully Heavy Tetraquarks Mass Spectra
  - $\bar{c}c\bar{c}c$ Mass Spectra
  - $\bar{b}b\bar{b}b$ Mass Spectra
- Summary

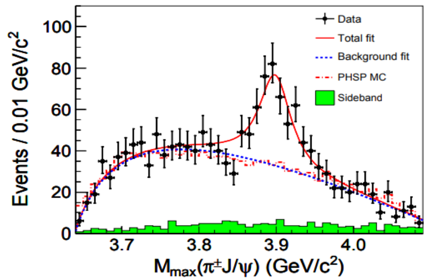
Background

■ New Hadronic States



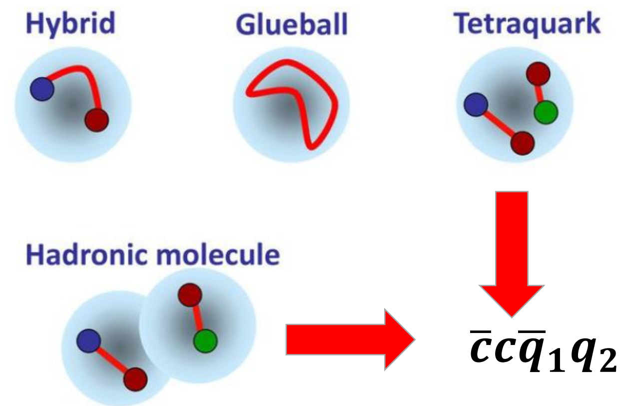
X(3872)

[Belle], PRL(2003).

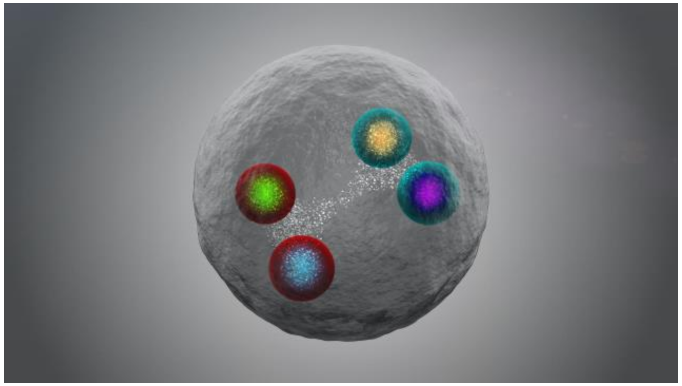


Zc(3900)

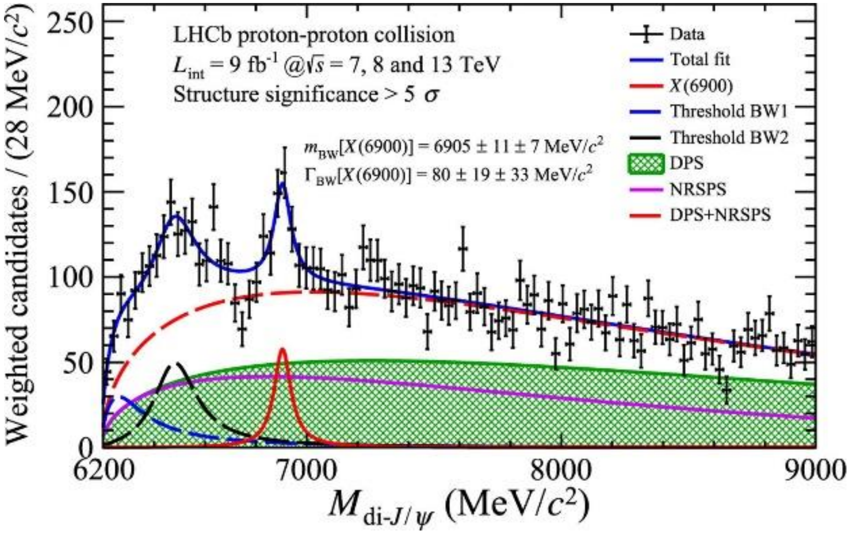
[BES III], PRL (2013)



- **X(6900)** *[LHCb], PRL(2020).*



$\bar{c}c\bar{c}c$



■ Theoretical Works for $\bar{Q}Q\bar{Q}Q$ mass spectra

◆ **Models and tools**

- QCD sum rules *W. Chen et. al., (2017); Z.G. Wang (2020); R.M. Albuquerque and S. Narison (2020) ……*
- Lattice QCD *C. Hughes et. al., (2017)*
- Potential Models *Y. Iwasaki (1975); K.T. Chao (1981); Richard J. Lloyd, et. al. (2004); J. Wu, et. al.,(2018); Y. Bai et. al., (2016); M. Karliner, et. al. (2017); V.R. Debastiani (2019); M.S. Liu et. al., (2019) ……*

◆ **Questions**

➢ **X(6900) is fully charmed tetraquark ($\bar{c}c\bar{c}c$)?** (J^{PC}? Mass?)

➢ **There exist bound states below corresponding threshold?**

- $\bar{c}c\bar{c}c$ System
  - Exist bound states below $J/\psi J/\psi$ *L. Heller, et. al., (1985); Z.G. Wang (2020);…*
  - Do not exist bound states below $J/\psi J/\psi$ *J. Ader, et. al., (1982); W. Chen, et. Al., (2019)…*
- $\bar{b}b\bar{b}b$ System
  - Exist bound states below $\eta_b\eta_b$ *Y. Bai, et. al., (2016); W. Chen, et. al., (2019)…*
  - Do not exist bound states below $\eta_b\eta_b$ *C. Hughes, et. al., (2017);*

