A theory overview of thermal radiation

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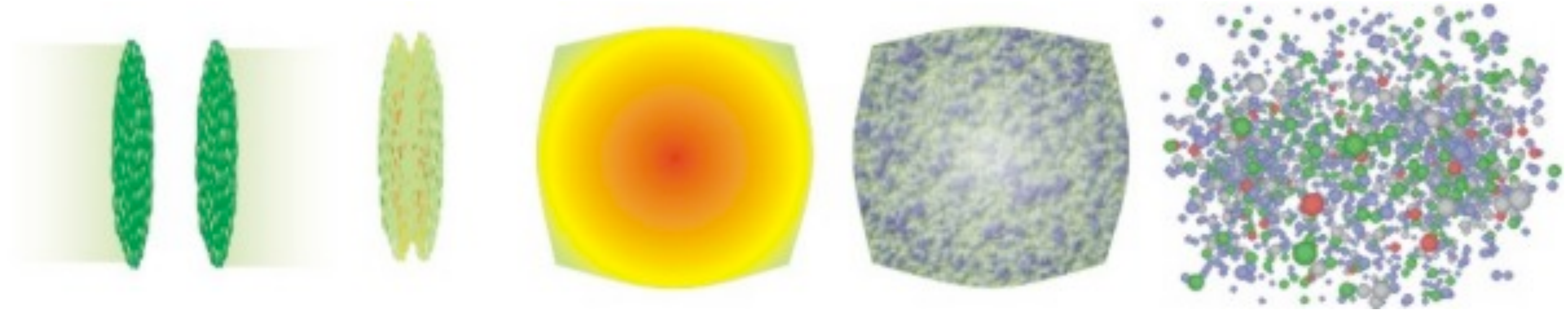
EMMI RRTF, “GSI”, September 14 2021

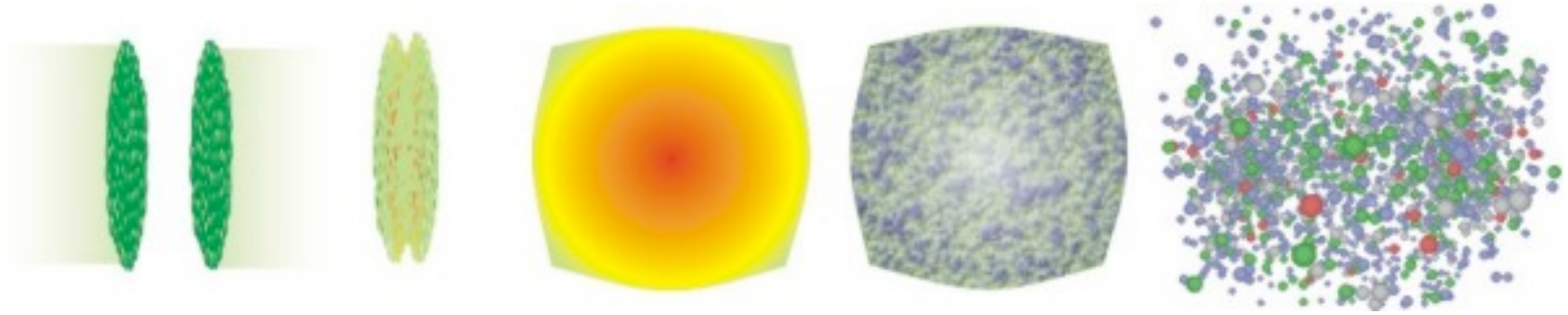
How EW probes are made

(and what they tell us)



- The hard partonic processes in the heavy ion collision produce quarks, gluons and *prompt photons and dileptons,* W and Z bosons. They can tell us about nPDFs
- At a later stage, quarks and gluons form a plasma.
  - Scatterings of thermal partons produce *QGP photons and dileptons*. *T,* hydro
  - A jet traveling can radiate *jet-thermal photons*. Jet quenching
- Later on, hadronization. *hadron gas photons and dileptons*. T, T_c, hydro
- (Some) hadrons decay into *decay photons and dileptons*

In this talk



- Theoretical description: convolution of microscopic rates over the macroscopic (hydro) evolution of the medium
- In this talk
  - overview and recent results on the microscopic rates, mostly for the *thermal phase*
  - Photons and dileptons in equilibrium from pQCD and the lattice

How to compute rates

- $\alpha \ll 1$ implies that photon production is a rare event and that rescatterings and back-reactions are negligible: medium is transparent to/not cooled by photons
- At leading order in QED and to all orders in QCD the photon and dilepton rates are given by

$$\frac{d\Gamma_{\gamma}(k)}{d^3k} = -\frac{\alpha}{4\pi^2 k}\int d^4X e^{iK\cdot X}\mathrm{Tr}\rho J^{\mu}(0)\, J_{\mu}(X)$$

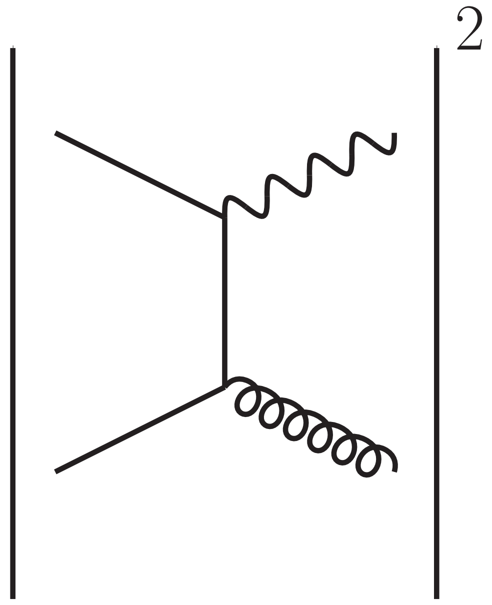
$$\frac{d\Gamma_{l^+l^-}(k)}{dk^0 d^3k} = -\frac{\alpha^2}{6\pi^3 K^2}\int d^4X e^{iK\cdot X}\mathrm{Tr}\rho J^{\mu}(0)\, J_{\mu}(X)$$

The ingredients

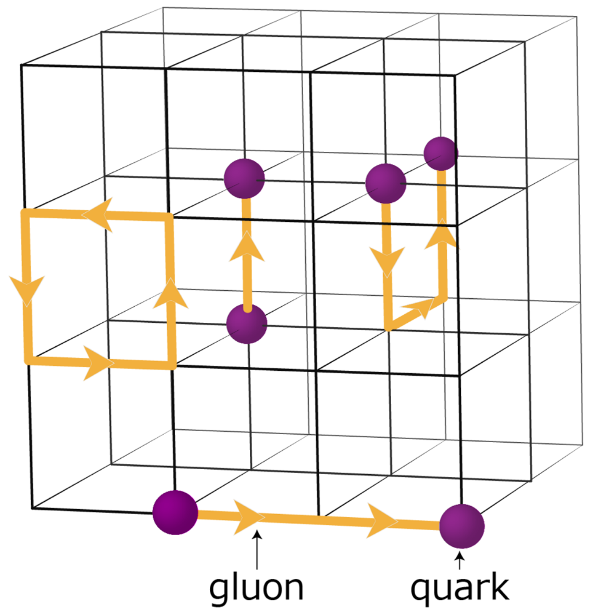
$$W^<(K) \equiv \int d^4X e^{iK\cdot X} \mathrm{Tr} \rho J^\mu(0) J_\mu(X)$$

- electromagnetic current *J*: how the d.o.f.s couple to photons
- density operator ρ. In the equilibrium (possibly just local) approximation it becomes the thermal density $\rho \propto e^{-\beta H}$ and the whole thing a thermal average
- The action *S*: how the d.o.f.s propagate and interact

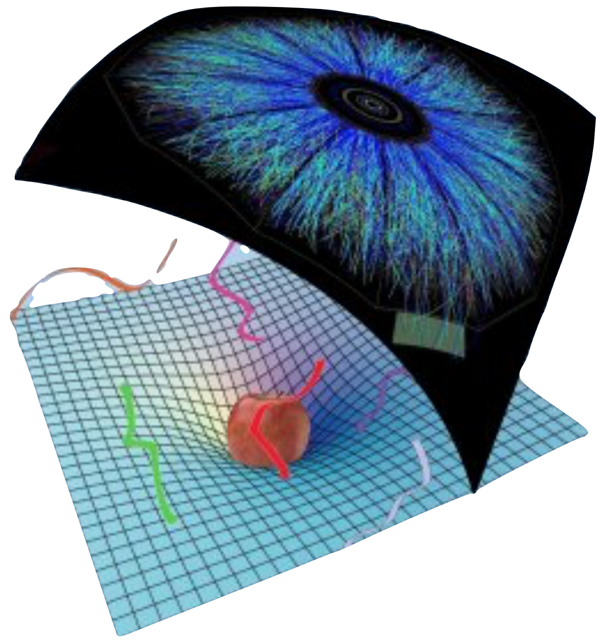
Theory approaches



pQCD: QCD action (and EFTs thereof), thermal average can be generalized to non-equilibrium. Real world: extrapolate from $g \ll 1$ to $\alpha_s \sim 0.3$



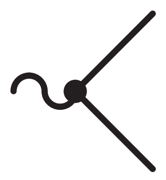
lattice QCD: Euclidean QCD action, pure thermal average. Real world: analytically continue to Minkowskian domain



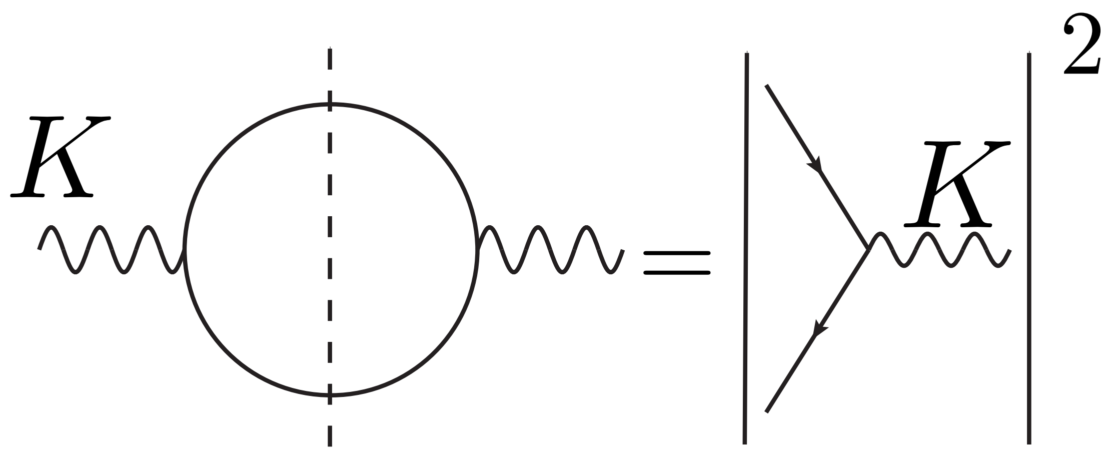
AdS/CFT: $\mathcal{N}=4$ action, in and out of equilibrium, weak and strong coupling. Real world: extrapolate to QCD

The basics of pQCD photons

$$\frac{d\Gamma_\gamma(k)}{d^3k} = -\frac{\alpha}{4\pi^2 k}\int d^4X e^{iK\cdot X}\,\mathrm{Tr}\rho J^\mu(0)\,J_\mu(X) \qquad J^\mu = \sum_{q=uds} e_q \bar{q}\gamma^\mu q :$$

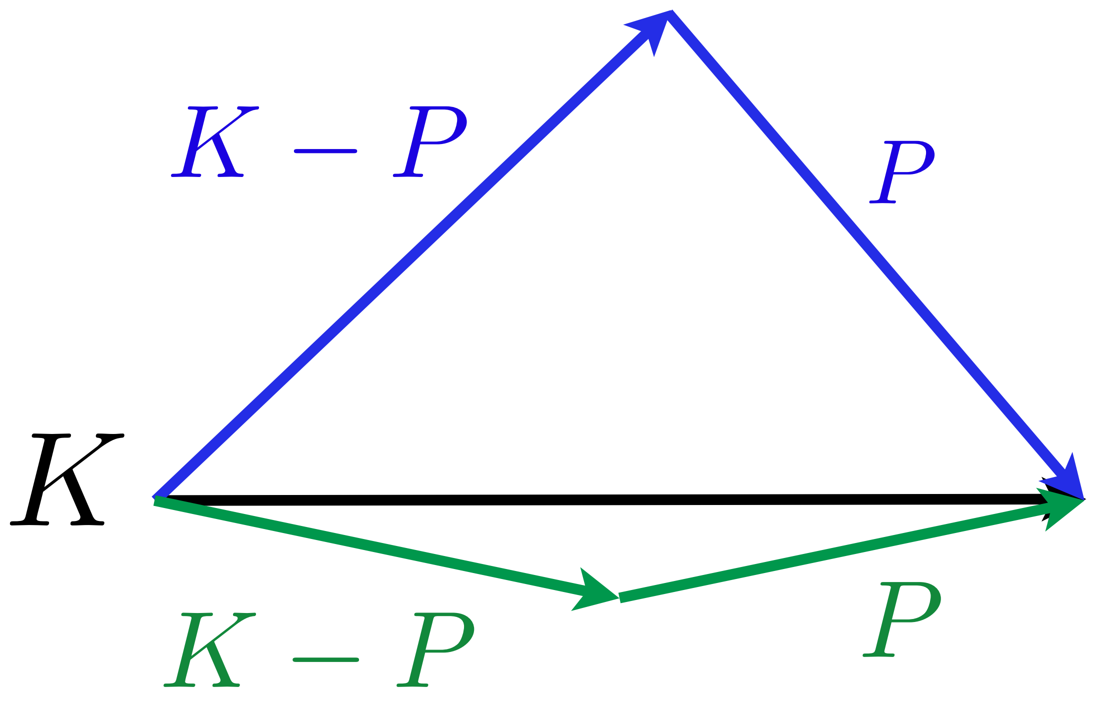


- Real, hard photon: $k^0 = k \gtrsim T$
- At one loop ($\alpha_{\mathrm{EM}}\, g^0$):



