SHELL-MODEL STUDY OF NUCLEAR ISOMERS IN SN AND PB REGION

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"The neutron-rich nuclei in the vicinity of ^{132}Sn and ^{208}Pb regions exhibit an abundance of nuclear isomers. The existence of the different isomers alludes to the dominance of proton or neutron excitations for low-lying states. Thus, the observed structure and transition probabilities can be easily described in terms of the seniority scheme for the low-lying structure near Sn and Pb region. In this meeting we are presenting our recent results of nuclear isomers for these two regions using large-scale shell model."

Sn-Region

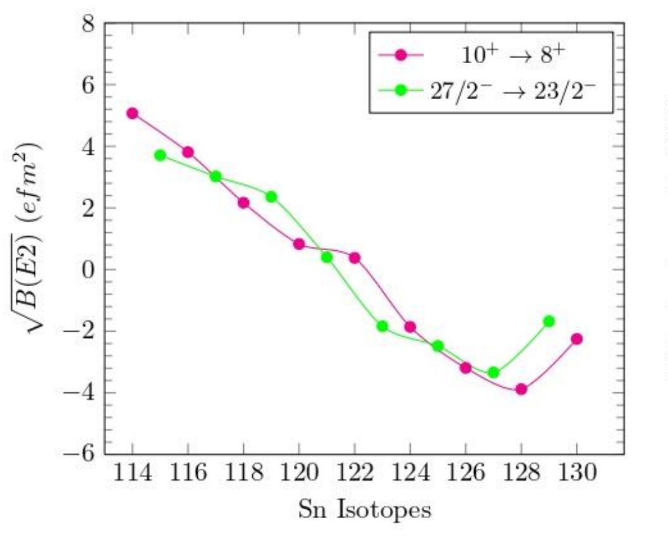
(A=119-126 Sn isotopes)

With the shell-model configurations of different high-spin states, we have analyzed different isomeric states in these nuclei, which can be described in terms of several broken neutron pairs occupying the $h_{11/2}$ orbital. The high-spin states of $^{119-126}$Sn isotopes with different seniority (v), including v = 4, 5, 6, and 7, have been interpreted.