■ $\bar{Q}Q\bar{Q}Q$ System studies in QCD sum rules - LO

● Moment QCD sum rules

J^{PC}	Currents	m_{X_c}(GeV)	m_{X_b}(GeV)
0^{++}	J_1	6.44 ± 0.15	18.45 ± 0.15
	J_2	6.59 ± 0.17	18.59 ± 0.17
	J_3	6.47 ± 0.16	18.49 ± 0.16
	J_4	6.46 ± 0.16	18.46 ± 0.14
	J_5	6.82 ± 0.18	19.64 ± 0.14
0^{-+}	J_1^+	6.84 ± 0.18	18.77 ± 0.18
	J_2^+	6.85 ± 0.18	18.79 ± 0.18
0^{--}	J_1^-	6.84 ± 0.18	18.77 ± 0.18
1^{++}	$J_{1\mu}^+$	6.40 ± 0.19	18.33 ± 0.17
	$J_{2\mu}^+$	6.34 ± 0.19	18.32 ± 0.18
1^{+-}	$J_{1\mu}^-$	6.37 ± 0.18	18.32 ± 0.17
	$J_{2\mu}^+$	6.51 ± 0.15	18.54 ± 0.15
1^{-+}	$J_{1\mu}^+$	6.84 ± 0.18	18.80 ± 0.18
	$J_{2\mu}^+$	6.88 ± 0.18	18.83 ± 0.18
1^{--}	$J_{1\mu}^-$	6.84 ± 0.18	18.77 ± 0.18
	$J_{2\mu}^-$	6.83 ± 0.18	18.77 ± 0.16
2^{++}	$J_{1\mu\nu}$	6.51 ± 0.15	18.53 ± 0.15
	$J_{2\mu\nu}$	6.37 ± 0.19	18.32 ± 0.17

W. Chen, et. al., 2019

● Laplace QCD sum rules

	M_Y(GeV)
$cc\bar{c}\bar{c}(0^{++})$	5.99 ± 0.08
$cc\bar{c}\bar{c}(1^{+-})$	6.05 ± 0.08
$cc\bar{c}\bar{c}(2^{++})$	6.09 ± 0.08
$bb\bar{b}\bar{b}(0^{++})$	18.84 ± 0.09
$bb\bar{b}\bar{b}(1^{+-})$	18.84 ± 0.09
$bb\bar{b}\bar{b}(2^{++})$	18.85 ± 0.09
$cc\bar{c}\bar{c}(1^{--})$	6.11 ± 0.08
$bb\bar{b}\bar{b}(1^{--})$	18.89 ± 0.09

Zhi-Gang Wang, 2018

	M_X (GeV)	M_X (GeV)
0^{++} case A	6.44 ± 0.11	18.38 ± 0.11
0^{++} case B	6.87 ± 0.10	18.50 ± 0.10
0^{++} case C	6.52 ± 0.11	18.44 ± 0.10
0^{++} case D	6.96 ± 0.11	18.59 ± 0.11

Bo-Cheng Yang et. al., 2020

J^{PC}	M_1(GeV)[7]
0^{++}	5.99 ± 0.08
1^{+-}	6.05 ± 0.08
2^{++}	6.09 ± 0.08
1^{--}	6.11 ± 0.08

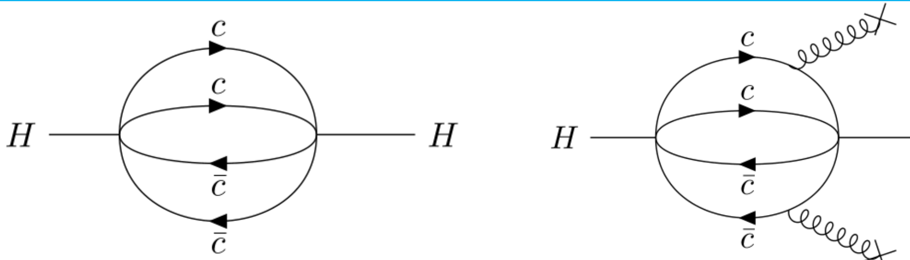
Zhi-Gang Wang, 2020

$\mathbf{0^{++}}$ case 1	$6.44^{+0.15}_{-0.16}$
$\mathbf{0^{++}}$ case 2	$6.45^{+0.14}_{-0.16}$
$\mathbf{0^{++}}$ case 3	$6.46^{+0.13}_{-0.17}$
$\mathbf{0^{++}}$ case 4	$6.47^{+0.12}_{-0.18}$

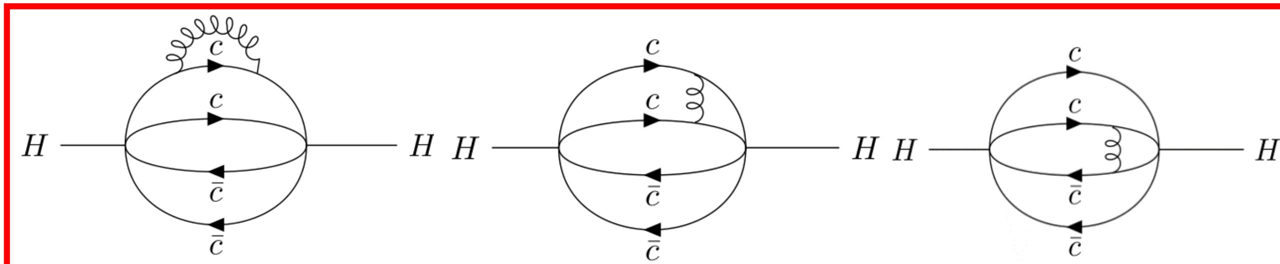
Jian-Rong Zhang, 2020

■ $\bar{Q}Q\bar{Q}Q$ System study in QCD sum rules - NLO

- **NLO corrections** are **non-negligible**
  - Ξ_{cc}^{++}: *C.Y. Wang, et. al., PRD(2019)*
  - Ω_{QQQ}: *R.H. Wu, et. al., CPC(2021)*
- **NLO corrections with $\bar{Q}Q\bar{Q}Q$ System**



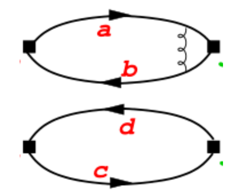
$C_1 - \text{LO}$ $\qquad$ $C_{GG} - \text{LO}$



$C_1 - \text{NLO}$

Lack of complete NLO corrections to C_1 !

