Signals of electrical conductivity

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Electrical conductivity from the hydrodynamic limit

- Assume a plasma under an external electromagnetic perturbation $A = (A^0, \vec{A})$

$$\Delta H(t) = -\int_{\vec{x}} [\delta A_0(t,\vec{x}) + \delta\mu(t,\vec{x})] n(t,\vec{x}) + \delta\vec{A}(t,\vec{x})\vec{J}(t,\vec{x})$$

$J^\mu = (n, \vec{J})$: charge density, electrical current
μ: chemical potential

- current in first order derivative expansion:

$$\tau u^\mu \partial_\mu J^\alpha + J^\alpha = n u^\alpha + \sigma \Delta^{\alpha\nu} E_\nu - D \Delta^{\alpha\nu} \partial_\nu n \tag{1}$$

$\Delta^{\alpha\beta}$: transverse projector
σ: electrical conductivity
τ: relaxation time
D: diffusion coefficient

- equations of motion for n and $\vec{J}$ given by $\partial^\mu n_\mu = 0$ and (1)

Electrical conductivity from the hydrodynamic limit

- in the linear response limit:

$$\delta\langle J^\mu(x)\rangle = \int_y G_R^{\mu\nu}(x-y)\,\delta A_\nu(y)$$

retarded propagator: $G_R^{\mu\nu} = i\theta(x^0 - y^0)\langle[J^\mu(x), J^\nu(y)]\rangle$

- derive spectral density $\rho = \rho^\mu_\mu = \mathrm{Im}\, G^\mu_{R,\mu}$

$$\rho(\omega, \mathrm{p}) = \frac{\sigma\omega(\omega^2 - \mathrm{p}^2)}{(\tau\omega^2 - D\mathrm{p}^2)^2 + \omega^2} + 2\frac{\sigma\omega}{\tau^2\omega^2 + 1} \qquad (2)$$

- to ensure relativistic causality from (2): $\tau > D$
- hydrodynamic approach only valid in the limit $\omega, |\mathrm{p}| \lesssim T_C$

Kubo relations

- spectral density

$$\rho(\omega, \mathrm{p}) = \frac{\sigma\omega(\omega^2 - \mathrm{p}^2)}{(\tau\omega^2 - D\mathrm{p}^2)^2 + \omega^2} + 2\frac{\sigma\omega}{\tau^2\omega^2 + 1}.$$

- Kubo relation for electrical conductivity

$$\lim_{\omega\to 0} \frac{1}{\omega}\rho(\omega, \mathrm{p})\Big|_{\mathrm{p}^2=\omega^2} = 2\sigma$$

or equivalently

$$\lim_{\omega\to 0} \frac{1}{\omega}\rho(\omega, \mathrm{p})\Big|_{\mathrm{p}=0} = 3\sigma$$

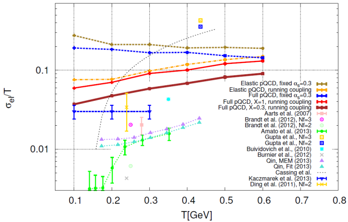
- Kubo relation for the relaxation time

$$\frac{1}{\omega}\frac{\partial^2\rho}{\partial\omega^2}\Big|_{\mathrm{p}=0} = -18\sigma\tau^2$$

Hydrodynamic spectral density

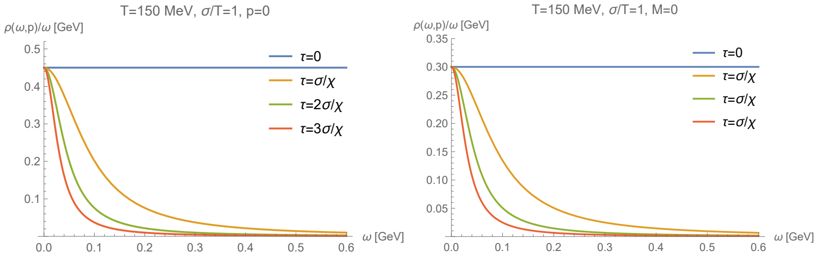
dimensional analysis and previous calculations lead to

- $\sigma = \hat{\sigma} T$, vary $\sigma / T \in (0.001, 2)$ [Greif et al. (2017)]



