Photon emission in $\pi\pi$ scattering and Low's theorem revised

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EMMI Workshop, Sept. 2021

1 Introduction

Some months ago Johanna Stachel, Peter Braun-Munzinger and collaborators initiated a seminar series on soft-photon production in hadronic reactions in connection with the plans for the ALICE 3 detector. I have been interested in soft photons for more than 30 years. Thus, I was glad to collaborate in these seminars. Antoni Szczurek and Piotr Lebiedowicz joined in and we found that we could probably say something interesting for high-energy exclusive diffractive reactions without and with photon emission. We started our investigations with the simplest processes we could think of:

$$\pi^- + \pi^0 \longrightarrow \pi^- + \pi^0$$

$$\pi^- + \pi^0 \longrightarrow \pi^- + \pi^0 + \gamma$$

Our results obtained so far are available from

P. Lebiedowicz, O.N., A. Szczurek,
"High-energy $\pi\pi$ scattering without and with photon radiation", arXiv: 2107.10829.

What I can present today relies on this article.

Of course, photon emission in hadronic reactions has been investigated in many theoretical and experimental papers before. In this short talk I cannot do justice to them. I shall only mention the important paper by Francis Low:

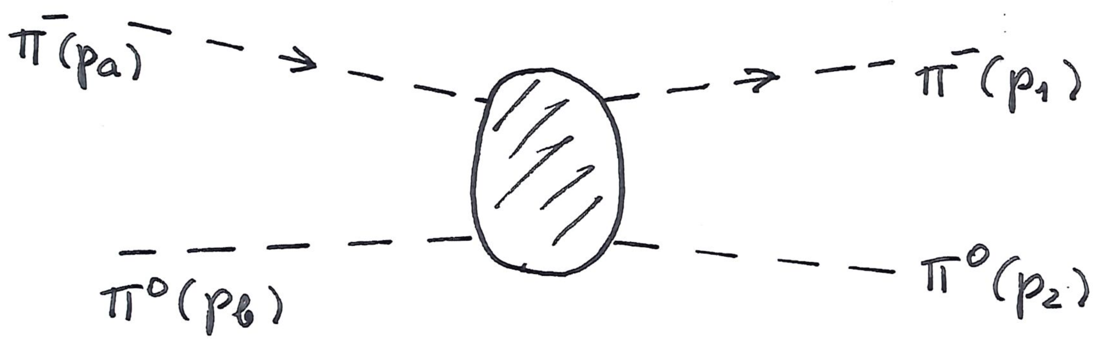
F. Low, "Bremsstrahlung of very low-energy quanta in elementary particle collisions", PR 110 (1958) 974,

which contains Low's theorem. We find that this theorem must be revised, as I shall try to convince you.

2 QFT analysis of $\pi\pi \longrightarrow \pi\pi$ and $\pi\pi \longrightarrow \pi\pi\gamma$

We consider the reaction, both on-shell and off shell,

$$\pi^-(p_a) + \pi^0(p_b) \longrightarrow \pi^-(p_1) + \pi^0(p_2)$$



We have always energy - momentum conservation

$$p_a + p_b = p_1 + p_2$$

As kinematic variables we have the masses of the, in general off shell, pions, an energy and a momentum transfer variable.

$$s_L = p_a \cdot p_b + p_1 \cdot p_2 \,, \qquad t = (p_a - p_1)^2 = (p_b - p_2)^2 \,,$$

$$m_a^2 = p_a^2 \,, \quad m_b^2 = p_b^2 \,, \quad m_1^2 = p_1^2 \,, \quad m_2^2 = p_2^2 \,.$$

Following Low we use here s_L instead of Mandelstam's variable

$$s = s_L + \frac{1}{2}\left(m_a^2 + m_b^2 + m_1^2 + m_2^2\right).$$

The scattering amplitude for $\pi^- \pi^0 \longrightarrow \pi^- \pi^0$ can only depend on the above variables

$$T(p_a, p_b, p_1, p_2) = \mathcal{M}^{(0)}(s_L, t, m_a^2, m_b^2, m_1^2, m_2^2)\,.$$

For the on-shell amplitude we have

$$m_a^2 = m_b^2 = m_1^2 = m_2^2 = m_\pi^2 \,.$$

Now we come to the photon-emission reaction (on shell)

$$\pi^-(p_a) + \pi^0(p_b) \longrightarrow \pi^-(p_1') + \pi^0(p_2') + \gamma(k,\varepsilon)$$

where we have from energy-momentum conservation

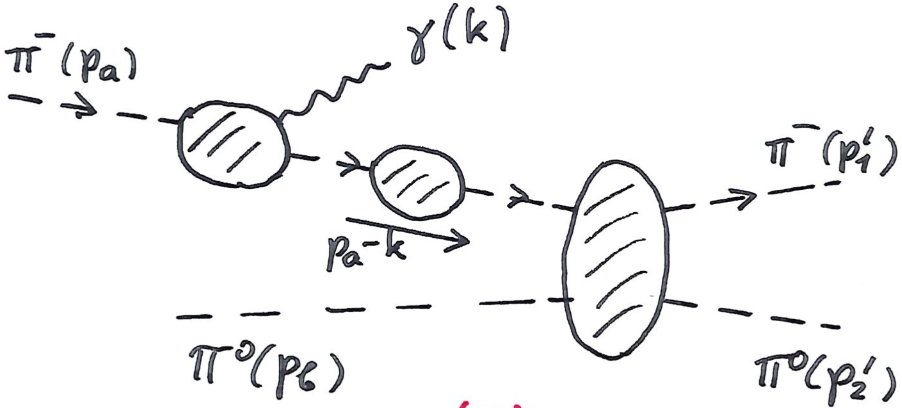
$$p_a + p_b = p_1' + p_2' + k .$$

Note that for $k \neq 0$ we must have a change of p_1', p_2' from the values p_1, p_2 for $k = 0$.

