Study of the Cluster Splitting Algorithm In EMC Reconstruction

Qing Pu[1,2] , Guang Zhao[2] , Chunxu Yu[1], Shengsen Sun[2]

[1]Nankai University
[2]Institute of High Energy Physics

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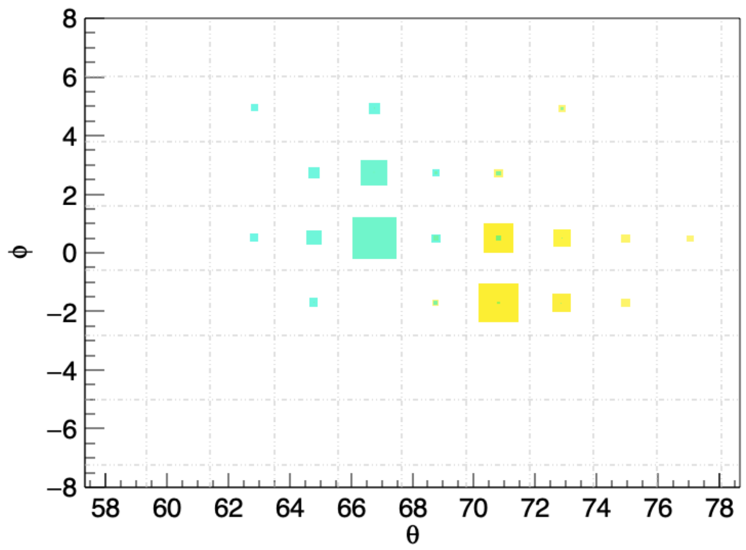
Outline

- Introduction
- Study of the cluster-splitting algorithm
  - Lateral development measurement
  - Seed energy correction
  - Reconstruction checks
- Summary

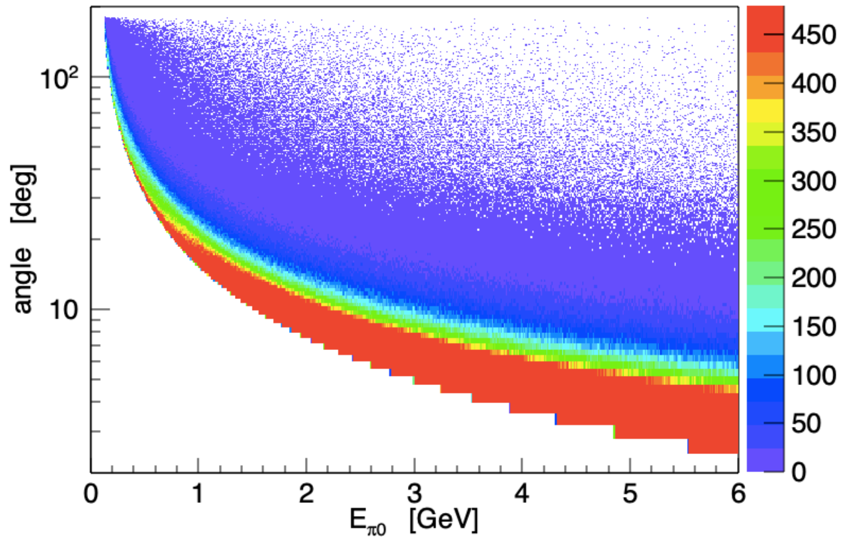
Introduction

- Cluster-splitting is an important algorithm in EMC reconstruction.
- The purpose of the cluster-splitting is to separate clusters that are close to each other.
- In this presentation, we improve the cluster-splitting algorithm in the following ways:
  - Update the lateral development formula
  - Correct the seed energy