- $D = \hat{D} / T$ [Banerjee et al. (2011)]
- Einstein relation for the static susceptibility $D\chi = \sigma$, $\chi = \hat{\chi} T^2$
  $\hat{\chi} \approx 0.6$ [HotQCD Collaboration (2012)]
- $\tau = \hat{\tau} / T$ [Heller & Janik (2007)],
  assume $\hat{\tau} = 2\hat{D}$ to ensure causality
- conductivity relaxation time τ has to be determined in further calculations

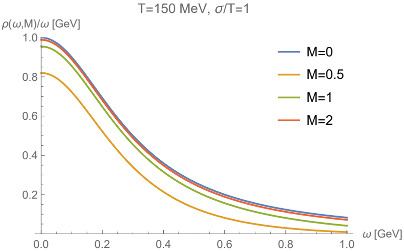
Hydrodynamic spectral density

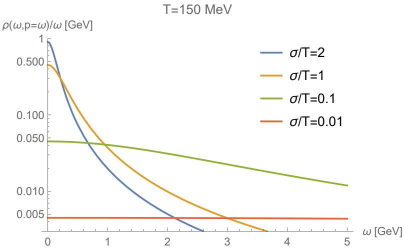


τ-dependence of the spectral function for the cases $\omega^2 = p^2$ and $p = 0$

- relaxation time narrows the spectral peak
- no influence on the value of electrical conductivity

Hydrodynamic spectral density





M-dependence and σ-dependence of the spectral density

- small dependence on invariant mass M
- spectral densities for different conductivities cross

Connection to photon and dilepton production rates

- Photon production rate $p = (\omega, \vec{p})$

$$\omega \frac{dN_\gamma}{d^3 p d^4 x} = \frac{1}{(2\pi)^3} n_B(\omega) \rho(\omega, |\vec{p}|)$$

- dilepton production rate $p = p_1 + p_2$, m: lepton mass

$$\frac{dN_{l^+l^-}}{d^4 p d^4 x} = \frac{\alpha}{12\pi^4} \frac{1}{p^2} \rho(\omega, |\vec{p}|) n_B(\omega) \left(1 + \frac{2m^2}{p^2}\right) \sqrt{1 - \frac{4m^2}{p^2}} \cdot \theta(p^2 - 2m)$$

n_B: Bose-Einstein-distribution

- can get information on conductivity from particle spectra

$$\Rightarrow \lim_{p_T \to 0} \frac{dN}{p_T dp_T} \propto \sigma$$

p_T: transverse momentum in Bjorken coordinates

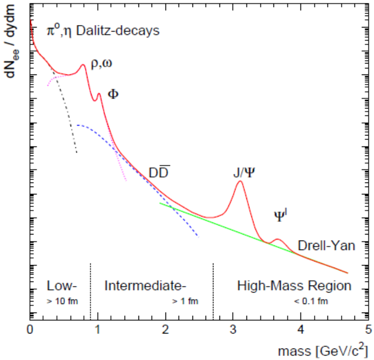
The spectral density

- analytical calculation of the spectral density $\rho(\omega, M)$ difficult
- calculation in the high energy limit [Aarts & Martinez Resco (2005)]
- approach for the limit $p = 0$ [Aarts & Nikolaev (2020)]
- NLO, LPM resummed calculations from [Ghigileri et al. (2013)] and [Jackson & Laine 2019]
- high limit $\omega \gg T_C \gg M$ given by thermal Drell Yan contribution [Carrington et al. (2008)] (NLO and LPM resummed)

$$\rho(\omega, M) = \alpha N_c \sum_f Q_f^2 m^2 \times \left[\frac{1}{2} \ln \left(\frac{4T^2\omega^2}{\frac{\pi^2}{4} m^4 + \left[m^2 + \frac{M^4}{2\omega^2} \right]^2} \right) + C_{\text{hard}} \right.$$
$$+ C_{\text{soft}} + \theta(M^2 - 4m^2) \left[\sqrt{1 - 4\frac{m^2}{M^2}} \left(2 + \frac{M^2}{m^2} \right) \right.$$
$$\left. \left. - 2 \ln \left(\frac{1 + \sqrt{1 - 4m^2/M^2}}{1 - \sqrt{1 - 4m^2/M^2}} \right) \right] \right]$$