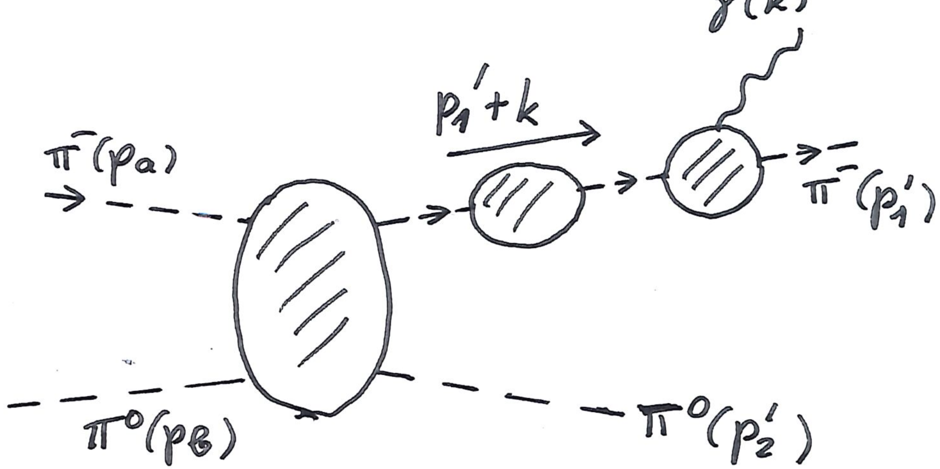
The amplitude for the above reaction is

$$\langle \gamma(k,\varepsilon), \pi^-(p_1'), \pi^0(p_2') | T | \pi^-(p_a), \pi^0(p_b) \rangle = \varepsilon^{\lambda *} \mathcal{M}_\lambda .$$

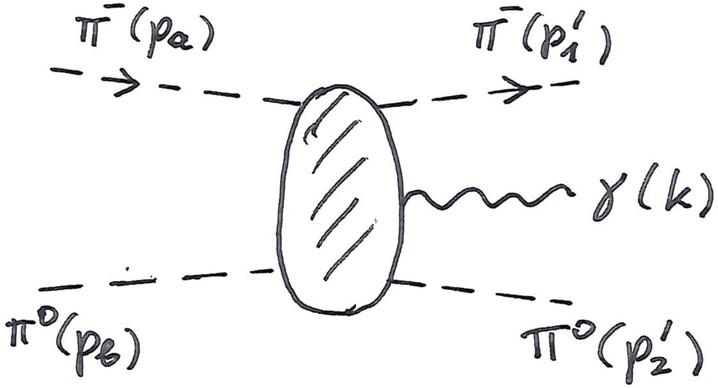
Diagrams for $\mathcal{M}_\lambda$



(a)



(b)

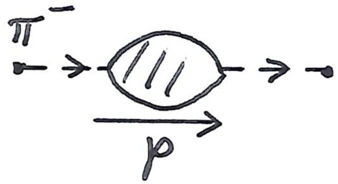


(c)

(a), (b) 1 particle reducible

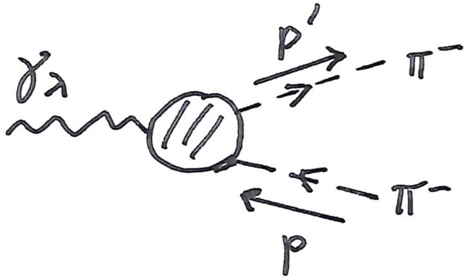
(c) 1 particle irreducible

pion propagator:



$i\Delta(p^2)$

vertex $\gamma\pi\pi$:



$ie\Gamma_\lambda(p',p)$

With the $\pi\pi \to \pi\pi$ off-shell amplitude, the pion propagator, and the $\gamma\pi\pi$ vertex we get

$$\mathcal{M}_\lambda^{(a)} = -e\, \mathcal{M}^{(0,a)}\, \Delta[(p_a-k)^2]\, \Gamma_\lambda(p_a-k, p_a),$$

$$\begin{aligned}\mathcal{M}^{(0,a)} &= T(p_a-k, p_b, p_1', p_2') \\ &= \mathcal{M}^{(0)}\left[(p_a-k, p_b) + p_1' \cdot p_2', (p_b-p_2')^2, (p_a-k)^2, m_\pi^2, m_\pi^2, m_\pi^2\right],\end{aligned}$$

$$\mathcal{M}_\lambda^{(b)} = -e\, \Gamma_\lambda(p_1', p_1'+k)\, \Delta[(p_1'+k)^2]\, \mathcal{M}^{(0,b)},$$

$$\begin{aligned}\mathcal{M}^{(0,b)} &= T(p_a, p_b, p_1'+k, p_2') \\ &= \mathcal{M}^{(0)}\left[p_a \cdot p_b + (p_1'+k, p_2'), (p_b-p_2')^2, m_\pi^2, m_\pi^2, (p_1'+k)^2, m_\pi^2\right].\end{aligned}$$

We shall now use the best tool from QFT: gauge invariance.

With this we get the Ward-Takahashi identity

$$(p'-p)^\lambda \Gamma_\lambda(p',p) = \Delta^{-1}(p'^2) - \Delta^{-1}(p^2)$$

and the condition

$$k^\lambda \left[m_\lambda^{(a)} + m_\lambda^{(b)} + m_\lambda^{(c)} \right] = 0.$$

As a consequence of these we find

$$k^\lambda m_\lambda^{(c)} = -e\, m^{(0,a)} + e\, m^{(0,b)}.$$

3 The soft-photon expansion

In this section we shall study the expansion of the amplitude $\mathcal{M}_\lambda$ of the reaction

$$\pi^-(p_a) + \pi^0(p_b) \longrightarrow \pi^-(p_1') + \pi^0(p_2') + \gamma(k,\varepsilon)$$

for small k where we set $k^0 = \omega$. For $k = 0$ we have $p_1' = p_1$, $p_2' = p_2$, with p_1, p_2 corresponding to the reaction $\pi\pi \to \pi\pi$. But as we go to $k \neq 0$ we also have to change p_1' and p_2'. We set:

$$p_1' = p_1 - l_1 \quad , \qquad p_2' = p_2 - l_2 .$$

Energy - momentum conservation gives:

$$p_a + p_b = p_1 + p_2 ,$$

$$p_a + p_b = p_1' + p_2' + k = p_1 + p_2 - l_1 - l_2 + k .$$

This gives the conditions:

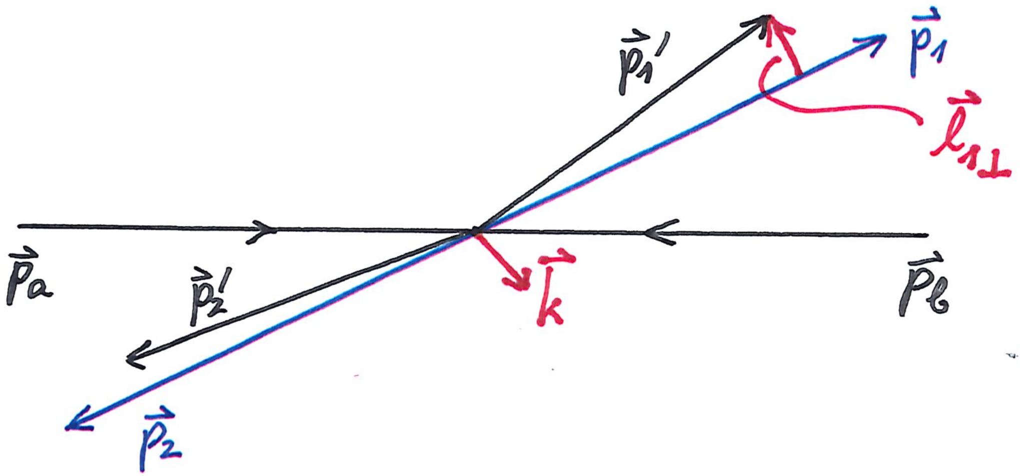
$$l_1 + l_2 = k,$$

$$(p_1 - l_1)^2 = p_1'^2 = m_\pi^2,$$

$$(p_2 - l_2)^2 = p_2'^2 = m_\pi^2.$$

For given k these are 6 equations for the 8 unknowns l_1, l_2. Therefore we get a 2 parameter solution.