Only factorized diagrams!



R.M. Albuquerque and S. Narison, (2020)

M_H [MeV]		
0^{++} Molecule		
$\eta_q\eta_q$	6029 ± 198	19259 ± 88
$J/\psi J/\psi, \Upsilon\Upsilon$	6376 ± 367	19430 ± 145
$\chi_{q1}\chi_{q1}$	6494 ± 66	19770 ± 137
$\chi_{q0}\chi_{q0}$	6675 ± 98	19653 ± 131
0^{++} Tetraquark		
Eq. 25		
S_qS_q	6411 ± 83	19217 ± 120
A_qA_q	6450 ± 75	19872 ± 156
V_qV_q	6462 ± 175	19489 ± 79
P_qP_q	6795 ± 268	19754 ± 79
Eq. 26		
A_qA_q	6471 ± 67	19717 ± 118

QCD sum rules

- Correlation function:

$$\Pi(p^2) = i\int d^4x\, e^{ip\cdot x}\, \langle\Omega|\, \mathcal{T}_1(x)\mathcal{T}_2^{\,+}(0)\,|\Omega\rangle \qquad \mathcal{T} = (\bar{Q}_i\Gamma_1 Q_i)(\,\bar{Q}_j\Gamma_2 Q_j)$$

- Källén − Lehmann representation

$$\Pi(p^2) = \int_0^\infty ds\,\frac{\rho(s)}{s-p^2-i\epsilon}$$

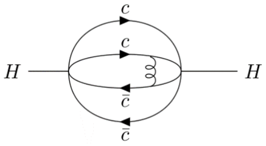
$$\rho(s) = \sum_i \lambda_i\,\delta\left(s-M_i^{\,2}\right) + \rho_{cont}(s)\,\theta(s-s_h)$$
$$= \lambda_H\,\delta\left(s-M_H^{\,2}\right) + \tilde{\rho}_{cont}(s)\,\theta(s-s_h)$$

Ground state — Higher resonances and continuum spectrum

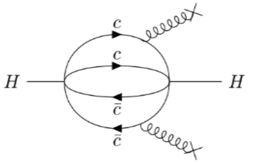
- The mass of ground state

(OPE, Borel transform, Quark-Hadron Duality…)

$$M_H^2 = \frac{\int_{s_{th}}^{s_0} ds\, Im[C_1(s)]\, s\, e^{-s/M_B^2} + \int_{s_{th}}^{\infty} ds\, Im[C_{GG}(s)]\, s\, e^{-s/M_B^2}\langle GG\rangle}{\int_{s_{th}}^{s_0} ds\, Im[C_1(s)]\, e^{-s/M_B^2} + \int_{s_{th}}^{\infty} ds\, Im[C_{GG}(s)]\, e^{-s/M_B^2}\langle GG\rangle}$$



C_1 − NLO



C_{GG} − LO

- **Calculation of Loop Integrals**

①. To reduce all loop integrals to a linear combination of master integrals ($\tilde{I}_i$):

$I = \sum_i a_i\,\tilde{I}_i$ 【REDUZE】 *A. von Manteuffel, et al, (2012)*

②. To calculate master integrals: Differential equation (DE) *A. V. Kotikov, (1991); Z. Bern, et al, (1993); E. Remiddi, et al, (1997); T. Gehrmann, et al, (2000)*

【AMFlow】: a package to calculate loop integrals **systematically** and **efficiently**.

arXiv:2201.11669[hep-ph]

$$M_H^2 = \frac{\int_{s_{th}}^{s_0} ds\, Im[C_1(s)]\, s\, e^{-s/M_B^2} + \int_{s_{th}}^{\infty} ds\, Im[C_{GG}(s)]\, s\, e^{-s/M_B^2}\langle GG\rangle}{\int_{s_{th}}^{s_0} ds\, Im[C_1(s)]\, e^{-s/M_B^2} + \int_{s_{th}}^{\infty} ds\, Im[C_{GG}(s)]\, e^{-s/M_B^2}\langle GG\rangle}$$

Borel Windows

- the validity of OPE
- the ground-state contribution dominance

➡ To constrain the range of s_0 and M_B^2

(called Borel windows)

$$r_{GG} = \left|\frac{\int_{s_{th}}^{\infty} ds\, Im[C_{GG}(s)]\, e^{-s/M_B^2}\langle GG\rangle}{\int_{s_{th}}^{\infty} ds\, Im[C_1(s)]\, e^{-s/M_B^2}}\right| \leq 30\%$$

$$r_{cont} = \left|\frac{\int_{s_0}^{\infty} ds\, Im[C_1(s)]\, e^{-s/M_B^2}}{\int_{s_{th}}^{\infty} ds\, Im[C_1(s)]\, e^{-s/M_B^2}}\right| \leq 30\%$$

Borel Platform

The point where the parameter dependence of M_H is weakest within Borel windows

$$\Delta(x,y) = \left(\frac{\partial M_H}{\partial x}\right)^2 + \left(\frac{\partial M_H}{\partial y}\right)^2 \qquad (x = s_0,\ y = M_B^2)$$

Fully Heavy Tetraquarks Mass Spectra

■ $J^{PC} = 0^{++}$

- Meson-Meson type Operators $\quad \mathcal{J}_{\mathrm{M-M}} = (\bar{Q}_i \Gamma_1 Q_i)(\bar{Q}_j \Gamma_2 Q_j)$
- Diquark- Antidiquark type Operators $\quad \mathcal{J}_{\mathrm{Di-Di}} = (Q_i^{\,T}\, \mathcal{C}\, \Gamma_1\, Q_j)(\bar{Q}_i\, \Gamma_2 \mathcal{C}\, \bar{Q}_j^{\,T})$

$$(\Gamma_1, \Gamma_2) = \begin{pmatrix} (\gamma^\mu, \gamma_\mu) \\ (\gamma^\mu\gamma^5, \gamma_\mu\gamma^5) \\ (1, 1) \\ (i\gamma^5, i\gamma^5) \\ (\sigma^{\mu\nu}, \sigma_{\mu\nu}) \end{pmatrix}$$

