Two-Loop QCD Corrections to Exclusive $J/\psi + \chi_{cJ}$ Production at B Factories

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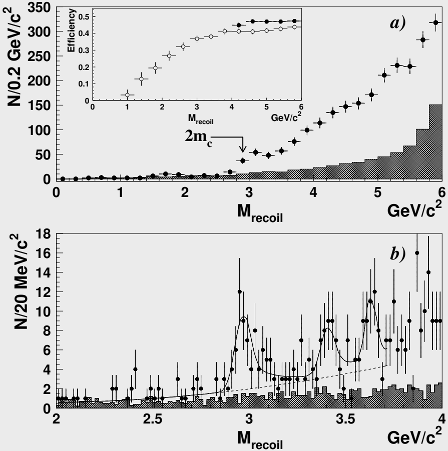
In collaboration with: Feng Feng(China University of Mining and Technology-Beijing), Yu Jia(IHEP, CAS), Zhewen Mo(IHEP, CAS), Wen-Long Sang(Southwest University)

Jia-Yue Zhang

Institute of High Energy Physics, Chinese Academy of Sciences

Motivation



The recoil mass distribution

Belle, PRL2002

$\sigma\,(J/\psi + \eta_c)$

- $\sigma \times \mathcal{B}_{>4} = 33^{+7}_{-6} \pm 9\,\text{fb}$ @Belle[1]
- $\sigma \times \mathcal{B}_{>2} = 25.6 \pm 2.8 \pm 3.4\,\text{fb}$ @Belle[2]
- $\sigma \times \mathcal{B}_{>2} = 17.6 \pm 2.8^{+1.5}_{-2.1}\,\text{fb}$ @BaBar[3]

$\sigma\,(J/\psi + \chi_{c0})$

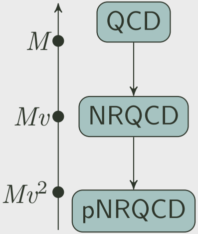
- $\sigma \times \mathcal{B}_{>2} = 6.4 \pm 1.7 \pm 1.0\,\text{fb}$ @Belle[2]
- $\sigma \times \mathcal{B}_{>2} = 10.3 \pm 2.5^{+1.4}_{-1.8}\,\text{fb}$ @BaBar[3]

$\sigma\,(J/\psi + \chi_{c1}) + \sigma\,(J/\psi + \chi_{c2})$

- $\sigma \times \mathcal{B}_{>2} < 5.3\,\text{fb}$ at 90%C.L. @Belle[2]

1 *Belle, PRL2002* 2 *Belle, PRD2004* 3 *BaBar, PRD2005*

NRQCD Factorization *Bodwin, Braaten, Lepage, PRD1995*

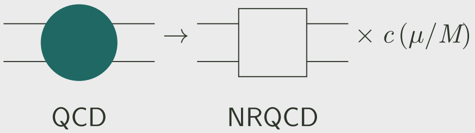


Vairo, Hadron 2011

- Quarkonium energy scale ***Braaten, 1997***

	$c\bar{c}$	$b\bar{b}$	$t\bar{t}$
M	$1.5\,\mathrm{GeV}$	$4.7\,\mathrm{GeV}$	$180\,\mathrm{GeV}$
Mv	$0.9\,\mathrm{GeV}$	$1.5\,\mathrm{GeV}$	$16\,\mathrm{GeV}$
Mv^2	$0.5\,\mathrm{GeV}$	$0.5\,\mathrm{GeV}$	$1.5\,\mathrm{GeV}$

- Integrate out the heavy($\sim M$) degrees of freedom



Qiu, 2011

NRQCD in Quarkonium Study (see *Brambilla et al., 2011* for review)

- Nowadays, NRQCD becomes standard approach to tackle various quarkonium production and decay processes including
  - Radiative and dileptonic decays of quarkonia, **e.g.**, $\chi_{c0} \to$ light hadrons
  - Exclusive charmonium production, **e.g.**, $e^+e^- \to J/\psi + \eta_c$ at B factories
  - Inclusive charmonium production, **e.g.**, J/ψ production at hadron colliders
- Most of the NRQCD successes based on the NLO QCD predictions.
- However, the NLO QCD corrections are often large:

process	K-factor	
$e^+e^- \to J/\psi + \eta_c$	$1.8 \sim 2.1$	*Zhang, Gao, Chao, PRL2006*
$e^+e^- \to J/\psi + J/\psi$	$-0.31 \sim 20.25$	*Gong, Wang, PRL2008*
$pp \to J/\psi + X$	~ 2	*Campbell , Maltoni, Tramontano, PRL2007*

The NNLO Corrections

Perturbative convergence of these processes seems to be rather poor.

$$\Gamma\left(J/\psi \to l^+ l^-\right) = \Gamma^{(0)}\left(1 - \frac{8}{3}\frac{\alpha_s}{\pi} - (44.55 - 0.41 n_L)\left(\frac{\alpha_s}{\pi}\right)^2 + (-2091 + 120.66 n_L - 0.82 n_L^2)\left(\frac{\alpha_s}{\pi}\right)^3\right)^2$$

Marquard, Piclum, et al., PRD2014

$$\Gamma\left(B_c \to l\nu\right) = \Gamma^{(0)}\left(1 - 1.39\frac{\alpha_s}{\pi} - 23.7\left(\frac{\alpha_s}{\pi}\right)^2\right)^2$$

Chen, Qiao, PLB2015

$$\Gamma\left(\eta_c \to \gamma\gamma\right) = \Gamma^{(0)}\left(1 - 1.69\frac{\alpha_s}{\pi} - 56.52\left(\frac{\alpha_s}{\pi}\right)^2\right)^2$$

Feng, Jia, Sang, PRL2017

So calculating the higher order QCD corrections is imperative to test the usefulness of NRQCD factorization.