Now we make the assumption that we consider only small deviations of p_1' from p_1 and p_2' from p_2. That is, we assume l_1, l_2 to be of order ω. Working in the overall c.m. system the above equation is easily solved with, e.g., $\vec{l}_{1\perp}$ playing the role of the 2 free parameters.



This gives us our first point:

- Expansion of the photon-emission amplitude in k alone around $k = 0$ is not a good idea, since k alone does not specify the final state completely. One possibility is to expand in k and $\vec{\ell}_{1\perp}$ which are independent and together specify the final state completely.

Now it is straightforward to expand the amplitude $\mathcal{M}_\lambda$ in powers of ω. For real photons ($k^2=0$) and neglecting gauge terms we get:

$$\mathcal{M}_\lambda = e\left[\frac{p_{a\lambda}}{(p_a\cdot k)} - \frac{p_{1\lambda}}{(p_1\cdot k)}\right]\mathcal{M}^{(0)}(s_L, t, m_\pi^2, m_\pi^2, m_\pi^2, m_\pi^2) \qquad \mathcal{O}\left(\frac{1}{\omega}\right)$$

$$+\, e\Big\{ -\frac{1}{(p_1\cdot k)^2}\left[p_{1\lambda}(l_1\cdot k) - l_{1\lambda}(p_1\cdot k)\right]$$

$$+\, 2\left[-p_{a\lambda}\frac{(p_b\cdot k)}{(p_a\cdot k)} + p_{b\lambda}\right]\frac{\partial}{\partial s_L} - 2\left[(p_a - p_1, k) - (p_a\cdot l_1)\right] \qquad \mathcal{O}(\omega^0)$$

$$\left[\frac{p_{a\lambda}}{(p_a\cdot k)} - \frac{p_{1\lambda}}{(p_1\cdot k)}\right]\frac{\partial}{\partial t}\Big\}\mathcal{M}^{(0)}(s_L, t, m_\pi^2, m_\pi^2, m_\pi^2, m_\pi^2)$$

$$+\, \mathcal{O}(\omega)$$

The terms of order ω^{-1} and ω^0 are determined by the on-shell amplitude $\mathcal{M}^{(0)}$

Low's result reads

$$
\begin{aligned}
\mathcal{M}_{k\lambda}^{\mathrm{Low}} &= e\left[\frac{p_{a\lambda}}{(p_a\cdot k)} - \frac{p_{1\lambda}}{(p_1\cdot k)}\right]\mathcal{M}^{(0)}(s_L, t, m_\pi^2, m_\pi^2, m_\pi^2, m_\pi^2) \qquad \mathcal{O}\left(\frac{1}{\omega}\right) \\
&+ e\left[-p_{a\lambda}\frac{(p_b\cdot k)}{(p_a\cdot k)} - p_{1\lambda}\frac{(p_2\cdot k)}{(p_1\cdot k)} + p_{b\lambda} + p_{2\lambda}\right] \qquad \mathcal{O}(\omega^0) \\
&\quad \frac{\partial}{\partial s_L}\mathcal{M}^{(0)}(s_L, t, m_\pi^2, m_\pi^2, m_\pi^2, m_\pi^2) \\
&+ \mathcal{O}(\omega)
\end{aligned}
$$

We agree with the ω^{-1} term but disagree with the ω^0 term.

What is the origin of this discrepancy?

We can reproduce Low's result by considering the reaction

$$\pi^-(p_a) + \pi^0(p_b) \longrightarrow \pi^-(p_1) + \pi^0(p_2) + \gamma(k,\varepsilon),$$

neglecting the fact that when going from $k=0$ to $k\neq 0$ we also have to go from $p_{1,2}$ to $p'_{1,2} \neq p_{1,2}$.

We <u>cannot</u> have for $k \neq 0$ simultanously:

$$p_a + p_b = p_1 + p_2$$

and

$$p_a + p_b = p_1 + p_2 + k.$$

- Low's result corresponds to the expansion of the above fictitious process where energy-momentum conservation is not respected.

4 The reactions $\pi\pi \to \pi\pi$ and $\pi\pi \to \pi\pi\gamma$ in the tensor-pomeron model

Now we shall construct an explicit model for these reactions at high energies and small momentum transfer:

$$\sqrt{s} \gg 1 \text{ GeV} \, , \qquad \sqrt{|t|} \lesssim 1 \text{ GeV}.$$

This is the kinematic region where the amplitudes are governed by Regge exchanges. We use the specific model developed in:

C. Ewerz, M. Maniatis, O. N., "A model for soft high-energy scattering: tensor pomeron and vector odderon", Annals Phys. 342 (2014) 31.

We consider the usual Regge exchanges with charge conjugation $C=+1$ and $C=-1$:

$C=+1$ $\quad$ $\mathbb{P}$ pomeron,

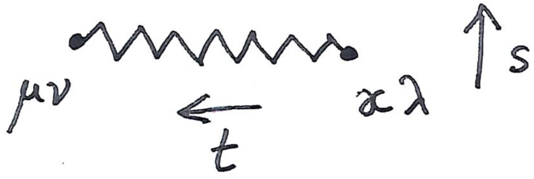
$\qquad\qquad$ f_{2R}, a_{2R} : f_2 and a_2 reggeons,

$C=-1$ $\quad$ $\mathbb{O}$ odderon,

$\qquad\qquad$ ω_R, ρ_R : ω and ρ reggeons.

We assume that all $C=+1$ exchange objects can be described as effective spin 2 symmetric tensors, all $C=-1$ exchanges as effective vectors.