- **Diagonalizable operators** $\quad \mathcal{J}_{\mathbf{Dia}} = \mathbf{T}.\mathcal{J}_{\mathbf{M-M}}$

Reducing renormalization scale μ dependence

$$\delta \begin{pmatrix} -6 & -2 & -12 & -12 & 0 \\ -2 & -6 & 12 & 12 & 0 \\ 0 & 0 & 26 & 6 & \frac{1}{3} \\ 0 & 0 & 6 & 26 & -\frac{1}{3} \\ 0 & 0 & -40 & 40 & -\frac{68}{3} \end{pmatrix}$$

The anomalous dimension matrix of $\mathcal{J}_{M-M}$

Diagonalization →

$$\frac{4}{3}\delta \begin{pmatrix} -6 & 0 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & 24 & 0 & 0 \\ 0 & 0 & 0 & -1+\sqrt{241} & 0 \\ 0 & 0 & 0 & 0 & -1-\sqrt{241} \end{pmatrix}$$

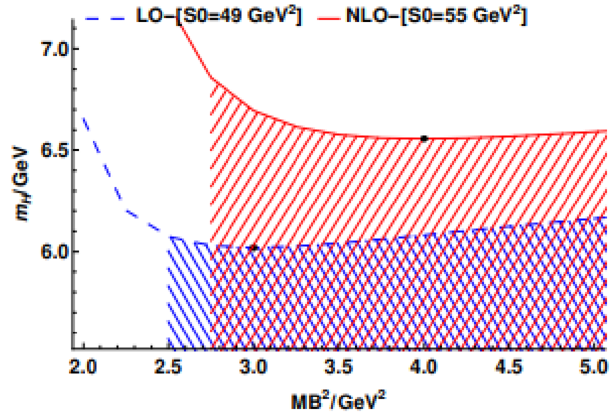
The anomalous dimension matrix of $\mathcal{J}_{Dia}$

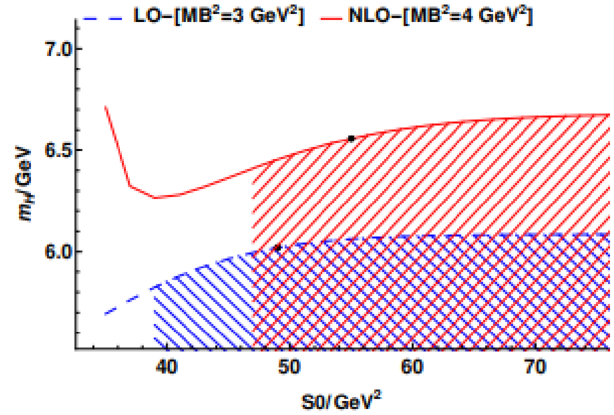
$$T = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & -\frac{15}{\sqrt{241}} & \frac{15}{\sqrt{241}} & \frac{1}{2} - \frac{8}{\sqrt{241}} \\ 0 & 0 & \frac{15}{\sqrt{241}} & -\frac{15}{\sqrt{241}} & \frac{1}{2} + \frac{8}{\sqrt{241}} \end{pmatrix}$$

The Borel Platform Curves - $\bar{c}c\bar{c}c$

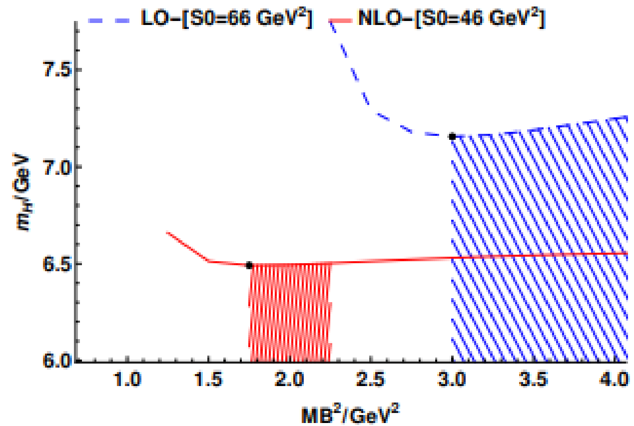
$J_{S,4}^{Dia}$

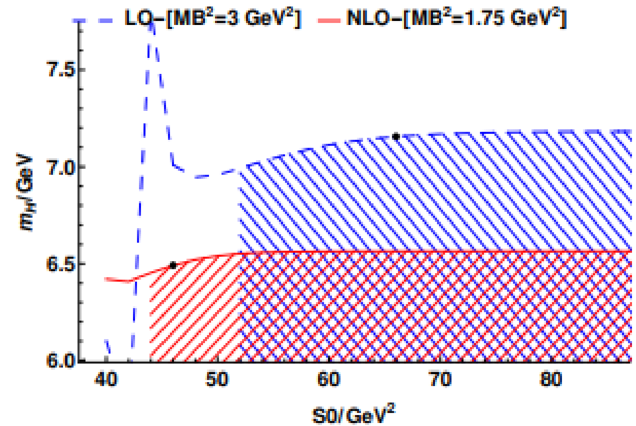
- $\overline{MS}$





- OS





➢ $J^{PC} = 0^{++}$ Diagonalized Operators- $\bar{c}c\bar{c}c$

- $\overline{MS}$

流算符	LO	NLO($\overline{MS}$)
$J^{Dia}_{S,1}$	$6.18^{+0.08}_{-0.10}$	$7.81^{+0.14}_{-0.16}$
$J^{Dia}_{S,2}$	$6.19^{+0.07}_{-0.12}$	$6.95^{+0.10}_{-0.12}$
$J^{Dia}_{S,3}$	$5.93^{+0.07}_{-0.10}$	$6.35^{+0.06}_{-0.13}$
$J^{Dia}_{S,4}$	$6.02^{+0.05}_{-0.06}$	$6.56^{+0.10}_{-0.12}$
$J^{Dia}_{S,5}$	$6.33^{+0.12}_{-0.14}$	$7.72^{+0.13}_{-0.14}$