$e^+e^- \to J/\psi + \chi_{cJ}$

- The tree-level results have been known long ago.
  Braaten, Lee, PRD2003; Liu, He, Chao, PLB2003; Hagiwara, Kou, Qiao, PLB2003
- More than a decade ago, the NLO perturbative corrections to $e^+e^- \to J/\psi + \chi_{cJ}$ have been computed by several groups.
  ***Zhang, Ma, Chao, PRD2008; Wang, Ma, Chao, PRD2011; Dong, Feng, Jia, JHEP2011.*.**
- Recently, the interference due to QED has also been investigated for these processes.
  ***Jiang, Sun, EPJC2018*.**
- There is also a comparative analysis of the J/ψ angular distributions in a number of double charmonium production processes has been conducted between the $\mathcal{O}(\alpha_s)$ NRQCD prediction and B factory data.
  Sun, JHEP2021.

Helicity Amplitudes *Jacob, Wick, Annals Phys.1959*

The process can be decomposed to the decay of a timelike photon and expressed as **helicity amplitudes**:

$$\frac{\mathrm{d}\sigma\left[e^+e^- \to J/\psi(\lambda_1)+\chi_{cJ}(\lambda_2)\right]}{\mathrm{d}\cos\theta} = \frac{\alpha|\mathbf{P}||\mathcal{A}^J_{\lambda_1,\lambda_2}|^2}{8s^{5/2}} \times \begin{cases} \frac{1+\cos^2\theta}{2}, \ \lambda=\pm 1, \\ 1-\cos^2\theta, \ \lambda=0, \end{cases}$$

λ_1, λ_2: helicities of $J/\psi, \chi_{cJ}$, $\quad \lambda = \lambda_1 - \lambda_2$

$|\mathbf{P}|$: magnitude of the 3-momentum of J/ψ (χ_{cJ}) in the CM frame

Helicity Selection Rule *Chernyak, Zhitnitsky, 1980; Brodsky, Lepage, 1981; Braaten, Lee, 2003*

$$\mathcal{A}^J_{\lambda_1,\lambda_2} \propto s^{-\frac{1}{2}(1+|\lambda_1+\lambda_2|)}, \quad s \gg m_c$$

Angular Distribution Parameter

- Unpolarized production rates:

$$\frac{\mathrm{d}\sigma(e^+e^- \to J/\psi + \chi_{cJ})}{\mathrm{d}\cos\theta} = A_J\left(1 + \alpha_J \cos^2\theta\right), \qquad J = 0, 1, 2$$

→ **Angular distribution parameter** α_J:

$$\alpha_0 = -\frac{|\mathcal{A}^0_{0,0}|^2 - |\mathcal{A}^0_{1,0}|^2}{|\mathcal{A}^0_{0,0}|^2 + |\mathcal{A}^0_{1,0}|^2}$$

$$\alpha_1 = \frac{|\mathcal{A}^1_{1,0}|^2 + |\mathcal{A}^1_{0,1}|^2 - 2|\mathcal{A}^1_{1,1}|^2}{|\mathcal{A}^1_{1,0}|^2 + |\mathcal{A}^1_{0,1}|^2 + 2|\mathcal{A}^1_{1,1}|^2}$$

$$\alpha_2 = -\frac{|\mathcal{A}^2_{0,0}|^2 - |\mathcal{A}^2_{1,0}|^2 - |\mathcal{A}^2_{0,1}|^2 + 2|\mathcal{A}^2_{1,1}|^2 - |\mathcal{A}^2_{1,2}|^2}{|\mathcal{A}^2_{0,0}|^2 + |\mathcal{A}^2_{1,0}|^2 + |\mathcal{A}^2_{0,1}|^2 + 2|\mathcal{A}^2_{1,1}|^2 + |\mathcal{A}^2_{1,2}|^2}$$

NRQCD Factorization Formula

NRQCD factorization is applicable at helicity amplitude level.

$$\mathcal{A}^J_{\lambda_1,\lambda_2} = \mathcal{C}^J_{\lambda_1,\lambda_2} \frac{\langle J/\psi|\psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}_{J/\psi}\chi(\mu_\Lambda)|0\rangle\langle\chi_{cJ}|\psi^\dagger\mathcal{K}_{^3P_J}\chi(\mu_\Lambda)|0\rangle}{m_c^3} + \mathcal{O}\left(v^2\right).$$

$$\mathcal{K}_{^3P_0} = \frac{1}{\sqrt{3}}\left(-\frac{\mathrm{i}}{2}\overleftrightarrow{\mathbf{D}}\cdot\boldsymbol{\sigma}\right), \quad \mathcal{K}_{^3P_1} = \frac{1}{\sqrt{2}}\left(-\frac{\mathrm{i}}{2}\overleftrightarrow{\mathbf{D}}\times\boldsymbol{\sigma}\right)\cdot\boldsymbol{\varepsilon}_{\chi_{c1}}, \quad \mathcal{K}_{^3P_2} = -\frac{\mathrm{i}}{2}\overleftrightarrow{D}^{(i}\sigma^{j)}\varepsilon^{ij}_{\chi_{c2}}$$

Through $\mathcal{O}(\alpha_s^2)$, **dimensionless** SDC is expected to take the following structure:

$$\mathcal{C}^J_{\lambda_1,\lambda_2}\left(r,\frac{\mu_R^2}{m_c^2},\frac{\mu_\Lambda^2}{m_c^2}\right) = \frac{64\pi e\alpha_s}{27\sqrt{3}} r^{(1+|\lambda_1+\lambda_2|)/2} \mathcal{C}^{J(\text{tree})}_{\lambda_1,\lambda_2}\left\{1 + \frac{\alpha_s(\mu_R)}{\pi}\left(\frac{1}{4}\beta_0\ln\frac{\mu_R^2}{m_c^2} + c^{J(1)}_{\lambda_1,\lambda_2}\right)\right.$$
$$+\frac{\alpha_s^2(\mu_R)}{\pi^2}\left(\frac{1}{16}\beta_0^2\ln^2\frac{\mu_R^2}{m_c^2} + \frac{1}{16}(8c^{J(1)}_{\lambda_1,\lambda_2}\beta_0+\beta_1)\ln\frac{\mu_R^2}{m_c^2}\right.$$
$$\left.\left.+(\gamma_{J/\psi}+\gamma_{\chi_{cJ}})\ln\frac{\mu_\Lambda^2}{m_c^2} + c^{J(2)}_{\lambda_1,\lambda_2}\right)\right\}, \quad r := \frac{4m_c^2}{s}$$