Cluster-splitting for a EmcCluster



Angles between the 2γ from pi0

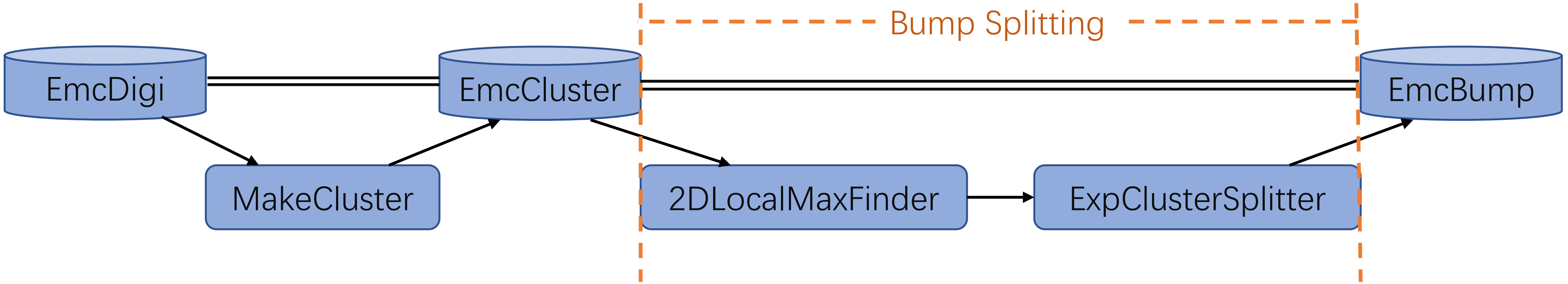


Cluster splitting is important for high energy pi0

EMC reconstruction overview

1. Cluster finding: a contiguous area of crystals with energy deposit.

2. The bump splitting
   - Find the local maximum: Preliminary split into seed crystal information
   - Update energy/position iteratively
     - The spatial position of a bump is calculated via a center-of-gravity method
     - The crystal weight for each bump is calculated by a formula.



The Cluster splitting algorithm

- Initialization:
  Place the bump center at the seed crystal.

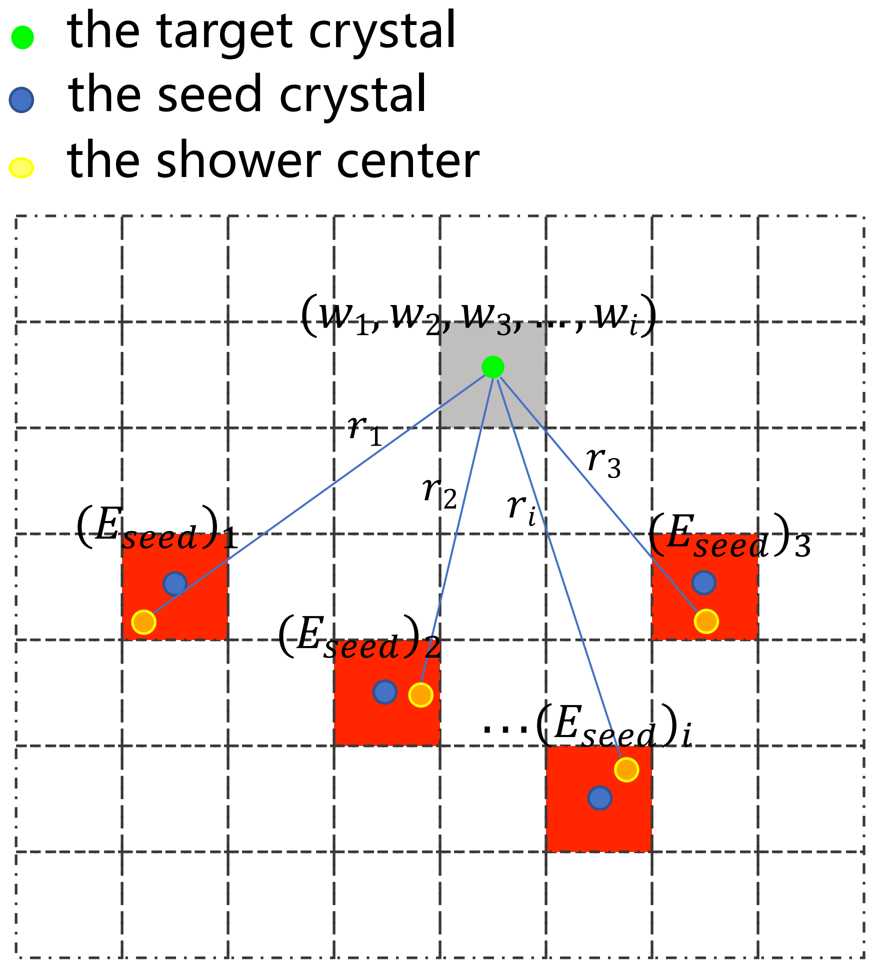
- Iteration:
  1.Traverse all digis to calculate w_i.