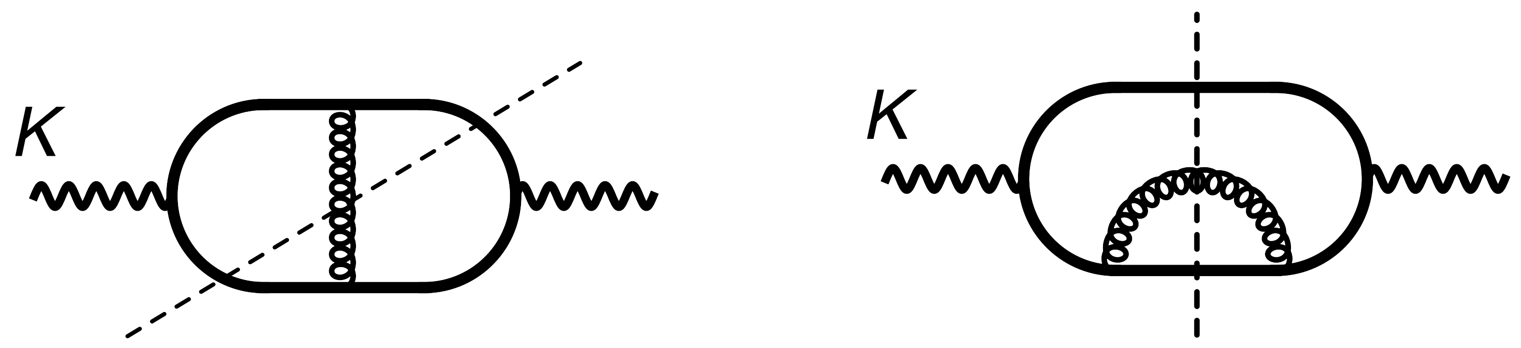
Kinematically forbidden. Need to kick one of the quarks off-shell. Works for dileptons

- Leading order photon is $\alpha_{\mathrm{EM}}\, g^2$
- Strength of the kick (virtuality) naturally divides the calculation in the distinct 2↔2 processes and collinear processes

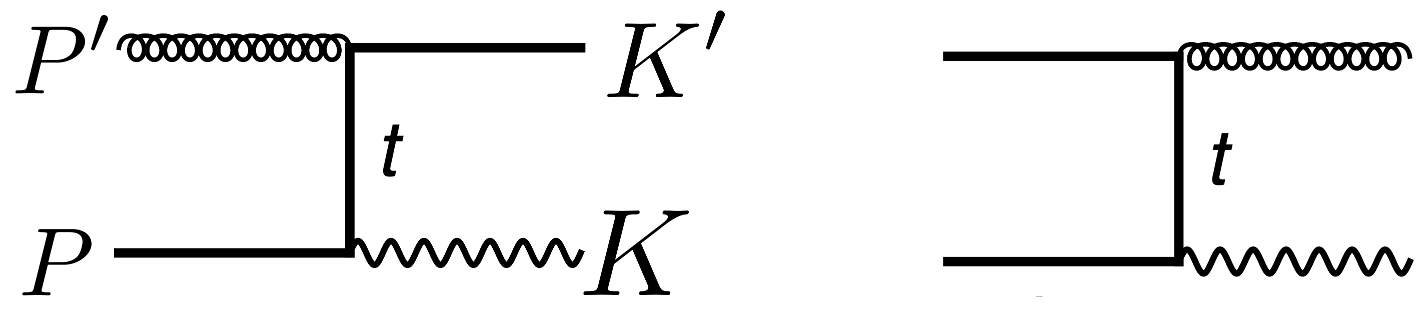


2↔2 processes

- Cut two-loop diagrams ($\alpha_{EM}\, g^2$)



2↔2 processes (with crossings and interferences):

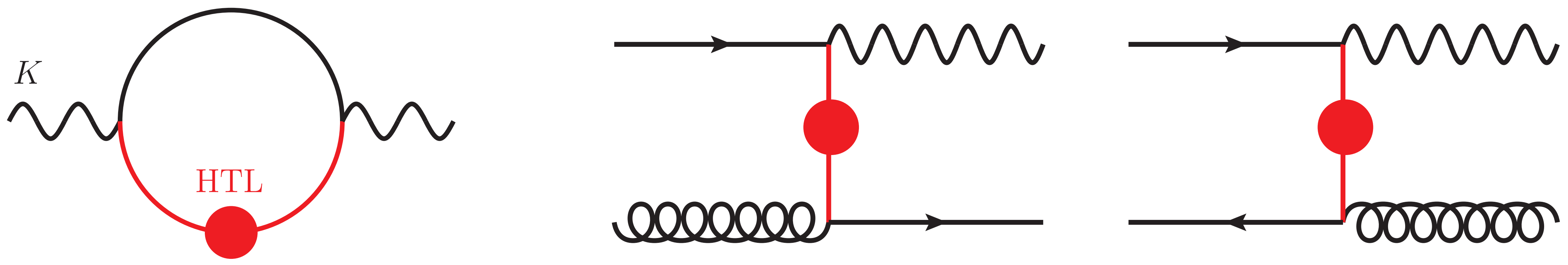


$$\int_{\text{ph. space}} f(p)f(p')(1\pm f(k'))|\mathcal{M}|^2\delta^4(P+P'-K-K')$$

- Equivalence with kinetic theory: distributions x matrix elements
- IR divergence (Compton) when t goes to zero

2↔2 processes

- The IR divergence disappears when **Hard Thermal Loop** resummation is performed Braaten Pisarski **NPB337** (1990)

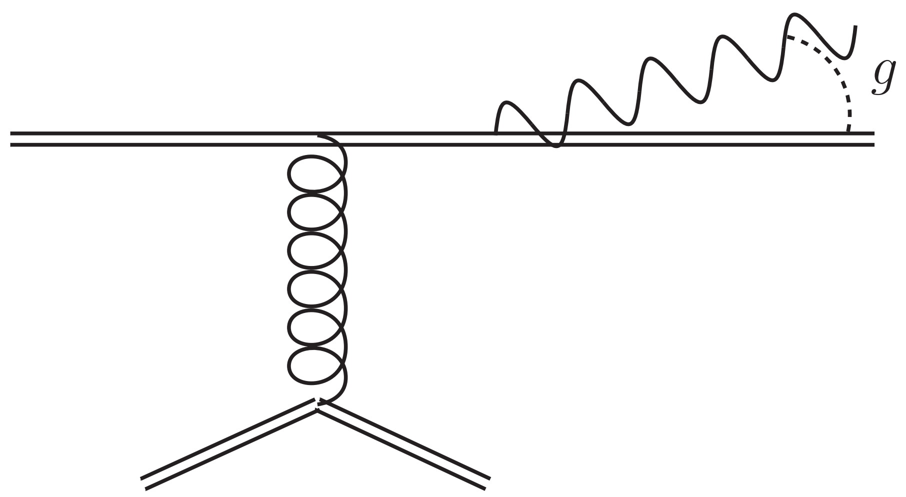


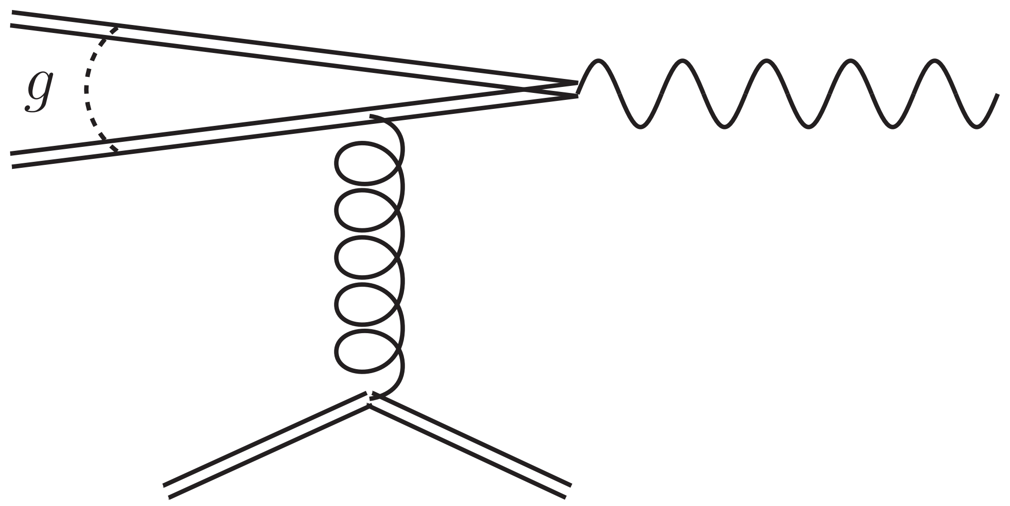
- In the end one obtains the result

$$\left.\frac{d\Gamma_\gamma}{d^3k}\right|_{2\leftrightarrow 2} \propto e^2 g^2 \left[\log\frac{T}{m_\infty} + C_{2\leftrightarrow 2}\left(\frac{k}{T}\right)\right]$$

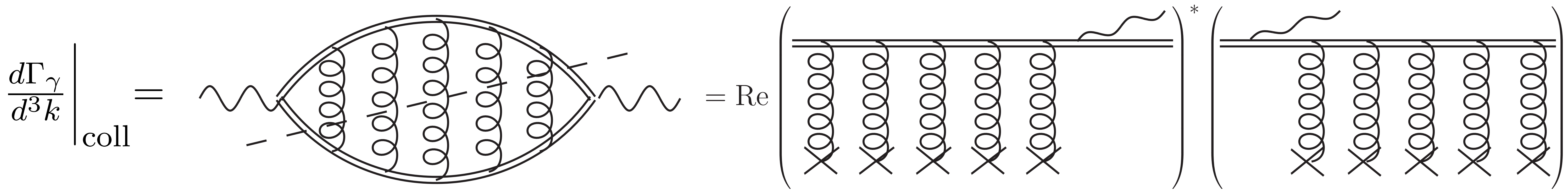
Kapusta Lichard Siebert **PRD44** (1991) Baier Nakkagawa Niegawa Redlich **ZPC53** (1992)

Collinear processes





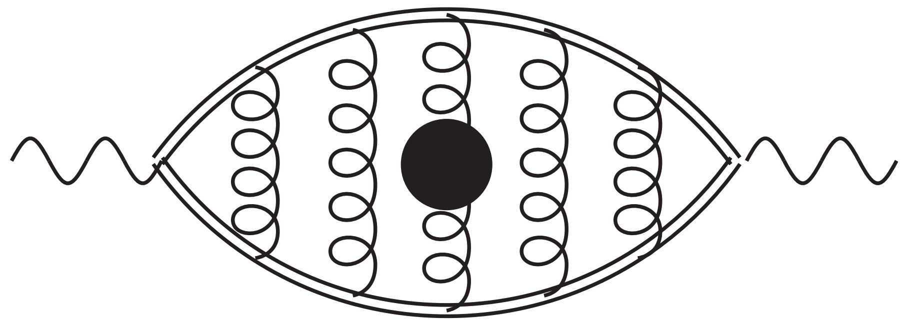
- These diagrams contribute to LO if small (g) angle radiation / annihilation Aurenche Gelis Kobes Petitgirard Zaraket 1998-2000
- Photon formation times is then of the same order of the soft scattering rate $\Rightarrow$ interference: *LPM effect*
- Requires resummation of infinite number of ladder diagrams



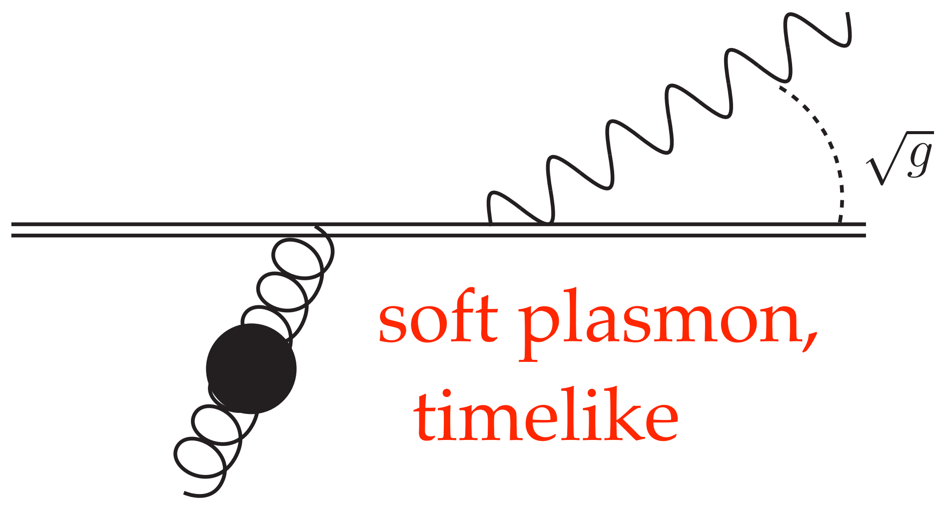
AMY (Arnold Moore Yaffe) **JHEP** 0111, 0112, 0226 (2001-02)