- OS

流算符	LO	NLO(OS)
$J^{Dia}_{S,1}$	$7.36^{+0.07}_{-0.10}$	$6.60^{+0.09}_{-0.12}$
$J^{Dia}_{S,2}$	$7.31^{+0.08}_{-0.12}$	$6.58^{+0.08}_{-0.11}$
$J^{Dia}_{S,3}$	$7.06^{+0.07}_{-0.10}$	$6.47^{+0.08}_{-0.10}$
$J^{Dia}_{S,4}$	$7.16^{+0.04}_{-0.05}$	$6.49^{+0.07}_{-0.10}$
$J^{Dia}_{S,5}$	$7.44^{+0.12}_{-0.14}$	$6.62^{+0.09}_{-0.13}$

◆ LO VS NLO

NLO corrections are **significant**.

- $|M_H^{NLO} - M_H^{LO}| > 0.5$ GeV
- Below or above $\eta_c\eta_c$, $J/\psi J/\psi$?

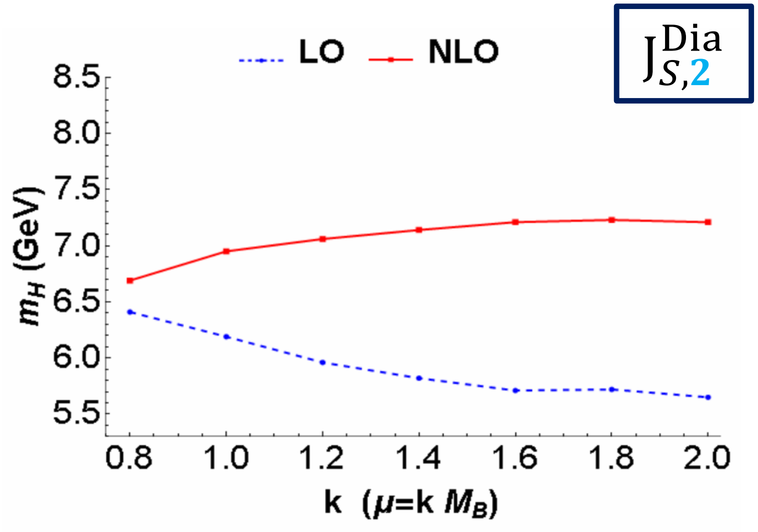
◆ $\overline{MS}$ VS OS

The **scheme(quark mass) dependence** is **reduced** observably.

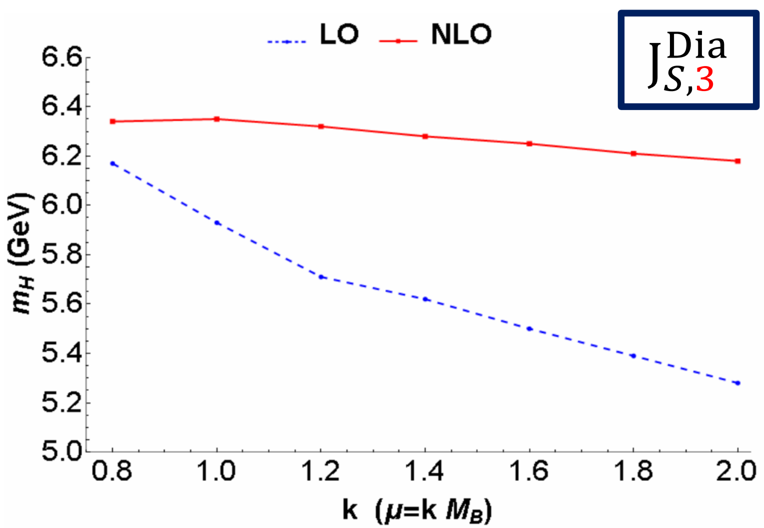
- $|M_H^{\overline{MS}, LO} - M_H^{OS, LO}| > 1$ GeV
- $|M_H^{\overline{MS}, NLO} - M_H^{OS,NLO}| \sim 0.5$ GeV

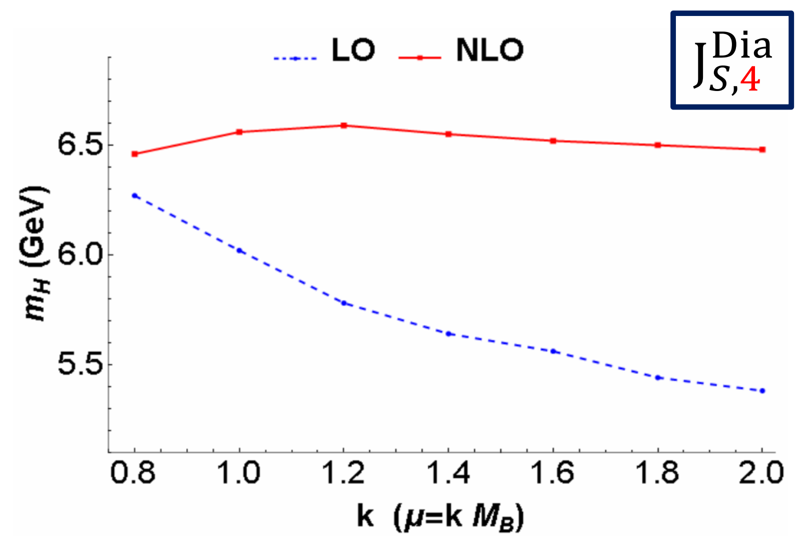
$|m_c^{\overline{MS}} - m_c^{OS}| \sim 0.3$ GeV