$C_{\text{hard}} = 0.584,\ C_{\text{soft}} = -0.69$

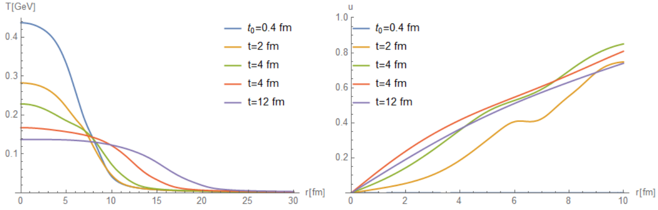
Energy regimes of the spectral functions



- Drell Yan equation reflects right continuum limit and a spectral peak
- compare spectra with Drell Yan calculation and hydrodynamic calculation in the following

Simulation of the QGP with FluiduM

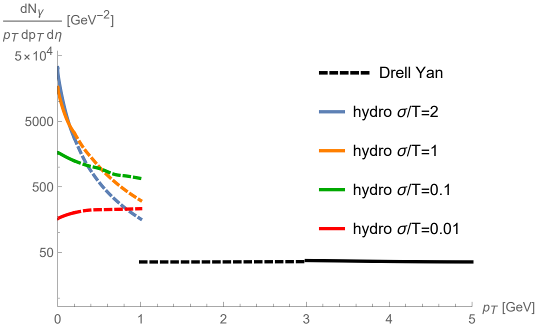
- simulate QGP using fluid dynamics with mode expansion [Flörchinger et al. 2019]
- solves fluid evolution equations
- FluiduM output: temperature $T(r,t)$ and fluid-velocity $u(r,t)$
- Pb-Pb-collision at $\sqrt{s} = 5\,TeV$, 0-5% centrality



- integrate particle rates up to freeze-out-surface at $T_{f0} = 140MeV$,

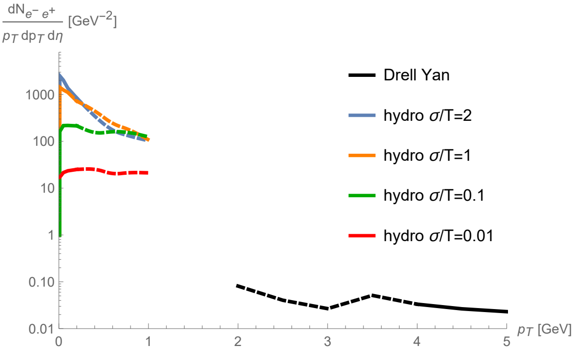
First results photons

- only thermal contributions in the hydrodynamic regime with electrical conductivity and from the high energy Drell-Yan limit



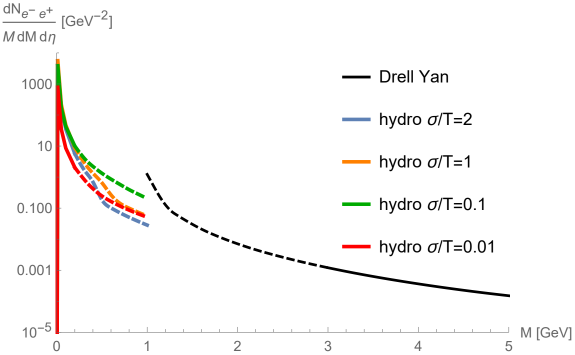
- curves not valid in the dashed areas
- hydro calculation valid for $\omega < T_C$, Drell Yan for $\omega \gg T_C \gg M$
- problem: blue shift in an expanding plasma $\omega = -u^\mu p_\mu$

First results dielectron p_T-spectrum



- curves not valid in the dashed areas, approximately valid in the solid areas

First results dielectron M-spectrum



- curves not valid in the dashed areas, approximately valid in the solid areas
- smaller signs of conductivity in the mass spectrum

Conclusion

- electrical conductivity can possibly be determined from the low thermal photon or dilepton spectra, at $p_T < 0.1 GeV$
- need to include calculations in all energy regions of the spectral density
- more precise information on transport coefficients τ and D
- no contributions from vector mesons included up to now
- experimental problem: decay particles could overlap thermal particle spectrum