$$w_i = \frac{(E_{seed})_i \exp(-2.5r_i/R_m)}{\sum_j (E_{seed})_j \exp(-2.5r_j / R_m)}$$

i or j : different seed crystals
R_m: Moliere radius
r_i: distance from the shower center to the target crystal

2. Update the position of the bump center.

3. Loop over 1 & 2 until the bump center stays stable within a tolerance of 1 mm or the number of iterations exceeds the maximum number of iterations.



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$$w_i = \frac{(E_{seed})_i \exp(-2.5 r_i / R_m)}{\sum_j (E_{seed})_j \exp(-2.5 r_j \,/\, R_m)}$$

Energy for the target crystal: E_{target}

i or j : different seed crystals
R_m: Moliere radius
r_i: distance from the shower center to the target crystal

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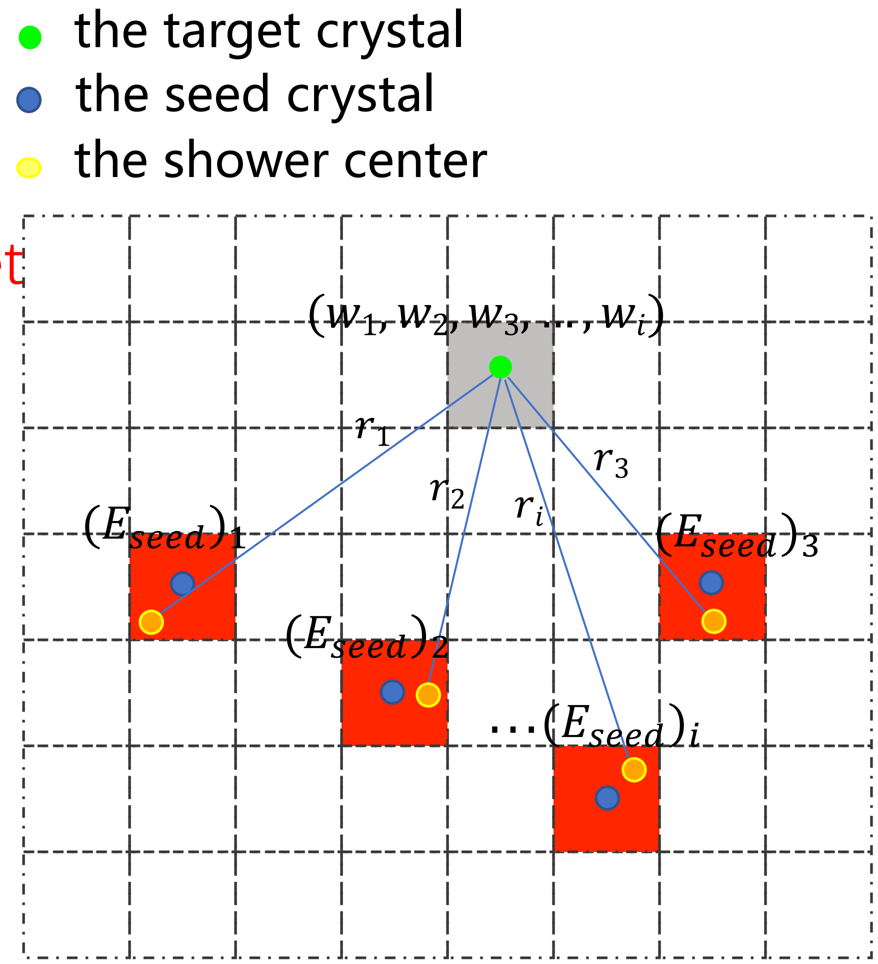