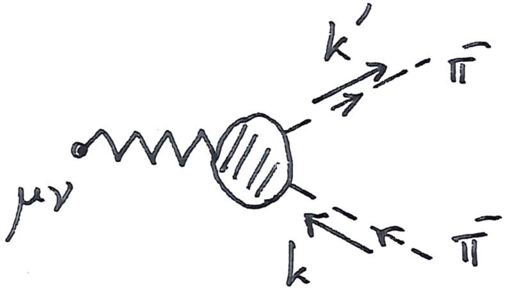
Example: $\mathbb{P}$ propagator and $\mathbb{P}\pi\pi$ coupling:



$$i\Delta^{(\mathbb{P})}_{\mu\nu,\kappa\lambda}(s,t) =$$

$$\frac{1}{4s}\left(g_{\mu\kappa}g_{\nu\lambda}+g_{\mu\lambda}g_{\nu\kappa}-\frac{1}{2}g_{\mu\nu}g_{\kappa\lambda}\right)\left(-is\alpha'_{\mathbb{P}}\right)^{\alpha_{\mathbb{P}}(t)-1},$$

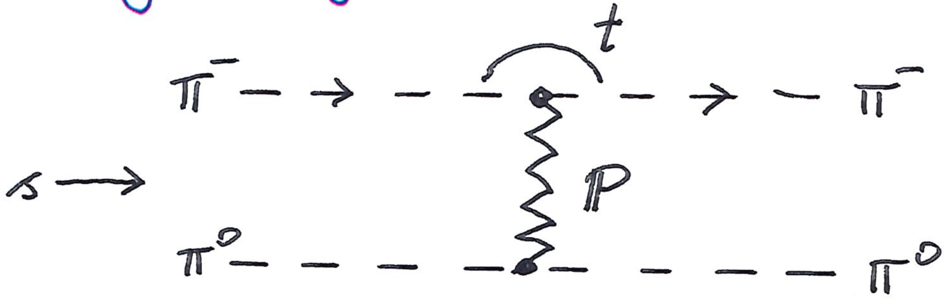
$$\alpha_{\mathbb{P}}(t)=\alpha_{\mathbb{P}}(0)+\alpha'_{\mathbb{P}}t\ ,\quad \alpha_{\mathbb{P}}(0)=1.0808\ ,\quad \alpha'_{\mathbb{P}}=0.25\ \mathrm{GeV}^{-2},$$



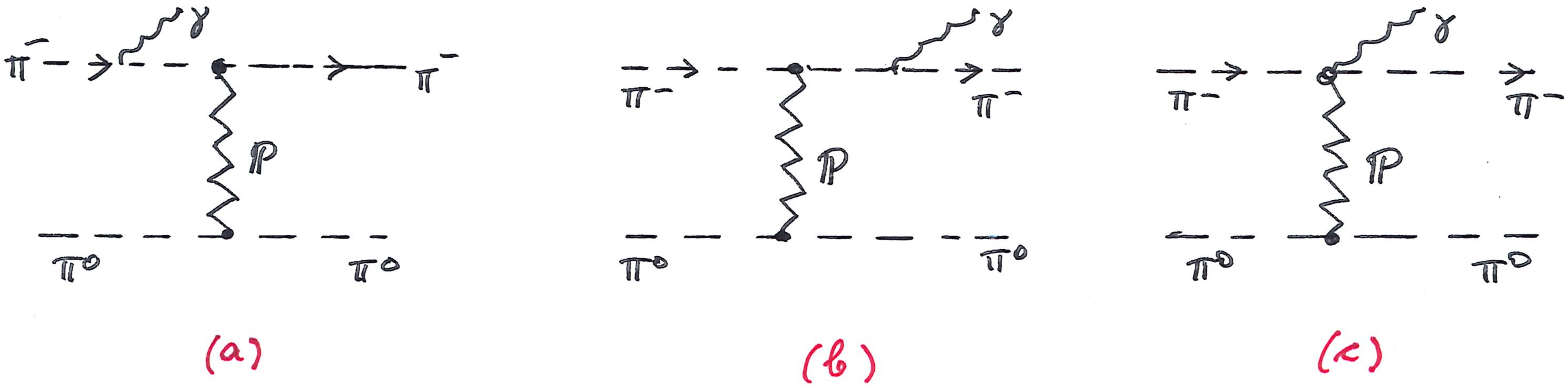
$$i\Gamma^{(\mathbb{P}\pi\pi)}_{\mu\nu}(k',k) = -i\,2\beta_{\mathbb{P}\pi\pi}F_M\left[(k'-k)^2\right]$$
$$\left[(k'+k)_\mu(k'+k)_\nu-\frac{1}{4}g_{\mu\nu}(k'+k)^2\right]$$

$$\beta_{\mathbb{P}\pi\pi}=1.76\ \mathrm{GeV}^{-1},\quad F_M(t)=\frac{m_0^2}{m_0^2-t}\ ,\quad m_0^2=0.50\ \mathrm{GeV}^2$$

$\mathbb{P}$ exchange diagrams for $\pi^-\pi^0 \to \pi^-\pi^0$ and $\pi^-\pi^0 \to \pi^-\pi^0\gamma$:



$$\mathcal{M}^{(0)} = i\left[2\beta_{\mathbb{P}\pi\pi} F_M(t)\right]^2 \frac{1}{4s}\left(-is\alpha'_{\mathbb{P}}\right)^{\alpha_{\mathbb{P}}(t)-1}$$
$$\left[2\left(2s+t-m_a^2-m_b^2-m_1^2-m_2^2\right)^2-\frac{1}{2}\left(-t+2m_a^2+2m_1^2\right)\left(-t+2m_b^2+2m_2^2\right)\right].$$



$\mathcal{M}_\lambda^{(a)}$ $\mathcal{M}_\lambda^{(b)}$ $\mathcal{M}_\lambda^{(c)}$