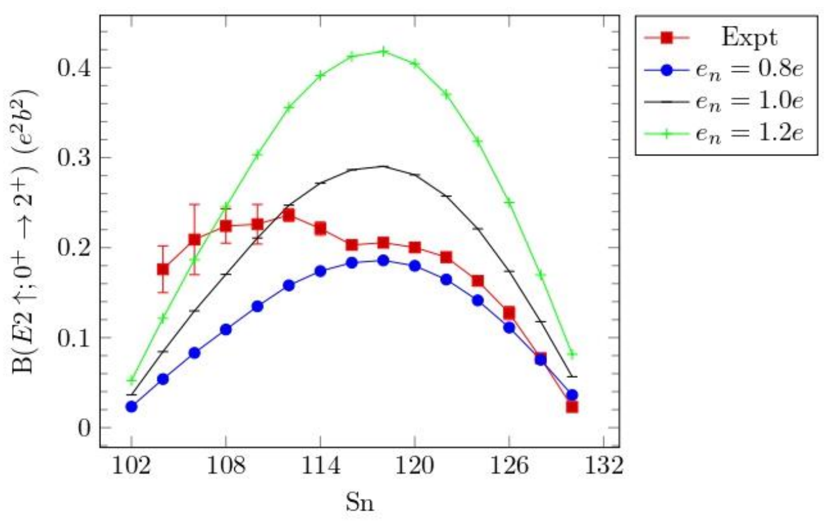
Spin	Seniority	^{120}Sn	^{122}Sn	^{124}Sn	^{126}Sn
10^+	$v=2\,(h^2_{11/2})$	$g^6_{7/2}d^6_{5/2}d^2_{3/2}h^6_{11/2}$ [10.13%]	$g^8_{7/2}d^6_{5/2}d^2_{3/2}h^6_{11/2}$ [15.07%]	$g^8_{7/2}d^6_{5/2}d^2_{3/2}s^2_{1/2}h^6_{11/2}$ [18.64%]	$g^8_{7/2}d^6_{5/2}d^2_{3/2}s^2_{1/2}h^8_{11/2}$ [37.24%]
5^-	$v=2\,(h^1_{11/2}s^1_{1/2})$	$g^8_{7/2}d^6_{5/2}s^1_{1/2}h^5_{11/2}$ [11.79%]	$g^8_{7/2}d^6_{5/2}d^2_{3/2}s^1_{1/2}h^5_{11/2}$ [12.26%]	$g^8_{7/2}d^6_{5/2}d^2_{3/2}s^1_{1/2}h^7_{11/2}$ [23.07%]	$g^8_{7/2}d^6_{5/2}d^2_{3/2}s^1_{1/2}h^9_{11/2}$ [27.67%]
7^-	$v=2\,(h^1_{11/2}d^1_{3/2})$	$g^8_{7/2}d^6_{5/2}d^1_{3/2}h^5_{11/2}$ [17.00%]	$g^8_{7/2}d^6_{5/2}d^1_{3/2}h^7_{11/2}$ [16.24%]	$g^8_{7/2}d^6_{5/2}d^1_{3/2}s^2_{1/2}h^7_{11/2}$ [22.14%]	$g^8_{7/2}d^6_{5/2}d^3_{3/2}s^2_{1/2}h^7_{11/2}$ [24.63%]
15^-	$v=4\,(h^3_{11/2}d^1_{3/2})$	$g^8_{7/2}d^6_{5/2}d^1_{3/2}h^5_{11/2}$ [27.60%]	$g^8_{7/2}d^6_{5/2}d^1_{3/2}h^7_{11/2}$ [23.21%]	$g^8_{7/2}d^6_{5/2}d^1_{3/2}s^2_{1/2}h^7_{11/2}$ [28.76%]	$g^8_{7/2}d^6_{5/2}d^3_{3/2}s^2_{1/2}h^7_{11/2}$ [30.86%]

Spin	Seniority	^{119}Sn	^{121}Sn	^{123}Sn	^{125}Sn
$27/2^-$	$v=3\,(h^3_{11/2})$	$g^8_{7/2}d^6_{5/2}h^5_{11/2}$ [13.80%]	$g^8_{7/2}d^6_{5/2}d^2_{3/2}h^5_{11/2}$ [14.26%]	$g^8_{7/2}d^6_{5/2}d^2_{3/2}h^7_{11/2}$ [19.86%]	$g^8_{7/2}d^6_{5/2}d^2_{3/2}s^2_{1/2}h^7_{11/2}$ [34.55%]
$19/2^+$	$v=3\,(h^2_{11/2}s^1_{1/2})$	$g^8_{7/2}d^6_{5/2}s^1_{1/2}h^4_{11/2}$ [11.77%]	$g^8_{7/2}d^6_{5/2}s^1_{1/2}h^6_{11/2}$ [13.81%]	$g^8_{7/2}d^6_{5/2}d^2_{3/2}s^1_{1/2}h^6_{11/2}$ [17.74%]	$g^8_{7/2}d^6_{5/2}d^2_{3/2}s^1_{1/2}h^8_{11/2}$ [31.15%]
$23/2^+$	$v=3\,(h^2_{11/2}d^1_{3/2})$	$g^8_{7/2}d^6_{5/2}d^1_{3/2}h^4_{11/2}$ [18.49%]	$g^8_{7/2}d^6_{5/2}d^1_{3/2}h^6_{11/2}$ [21.77%]	$g^8_{7/2}d^6_{5/2}d^1_{3/2}s^2_{1/2}h^6_{11/2}$ [20.19%]	$g^8_{7/2}d^6_{5/2}d^1_{3/2}s^2_{1/2}h^8_{11/2}$ [25.57%]
$35/2^+$	$v=5\,(h^4_{11/2}d^1_{3/2})$			$g^8_{7/2}d^6_{5/2}d^3_{3/2}s^2_{1/2}h^4_{11/2}$ [2.50%]	

The probability of different configurations is not so large, but it increases with neutron number for heavier Sn isotopes.