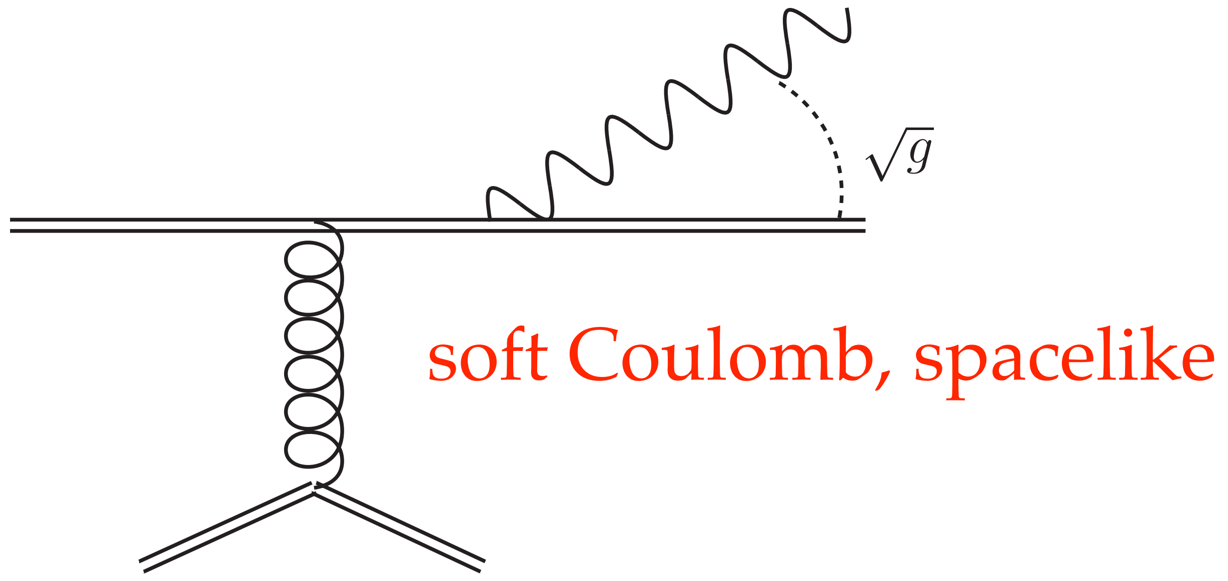
Beyond leading order

- The soft scale gT introduces $O(g)$ corrections
- In the collinear sector: 1-loop rungs (related to NLO qhat). Euclidean (EQCD) evaluation Caron-Huot **PRD79**

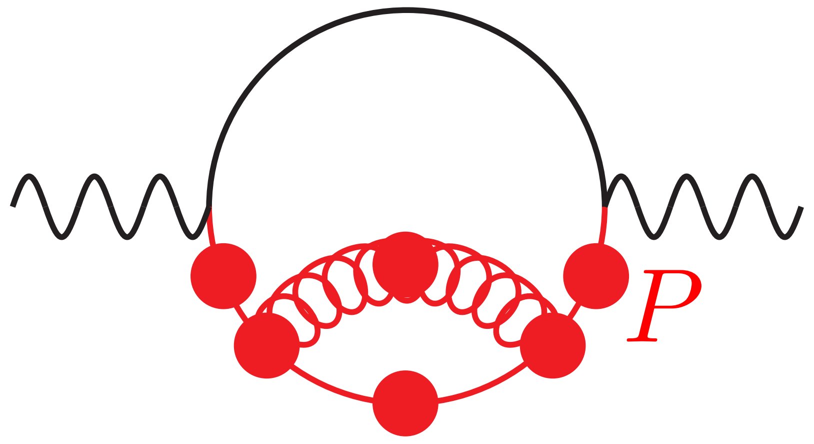


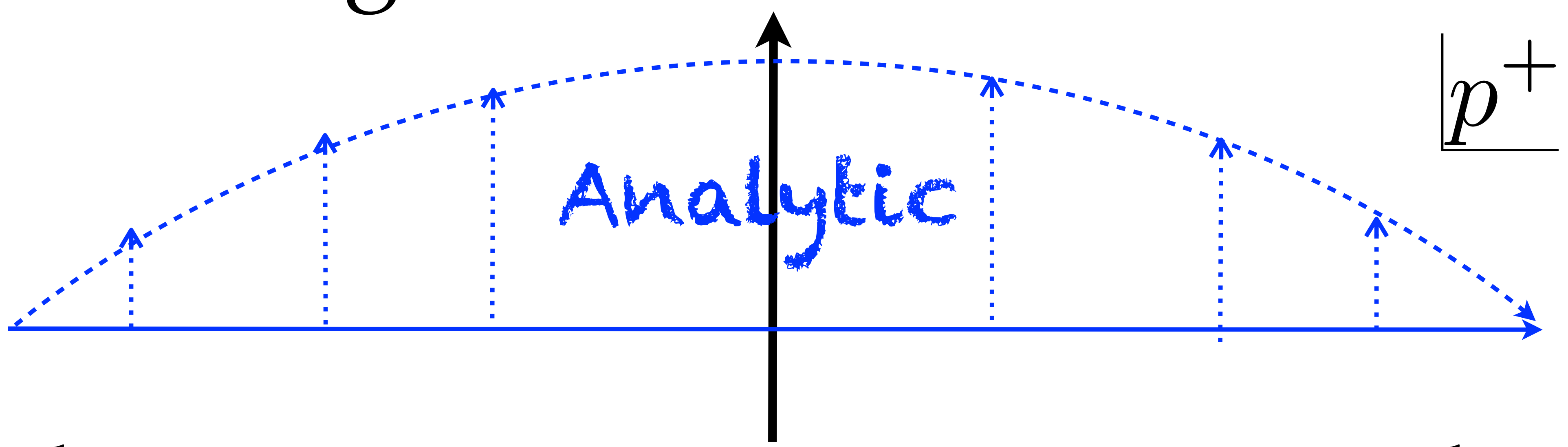
- New semi-collinear processes: larger angle radiation, NLO in collinear radiation approx. Requires a *“modified qhat”*, relevance for jets





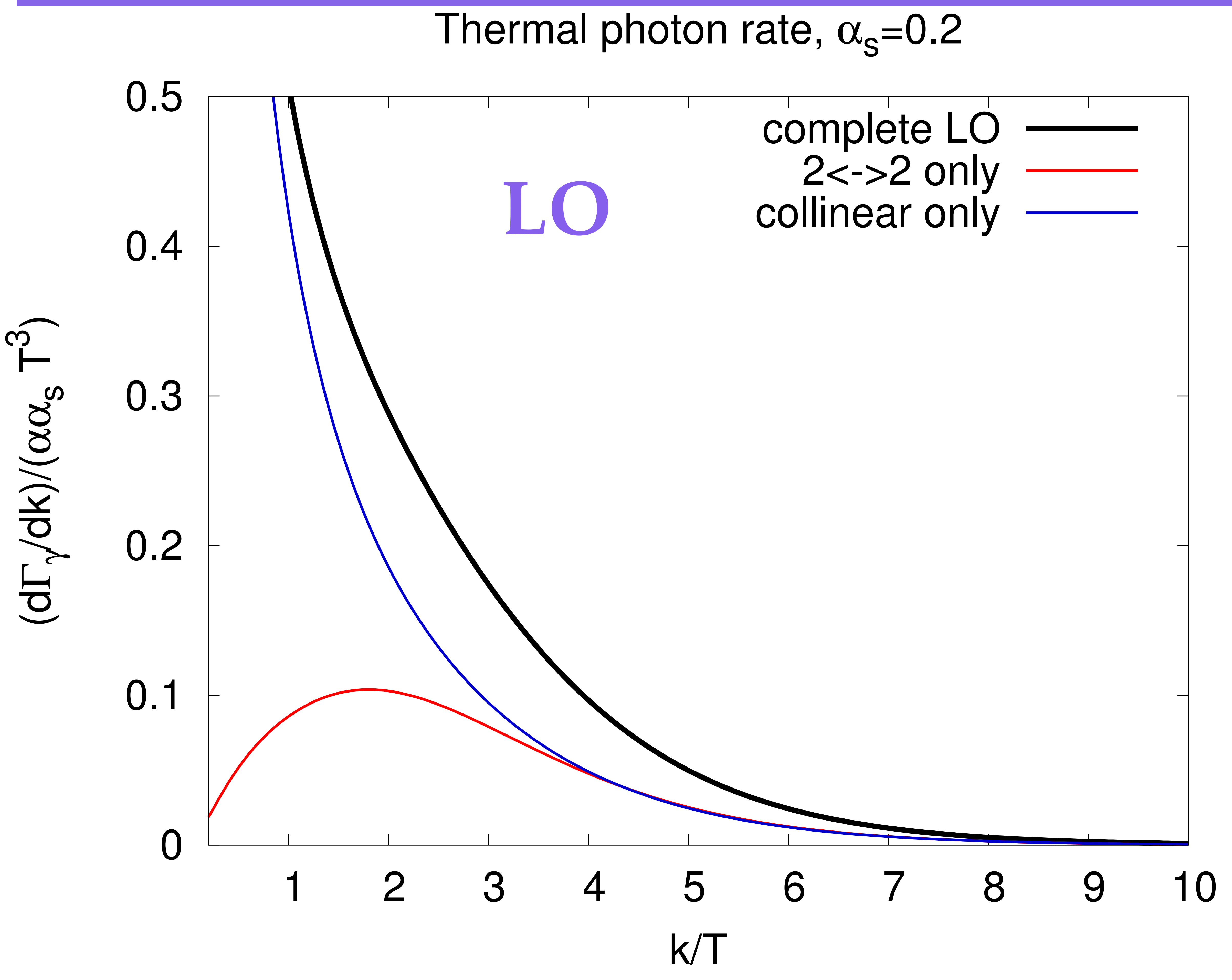
- Add soft gluons to soft quarks: nasty all-HTL region

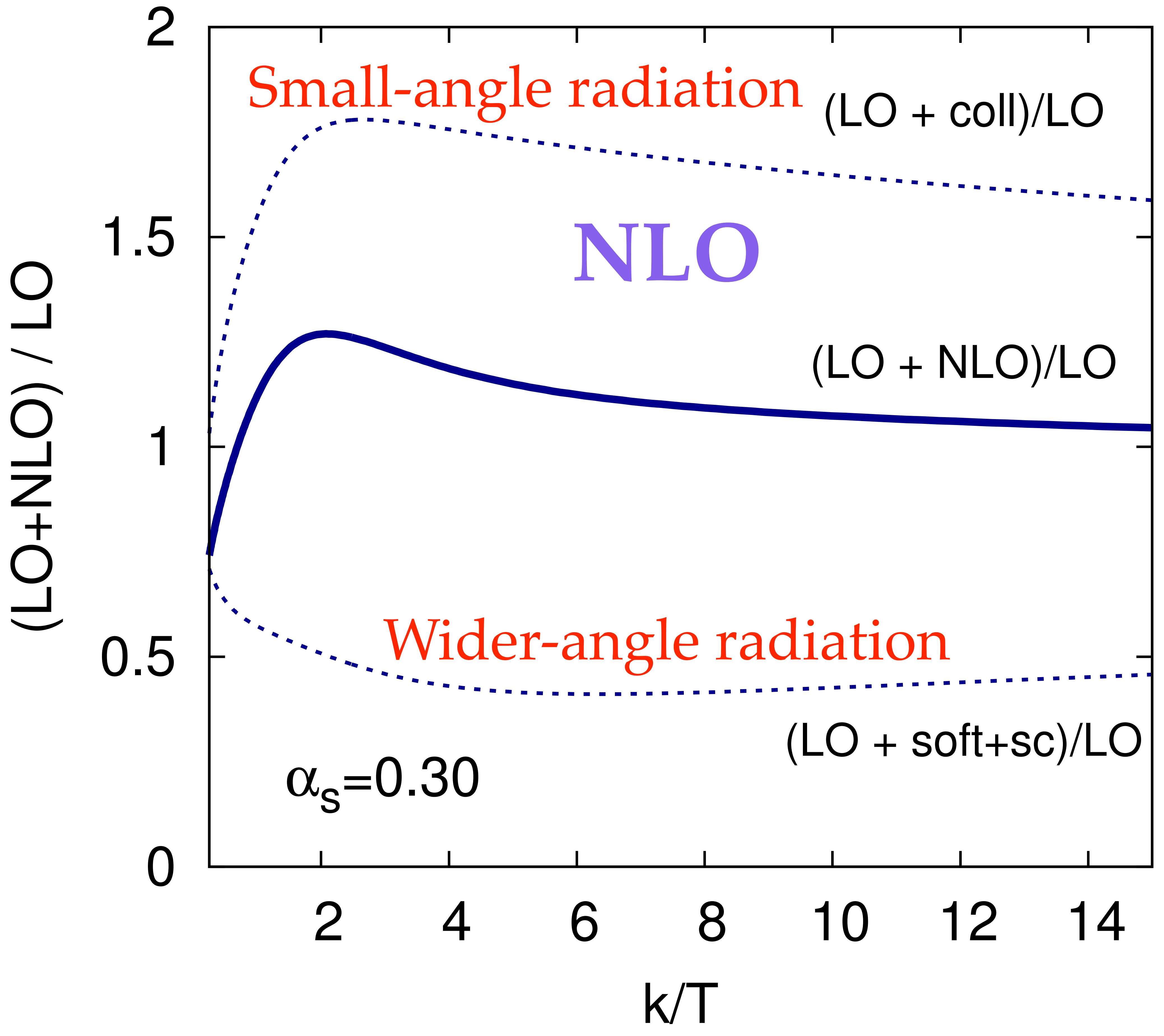




Analyticity allows us to take a detour in the complex plane away from the nasty region $\Rightarrow$ compact expression

pQCD photons





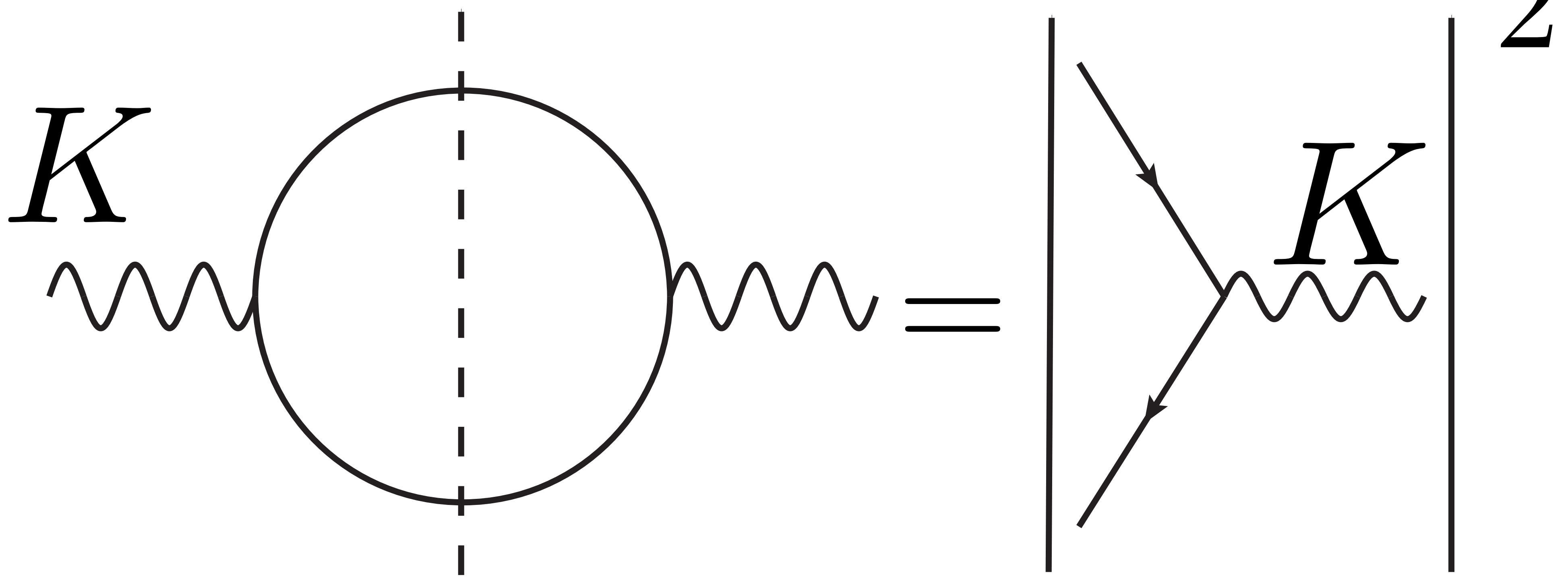
LO: AMY (2001-02) **NLO:** JG Hong Kurkela Lu Moore Teaney **JHEP0503** (2013)