Tree-Level SDCs *Braaten, Lee, PRD2003*

$\sigma\left(J/\psi+\chi_{c0}\right)$

$$\mathcal{C}_{0,0}^{0(\text{tree})}=1+10r-12r^2,\quad \mathcal{C}_{1,0}^{0(\text{tree})}=9-14r,$$

$\sigma\left(J/\psi+\chi_{c1}\right)$

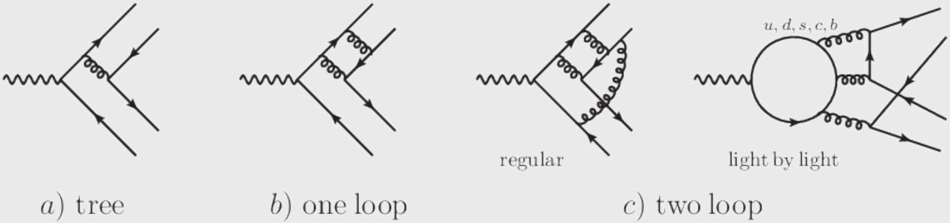
$$\mathcal{C}_{1,0}^{1(\text{tree})}=-\sqrt{6}r,\quad \mathcal{C}_{0,1}^{1(\text{tree})}=-\sqrt{6}(2-7r),\quad \mathcal{C}_{1,1}^{1(\text{tree})}=-2\sqrt{6}(1-3r),$$

$\sigma\left(J/\psi+\chi_{c2}\right)$

$$\mathcal{C}_{0,0}^{2(\text{tree})}=-\sqrt{2}(1-2r-12r^2),\quad \mathcal{C}_{1,0}^{2(\text{tree})}=-\sqrt{2}(3-11r),$$
$$\mathcal{C}_{1,1}^{2(\text{tree})}=-2\sqrt{6}(1-3r),\quad \mathcal{C}_{0,1}^{2(\text{tree})}=-\sqrt{6}(1-5r),\quad \mathcal{C}_{1,2}^{2(\text{tree})}=-2\sqrt{3}.$$

Calculating the SDCs

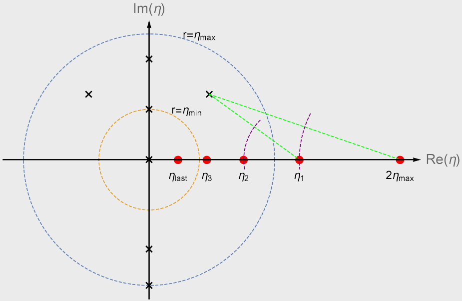
- In principle the SDCs can be inferred by the perturbative matching procedure.
- Since we only concerned with the lowest order in v, we directly extract the SDCs in the context of method of region. For simplicity, we ultilize the well-known covariant color/spin/orbital projector technique. ***Beneke, Smirnov, NPB1997; Petrelli, Cacciari, et al., NPB1998***
- Nearly 2000 two-loop diagrams survive for the $\gamma \to c\bar{c}\left({}^3S_1^{(1)}\right) + c\bar{c}\left({}^3P_J^{(1)}\right)$ processes, generated by `Qgraf` and `FeynArts`.



- $e_u + e_d + e_s = 0 \rightsquigarrow$ "Light-by-light" amplitudes stemming from the light quark loops cancels.

Strategy of Calculation

- Trace && Contraction: `FenyCalc` and `FormLink`
- Partial Fraction && IBP Reduction: `Apart` and `FIRE` $\rightsquigarrow \sim$ 600 master integrals (MIs)
- MIs by **Auxiliary-Mass-Flow** method: `AMFlow` ***Liu, Ma, 2201.11669***



Liu, Ma, Wang, PLB2018

The biggest challenge of this work is to precisely compute MIs, many of which bears rather complicated topology and are generally complex-valued.

IR Divergences

- Renormalized quark helicity amplitude is left with a **single IR pole**, whose coefficients are exactly identical to $(\gamma_{J/\psi}+\gamma_{\chi_{cJ}})/2$. ***Czarnecki, Melnikov, PRL1998; Beneke, Signer, Smirnov, PRL1998; Hoang, Ruiz-Femenia, PRD2006; Sang, Feng, et al., PRD2016***

$$\mathcal{O}_{J/\psi} = \psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}_{J/\psi}\chi, \qquad \gamma_{J/\psi} = -\pi^2\left(\frac{C_A C_F}{4}+\frac{C_F^2}{6}\right),$$

$$\mathcal{O}_{\chi_{c0}} = \frac{1}{\sqrt{3}}\psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\boldsymbol{\sigma}\right)\chi, \qquad \gamma_{\chi_{c0}} = -\pi^2\left(\frac{C_A C_F}{12}+\frac{C_F^2}{3}\right),$$

$$\mathcal{O}_{\chi_{c1}} = \frac{1}{\sqrt{2}}\psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\times\boldsymbol{\sigma}\right)\cdot\boldsymbol{\varepsilon}_{\chi_{c1}}\chi \qquad \gamma_{\chi_{c1}} = -\pi^2\left(\frac{C_A C_F}{12}+\frac{5C_F^2}{24}\right),$$

$$\mathcal{O}_{\chi_{c2}} = \psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{D}^{(i}\sigma^{j)}\varepsilon^{ij}_{\chi_{c2}}\right)\chi, \qquad \gamma_{\chi_{c2}} = -\pi^2\left(\frac{C_A C_F}{12}+\frac{13C_F^2}{120}\right).$$

- These IR divergences give a **highly nontrivial success of NRQCD factorization for exclusive $S+P$ -wave charmonium production at two loop order**.