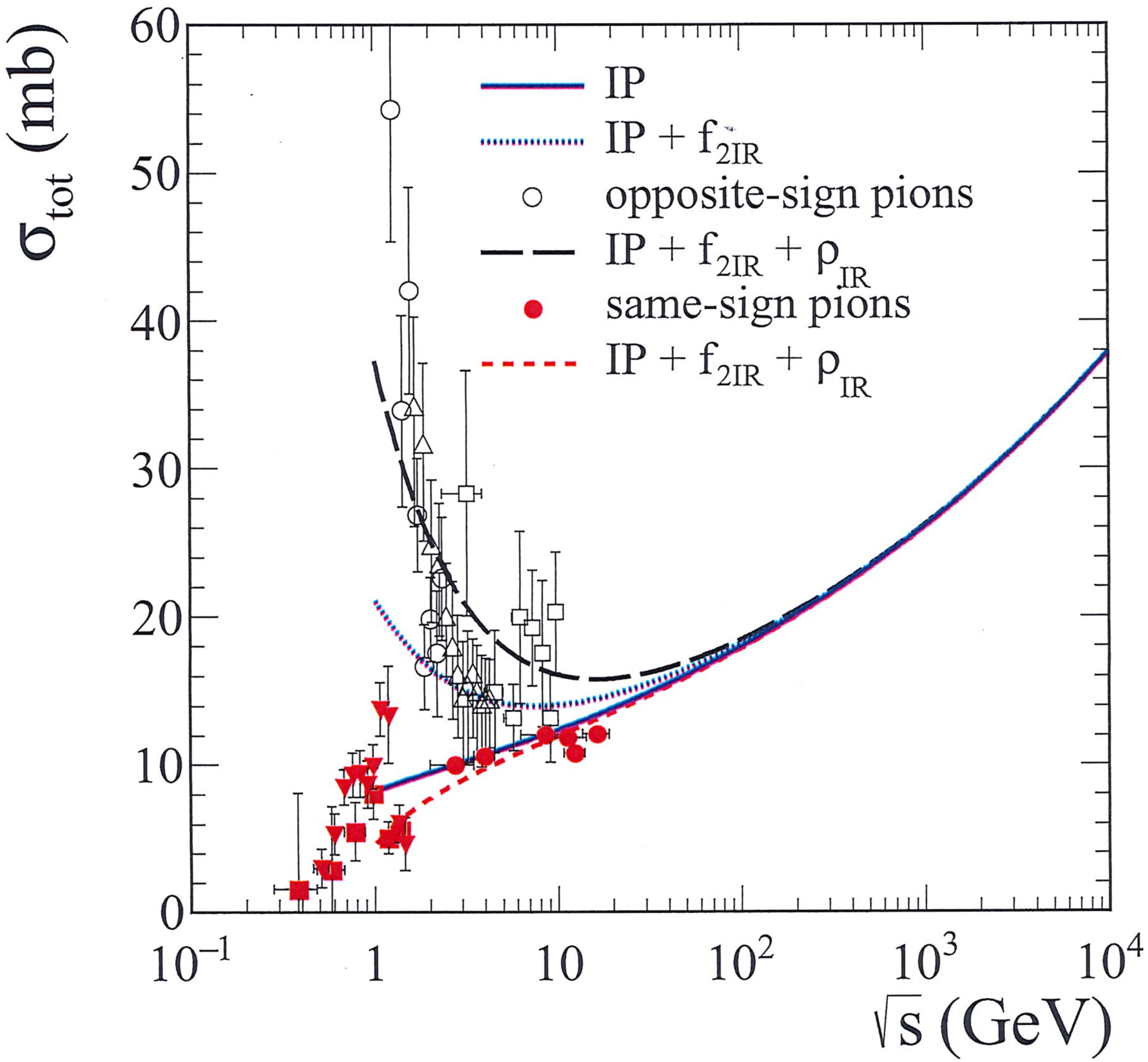
Now we can calculate the amplitudes $\mathcal{M}_\lambda^{(a)}$ and $\mathcal{M}_\lambda^{(b)}$ in terms of $\mathcal{M}^{(0)}$. For $\mathcal{M}_\lambda^{(c)}$ we have the QFT relation

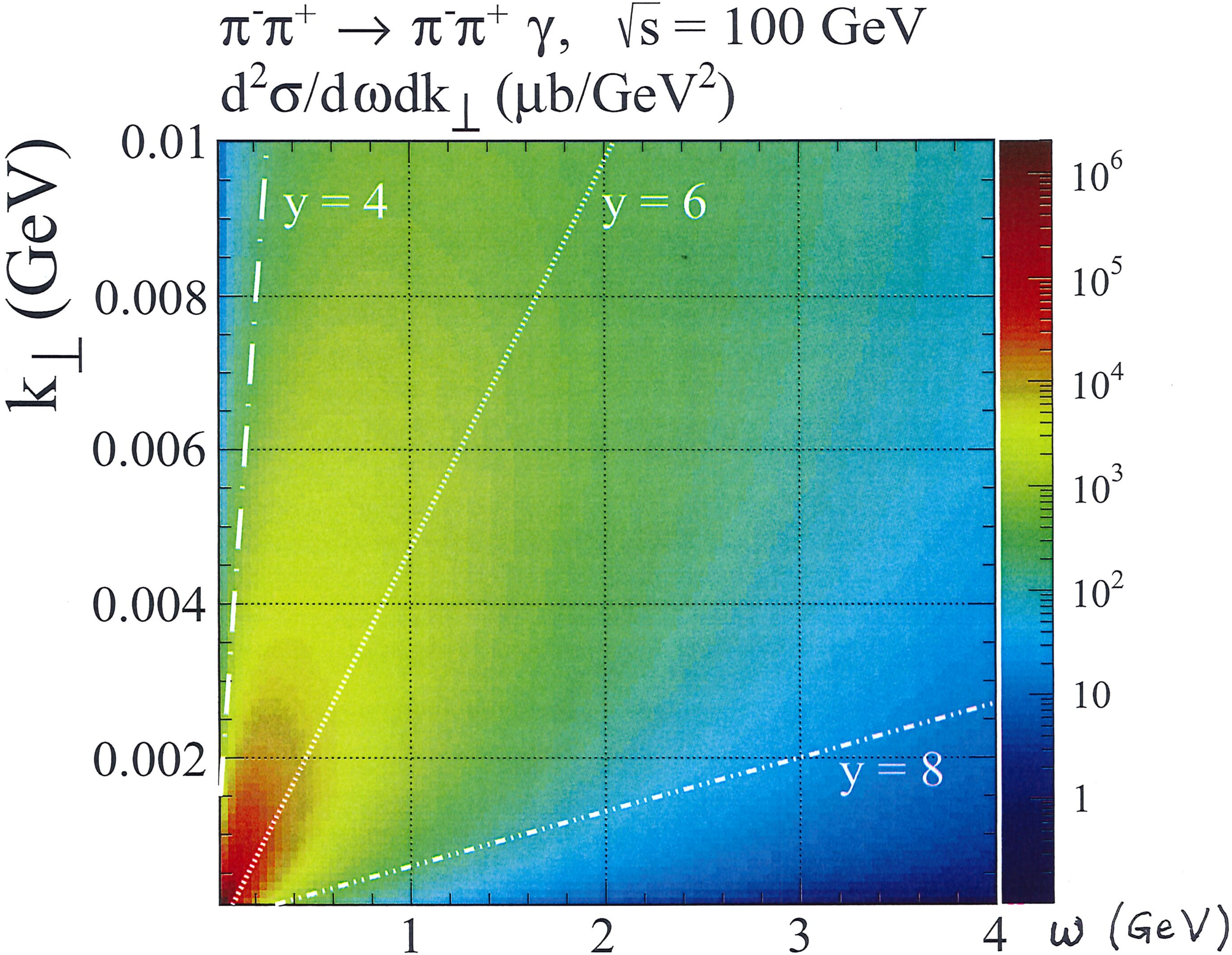
$$k^\lambda \mathcal{M}_\lambda^{(c)} = - k^\lambda \mathcal{M}_\lambda^{(a)} - k^\lambda \mathcal{M}_\lambda^{(b)} .$$

To order ω^0 this equation determined $\mathcal{M}_\lambda^{(c)}$ uniquely. In general this is no longer the case. We choose for our "standard" model the simplest solution. But clearly, other solutions are possible. This is a place where <u>anomalous</u> contributions to soft-photon production easily could come up.

Next I show some results of our calculations.