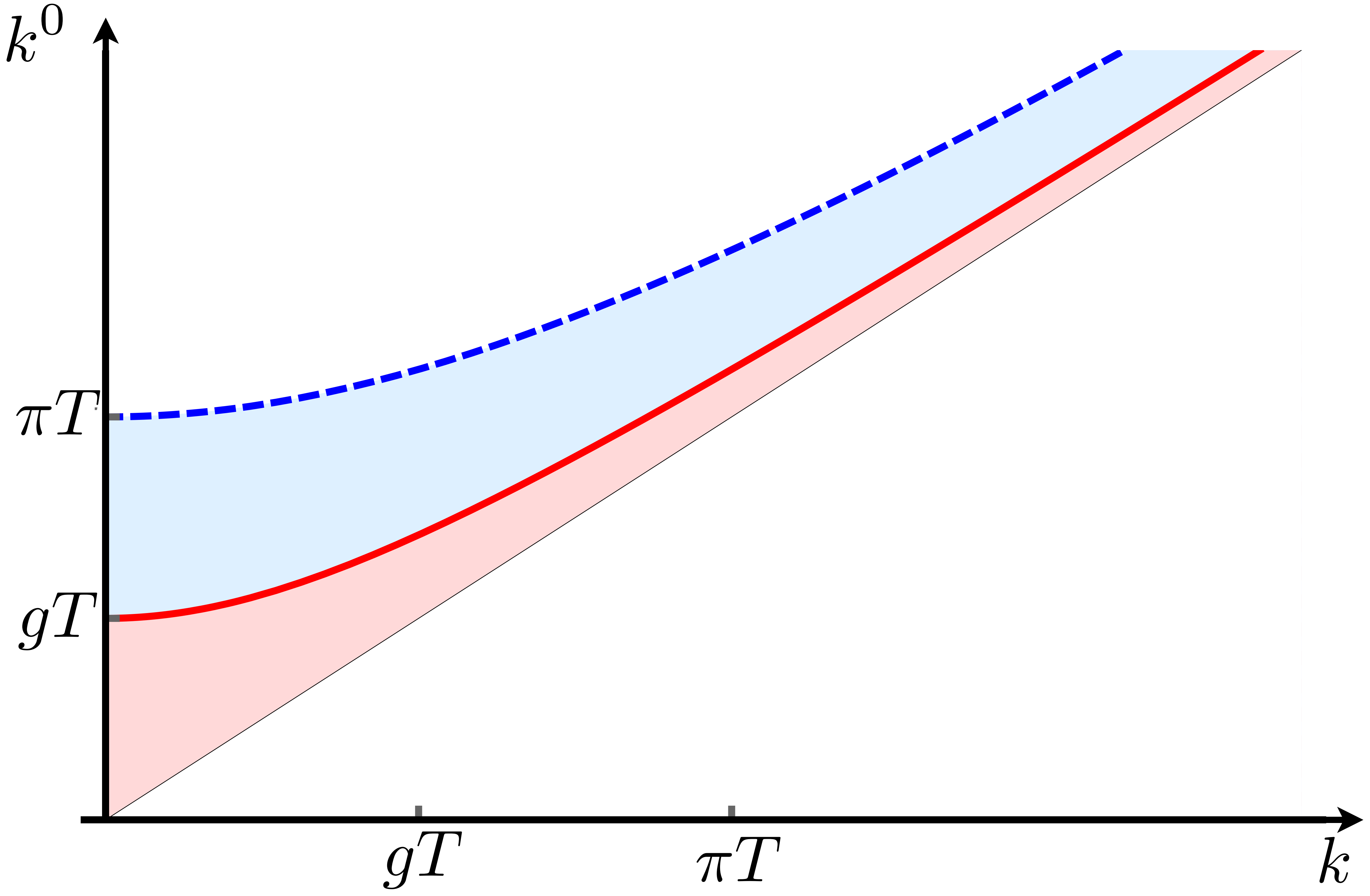
pQCD dileptons

- Consider non-zero virtuality $k^0 > k \geq 0$.

$$\frac{d\Gamma_{l^+l^-}(k)}{dk^0 d^3k} = -\frac{\alpha^2}{6\pi^3 K^2} \int d^4X e^{iK\cdot X} \mathrm{Tr}\rho J^\mu(0)\, J_\mu(X)$$

- Born contribution present, gets larger as $M^2 = K^2$ grows





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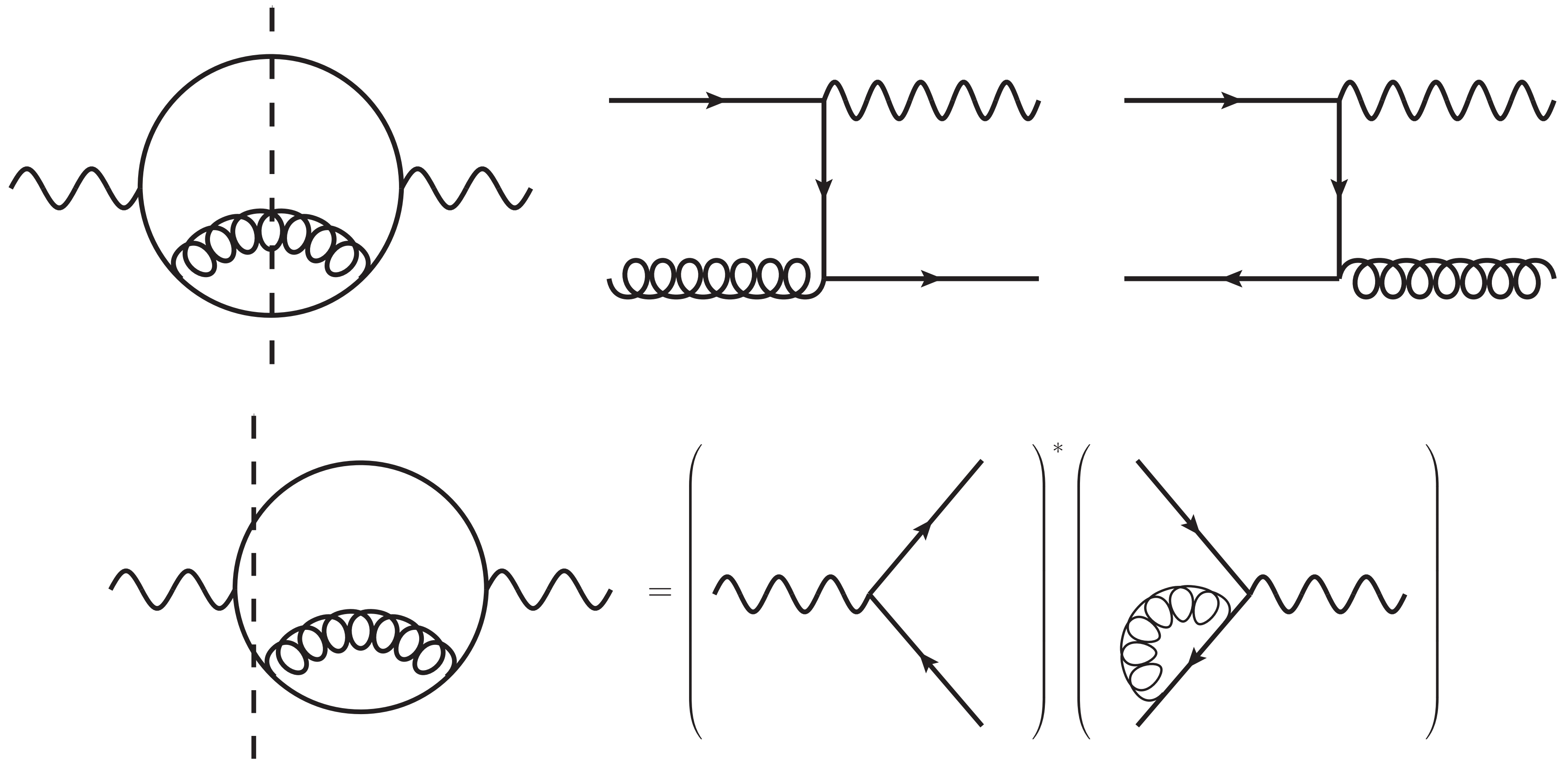
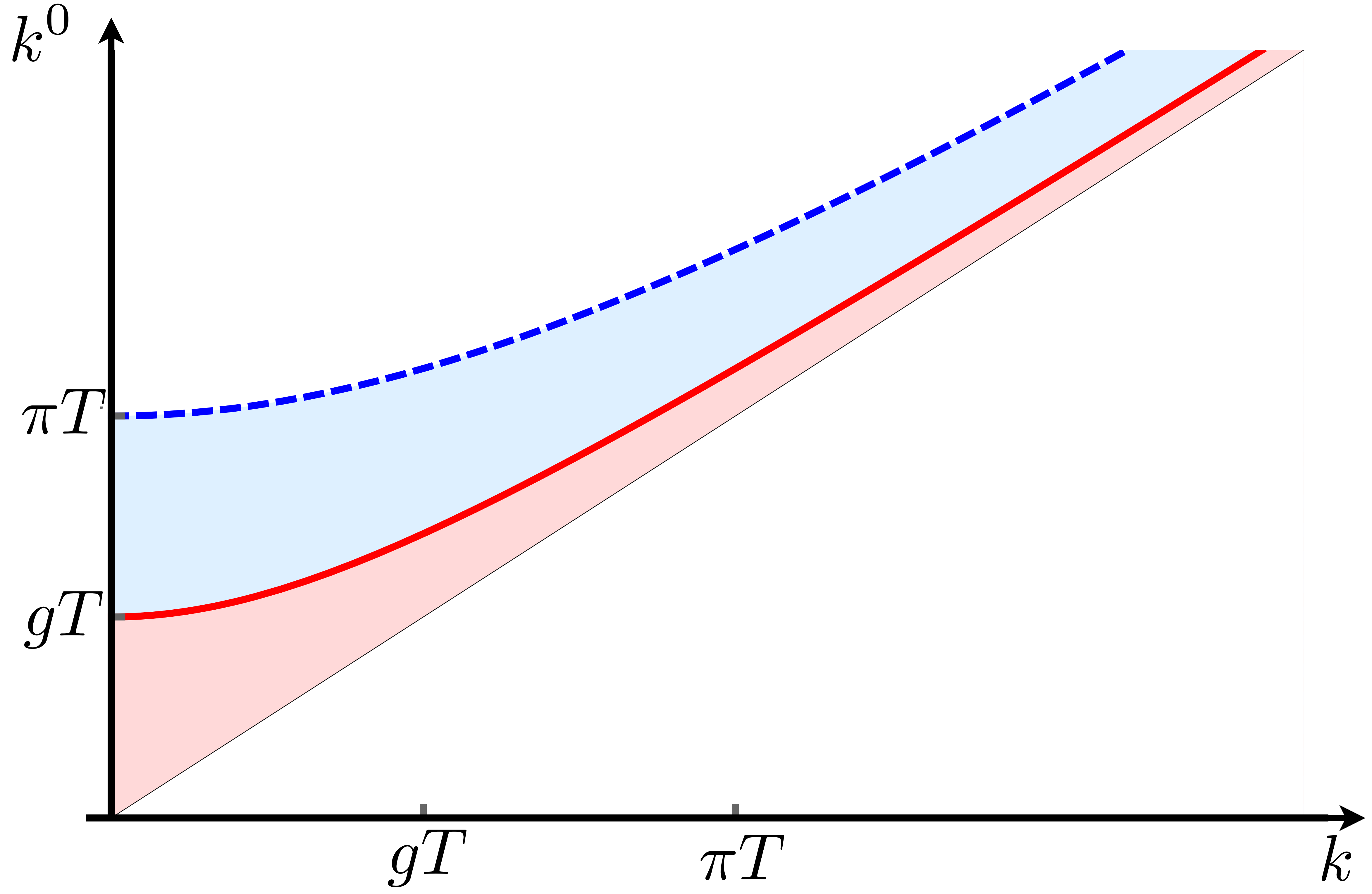
- If $K^2 \sim T^2$ loop corrections: real and virtual (with IR cancellations)