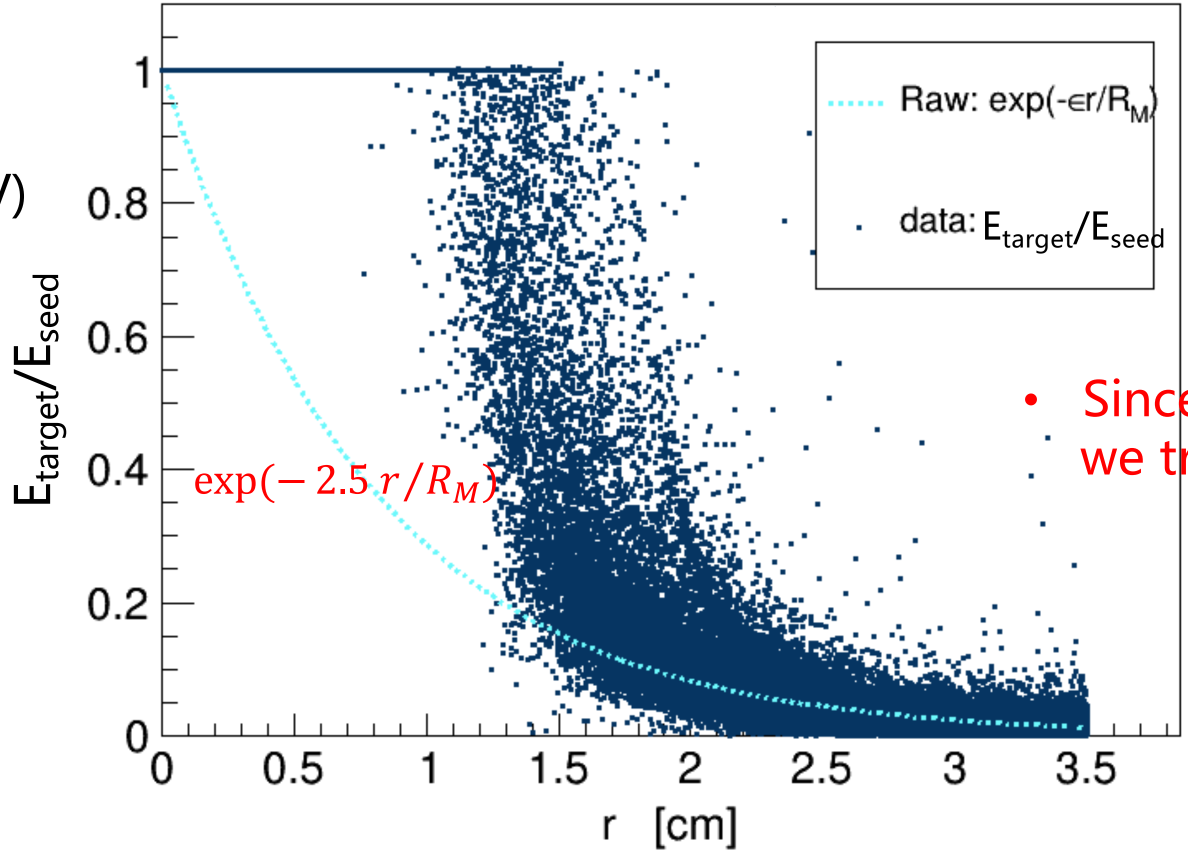
The E_{target} is calculated using the lateral development formula

The lateral development (old PandaRoot)

Energy deposition from a seed:

$$E_{target} = E_{seed} \exp(-2.5\, r/R_M)$$ → $$\frac{E_{target}}{E_{seed}} = \exp(-2.5\, r/R_M) \quad , R_M = 2.00\ cm$$

- Gamma (0~6GeV)
- Events 10000
- Geant4
- Generator: Box
- Phi(0, 360)
- Theta(22, 140)



- Since exp does not fit the data well, we try to improve the formula.