■ The renormalization scale μ dependence- $\bar{c}c\bar{c}c$



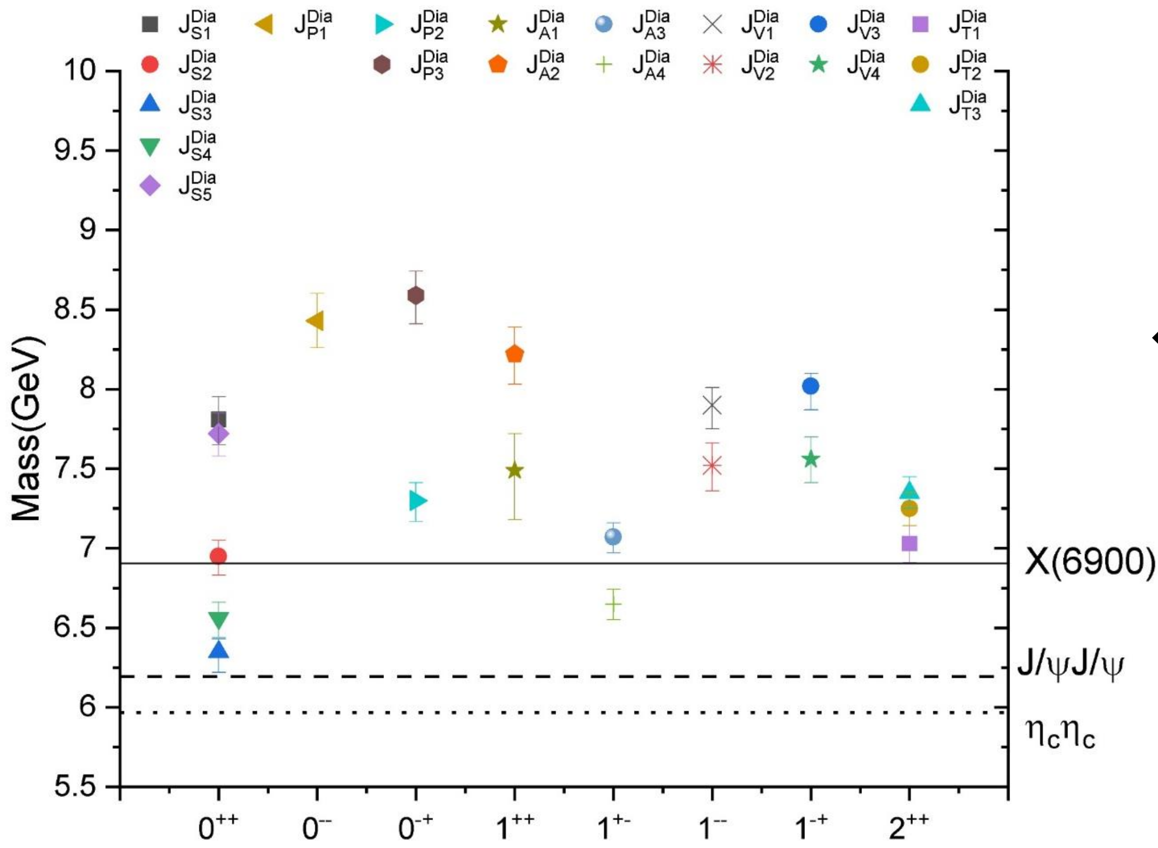
The NLO contributions significantly improve μ dependence of hadron mass m_H





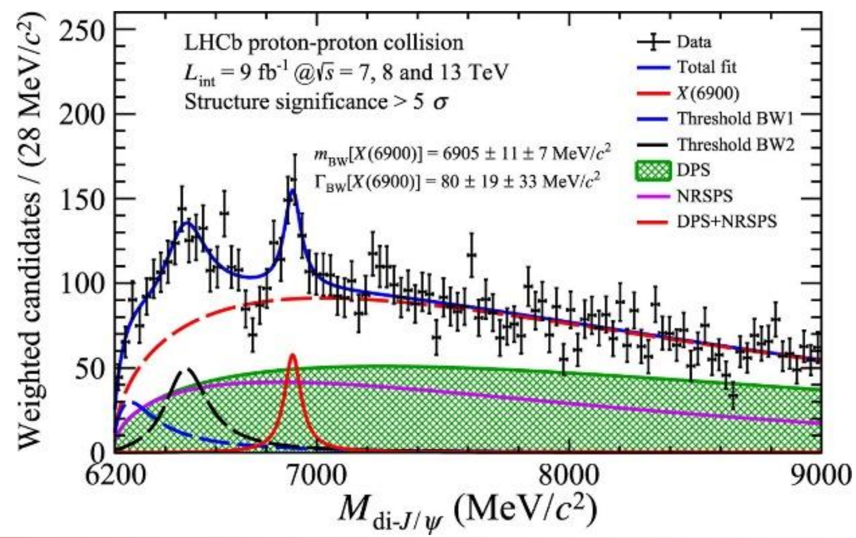
■ $\bar{c}c\bar{c}c$ Mass Spectra

$J_{\mathrm{Dia}}^{\overline{MS},\mathrm{NLO}}$



- May not exist bound states below J/ψJ/ψ.
- $J_{S,2}^{\mathrm{Dia}}$ with $J^{PC} = 0^{++}$ may be assigned to be X(6900) .