Amplitudes of the reduced transition probabilities for E2 isomeric decay in Sn isotopes corresponding to isomeric states 10^+ and $27/2^-$



Comparison for $B(E2; 0^+ \rightarrow 2^+)$ values for effective charge $e_n = 0.8e$, $e_n = 1.0e$, $e_n = 1.2e$

Probability	Wave functions
8.4%	$\nu(g^8_{7/2}d^6_{5/2}d^2_{3/2}h^4_{11/2})$
5.9%	$\nu(g^8_{7/2}d^6_{5/2}s^2_{1/2}h^4_{11/2})$
9.6%	$\nu(g^8_{7/2}d^6_{5/2}h^6_{11/2})$
3.1%	$\nu(g^8_{7/2}d^4_{5/2}d^2_{3/2}\ s^2_{1/2}h^4_{11/2})$
6.6%	$\nu(g^8_{7/2}d^4_{5/2}d^2_{3/2}h^6_{11/2})$
4.0%	$\nu(g^8_{7/2}d^4_{5/2}s^2_{1/2}h^6_{11/2})$
2.4%	$\nu(g^8_{7/2}d^4_{5/2}h^8_{11/2})$
1.9%	$\nu(g^8_{7/2}d^2_{5/2}d^2_{3/2}s^2_{1/2}h^6_{11/2})$
1.8%	$\nu(g^6_{7/2}d^6_{5/2}d^4_{3/2}h^4_{11/2})$
4.7%	$\nu(g^6_{7/2}d^6_{5/2}d^2_{3/2}s^2_{1/2}h^4_{11/2})$
10.1%	$\nu(g^6_{7/2}d^6_{5/2}d^2_{3/2}h^6_{11/2})$
4.9%	$\nu(g^6_{7/2}d^6_{5/2}s^2_{1/2}h^6_{11/2})$
2.6%	$\nu(g^6_{7/2}d^6_{5/2}h^8_{11/2})$
1.8%	$\nu(g^6_{7/2}d^4_{5/2}d^4_{3/2}h^6_{11/2})$
4.5%	$\nu(g^6_{7/2}d^4_{5/2}d^2_{3/2}s^2_{1/2}h^6_{11/2})$
2.9%	$\nu(g^6_{7/2}d^4_{5/2}d^2_{3/2}h^8_{11/2})$
1.4%	$\nu(g^6_{7/2}d^4_{5/2}s^2_{1/2}h^8_{11/2})$
1.3%	$\nu(g^4_{7/2}d^6_{5/2}d^2_{3/2}s^2_{1/2}h^6_{11/2})$

The percentage of the configurations increases by increasing the neutron number. This shows the dominant role of the $h_{11/2}$ orbital. The wave functions are very fragmented. The 10^+ states of all even isotopes and $27/2^-$ states of all odd isotopes are formed by breaking pairs in a pure $\nu(h_{11/2})$ orbital with v = 2 and v = 3, respectively. The $d_{3/2}$ and $s_{1/2}$ orbitals also participate in the formation of other isomeric states.

> The high-spin isomers in Sn isotopes are due to seniority (v) = 2, 3, 4, and 5. For the ^{120,122,124,126}Sn isotopes, the seniority of isomeric states 10^+, 5^-, and 7^- is two (v = 2); the seniority of isomeric state 15^- is four (v = 4); the seniority of the 19^- state is six (v = 6).

> For the ^{119,121,123,125}Sn isotopes, the seniority of isomeric states $19/2^+$, $23/2^+$, and $27/2^-$ is three (v = 3); the seniority of isomeric state $35/2^+$ is five (v = 5); the seniority of the $39/2^+$ state is seven (v = 7).