NLO results Laine **JHEP1311** (2013) extended to spacelike region in Jackson **PRD100** (2019)

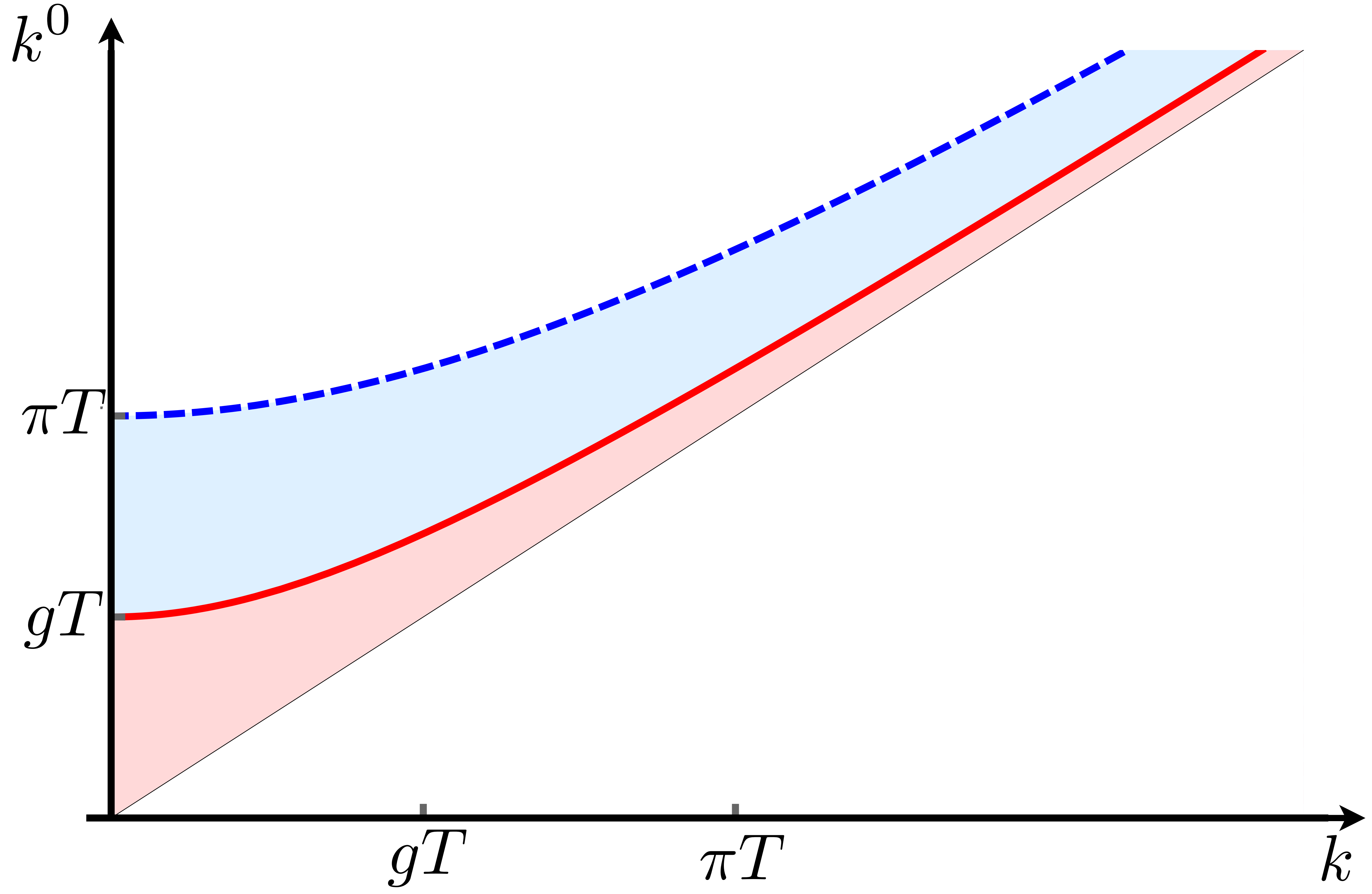
pQCD dileptons

$$\frac{d\Gamma_{l+l-}(k)}{dk^0 d^3k} = -\frac{\alpha^2}{6\pi^3 K^2}\int d^4X\, e^{iK\cdot X}\,\mathrm{Tr}\rho J^\mu(0)\,J_\mu(X)$$

- Consider non-zero virtuality $k^0 > k \geq 0$.

- If $K^2 \ll T^2$ LPM and/or HTL resummations are again necessary, similar to $K^2=0$
  Braaten Pisarski Yuan **PRL64** (1990),
  Aurenche Gelis Moore Zaraket **JHEP0212** (2002)
  NLO results JG Moore **JHEP1412** (2014)

- Finite-k rate available at NLO for all $K^2 \geq 0$
  Ghisoiu Laine **JHEP1014** (2014) JG Moore (2014)
  JG Laine, in progress



And the lattice?

- What is measured directly is the Euclidean correlator

$$G_E(\tau, k) = \int d^3x J_\mu(\tau, \mathbf{x}) J_\mu(0,0) e^{i\mathbf{k}\cdot\mathbf{x}}$$

- Analytical continuation $G_E(\tau, k) = G^<(i\tau, k)$

$$G_E(\tau, k) = \int_0^\infty \frac{dk^0}{2\pi} \rho_V(k^0, k) \frac{\cosh\left(k^0(\tau - 1/2T)\right)}{\sinh\left(\frac{k^0}{2T}\right)} \qquad W^<(K) = n_B(k^0)\, \rho_V(k^0, k)$$

- It contains much more info (full spectral function), but hidden in the convolution. Inversion tricky, discrete dataset with errors

And the lattice?

- What is measured directly is the Euclidean correlator

$$G_E(\tau, k) = \int d^3x J_\mu(\tau, \mathbf{x}) J_\mu(0,0) e^{i\mathbf{k}\cdot\mathbf{x}}$$

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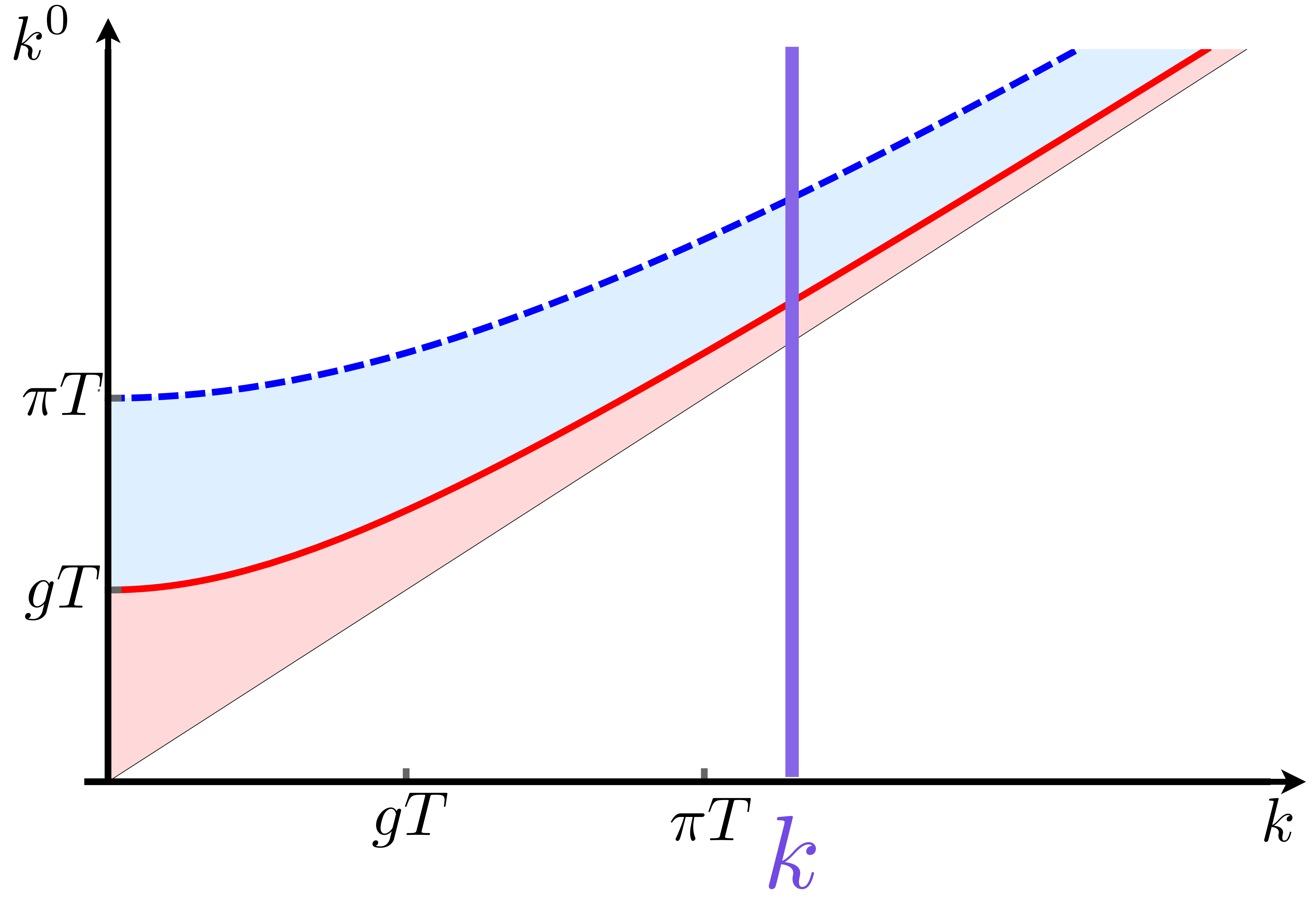
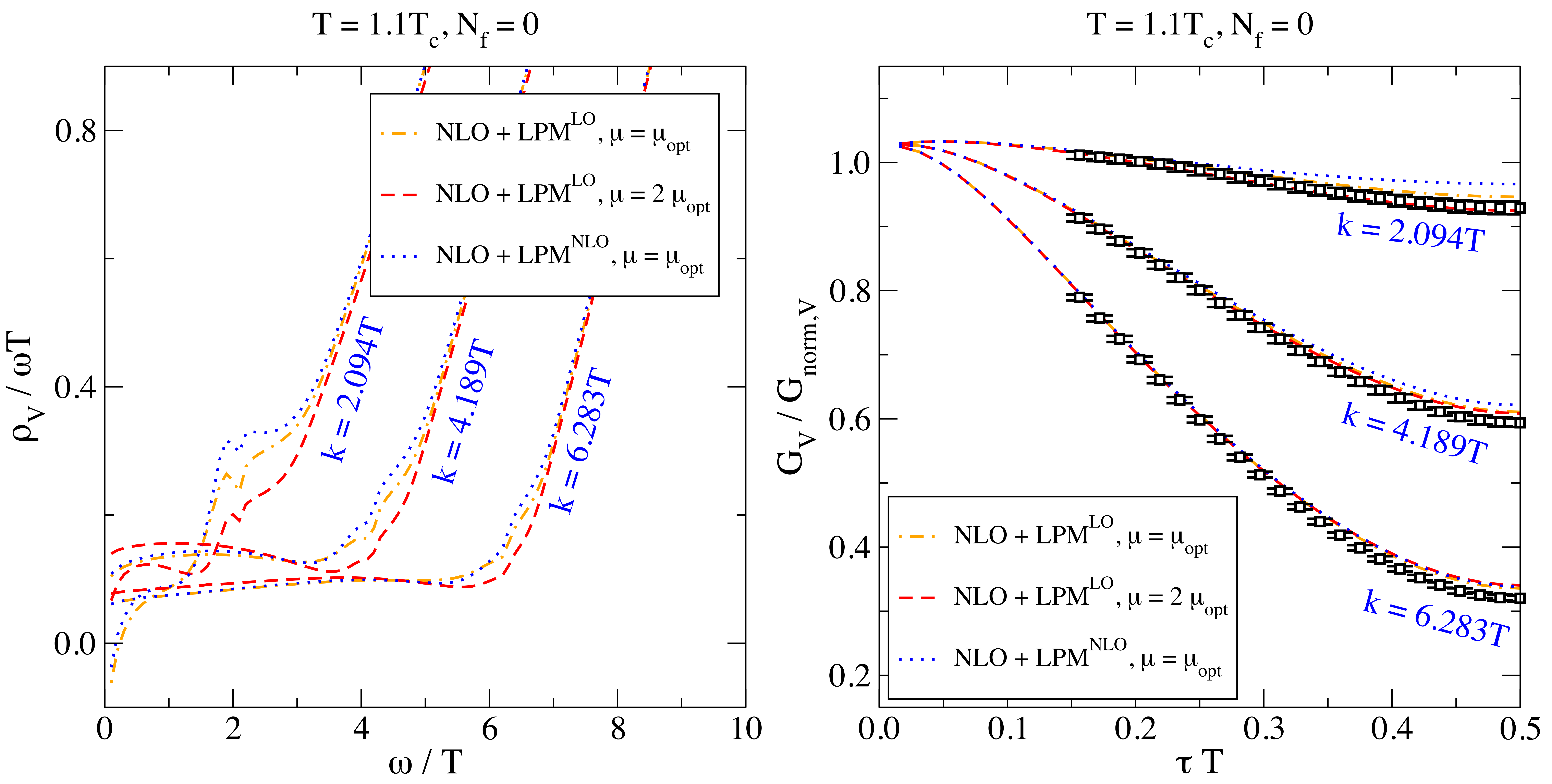
$$W^<(K) = n_B(k^0)\, \rho_V(k^0, k)$$