SDCs

The finite SDCs with $\sqrt{s} = 10.58\,\mathrm{GeV}$ and $m_c = 1.5\,\mathrm{GeV}$, $m_b = 4.7\,\mathrm{GeV}$ are

H	(λ_1, λ_2)	$c^{(2)}_{\lambda_1,\lambda_2}$
χ_{c0}	$(1,0)$	$-29.59 + 8.53\mathrm{i} + (-0.2621 - 0.1190\mathrm{i})n_L^2 + (-1.323 + 0.709\mathrm{i})n_L$ $+(-0.1994 + 0.1361\mathrm{i})_{\mathrm{lbl},c} + (-0.0559 + 0.1737\mathrm{i})_{\mathrm{lbl},b}$
	$(0,0)$	$-44.93 + 20.09\mathrm{i} + (-0.2755 - 0.0694\mathrm{i})n_L^2 + (0.1312 + 0.1762\mathrm{i})n_L$ $+(-0.2735 + 0.2099\mathrm{i})_{\mathrm{lbl},c} + (-0.0711 + 0.1690\mathrm{i})_{\mathrm{lbl},b}$
χ_{c1}	$(1,1)$	$-67.21 - 43.30\mathrm{i} + (-0.2751 - 0.0710\mathrm{i})n_L^2 + (2.709 + 6.129\mathrm{i})n_L$ $+(0.2348 - 0.0348\mathrm{i})_{\mathrm{lbl},c} + (0.4232 + 0.1602\mathrm{i})_{\mathrm{lbl},b}$
	$(1,0)$	$-769.4 - 585.4\mathrm{i} + (-0.2539 - 0.1491\mathrm{i})n_L^2 + (72.27 + 39.43\mathrm{i})n_L$ $+(-0.129 + 1.425\mathrm{i})_{\mathrm{lbl},c} + (2.890 + 3.365\mathrm{i})_{\mathrm{lbl},b}$
	$(0,1)$	$-37.97 - 16.38\mathrm{i} + (-0.2763 - 0.0667\mathrm{i})n_L^2 + (-0.217 + 3.406\mathrm{i})n_L$ $+(0.12717 - 0.02899\mathrm{i})_{\mathrm{lbl},c} + (0.1941 + 0.0208\mathrm{i})_{\mathrm{lbl},b}$
χ_{c2}	$(1,2)$	$-36.04 - 27.64\mathrm{i} + (-0.2539 - 0.1491\mathrm{i})n_L^2 + (4.426 + 3.230\mathrm{i})n_L$ $+(-0.7413 + 0.1567\mathrm{i})_{\mathrm{lbl},c} + (-0.6149 + 0.2857\mathrm{i})_{\mathrm{lbl},b}$
	$(1,1)$	$-69.87 - 5.01\mathrm{i} + (-0.2751 - 0.0710\mathrm{i})n_L^2 + (2.967 + 3.058\mathrm{i})n_L$ $+(-0.2400 + 0.0725\mathrm{i})_{\mathrm{lbl},c} + (-0.1554 + 0.1515\mathrm{i})_{\mathrm{lbl},b}$
	$(1,0)$	$-55.16 - 11.06\mathrm{i} + (-0.2843 - 0.0371\mathrm{i})n_L^2 + (1.716 + 2.894\mathrm{i})n_L$ $+(-0.2349 + 0.1069\mathrm{i})_{\mathrm{lbl},c} + (-0.1166 + 0.1127\mathrm{i})_{\mathrm{lbl},b}$
	$(0,1)$	$-15.87 - 18.98\mathrm{i} + (-0.3077 + 0.0489\mathrm{i})n_L^2 + (0.813 + 1.989\mathrm{i})n_L$ $+(0.1442 + 0.0718\mathrm{i})_{\mathrm{lbl},c} + (0.2558 - 0.0429\mathrm{i})_{\mathrm{lbl},b}$
	$(0,0)$	$-67.22 - 28.33\mathrm{i} + (-0.3030 + 0.0314\mathrm{i})n_L^2 + (4.399 + 2.106\mathrm{i})n_L$ $+(-0.1578 + 0.1796\mathrm{i})_{\mathrm{lbl},c} + (0.012572 + 0.005615\mathrm{i})_{\mathrm{lbl},b}$

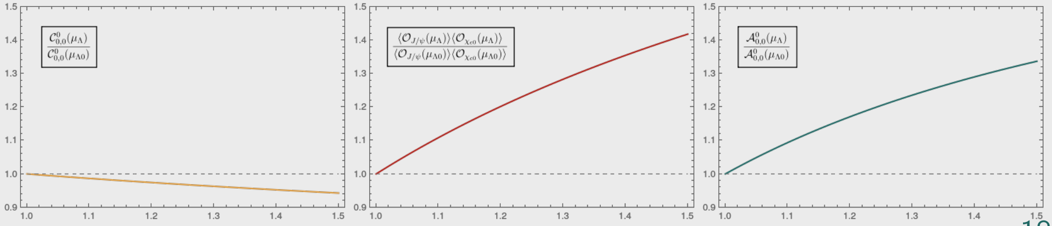
μ_Λ Evolution

- Both SDCs and long-distance matrix elements(LDME) are dependent on factorization scale μ_Λ.

$$\frac{\mathrm{d}\ln\langle\mathcal{O}(\mu_\Lambda)\rangle}{\mathrm{d}\ln\mu_\Lambda^2} = -\left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^2\gamma^{(2)} + \mathcal{O}\left(\alpha_s^3\right) \Rightarrow \frac{\langle\mathcal{O}(\mu_\Lambda)\rangle}{\langle\mathcal{O}(\mu_{\Lambda 0})\rangle} = \exp\left\{\frac{4\gamma^{(2)}}{\beta_0}\left[\frac{\alpha_s(\mu_\Lambda)}{\pi} - \frac{\alpha_s(\mu_{\Lambda 0})}{\pi}\right]\right\}$$

- The scale dependence is expected to be cancelled in the product.
- However, at a fixed order of α_s, the combined result may still suffer from severe μ_Λ dependence.