Pb-Region

(A=207-216 Rn isotopes)

Comprehensive shell-model calculations of $^{207-216}$Rn isotopes to cover nuclei below and above N = 126 shell gap. The isomeric states are described in terms of the shell-model configuration, half-life and seniority quantum number (v).

Nucleus	J^π	Seniority	Wave-function	Probability
^{207}Rn	$13/2^+_1$	$v=1$	$\nu(i_{13/2})^{-1}$	25.28%
^{209}Rn	$13/2^+_1$	$v=1$	$\nu(i_{13/2})^{-1}$	26.33%
	$29/2^-_1$	$v=3$	$\pi(h_{9/2})^4 \otimes \nu(f_{5/2})^{-1}$	73.43%
	$35/2^+_1$	$v=5$	$\pi(h_{9/2})^3(i_{13/2}) \otimes \nu(f_{5/2})^{-1}$ $\pi(h_{9/2})^3(i_{13/2}) \otimes \nu(p_{1/2})^{-1}$	58.53% 11.45%
	$41/2^-_1$	$v=5$	$\pi(h_{9/2})^2(i_{13/2})^2 \otimes \nu(f_{5/2})^{-1}$ $\pi(h_{9/2})^2(i_{13/2})^2 \otimes \nu(p_{1/2})^{-1}$	49.48% 17.27%
^{211}Rn	$17/2^-_1$	$v=3$	$\pi(h_{9/2})^4 \otimes \nu(p_{1/2})^{-1}$	69.04%
	$35/2^+_1$	$v=5$	$\pi(h_{9/2})^3(i_{13/2}) \otimes \nu(p_{1/2})^{-1}$	85.45%
	$43/2^-_1$	$v=5$	$\pi(h_{9/2})^2(i_{13/2})^2 \otimes \nu(f_{5/2})^{-1}$	91.30%
	$49/2^+_1$	$v=5$	$\pi(h_{9/2})^2(i_{13/2})^2 \otimes \nu(g_{9/2})$	61.78%
	$63/2^-_1$	$v=7$	$\pi(h_{9/2})^2(i_{13/2})^2\otimes \nu(g_{9/2})(i_{11/2})(f_{5/2})^{-1}$	88.04%
^{213}Rn	$15/2^-_1$	$v=1$	$\nu(j_{15/2})$	41.94%
	$21/2^+_1$	$v=3$	$\pi(h_{9/2})^4 \otimes \nu(g_{9/2})$	51.79%
	$25/2^+_1$	$v=3$	$\pi(h_{9/2})^4 \otimes \nu(g_{9/2})$	48.37%
	$31/2^-_1$	$v=3$	$\pi(h_{9/2})^3(i_{13/2}) \otimes \nu(g_{9/2})$	63.64%
	$37/2^+_1$	$v=5$	$\pi(h_{9/2})^3(f_{7/2}) \otimes \nu(g_{9/2})$	95.82%
	$43/2^-_1$	$v=5$	$\pi(h_{9/2})^3(i_{13/2}) \otimes \nu(g_{9/2})$	92.94%
	$49/2^+_1$	$v=5$	$\pi(h_{9/2})^2(i_{13/2})^2 \otimes \nu(g_{9/2})$	91.62%
	$55/2^+_1$	$v=5$	$\pi(h_{9/2})(i_{13/2})^3 \otimes \nu(j_{15/2})$	99.95%
^{208}Rn	8^+_1	$v=2$	$\pi(h_{9/2})^4$	38.74%
	10^-_1	$v=2$	$\pi(h_{9/2})^3(i_{13/2})$	32.88%
	16^-_1	$v=2$	$\pi(h_{9/2})^3(i_{13/2})$	43.65%
^{210}Rn	8^+_1	$v=2$	$\pi(h_{9/2})^4$	46.63%
	11^-_1	$v=2$	$\pi(h_{9/2})^3(i_{13/2})$	42.72%
	14^+_1	$v=4$	$\pi(h_{9/2})^3(f_{7/2})$	49.23%
	17^-_1	$v=4$	$\pi(h_{9/2})^3(i_{13/2})$	50.80%
	20^+_1	$v=4$	$\pi(h_{9/2})^2(i_{13/2})^2$	42.73%
^{212}Rn	6^+_1	$v=2$	$\pi(h_{9/2})^4$	67.34%
	8^+_1	$v=2$	$\pi(h_{9/2})^4$	68.42%
	14^+_1	$v=4$	$\pi(h_{9/2})^3(f_{7/2})$	98%
	17^-_1	$v=4$	$\pi(h_{9/2})^3(i_{13/2})$	99.59%
	22^+_1	$v=6$	$\pi(h_{9/2})^3(i_{13/2})\otimes \nu(p_{1/2})^{-1}(g_{9/2})$	80.48%
	25^-_1	$v=6$	$\pi(h_{9/2})^2(i_{13/2})^2\otimes \nu(p_{1/2})^{-1}(g_{9/2})$	73.45%
^{214}Rn	18^+_1	$v=4$	$\pi(h_{9/2})^3(i_{13/2}) \otimes \nu(g_{9/2})^2$	60%
	22^+_1	$v=4$	$\pi(h_{9/2})^3(i_{13/2})\otimes \nu(g_{9/2})(j_{15/2})$	83%