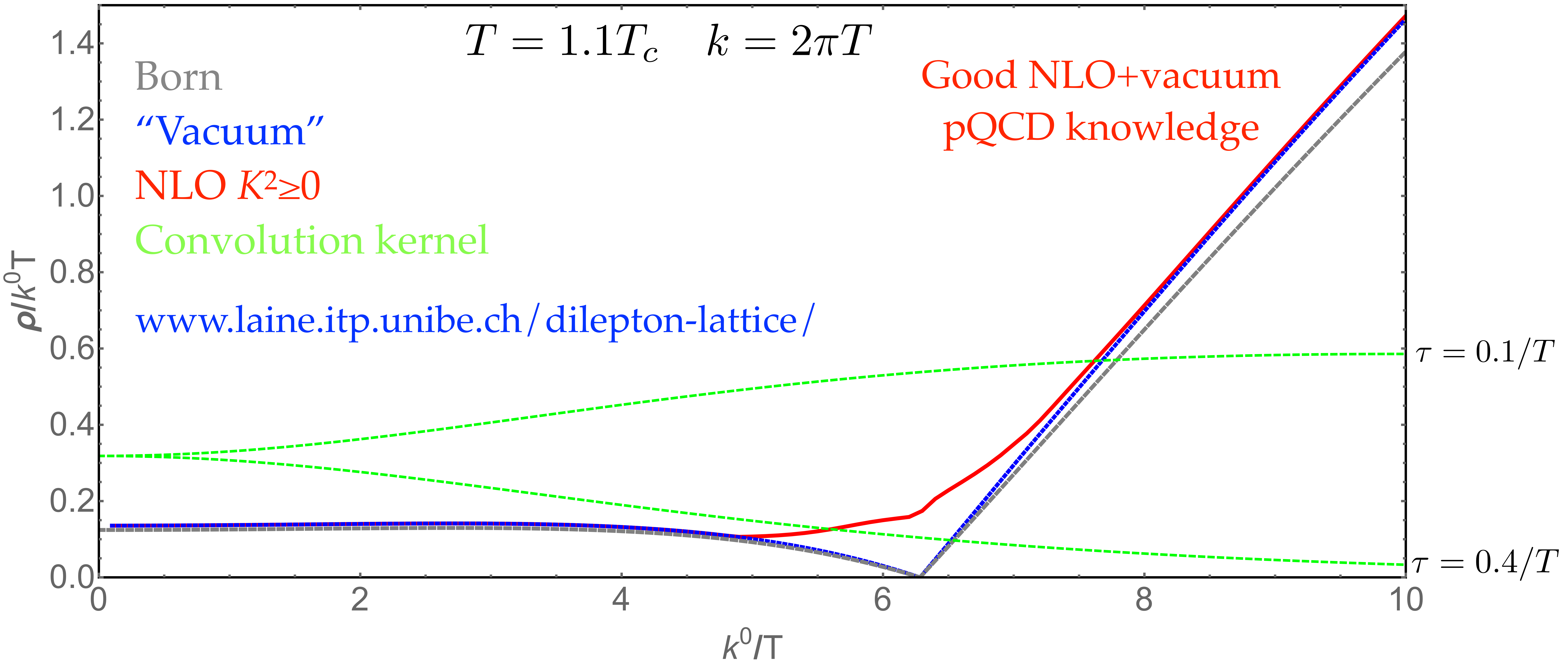
- If k>0 spf describes DIS ($k^0<k$), photons ($k^0=k$) and dileptons ($k^0>k$)



- Plots and spectral function from Jackson Laine **JHEP1109** (2019)
  Quenched lattice from JG Kaczmarek Laine F.Meyer **PRD94** (2016)

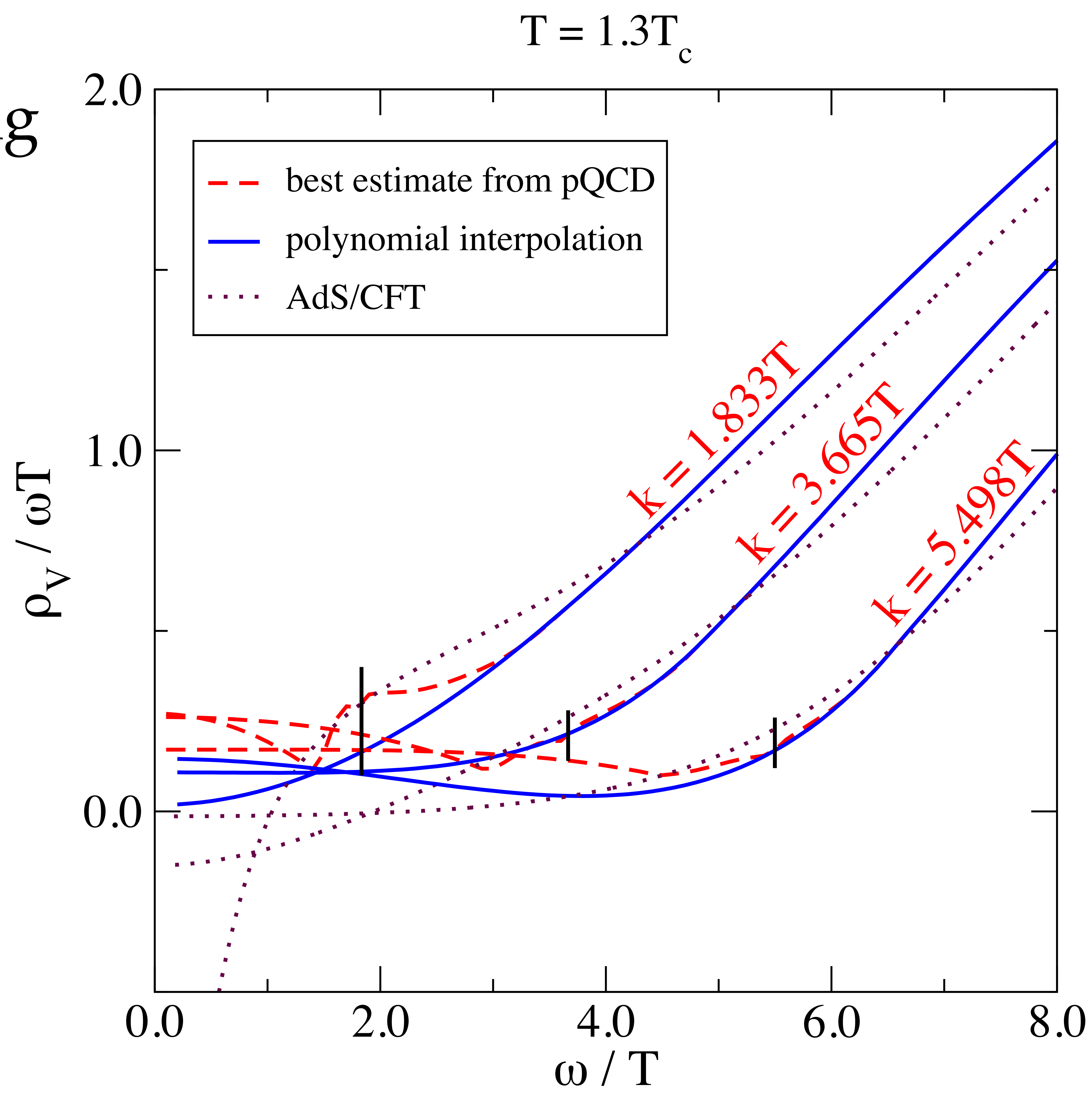
And the lattice?

- If k>0 spf describes DIS ($k^0<k$), photons ($k^0=k$) and dileptons ($k^0>k$)



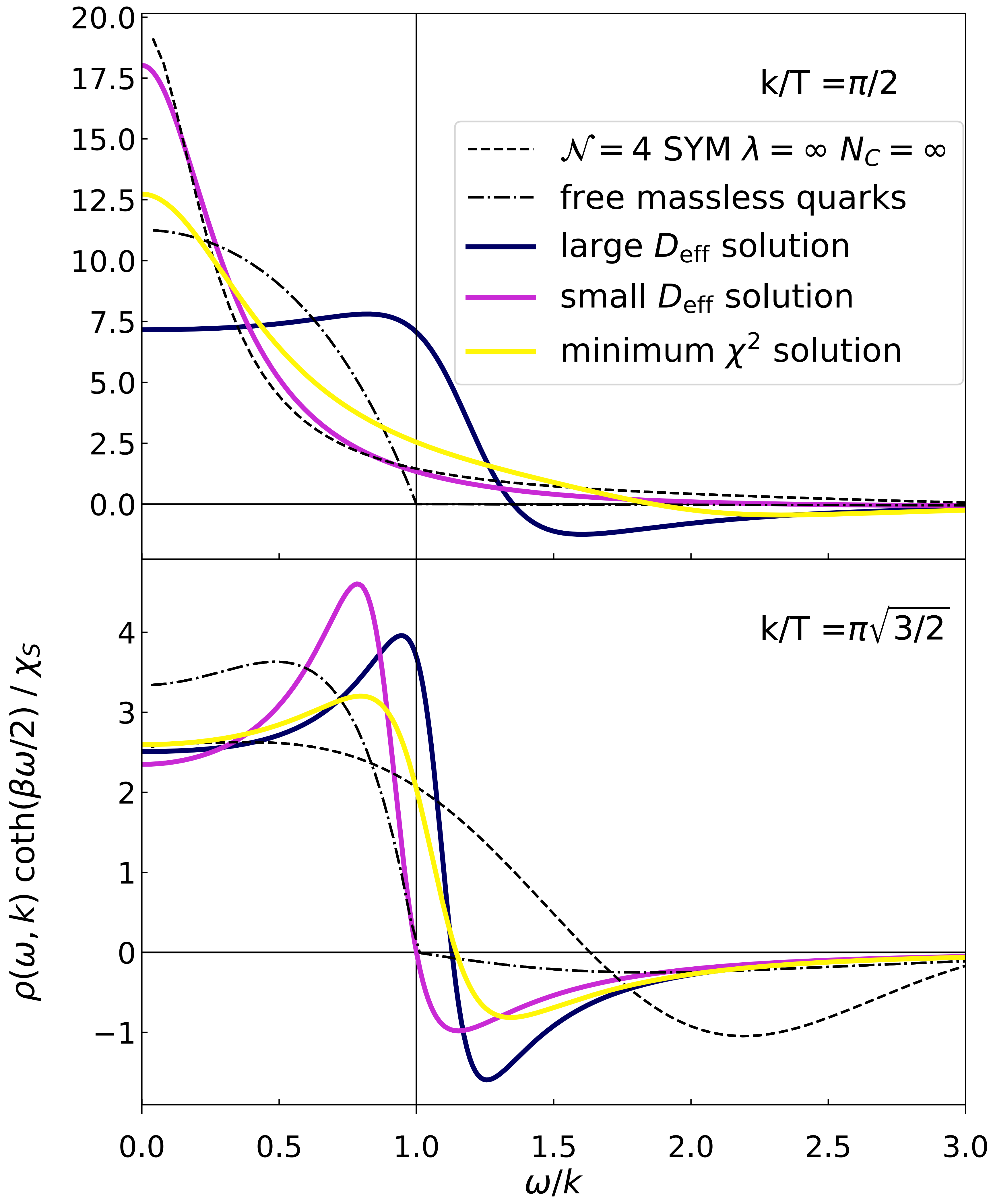
Fitting to the lattice

- Main idea: assume a fitting form for the spf, relying on Ansätze
- Get the Euclidean correlator from this ansatz spf and fit the spf coeffs to the lattice data
- Two approaches so far
  - Quenched, continuum extrapolated lattice data, **standard vector spf** $\rho_V = 2\rho_T + \rho_L$
  - Convolution dominated by (well-understood) vacuum physics at $\omega \gg k$
    JG Kaczmarek Laine F.Meyer **PRD94** (2016)