The lateral development (new measurement)

Define the lateral development:

$$f(r) = \frac{E_{target}}{E_{shower}}$$

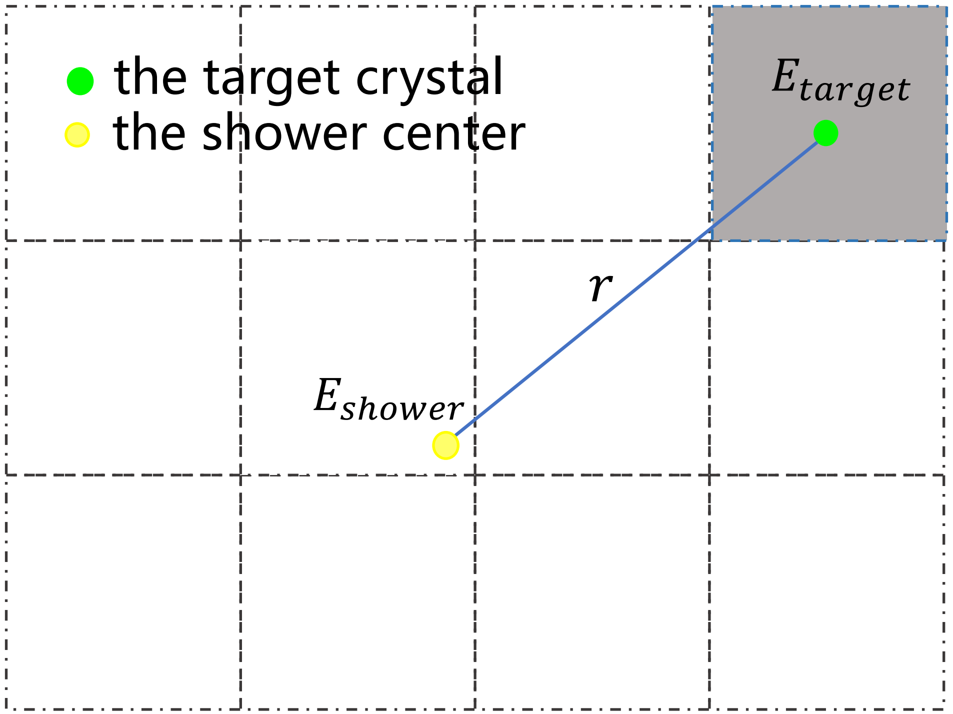
E_{shower} is the total energy of the single-particle shower.

The lateral development $f(r)$ can from obtained Geant4 simulation.

In this measurement the crystal dimension is considered

Control sample:

- Gamma (0 ~ 6GeV)
- Geant4
- Phi(0, 360)
- Events 10000
- Generator: Box
- Theta(22, 140)