The orbitals $h_{9/2}$ and $i_{13/2}$ are responsible for the isomeric states in the Rn isotopes. The high-spin isomers in Rn isotopes are due to seniority (v) = 1, 2, 3, 4, 5, 6 and 7.

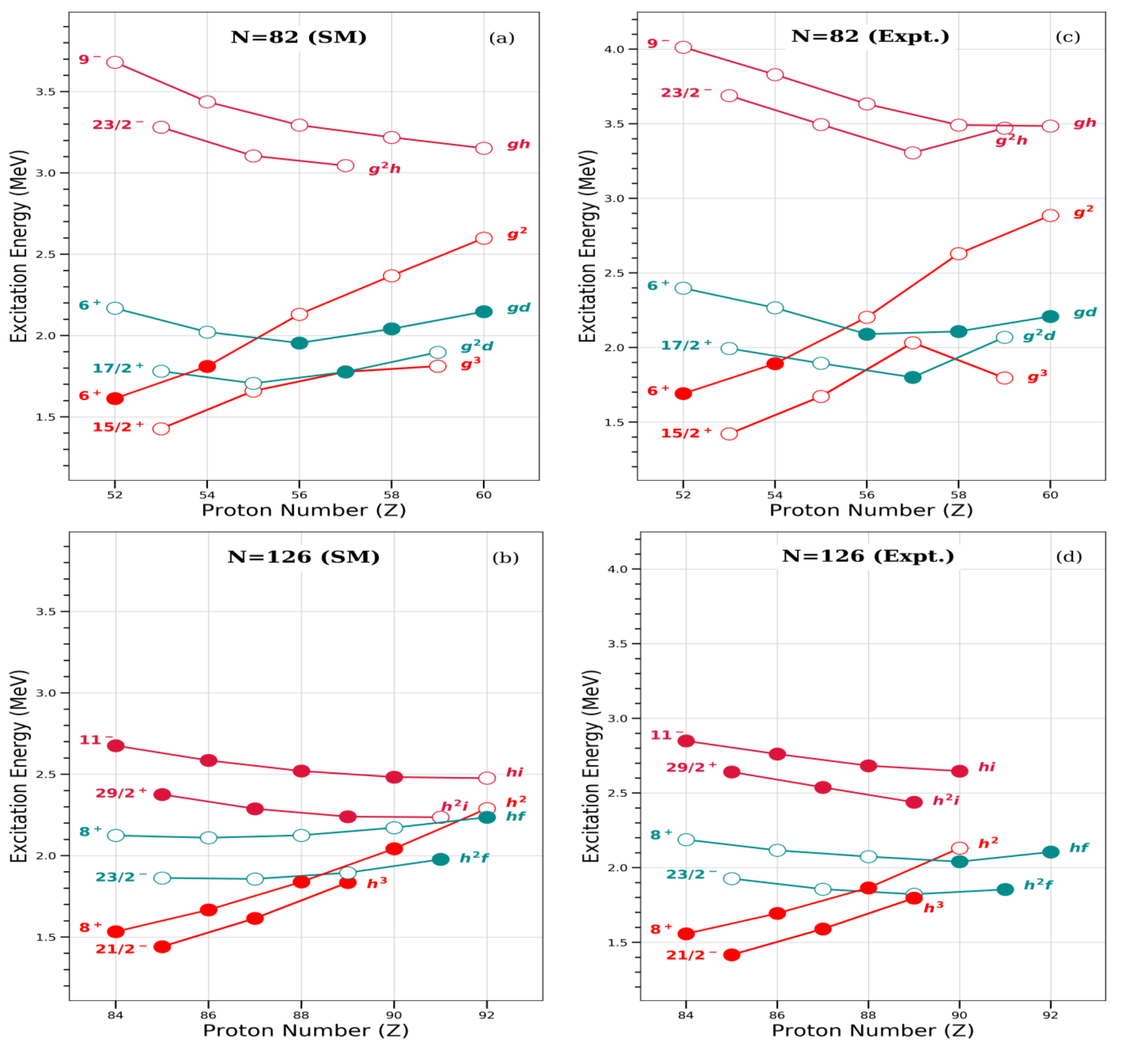
J^π	E_γ (MeV)	$B(E\lambda)$ or $B(M\lambda)$	$B(E\lambda)$ ($e^2\mathrm{fm}^{2\lambda}$)	$B(M\lambda)$ ($\mu_N^2\mathrm{fm}^{2\lambda-2}$)	Expt. $T_{1/2}$	SM $T_{1/2}$
^{208}Rn						
8^+_1	6×10^{-3}	$B(E2)$	32.90		487(12) ns	1076 ns
10^-_1	0.296	$B(M1)$		2×10^{-4}	11.8(7) ns	4.69 ns
16^-_1	0.143	$B(M1)$		0.0819	18.3(4) ns	0.03 ns
^{209}Rn						
$29/2^-_1$	2.29×10^{-3}	$B(M1)+B(E2)$	6×10^7	1.09×10^4	13.9(21) ns	55.55 ns
^{210}Rn						
8^+_1	0.019	$B(E2)$	2.55×10^4		644(40) ns	344 ns
14^+_1	0.274	$B(E2)$	1.82×10^{-1}		76(7) ns	1049 ns
23^+_1	0.843	$B(E2)$	1.14×10^{-2}		1.04(7) ns	0.61 ns
23^+_1	0.445	$B(E2)$	4.67×10^{-2}		1.04(7) ns	13.04 ns