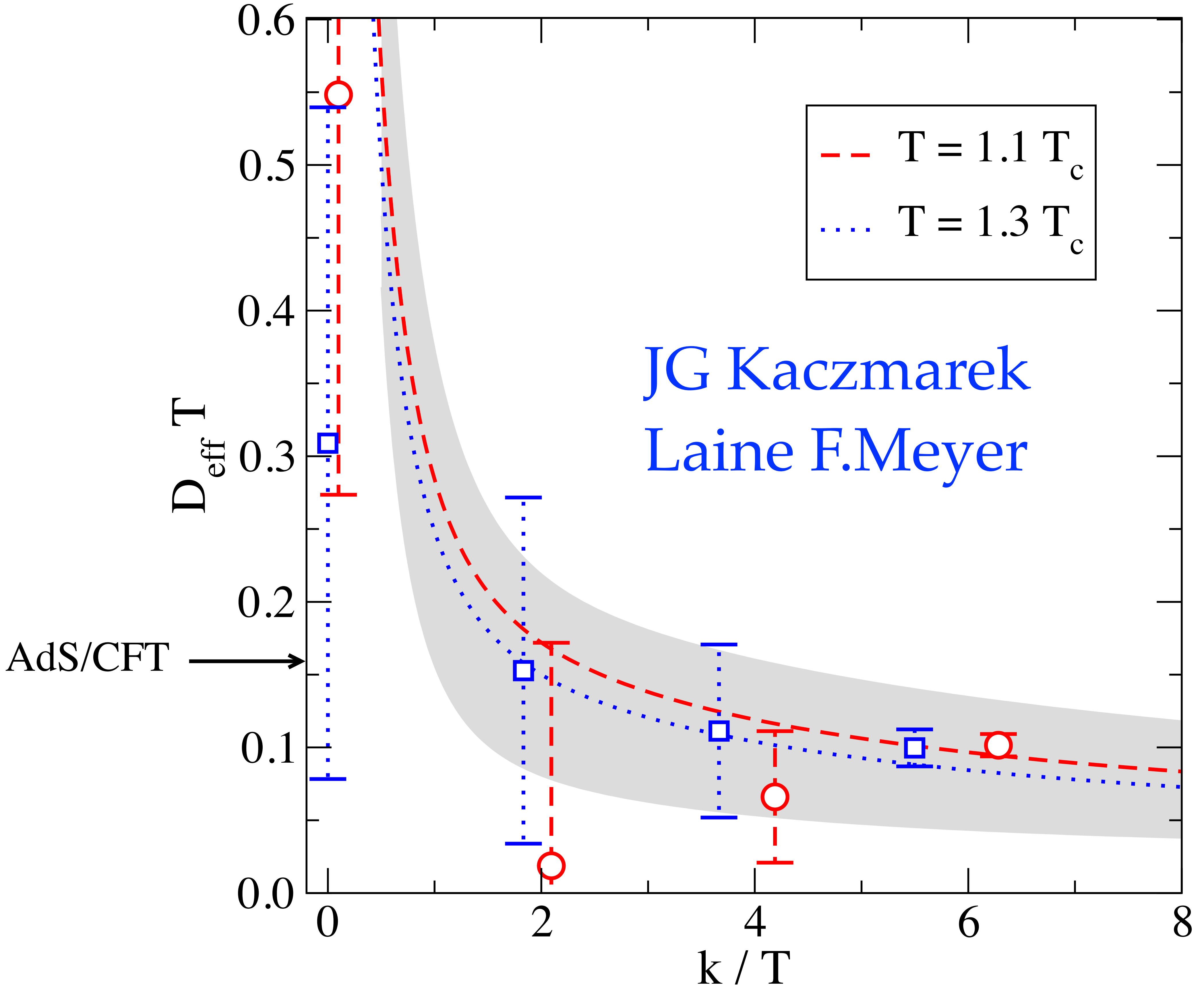
Fitting to the lattice

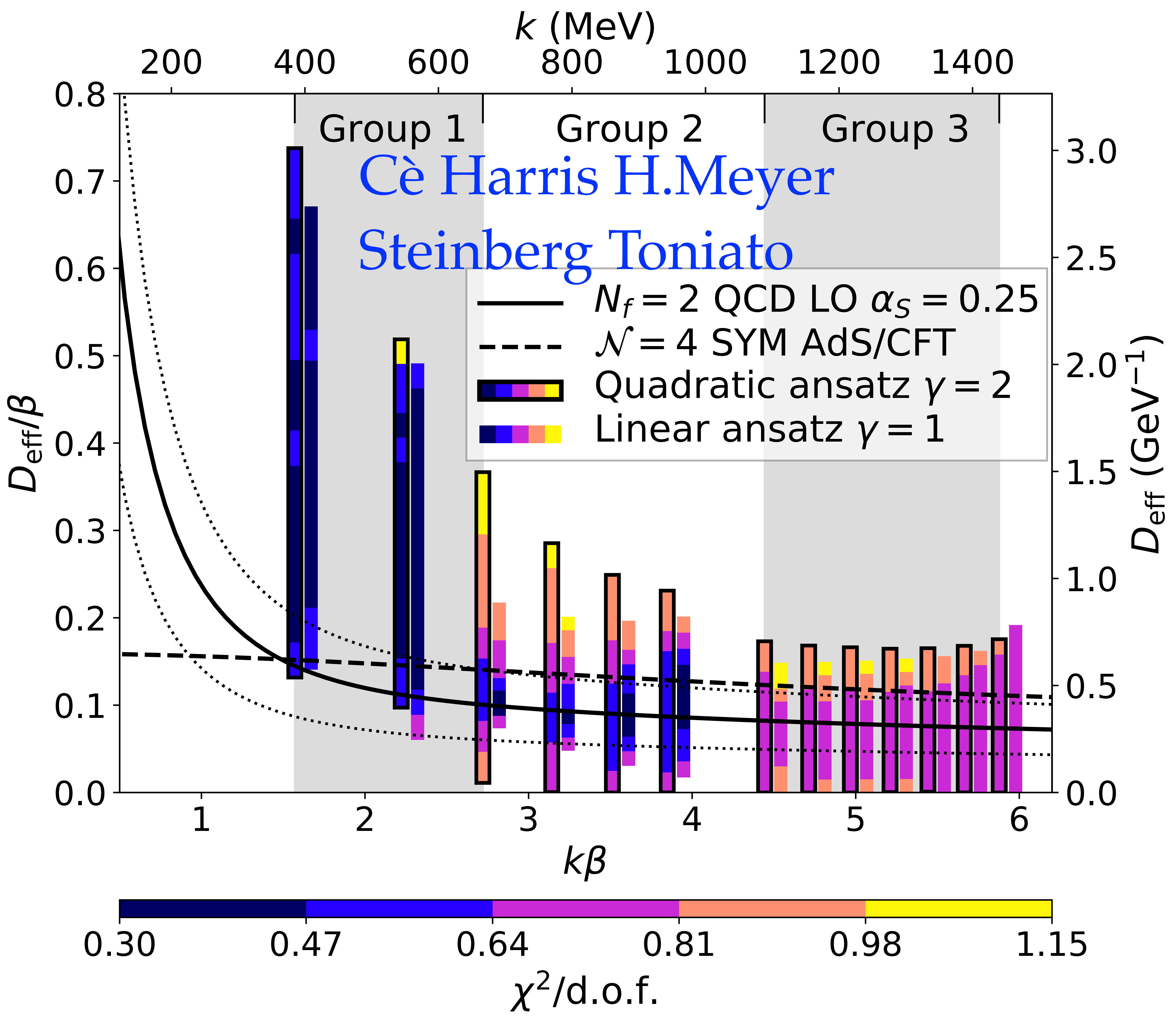
- Main idea: assume a fitting form for the spf, relying on Ansätze
- Get the Euclidean correlator from this ansatz spf and fit the spf coeffs to the lattice data
- Two approaches so far
  - N_f=2 continuum extrapolated, **modified spf** $\rho_{\mathrm{Mainz}}=2\rho_T-2\rho_L$
  - Vacuum contribution vanishes identically (Lorentz invariance). $\boldsymbol{\rho_{\mathrm{Mainz}}(\omega=k)=\rho_V(\omega=k)}$
    Brandt Francis Harris H.Meyer Steinberg
    1710.07050 Cè Harris H.Meyer Steinberg Toniato
    PRD102 (2020)



Fitting to the lattice

- Define $D_{\rm eff}(k) \equiv \begin{cases} \dfrac{\rho_{\rm V}(k,\mathbf{k})}{2\chi_{\rm q}k} & , \quad k>0 \\ \lim\limits_{\omega\to 0^+} \dfrac{\rho^{ii}(\omega,\mathbf{0})}{3\chi_{\rm q}\omega} & , \quad k=0 \end{cases}$
- In the hydro limit $k \ll T$ $D_{\rm eff} \to D$ $\quad \sigma = e^2 \sum_{f=1}^{N_{\rm f}} Q_f^2 \chi_{\rm q} D$



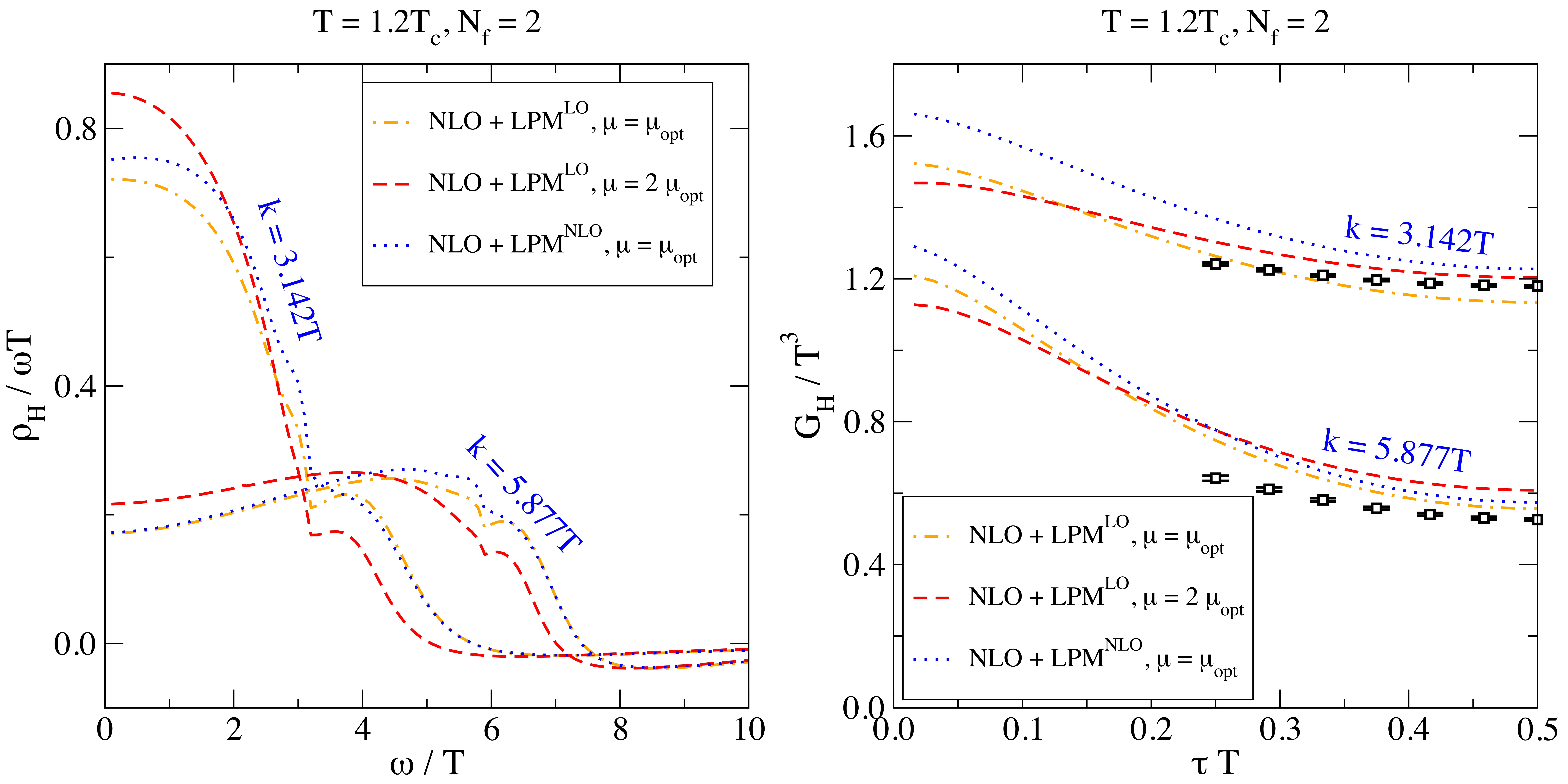


Summary

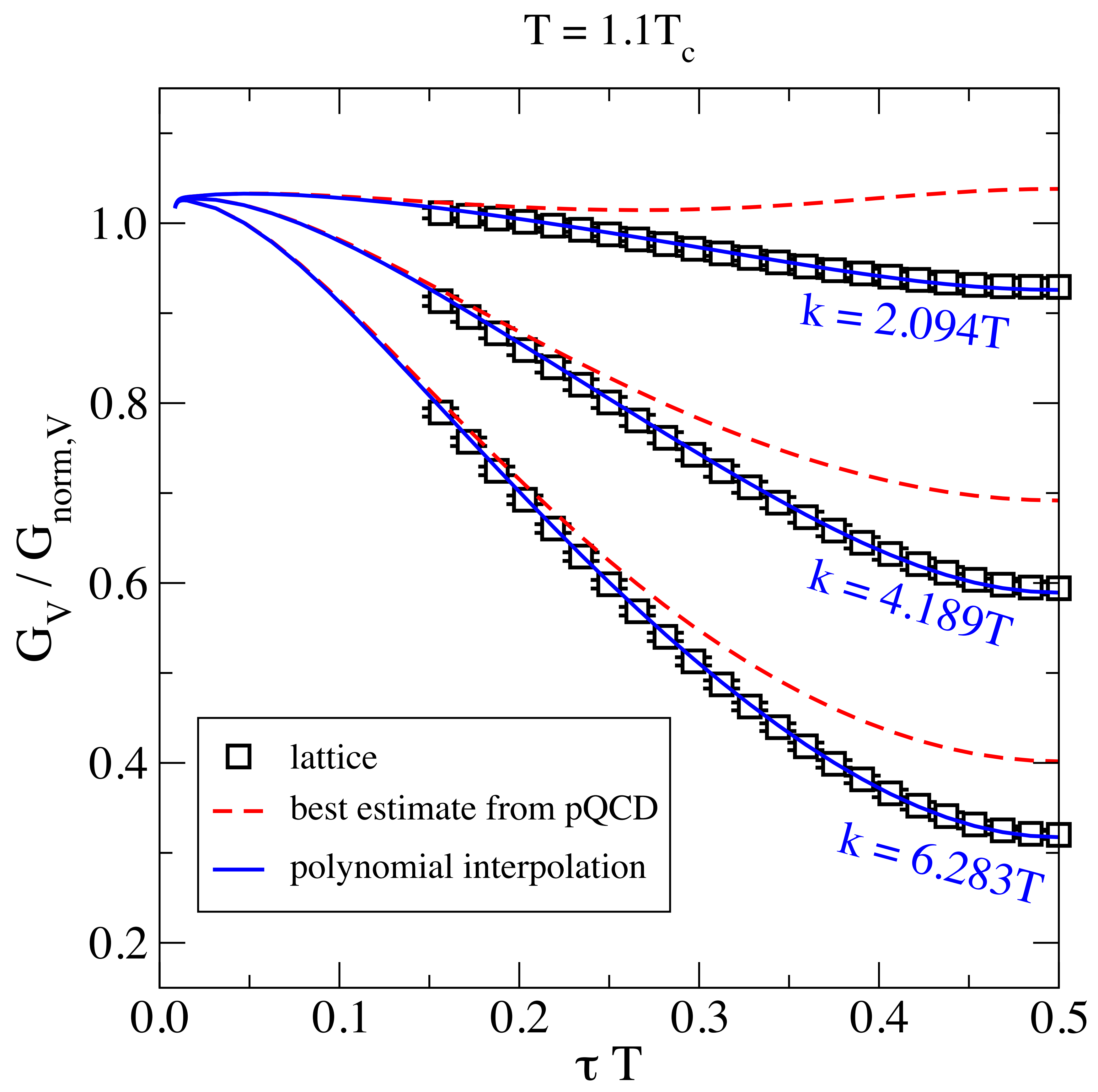
- Precise knowledge of the rates of the associated error uncertainty is very important for phenomenology
- In equilibrium, at $k \gtrsim \pi T$, NLO pQCD calculations, hybrid pQCD/lattice approaches and lattice reconstructed spf are now becoming available and can be used to constrain the uncertainty.
- Equilibrium pQCD photon rate reliable to O(50)% or less now. Transition to low-mass dileptons smooth, theory interpolation will be improved in the near future
- Elsewhere in this workshop: beyond-equilibrium rates