  $J_{T,1}^{\mathrm{Dia}}$ with $J^{PC} = 2^{++}$ may also be a candidate for the X(6900) .
- $J_{S,3}^{\mathrm{Dia}}$ and $J_{S,4}^{\mathrm{Dia}}$ may explain the broad structure (6.2∼6.8 GeV)



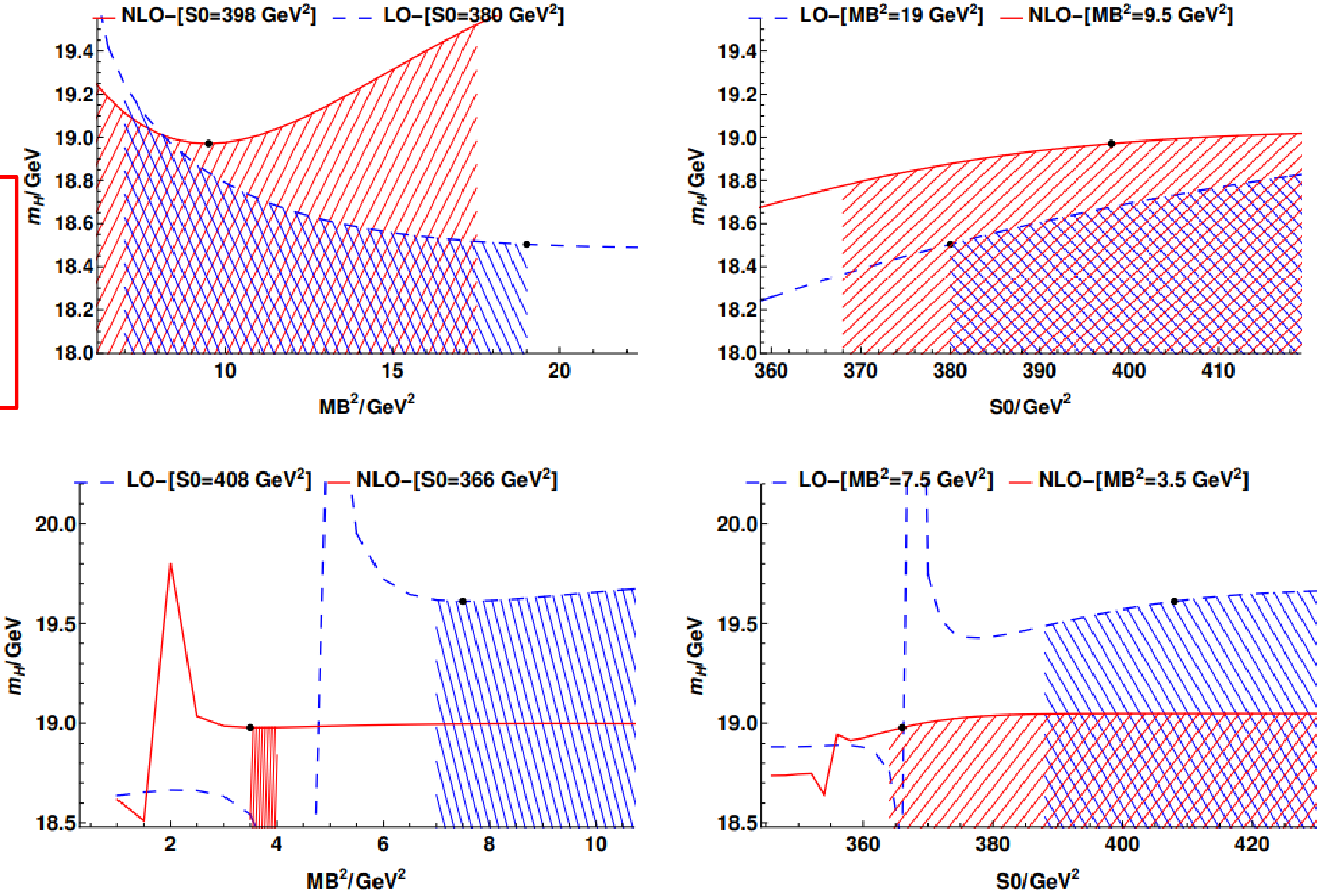
■ The Borel Platform Curves - $\bar{b}b\bar{b}b$

$J_{S,4}^{Dia}$

- $\overline{MS}$

NLO contributions improve the quality of Borel platform evidently

- OS



➢ $J^{PC} = 0^{++}$ Diagonalized Operators - $\bar{b}b\bar{b}b$

- $\overline{\text{MS}}$

流算符	LO	NLO($\overline{MS}$)
$J^{Dia}_{S,1}$	$18.51^{+0.17}_{-0.26}$	$19.01^{+0.05}_{-0.10}$
$J^{Dia}_{S,2}$	$18.51^{+0.17}_{-0.26}$	$18.97^{+0.06}_{-0.11}$
$J^{Dia}_{S,3}$	$18.50^{+0.18}_{-0.26}$	$18.96^{+0.05}_{-0.11}$
$J^{Dia}_{S,4}$	$18.50^{+0.17}_{-0.26}$	$18.97^{+0.06}_{-0.11}$
$J^{Dia}_{S,5}$	$18.51^{+0.17}_{-0.26}$	$18.95^{+0.08}_{-0.14}$

- OS

流算符	LO	NLO(OS)
$J^{Dia}_{S,1}$	$19.68^{+0.04}_{-0.10}$	$18.98^{+0.07}_{-0.28}$
$J^{Dia}_{S,2}$	$19.67^{+0.04}_{-0.10}$	$18.98^{+0.07}_{-0.28}$
$J^{Dia}_{S,3}$	$19.64^{+0.02}_{-0.06}$	$18.98^{+0.07}_{-0.36}$
$J^{Dia}_{S,4}$	$19.61^{+0.07}_{-0.14}$	$18.98^{+0.07}_{-0.33}$
$J^{Dia}_{S,5}$	$19.66^{+0.08}_{-0.15}$	$18.98^{+0.07}_{-0.26}$

◆ **LO** VS **NLO**

NLO corrections are significant.