^{211}Rn						
$17/2^-_1$	0.015	$B(E2)$	20.72		596 (28) ns	1660 ns
$35/2^+_1$	0.043	$B(E2)$	185.39		40.2(14) ns	44.74 ns
^{212}Rn						
6^+_1	0.093	$B(E2)$	53.91		118(14) ns	122 ns
8^+_1	8×10^{-3}	$B(E2)$	17.24		0.91(3) μs	2.04 μs
14^+_1	0.273	$B(E2)$	0.40		7.4(9) ns	800 ns
17^-_1	2×10^{-3}	$B(E2)$	187.04		28.9(14) ns	807 ns
^{213}Rn						
$15/2^-_1$	0.478	$B(M2)$		0.28×10^3	26(1) ns	4.84 ns
$15/2^-_1$	1.158	$B(E3)$	0.66×10^4		26(1) ns	64.79 ns
$21/2^+_1$	0.574	$B(E3)$	0.24×10^{-1}		29(2) ns	2.29 s
$21/2^+_1$	0.08	$B(E2)$	115.81		29(2) ns	61.81 ns
$31/2^-_1$	0.269	$B(E3)$	0.33×10^4		1.36(7) μs	1559 μs
$31/2^-_1$	0.168	$B(E3)$	0.17×10^5		1.36(7) μs	1326 μs
$37/2^+_1$	0.100	$B(E2)$	281.99		26(1) ns	122.6 ns
$37/2^+_1$	0.078	$B(E2)$	19.58		26(1) ns	369.6 ns
$49/2^+_1$	0.789	$B(E3)$	0.82×10^{-1}		12(1) ns	7.50×10^{-2} s
^{214}Rn						
18^+_1	0.114	$B(E2)$	5.7145		44(3) ns	894 ns