Results for $\pi^-\pi^+$ and $\pi^+\pi^+$ or $\pi^-\pi^-$ total cross sections





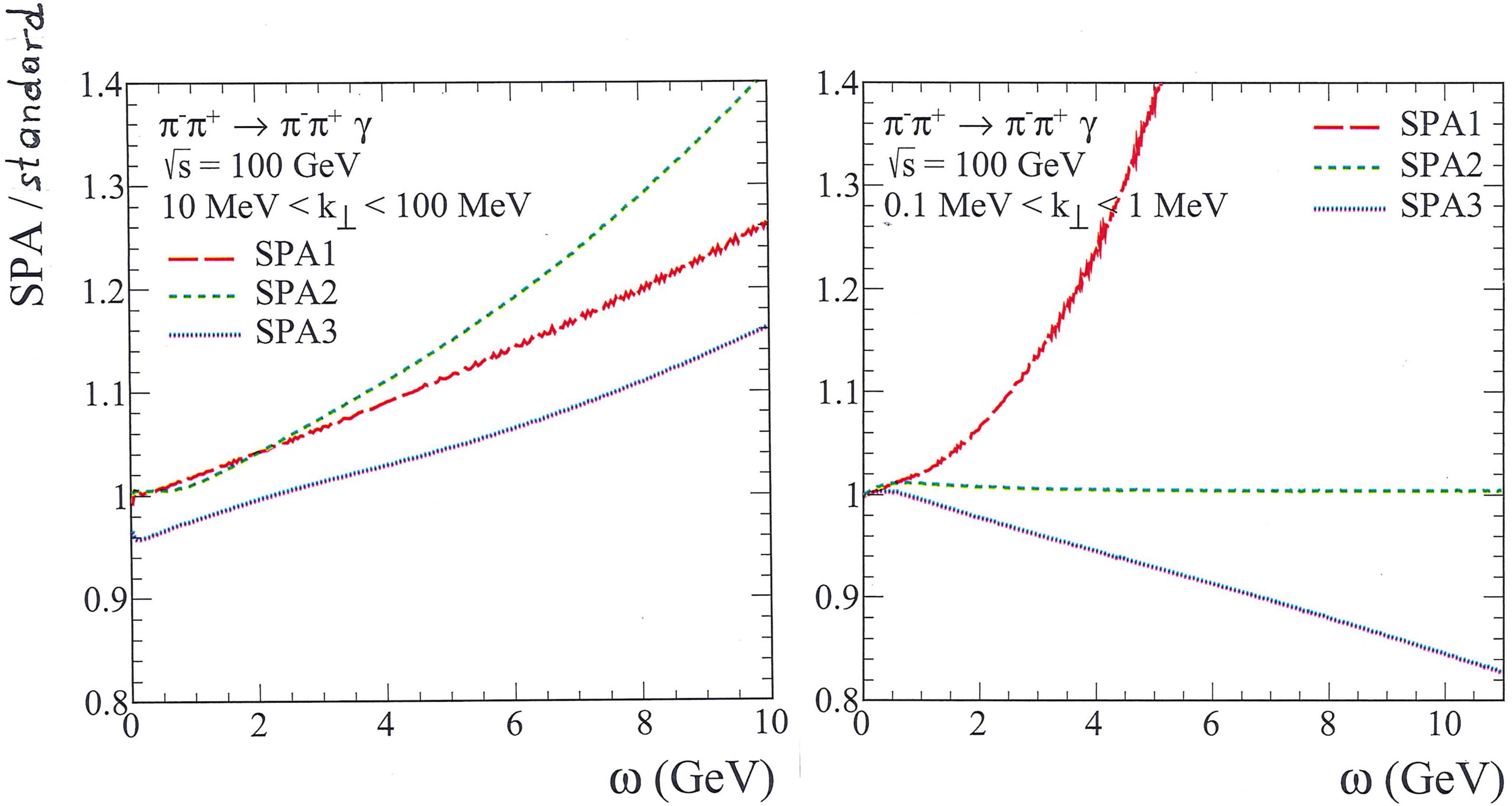
$\pi^-\pi^+ \to \pi^-\pi^+\gamma$, $\sqrt{s} = 100$ GeV
$d^2\sigma/d\omega dk_\perp$ (μb/GeV^2)
$k_\perp$ (GeV)
0.01
0.008
0.006
0.004
0.002
y = 4
y = 6
y = 8
1
2
3
4
ω (GeV)
10^6
10^5
10^4
10^3
10^2
10
1

It is also of great interest to compare our "standard" result for $\pi^- \pi^+ \to \pi^- \pi^+ \gamma$ to various soft-photon approximations (SPAs). I only show you what we call SPA1, keeping in the photon-emission amplitude only the term $\propto 1/\omega$.

I show you the ratio

$$R(\omega) = \frac{(d\sigma/d\omega)_{SPA}}{(d\sigma/d\omega)_{standard}}$$

for $\sqrt{s} = 100$ GeV and two $k_\perp$ intervals.


SPA / standard
$\pi^-\pi^+ \to \pi^-\pi^+ \gamma$
$\sqrt{s}$ = 100 GeV
10 MeV < $k_\perp$ < 100 MeV
SPA1
SPA2
SPA3
ω (GeV)
$\pi^-\pi^+ \to \pi^-\pi^+ \gamma$
$\sqrt{s}$ = 100 GeV
0.1 MeV < $k_\perp$ < 1 MeV
SPA1
SPA2
SPA3
ω (GeV)