Parametrization

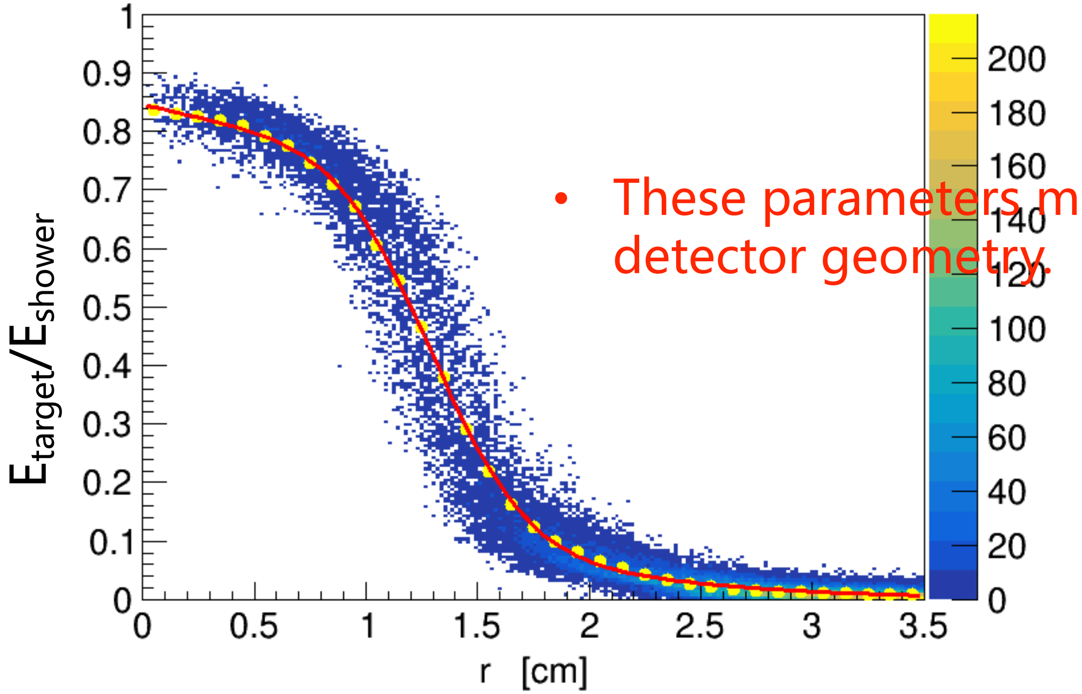
The function form used for fitting:

$$f(r) = p_0 \exp\left[-\frac{p_1}{R_M}\xi(r)\right], \qquad \xi(r) = r - p_2 r \exp[-\left(\frac{r}{p_3 R_M}\right)^{p_4}]$$

Where p_0, p_1, p_2, p_3 and p_4 are parameters.

p_0 represents the energy of the seed

- Gamma (1GeV)
- Events 10000
- Geant4
- Generator: Box
- Phi(0, 360)
- Theta(70.8088, 72.8652)



- These parameters may depend on the detector geometry.

Parametrization (II)

- The characteristic of the cluster has dependency on the energy and the detector geometry
- We consider the dependency of the parameters on energy and polar angle

$p_1(E_\gamma, \theta)$ $p_2(E_\gamma, \theta)$ $p_3(E_\gamma, \theta)$ $p_4(E_\gamma, \theta)$

- Assuming no phi-directed dependency.