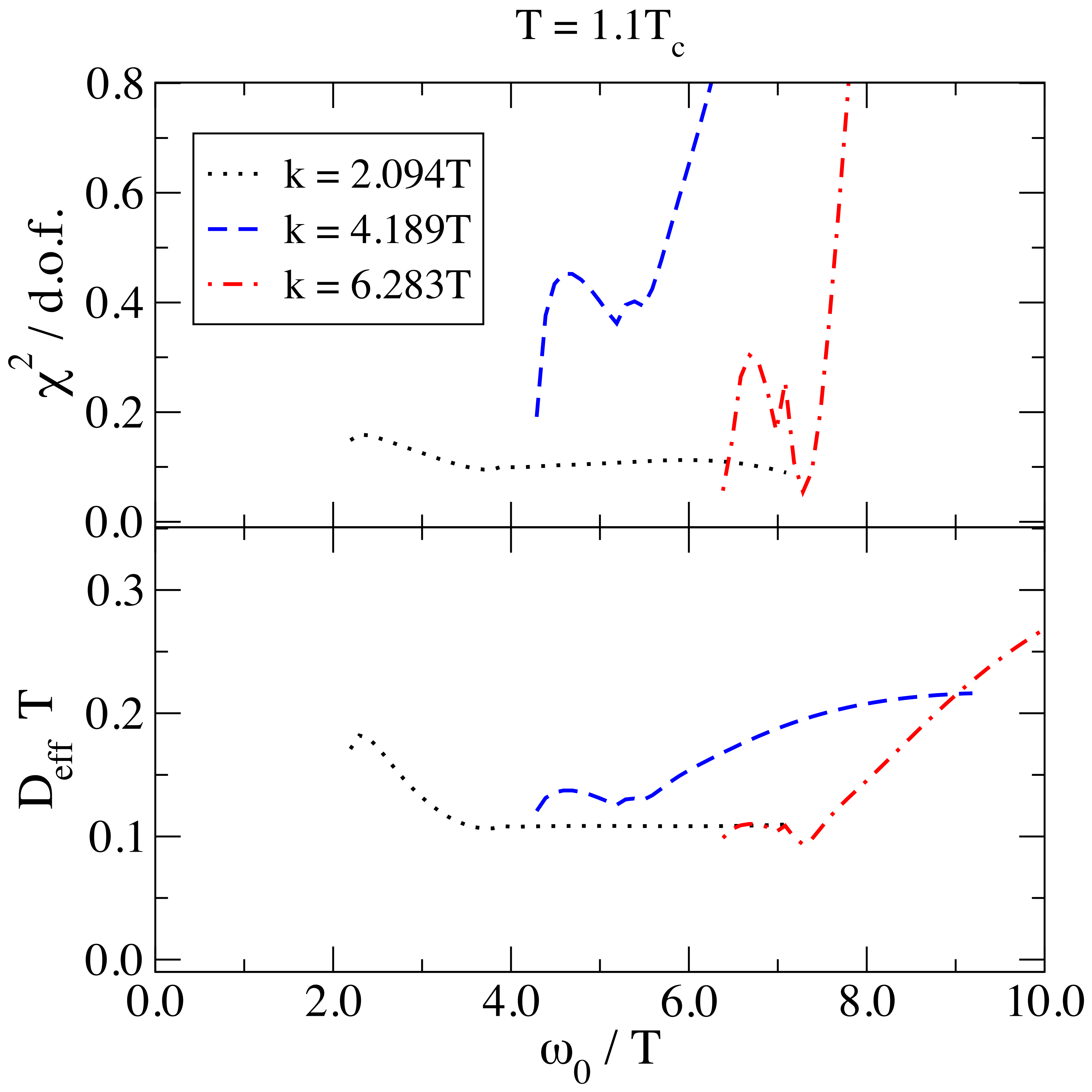
Backup

- If k>0 spf describes DIS ($k^0<k$), photons ($k^0=k$) and dileptons ($k^0>k$)



- Plots and spectral function from Jackson Laine **JHEP1109** (2019)
  Lattice from Cè Harris H.Meyer Steinberg Toniato **PRD102** (2020)

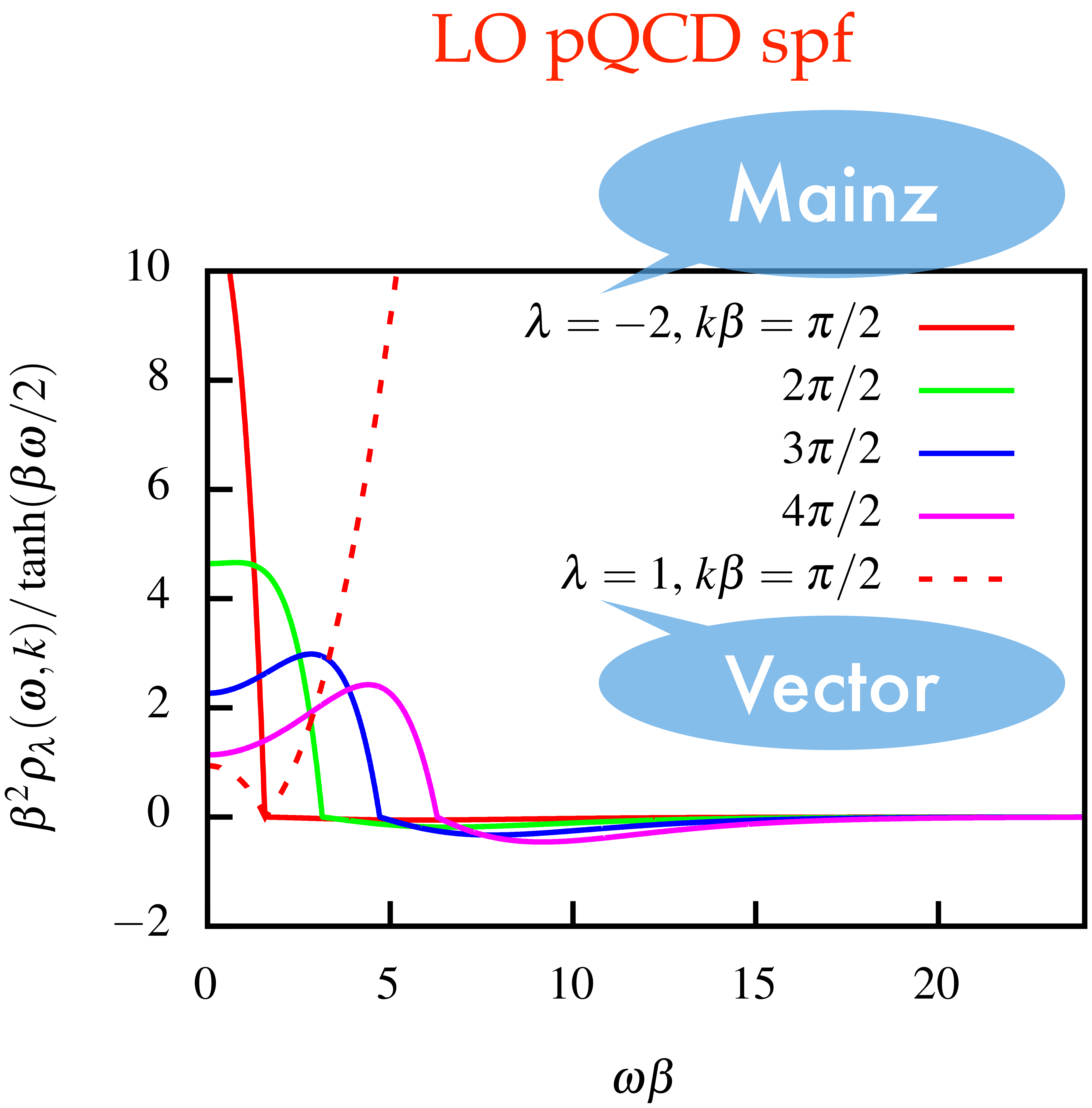




T/T_c	k/T	α/T	β/T^2	γ/T	$TD_{\text{eff}}\vert_{n_{\max}=0}$	$TD_{\text{eff}}\vert_{n_{\max}=1}$
1.1	2.094	0.028(15)	2.072	1.611	0.108(4)	0.019(153)
	4.189	0.091(8)	2.325	1.963	0.130(1)	0.066(45)
	6.283	0.105(4)	2.498	2.331	0.109(1)	0.102(8)
1.3	1.833	0.024(17)	2.038	1.558	0.093(5)	0.153(119)
	3.665	0.112(10)	2.229	1.984	0.119(1)	0.111(59)
	5.498	0.141(6)	2.367	2.438	0.094(1)	0.097(13)

JG Kaczmarek
Laine F.Meyer

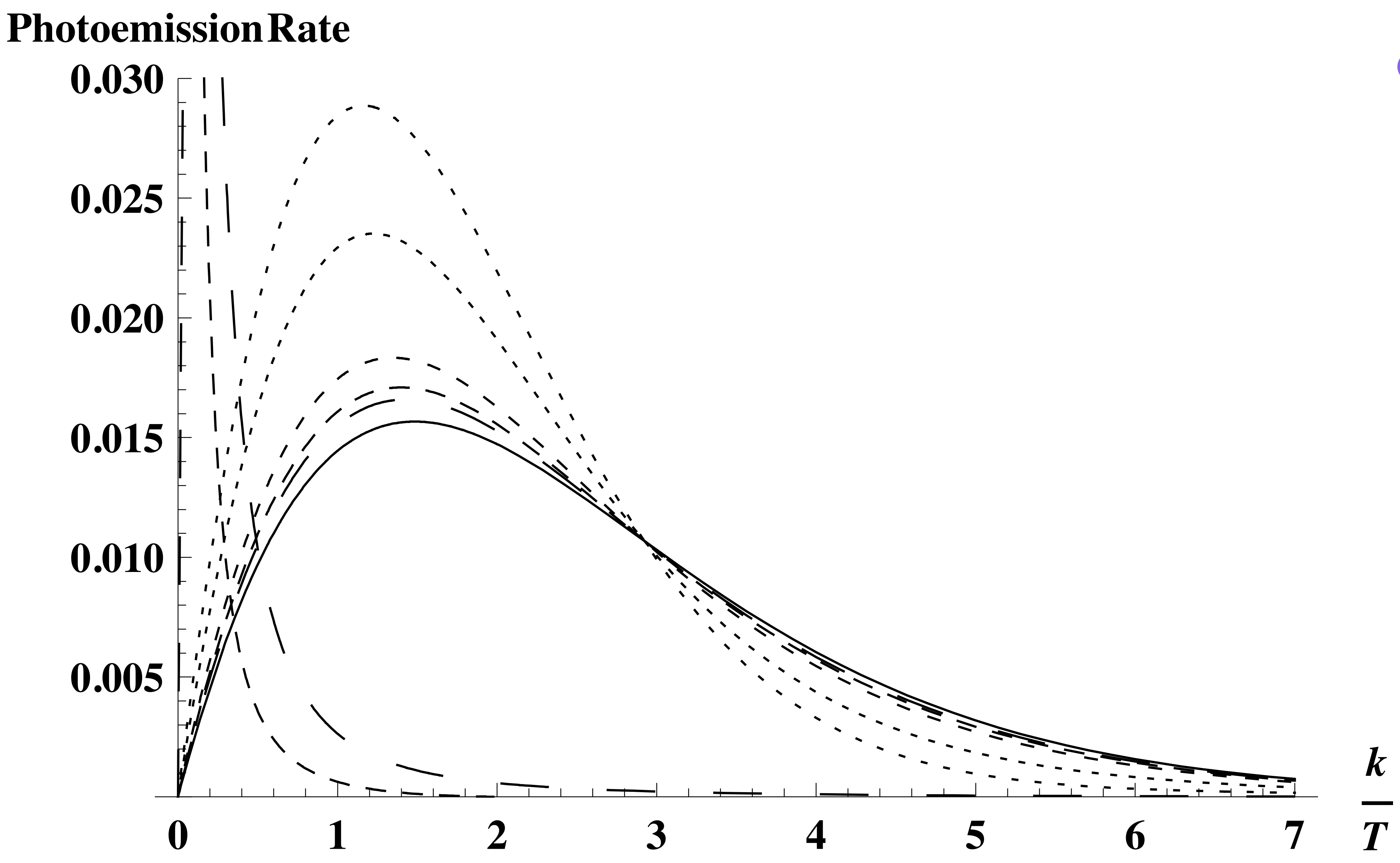
- Backus-Gilbert method: linear map from the space of functions in the time domain, G, to the space of functions on the frequency domain, ρ_{BG}
- It is exact for constant spfs and advantageous for a slowly varying spf
- The Mainz spf might indeed be slowly varying, or at least much slower than the vector one



Brandt Francis Harris H.Meyer Steinberg

AdS/CFT approaches

- Gauge a U(1) subgroup of $\mathcal{N}=4$: that's your photon
- LO at weak coupling, $\lambda \to \infty$ at strong coupling in equilibrium
  Caron-Huot Kovtun Moore Starinets Yaffe **JHEP06012** (2006)
- $1/\lambda$ corrections Hassanain Schvellinger **JHEP1212** (2012)
- Holographic thermalizations (out of equilibrium) Baier Stricker Taanila Vuorinen (2012), Steineder Stricker Vuorinen (2013)



- Hassanain Schvellinger strong coupling for decreasing lambda (finer dashing) compared with LO weak coupling (leftmost curves)

- Steineder *et al* strong coupling e.m. spectral function at equilibrium (dashed) and in the thermalizing metric (cont.). c=k/ω