- $|M_H^{NLO} - M_H^{LO}| \sim 0.5$ GeV
- NLO results are above $\eta_b\eta_b$

◆ $\overline{MS}$ VS **OS**

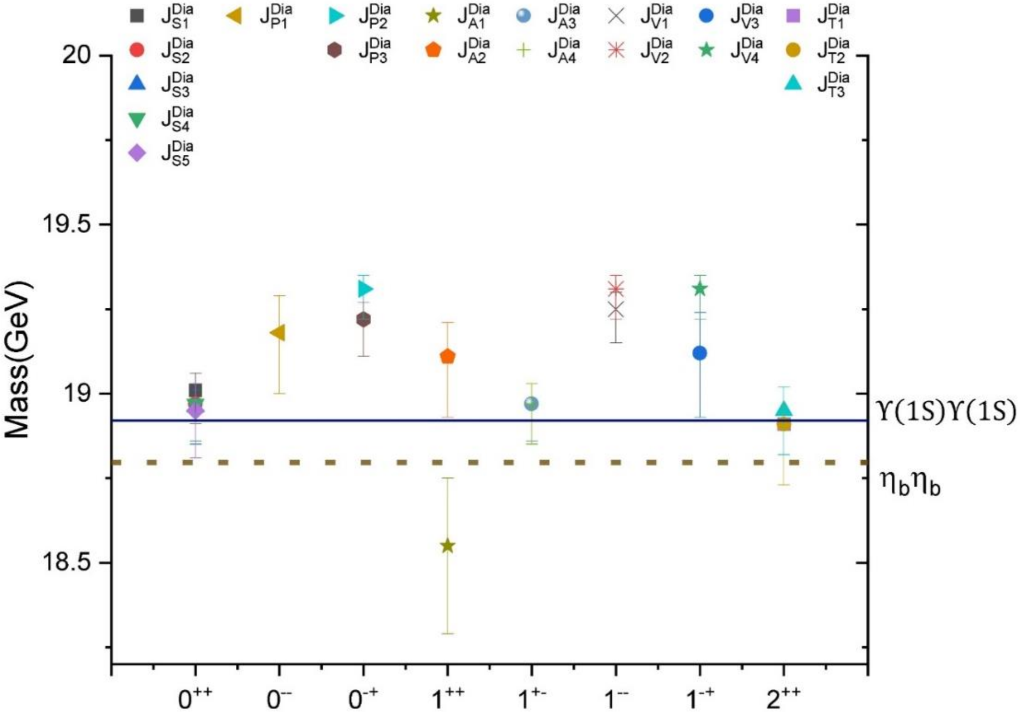
The scheme(quark mass) dependence is reduced.

- $\left|M_H^{\overline{MS},LO} - M_H^{OS,LO}\right| \sim \mathbf{1}$ **GeV**
- $\left|M_H^{\overline{MS},NLO} - M_H^{OS,NLO}\right| \sim \mathbf{0.1}$**GeV**

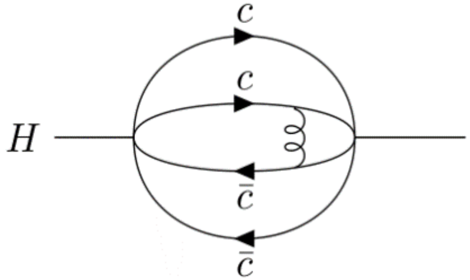
◆ Problem

Bad Perturbative Convergence

■ $\bar{b}b\bar{b}b$ Mass Spectra

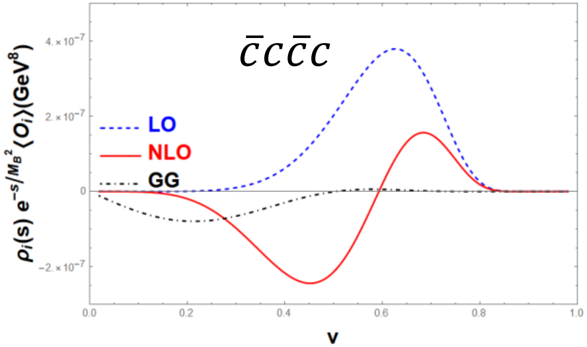


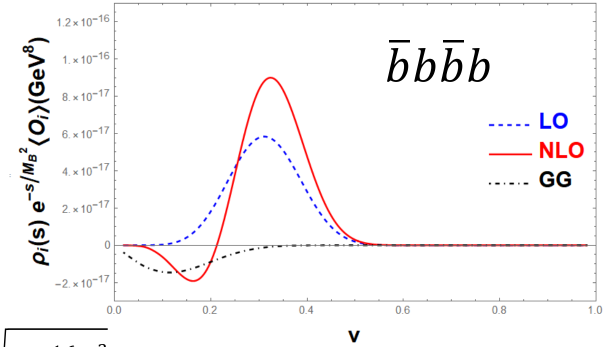
- The perturbative convergence is bad in $\bar{b}b\bar{b}b$ system.
  - Coulomb divergence
  - More nonrelativistic than $\bar{c}c\bar{c}c$
- The errors are still large (before resumming near-threshold divergence).
- Current results should be treated cautiously



$J^{\mathrm{Dia}}_{S,3}$

Coulomb divergence: $\frac{\alpha_s}{v}$





$v=\sqrt{1-\frac{16m^2}{s}}$

Summary

- NLO corrections and operator mixing are important and non-negligibel
  - Large corrections to hadron masses M_H $\left(\left|M_H^{NLO}-M_H^{LO}\right|>0.5\ GeV\right)$.
  - Improving the quality of the Borel platform evidently. ($\bar{b}b\bar{b}b$ system)
  - Reducing the renormalization scale μ dependence.
  - Reducing the scheme dependence. $\begin{cases}\left|M_H^{\overline{MS},LO}-M_H^{OS,LO}\right|>1\text{ GeV}\\ \left|M_H^{\overline{MS},NLO}-M_H^{OS,NLO}\right|\sim0.5\text{ GeV}\end{cases}$
- $\bar{c}c\bar{c}c$ Mass Spectra
  - May not exist bound states below $J/\psi J/\psi$
  - $J_{S,3}^{Dia}$ and $J_{S,4}^{Dia}$ with $J^{PC}=0^{++}$ may explain the broad structure
  - $J_{S,2}^{Dia}$ with $J^{PC}=0^{++}$ and $J_{T,1}^{Dia}$ with $J^{PC}=2^{++}$ may be candidates of the X(6900) .
- $\bar{b}b\bar{b}b$ Mass Spectra
  - Bad perturbative convergence and large errors . (near-threshold divergence resummation?)
  - There may not exist bound states below $\eta_b\eta_b$ (Based on current results without resummation).

Thanks！