E.g. $\mathcal{A}^0_{0,0}[\gamma^* \to J/\psi(0) + \chi_{c0}(0)] = \mathcal{C}^0_{0,0}(\mu_\Lambda)\frac{\langle\mathcal{O}_{J/\psi}(\mu_\Lambda)\rangle\langle\mathcal{O}_{\chi_{c0}}(\mu_\Lambda)\rangle}{m_c^3}, \quad \mu_{\Lambda 0} = 1\,\mathrm{GeV} \sim m_c v$



Long-Distance Matrix Elements(LDMEs)

- The **NRQCD LDMEs** should be calculated in lattice QCD in principle since they are non-perturbative.
- In phenomenological analysis, the long-distance NRQCD matrix elements are often approximated by **the radial Schrödinger wave functions at the origin (J/ψ) and the first derivative of the P-wave radial wave functions at the origin (χ_{cJ})**:

$$\langle J/\psi|\psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}_{J/\psi}\chi(\mu_\Lambda)|0\rangle \approx \sqrt{\frac{N_c}{2\pi}}\overline{R_{J/\psi}}(\mu_\Lambda), \quad \left|R_{J/\psi}\right|^2 = 0.81\,\mathrm{GeV}^3(\text{B-T})$$

$$\langle \chi_{cJ}|\psi^\dagger \mathcal{K}_{^3P_J}\chi(\mu_\Lambda)|0\rangle \approx \sqrt{\frac{3N_c}{2\pi}}\overline{R'_{\chi_{cJ}}}(\mu_\Lambda), \quad \left|R'_{\chi_{cJ}}\right|^2 = 0.075\,\mathrm{GeV}^5(\text{B-T})$$

- We tacitly assume $\overline{R'_{\chi_{c0}}} \approx \overline{R'_{\chi_{c1}}} \approx \overline{R'_{\chi_{c2}}}$ by appealing to approximate **heavy quark spin symmetry**.

Polarized Cross Sections $\sigma^{(\lambda_1,\lambda_2)}$ [fb]

Central values are obtained by setting $\alpha(\sqrt{s}) = 1/130.9$, $m_c = 1.5$ GeV, $\mu_R = \sqrt{s}/2$. First error is estimated by setting $m_c = 1.3$ and 1.7 GeV, while second error is given by varying $\mu_R = 2m_c \sim \sqrt{s}$.

		$\sigma^{(0,0)}$	$\sigma^{(1,0)}$	$\sigma^{(0,1)}(\times 10^{-1})$	$\sigma^{(1,1)}(\times 10^{-2})$	$\sigma^{(1,2)}(\times 10^{-3})$
$J/\psi + \chi_{c0}$	LO	$1.10^{+0.59+0.51}_{-0.35-0.32}$	$1.85^{+0.94+0.85}_{-0.62-0.54}$	–	–	–
	NLO	$2.03^{+1.26+0.51}_{-0.71-0.40}$	$3.59^{+2.27+0.97}_{-1.36-0.74}$	–	–	–
	NNLO	$2.12^{+1.34+0.07}_{-0.76-0.15}$	$4.09^{+2.89+0.36}_{-1.64-0.43}$	–	–	–
$J/\psi + \chi_{c1}$	LO	–	$0.00116^{+0.00022+0.00053}_{-0.00024-0.00034}$	$3.69^{+2.57+1.69}_{-1.58-1.09}$	$3.31^{+0.77+1.52}_{-0.76-0.97}$	–
	NLO	–	$0.0374^{+0.0307+0.0468}_{-0.0160-0.0199}$	$4.86^{+4.23+0.56}_{-2.32-0.64}$	$2.94^{+0.94+0.009}_{-0.81-0.15}$	–
	NNLO	–	$0.112^{+0.101+0.086}_{-0.051-0.047}$	$4.12^{+4.24+0.14}_{-2.14-0.49}$	$1.36^{+0.59+0.52}_{-0.46-0.70}$	–
$J/\psi + \chi_{c2}$	LO	$0.430^{+0.539+0.197}_{-0.245-0.126}$	$0.267^{+0.191+0.122}_{-0.117-0.078}$	$0.639^{+0.587+0.293}_{-0.336-0.188}$	$3.31^{+0.77+1.52}_{-0.76-0.97}$	$2.31^{+0.45+1.06}_{-0.48-0.68}$
	NLO	$0.452^{+0.610+0.020}_{-0.266-0.040}$	$0.328^{+0.269+0.030}_{-0.153-0.039}$	$0.808^{+0.895+0.079}_{-0.458-0.10}$	$4.04^{+1.10+0.42}_{-1.02-0.49}$	$2.71^{+0.58+0.28}_{-0.61-0.32}$
	NNLO	$0.264^{+0.330+0.060}_{-0.151-0.099}$	$0.236^{+0.199+0.026}_{-0.111-0.055}$	$0.806^{+0.951+0.010}_{-0.461-0.026}$	$2.79^{+0.67+0.31}_{-0.66-0.58}$	$2.39^{+0.47+0.04}_{-0.42-0.17}$

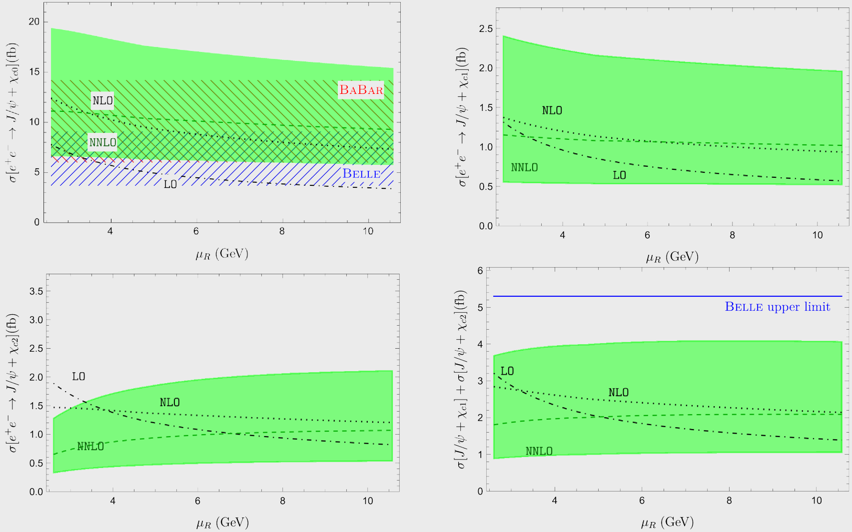
Recall $\mathcal{C}^{1(\text{tree})}_{1,0} = -\sqrt{6}r$