The half-lives of the isomeric states are well reproduced in comparison with the experimental data. The calculated *B(E2)* value supports the behavior of these isomeric.

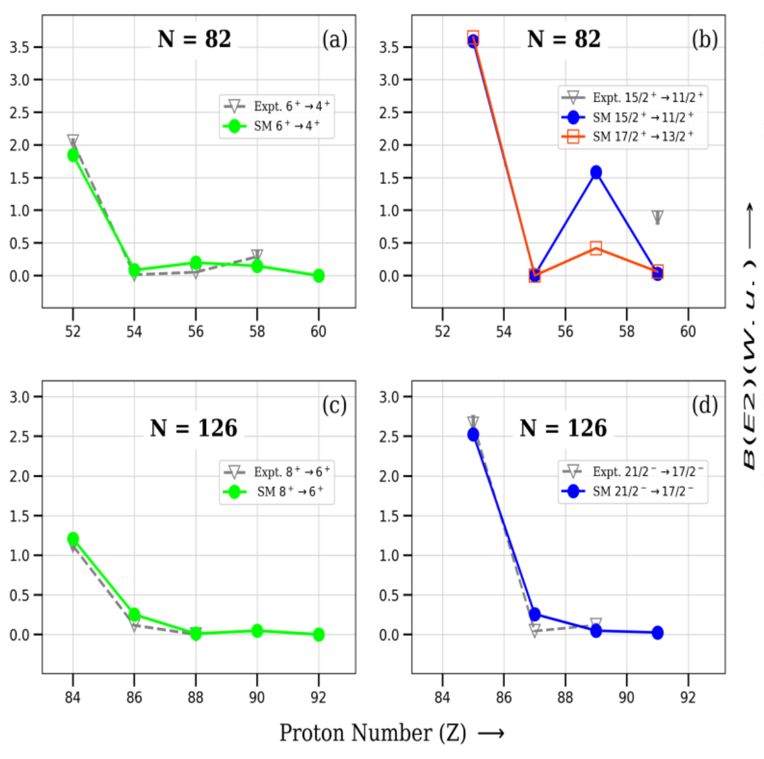
Similarity Between Sn And Pb Region

($52 \le Z \le 60$ and $84 \le Z \le 92$ isotones)