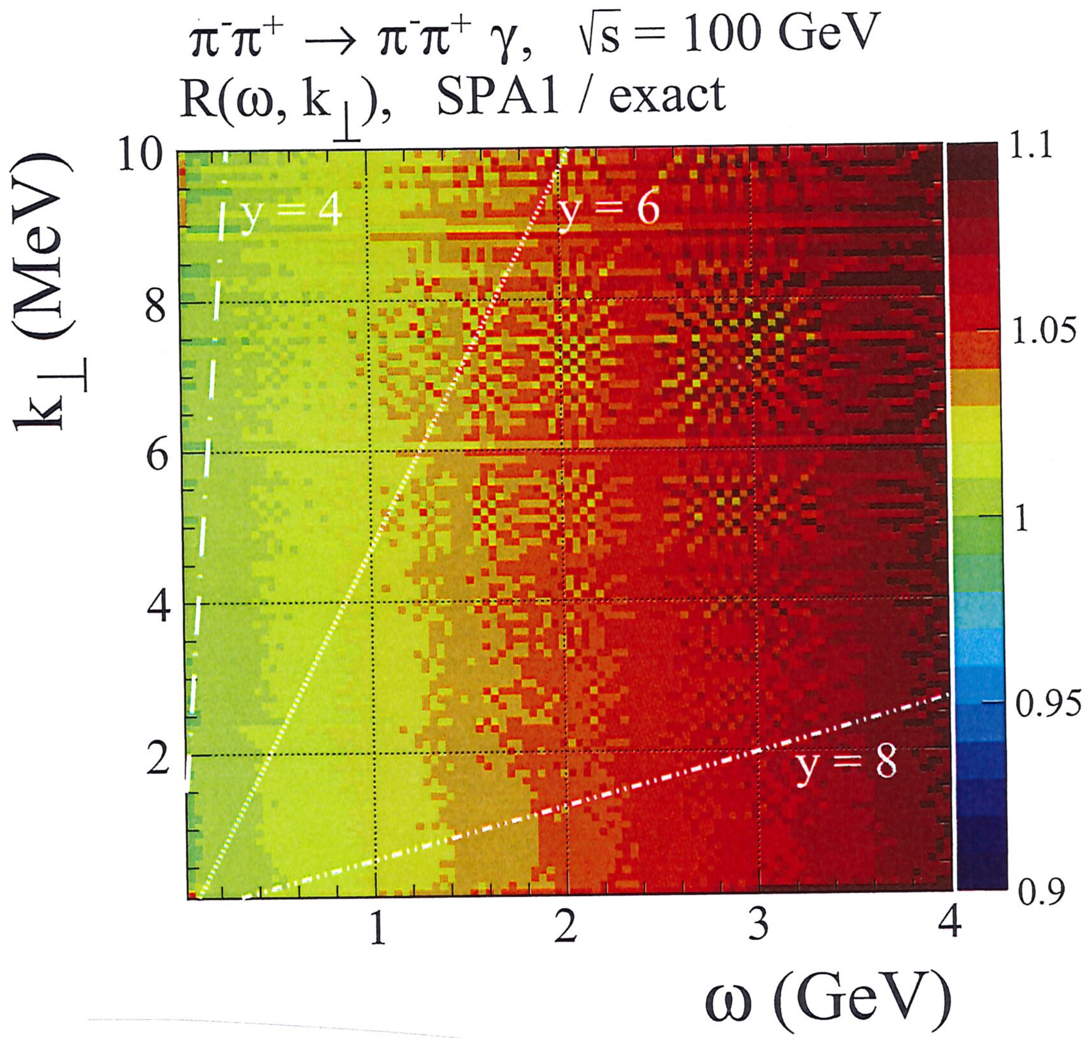
5 Conclusions

In this talk we have discussed the emission of soft photons in $\pi\pi$ elastic scattering.

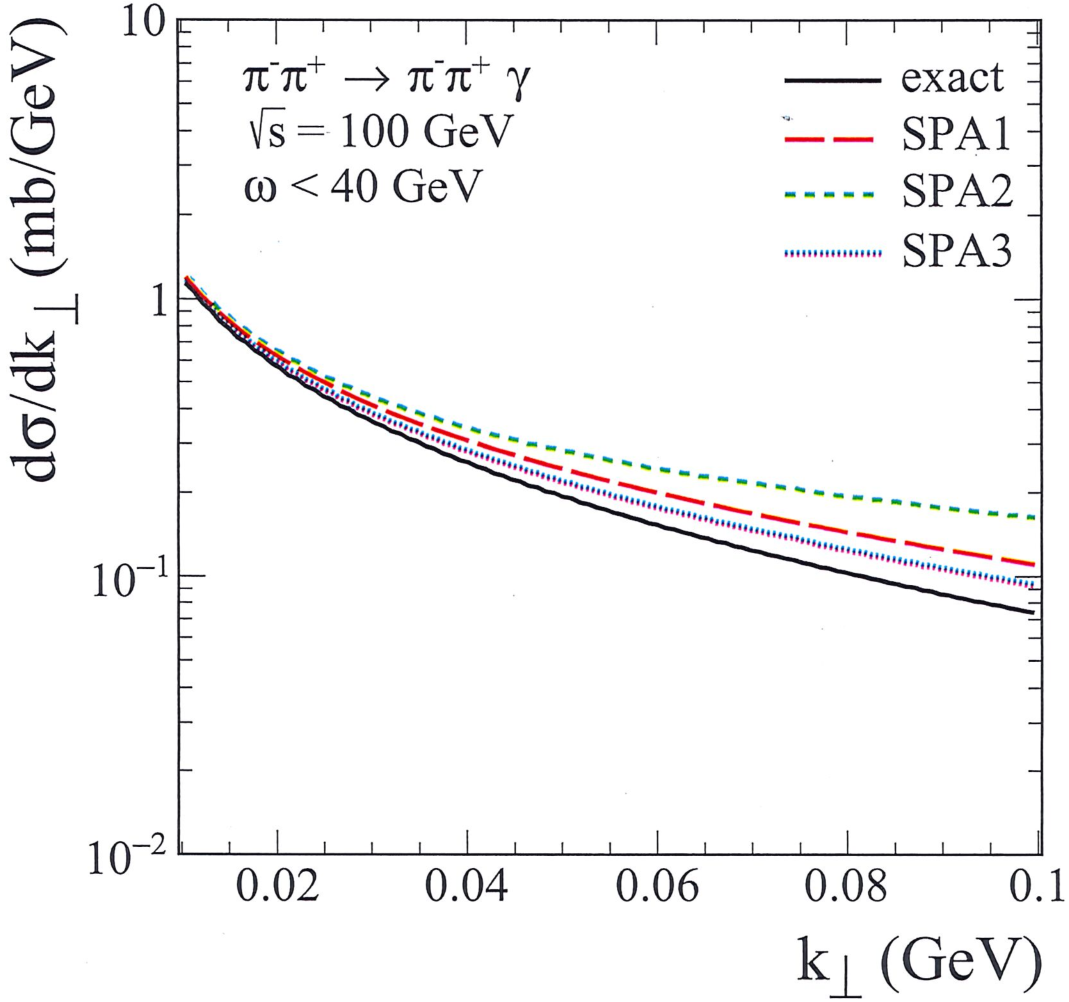
- For photon energy $\omega \to 0$ we examined Low's theorem. We found that the term of order ω^0 needs revision.

- We constructed a model for $\pi\pi \longrightarrow \pi\pi$ and $\pi\pi \longrightarrow \pi\pi\gamma$ for high energies and small momentum tranfers using the tensor-pomeron approach. We compared frequently used soft-photon approximations to our "standard" model results. For $\sqrt{s} = 100$ GeV we find reasonable agreement for $k_\perp \lesssim 10$ MeV and $\omega \lesssim 0.5$ GeV.

Thank you for
your attention!



$\pi^-\pi^+ \to \pi^-\pi^+\gamma$, $\sqrt{s} = 100$ GeV
$R(\omega, k_\perp)$, SPA1 / exact
$k_\perp$ (MeV)
ω (GeV)
y = 4
y = 6
y = 8
10
8
6
4
2
1
2
3
4
1.1
1.05
1
0.95
0.9


$\pi^-\pi^+ \to \pi^-\pi^+ \gamma$
$\sqrt{s}$ = 100 GeV
ω < 40 GeV
exact
SPA1
SPA2
SPA3
$d\sigma/dk_\perp$ (mb/GeV)
$k_\perp$ (GeV)
10
1
10^{-1}
10^{-2}
0.02
0.04
0.06
0.08
0.1

2 Applications of the tensor-pomeron model

By now we have made quite a number of applications of our tensor-pomeron model.