Unpolarized Cross Sections

- Comparison between our finest predictions to the unpolarized cross sections and the measurements in two B factories (in units of fb).
- $\left|R_{\psi(2S)}(0)\right|^2 = 0.529\,\mathrm{GeV}^3$

	LO	NLO	NNLO	Belle $\sigma\times\mathcal{B}_{>2(0)}$	BaBar $\sigma\times\mathcal{B}_{>2}$
$\sigma(J/\psi+\chi_{c0})$	$4.80^{+2.47+2.20}_{-1.58-1.41}$	$9.20^{+5.81+2.45}_{-3.43-1.87}$	$10.3^{+7.1+0.8}_{-4.0-1.0}$	$6.4\pm1.7\pm1.0$	$10.3\pm2.5^{+1.4}_{-1.8}$
$\sigma(J/\psi+\chi_{c1})$	$0.807^{+0.528+0.370}_{-0.331-0.237}$	$1.11^{+0.93+0.21}_{-0.51-0.17}$	$1.07^{+1.06+0.06}_{-0.54-0.06}$	–	–
$\sigma(J/\psi+\chi_{c2})$	$1.16^{+1.05+0.53}_{-0.56-0.34}$	$1.36^{+1.35+0.11}_{-0.68-0.15}$	$0.958^{+0.931+0.113}_{-0.478-0.220}$	–	–
$\sigma(J/\psi+\chi_{c1})+\sigma(J/\psi+\chi_{c2})$	$1.97^{+1.58+0.90}_{-0.89-0.58}$	$2.46^{+2.27+0.31}_{-1.20-0.32}$	$2.03^{+1.99+0.06}_{-1.02-0.16}$	< 5.3 at 90% C.L.	–
$\sigma(\psi(2S)+\chi_{c0})$	$3.13^{+1.61+1.44}_{-1.03-0.92}$	$6.01^{+3.80+1.60}_{-2.24-1.22}$	$6.72^{+4.65+0.51}_{-2.64-0.66}$	$12.5\pm3.8\pm3.1$	–
$\sigma(\psi(2S)+\chi_{c1})$	$0.527^{+0.345+0.241}_{-0.216-0.155}$	$0.722^{+0.605+0.134}_{-0.335-0.112}$	$0.702^{+0.694+0.040}_{-0.352-0.037}$	–	–
$\sigma(\psi(2S)+\chi_{c2})$	$0.759^{+0.688+0.348}_{-0.366-0.223}$	$0.886^{+0.880+0.069}_{-0.445-0.097}$	$0.625^{+0.608+0.073}_{-0.312-0.144}$	–	–
$\sigma(\psi(2S)+\chi_{c1})+\sigma(\psi(2S)+\chi_{c2})$	$1.29^{+1.03+0.59}_{-0.58-0.38}$	$1.61^{+1.48+0.20}_{-0.78-0.21}$	$1.33^{+1.30+0.04}_{-0.66-0.10}$	< 8.6 at 90% C.L.	–

μ_R Dependence



Angular Distribution Parameter α_J

NRQCD predictions for the angular distribution parameter α_J (defined before) at various perturbative accuracy.

	LO	NLO	NNLO	Belle
$J/\psi + \chi_{c0}$	$0.252^{+0.0005}_{-0.014}$	$0.278^{+0.003+0.008}_{-0.020-0.006}$	$0.318^{+0.020+0.023}_{-0.032-0.016}$	$-1.01^{+0.38}_{-0.33}$
$J/\psi + \chi_{c1}$	$0.697^{+0.073}_{-0.083}$	$0.798^{+0.055+0.033}_{-0.066-0.024}$	$0.901^{+0.027+0.053}_{-0.032-0.044}$	—
$J/\psi + \chi_{c2}$	$-0.197^{+0.070}_{-0.090}$	$-0.128^{+0.057+0.018}_{-0.077-0.014}$	$-0.00170^{+0.02172+0.11780}_{-0.03785-0.05816}$	—

Recall

$$\alpha_0 = -\frac{|\mathcal{A}^0_{0,0}|^2 - |\mathcal{A}^0_{1,0}|^2}{|\mathcal{A}^0_{0,0}|^2 + |\mathcal{A}^0_{1,0}|^2}$$

Summary

- We compute the NNLO perturbative corrections to $e^+e^- \to J/\psi + \chi_c$ at B factories within NRQCD. With the aid of AMF method, the SDCs are presented with high numerical accuracy.
- At $\mathcal{O}(\alpha_s^2)$, the μ_R dependence for $\sigma(J/\psi + \chi_{c0,1})$ are significantly reduced, while get slightly worsen for $\sigma(J/\psi + \chi_{c2})$.
- NNLO predictions of $\sigma(J/\psi + \chi_{cJ})$ are consistent with experimental measurements.
- There are also severe discrepancy between the most refined NRQCD predictions and the measurements.
- We hope that future `Belle 2` experiment will shed crucial light on the mechanism of exclusive double charmonium production and the applicability of NRQCD factorization.