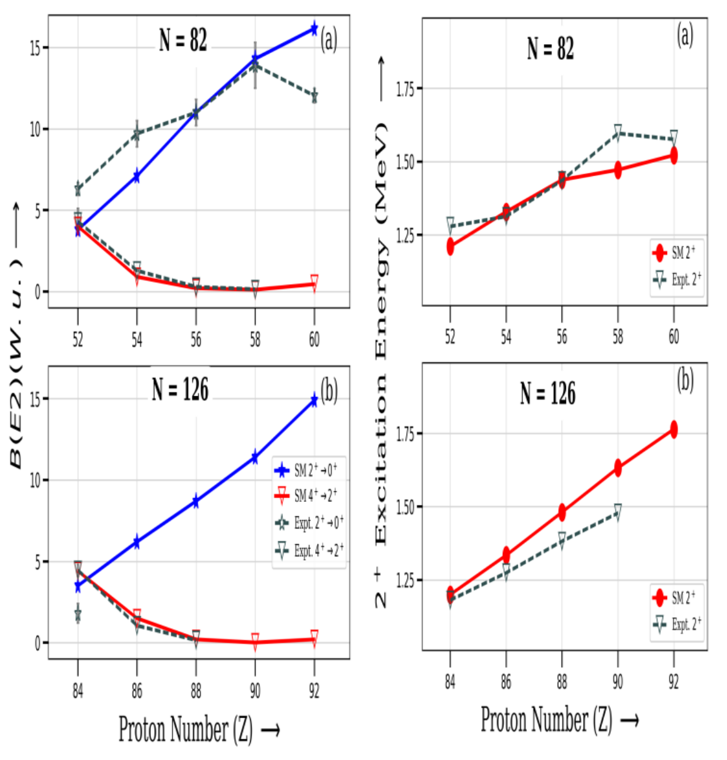
Study of similarity between the shell structures, using the strong resemblance between the high-*j* orbitals in the ^{132}Sn and ^{208}Pb regions for the fully-aligned states with one broken proton pair, in the N = 82 isotones, with the three orbitals above the Z = 50 gap, $\pi\, g_{7/2}$, $\pi\, d_{5/2}$, and $\pi\, h_{11/2}$, and in the N = 126 isotones, the three orbitals above the Z = 82 gap, $\pi\, h_{9/2}$, $\pi\, f_{7/2}$, and $\pi\, i_{13/2}$.



In these two regions the orbital angular momentum differ by one unit, with the same orientation of the intrinsic spin, $\pi\, g_{7/2} \rightarrow \pi\, h_{9/2}$, $\pi\, d_{5/2} \rightarrow \pi\, f_{7/2}$, and $\pi\, h_{11/2} \rightarrow \pi\, i_{13/2}$. Similar evolution can be seen for the same seniority v = 2 states in the two 6^+ states of the N = 82 isotones and the two 8^+ states of the N = 126 isotones. Similarly for the odd isotones with seniority v = 3 the behavior between the $15/2^+$ and $17/2^+$ states of the N = 82 isotones and the $21/2^-$ and $23/2^-$ states of the N = 126 isotones.



> *B(E2)* value decreases with the increase in proton number for both the N = 82 and 126 isotones.
> For the spins, $J \ge 2$ the decrement of *B(E2)* values away from the shell-closure is an outcome of the seniority scheme.
> The orbitals involved in the wave-functions of the initial and final states differ by one-unit angular momentum and the formation of the involved states in transition follows the same seniority scheme.
> The *B(E2)* transition rates for the same seniority states in the two different closed-shell regions N = 82 and 126 follow the same trend.



> The increment of the $B(E2$: $2^+ \rightarrow 0^+)$ transition probabilities as the proton number increases.
> A weak $B(E2$: $2^+ \rightarrow 0^+)$ transition rate is observed for the ^{134}Te in the Sn-region similar to ^{210}Po in the Pb-region.
> The analogous increase in the first 2^+ excitation energy is observed for N = 82 and 126 region as the proton number increases.
> The increase in $B(E2$: $2^+ \rightarrow 0^+)$ values is very rapid. In contrast, 2^+ excitation energy shows a gradual increment. This behavior indicates the emergence of collectivity

> In the N =82 region, we have mainly discussed properties of the 6^+ and $17/2^+$ isomers, while in N=126 region for 8^+, 11^-, $21/2^-$ and $29/2^+$ isomers.

> The 6^+ and 9^- states in the Sn region and their counterpart 8^+ and 11^- states in the Pb region show several similarities in the structural formation.

> Several isomeric states showing similar evolution in N=82 and N=126 isotones are due to the breaking of high-*j* nucleon pairs and well described in terms of seniority quantum number.

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