- $\gamma p \to \pi^+ \pi^- p$ Bolz, Ewerz, Maniatis, O.N., Sauter, Schöning, JHEP 01 (2015) 151.
- $p p \to p p$, spin dependence. Ewerz, Lebiedowicz, O.N., Szczurek, PL B 763 (2016) 382.

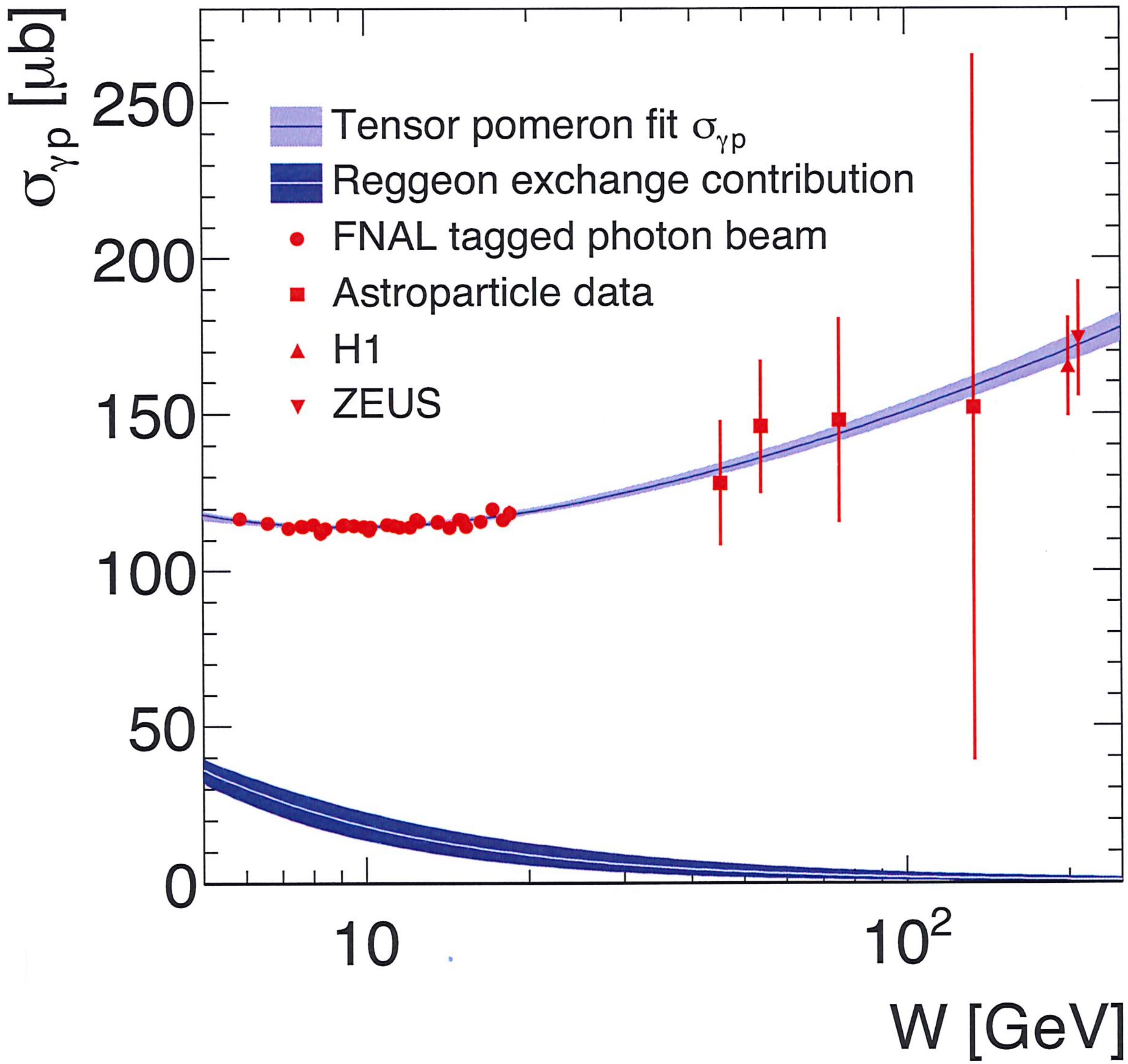
In this paper we could show that the data on the single spin flip in pp elastic scattering from the STAR experiment at $\sqrt{s} = 200$ GeV exclude a scalar character of the pomeron, but are perfectly compatible with our tensor-pomeron ansatz.

- photo production and low x DIS,

  Britzger, Ewerz, Glazov, O.N., Schmitt, PRD 100 (2019) 114007.

In this paper we could show that a vector pomeron decouples completely in the total photoabsorption cross section and in the structure functions of DIS. In contrast, with the tensor pomeron we get excellent fits. I show this for the case of $\sigma_{tot}(\gamma p)$ for real photons.

Fit results: photoproduction



We hope that ALice 3 will be able to measure $\sigma_{\gamma p}$ at even higher W in ultraperipheral Ap collisions.

- $pp \longrightarrow pXp$ **CEP reactions**

X		
η, η', f_0	Lebiedowicz, O.N., Szczurek, Ann. Phys. 344 (2014) 301	
ρ^0	— " —	, PRD 91 (2015) 074023
$\pi^+\pi^-, f_0, f_2$	— " —	, PRD 93 (2016) 054015
$\pi^+\pi^-\pi^+\pi^-$	— " —	, PRD 94 (2016) 034017
ρ^0 with proton diss.	— " —	, PRD 95 (2017) 034036
$p\bar{p}$	— " —	, PRD 97 (2018) 094027
K^+K^-	— " —	, PRD 98 (2018) 014001
$K^+K^-K^+K^-$ via $\phi\phi$	— " —	, PRD 99 (2019) 094034
$f_2 \longrightarrow \pi^+\pi^-$	— " —	, PRD 101 (2020) 034008
$\phi \longrightarrow K^+K^-, \mu^+\mu^-$	— " —	, PRD 101 (2020) 094012
$f_1(1285), f_1(1420)$	Lebiedowicz, Leutgeb, O.N., Rebhan, Szczurek, PRD 102 (2020) 114003	

- CEP of $p\bar{p}$ in ultraperipheral heavy ion collisions,
  Klusek-Gawenda, Lebiedowicz, O.N., Szczurek, PRD 96 (2017) 094029

Many thanks go to all colleagues with whom I had the pleasure to collaborate on these projects.

In my opinion CEP reactions deserve detailed studies with the new detector, which is studied here, for the LHC. Let me just mention the good possibilities to search for odderon effects in CEP of single ϕ and double ϕ final states X.
The ϕ's come out at low p_T and their K^+, K^- decay products are then also at low p_T. If I understand it correctly, this is the region where the new detector will have high sensitivities.