Thank you

NRQCD Lagrangian

$$\begin{aligned}
\mathcal{L}_{\text{NRQCD}} &= \mathcal{L}_{\text{light}} + \mathcal{L}_{\text{heavy}} + \delta\mathcal{L} \\
\mathcal{L}_{\text{light}} &= \frac{1}{2}\operatorname{tr} G_{\mu\nu}^2 + \sum \bar{q}\,\mathrm{i}\not{D} q, \\
\mathcal{L}_{\text{heavy}} &= \psi^\dagger \left(\mathrm{i}D_t + \frac{\mathbf{D}^2}{2M}\right)\psi + \chi^\dagger\left(\mathrm{i}D_t - \frac{\mathbf{D}^2}{2M}\right)\chi \\
\delta\mathcal{L}_{\text{bilinear}} &= \frac{c_1}{8M^3}\psi^\dagger\left(\mathbf{D}^2\right)^2\psi + \frac{c_2}{8M^2}\psi^\dagger\left(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D}\right)\psi \\
&\quad + \frac{c_3}{8M^2}\psi^\dagger\left(\mathrm{i}\mathbf{D}\times g\mathbf{E} - \mathrm{i}g\mathbf{E}\times\mathbf{D}\right)\psi + \frac{c_4}{2M}\psi^\dagger\left(g\mathbf{B}\cdot\boldsymbol{\sigma}\right)\psi \\
&\quad + \text{charge conjugation terms}
\end{aligned}$$

ψ and χ: Pauli spinor fields; $\mathbf{E}, \mathbf{B}$: QCD field strengths

Helicity Amplitudes

The pro-
cess can be decomposed to the decay of a timelike photon and then expressed as the helicity amplitudes:

$$\frac{d\sigma\left[e^+e^- \to J/\psi(\lambda_1)+\chi_{cJ}(\lambda_2)\right]}{d\cos\theta} = \frac{2\pi\alpha}{s^{3/2}} \sum_{S_z=\pm 1} \frac{d\Gamma\left[\gamma^*(S_z) \to J/\psi(\lambda_1)+\chi_{cJ}(\lambda_2)\right]}{d\cos\theta}$$

$$= \frac{2\pi\alpha}{s^{3/2}} \sum_{S_z=\pm 1} \frac{|\mathbf{P}|}{16\pi s} \left|d^1_{S_z,\lambda}(\theta)\right|^2 \left|\mathcal{A}^J_{\lambda_1,\lambda_2}\right|^2$$

$$= = \frac{\alpha}{8s^2}\left(\frac{|\mathbf{P}|}{\sqrt{s}}\right)\left|\mathcal{A}^J_{\lambda_1,\lambda_2}\right|^2 \times \begin{cases} \dfrac{1+\cos^2\theta}{2}, & \lambda = \pm 1, \\ 1-\cos^2\theta, & \lambda = 0, \end{cases}$$

S_z:magnetic number of the photon, λ_1, λ_2: helicities of $J/\psi, \chi_{cJ}$

$|\mathbf{P}|$: magnitude of the 3-momentum of J/ψ (χ_{cJ}) in the CM frame

Properties of Helicity Amplitudes

- Parity invariance:

$$\mathcal{A}^J_{\lambda_1,\lambda_2} = (-)^J \mathcal{A}^J_{-\lambda_1,-\lambda_2}.$$

- Helicity selection rule:

$$A^J_{\lambda_1,\lambda_2} \propto s^{-\frac{1}{2}(1+|\lambda_1+\lambda_2|)}$$
$$\Rightarrow \sigma(J/\psi(\lambda_1) + \chi_{cJ}(\lambda_2)) \propto s^{-3-|\lambda_1+\lambda_2|}$$

Expression of A_J and α_J

We can explicitly write A_J and α_J as:

$$A_0 = \frac{\alpha}{8s^2}\left(\frac{|\mathbf{P}|}{\sqrt{s}}\right)\{|\mathcal{A}^0_{0,0}|^2 + |\mathcal{A}^0_{1,0}|^2\}, \qquad \alpha_0 = -\frac{|\mathcal{A}^0_{0,0}|^2 - |\mathcal{A}^0_{1,0}|^2}{|\mathcal{A}^0_{0,0}|^2 + |\mathcal{A}^0_{1,0}|^2}$$

$$A_1 = \frac{\alpha}{8s^2}\left(\frac{|\mathbf{P}|}{\sqrt{s}}\right)\{|\mathcal{A}^1_{1,0}|^2 + |\mathcal{A}^1_{0,1}|^2 + 2|\mathcal{A}^1_{1,1}|^2\},$$

$$\alpha_1 = \frac{|\mathcal{A}^1_{1,0}|^2 + |\mathcal{A}^1_{0,1}|^2 - 2|\mathcal{A}^1_{1,1}|^2}{|\mathcal{A}^1_{1,0}|^2 + |\mathcal{A}^1_{0,1}|^2 + 2|\mathcal{A}^1_{1,1}|^2}$$

$$A_2 = \frac{\alpha}{8s^2}\left(\frac{|\mathbf{P}|}{\sqrt{s}}\right)\{|\mathcal{A}^2_{0,0}|^2 + |\mathcal{A}^2_{1,0}|^2 + |\mathcal{A}^2_{0,1}|^2 + 2|\mathcal{A}^2_{1,1}|^2 + |\mathcal{A}^2_{1,2}|^2\}$$

$$\alpha_2 = -\frac{|\mathcal{A}^2_{0,0}|^2 - |\mathcal{A}^2_{1,0}|^2 - |\mathcal{A}^2_{0,1}|^2 + 2|\mathcal{A}^2_{1,1}|^2 - |\mathcal{A}^2_{1,2}|^2}{|\mathcal{A}^2_{0,0}|^2 + |\mathcal{A}^2_{1,0}|^2 + |\mathcal{A}^2_{0,1}|^2 + 2|\mathcal{A}^2_{1,1}|^2 + |\mathcal{A}^2_{1,2}|^2}$$

Total Cross Section

Integrating the differential decay rate over the polar angle, the total unpolarized cross sections read:

$$\sigma(J/\psi+\chi_{c0}) = \frac{\alpha}{6s^2}\frac{|\mathbf{P}|}{\sqrt{s}}\left(|\mathcal{A}^0_{0,0}|^2+2|\mathcal{A}^0_{1,0}|^2\right),$$

$$\sigma(J/\psi+\chi_{c1}) = \frac{\alpha}{6s^2}\frac{|\mathbf{P}|}{\sqrt{s}}\left(2|\mathcal{A}^1_{1,0}|^2+2|\mathcal{A}^1_{0,1}|^2+2|\mathcal{A}^1_{1,1}|^2\right),$$

$$\sigma(J/\psi+\chi_{c2}) = \frac{\alpha}{6s^2}\frac{|\mathbf{P}|}{\sqrt{s}}\left(|\mathcal{A}^2_{0,0}|^2+2|\mathcal{A}^2_{1,0}|^2+2|\mathcal{A}^2_{0,1}|^2+2|\mathcal{A}^2_{1,1}|^2+2|\mathcal{A}^2_{1,2}|^2\right).$$

Calculating SDCs

- We use the standard covariant projection methods.

$$v\bar{u} \to \Pi$$

- The relativistically normalized color-singlet and spin-triplet projectors for $J/\psi(c(\frac{P_1}{2})\bar{c}(\frac{P_1}{2}))$ and $\chi_{cJ}(c(p)\bar{c}(\bar{p}))$ read:

$$\Pi_{10}^{\mu} = \frac{1}{\sqrt{2}}\gamma^{\mu}(\frac{\not{P}_1}{2} + m_c) \otimes \frac{\mathbf{1}_c}{\sqrt{N_c}},$$

$$\Pi_{11}^{\nu} = \frac{-1}{8\sqrt{2}m_c^2}(\not{\bar{p}} - m_c)\gamma^{\nu}(\not{P}_2 + 2m_c)(\not{p} + m_c) \otimes \frac{\mathbf{1}_c}{\sqrt{N_c}}.$$

$$p = \frac{P_2}{2} + q, \quad \bar{p} = \frac{P_2}{2} - q,$$

Calculating SDCs

- The χ_{cJ} states can be read off by projecting out the diagonal, antisymmetric and symmetric traceless components w.r.t. the vector indeces for spin and orbital momentum:

$$\epsilon^{*}_{\mu,J/\psi}\mathcal{J}^{J}_{\nu\alpha}\frac{\mathrm{d}}{\mathrm{d}q_\alpha}\mathrm{tr}[\Pi^{\nu}_{11}\mathcal{A}\Pi^{\mu}_{10}]\Big|_{q=0}, \quad \eta^{\mu\nu}(P) := -g^{\mu\nu} + \frac{P^\mu P^\nu}{P\cdot P}$$

$$\mathcal{J}^{0}_{\mu\nu} = \frac{1}{\sqrt{3}}\eta_{\mu\nu}(P), \quad \mathcal{J}^{1}_{\mu\nu}(\epsilon) = -\frac{\mathrm{i}}{\sqrt{2P^2}}\varepsilon_{\mu\nu\rho\sigma}\epsilon^{\rho}P^{\sigma},$$

$$\mathcal{J}^{2}_{\mu\nu}(\epsilon) = \epsilon^{\rho\sigma}\left\{\frac{1}{2}\left[\eta_{\mu\rho}(P)\eta_{\nu\sigma}(P) + \eta_{\mu\sigma}(P)\eta_{\nu\rho}(P)\right] - \frac{1}{3}\eta_{\mu\nu}(P)\eta_{\rho\sigma}(P)\right\}$$

  where $\mathcal{A}$ represents the quark-level amplitude with the external quark spinors truncated.
- We use the method of region to extract the hard contributions directly.

Charm Mass

Various values of the heavy quark pole mass are adopted for charmonium production in literature.

m_c(GeV)	
1.5	Bodwin, et al., *Fragmentation contributions to hadroproduction of prompt J/ψ, χ_{cJ}, and $\psi(2S)$ states* He, et al., *Inclusive J/ψ and η_c production in Υ decay at $\mathcal{O}(\alpha_s^5)$ in non-relativistic QCD factorization* Butenschoen, et al., *Next-to-leading-order tests of NRQCD factorization with J/ψ yield and polarization* Butenschoen, et al., *η_c production at the LHC challenges nonrelativistic-QCD factorization* Kniehl, et al., *Complete Nonrelativistic-QCD Prediction for Prompt Double J/ψ Hadroproduction* Wang, et al., *Polarization for Prompt J/ψ and $\psi(2S)$ Production at the Tevatron and LHC*
1.5 ± 0.1	Ma, et al., *$J/\psi(\psi')$ production at the Tevatron and LHC at $\mathcal{O}(\alpha_s^4 v^4)$ in nonrelativistic QCD*
$1.4 - 1.5$	Chen, et al., *NNLO QCD corrections to $\Upsilon + \eta_c(\eta_b)$ exclusive production in electron-positron collision*
1.4 ± 0.2	Bodwin, et al., *Resummation of Relativistic Corrections to $e^+e^- \to J/\psi + \eta_c$*
1.483 ± 0.029	Bodwin, et al., *Relativistic corrections to Higgs boson decays to quarkonia*
1.67 ± 0.07	Bodwin, et al., *Higgs boson decays to quarkonia and the $H\bar{c}c$ coupling*
1.3	Kniehl, et al., *Inclusive J/ψ and $\psi(2S)$ production from b-hadron decay in p anti-p and pp collisions*

LDMEs

- The values of $R_{J/\psi}(0)$ in literature.

	Cornell	Power Law	Log	Coul. + power	pNRQCD
$\lvert R_{J/\psi}(0)\rvert^2\,(\mathrm{GeV}^3)$	1.454	0.999	0.815	$0.610 \sim 1.850$	1.271
	Screened	Lattice	BT	Modified NR	Semi-relativistic
$\lvert R_{J/\psi}(0)\rvert^2\,(\mathrm{GeV}^3)$	1.19	1.1184	0.810	1.9767	0.478

- The values of $R'_{\chi_c}(0)$ in literature.

	Cornell	Power Law	Log	Coul. + power	pNRQCD	BT	Modified NR
$\lvert R'_{\chi_c}(0)\rvert^2$ (GeV^5)	0.131	0.125	0.078	$0.047 \sim 0.839$	$0.1124(\chi_{c0})$ $0.1174(\chi_{c1})$ $0.1189(\chi_{c2})$	0.075	$0.4074(\chi_{c0})$ $0.3334(\chi_{c1})$ $0.3021(\chi_{c2})$