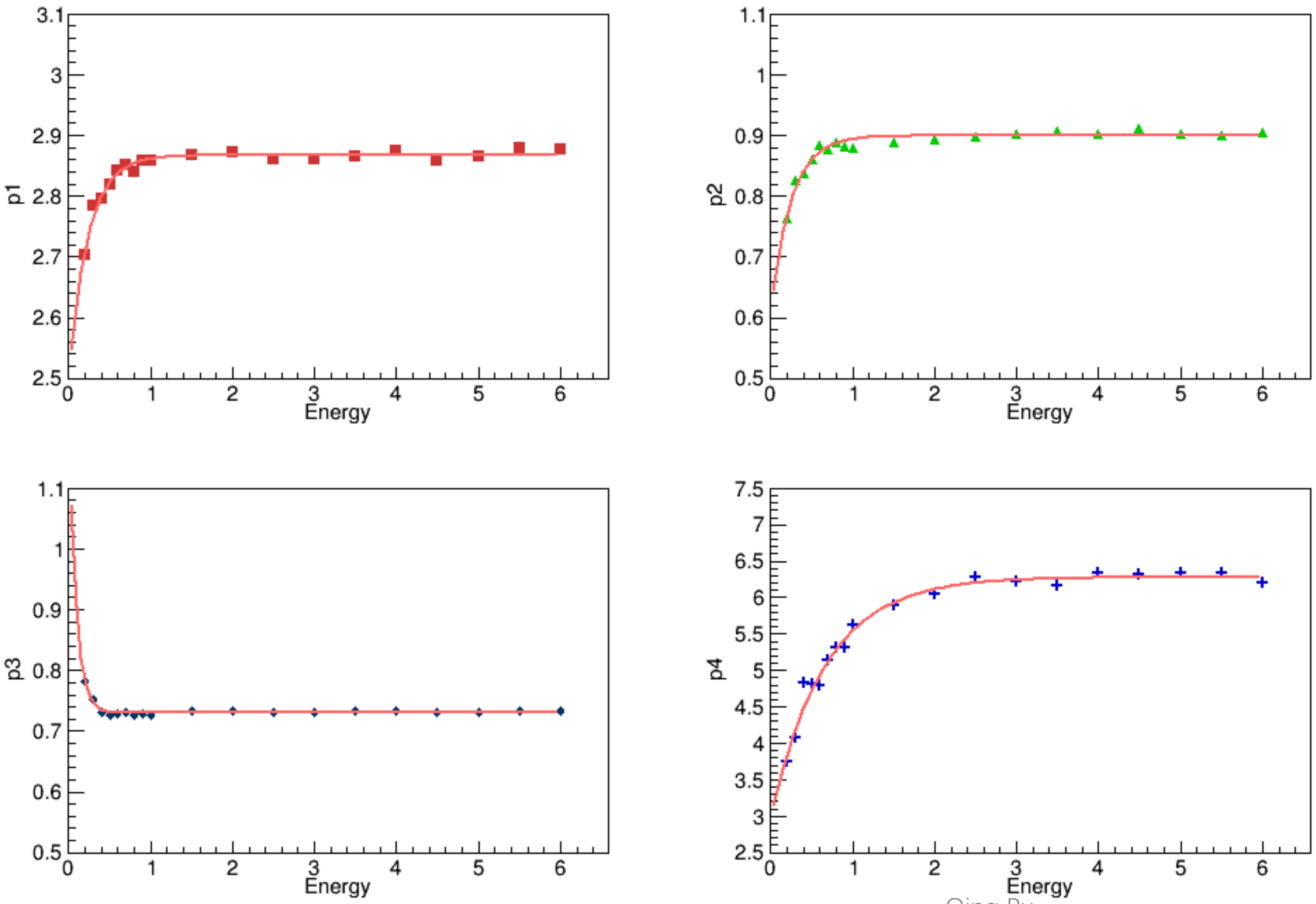
Energy dependency

$$p_1 = A\exp(-\kappa E_\gamma) + h$$
$$p_2 = B\exp(-\mu E_\gamma) + m$$
$$p_3 = C\exp(-\tau E_\gamma) + n$$
$$p_4 = D\exp(-\lambda E_\gamma) + q$$

Theta dependency

$$A = g_A(\theta) \quad \kappa = g_\kappa(\theta) \quad h = g_h(\theta)$$
$$B = g_B(\theta) \quad \mu = g_\mu(\theta) \quad m = g_m(\theta)$$
$$C = g_C(\theta) \quad \tau = g_\tau(\theta) \quad n = g_n(\theta)$$
$$D = g_D(\theta) \quad \lambda = g_\lambda(\theta) \quad q = g_q(\theta)$$

Parameters (energy dependency)



Range of simulated samples:

- Theta

Range4: 32.6536 ~ 33.7759

- Phi

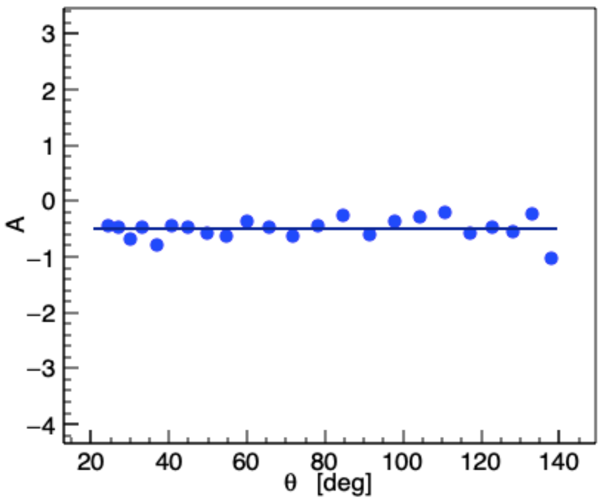
0~360

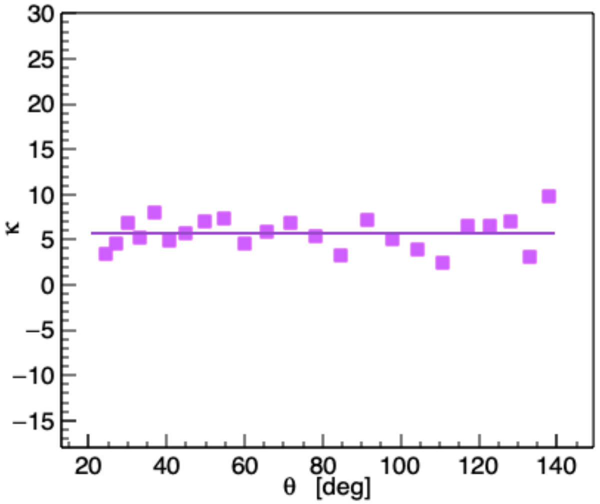
Fitting function:

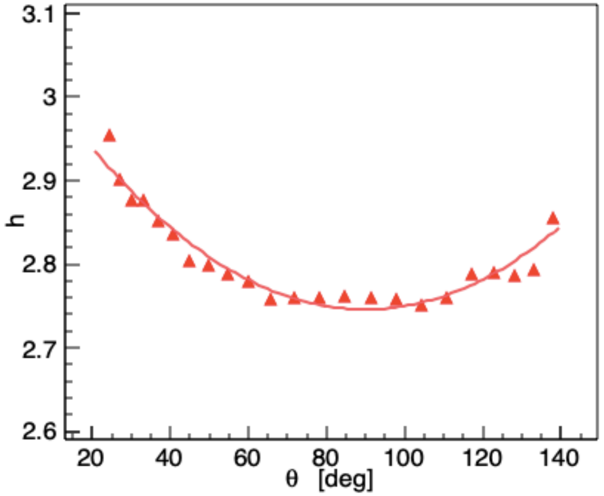
$$p_1 = A\exp(-\kappa E_\gamma) + h$$
$$p_2 = B\exp(-\mu E_\gamma) + m$$
$$p_3 = C\exp(-\tau E_\gamma) + n$$
$$p_4 = D\exp(-\lambda E_\gamma) + q$$

Parameters (angle dependency)

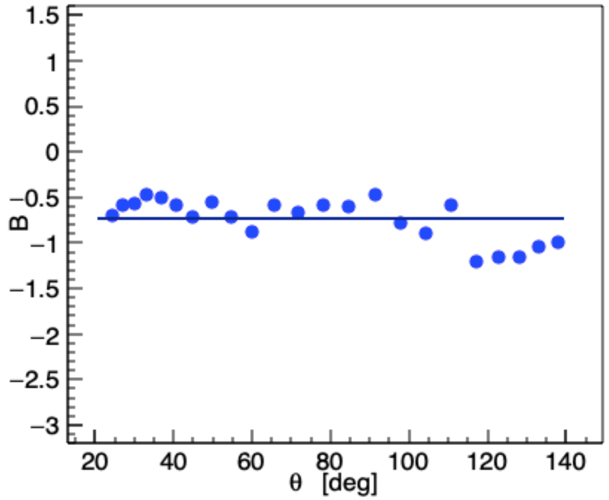
$p_1 =$
$A \exp(-\kappa E_\gamma) + h$

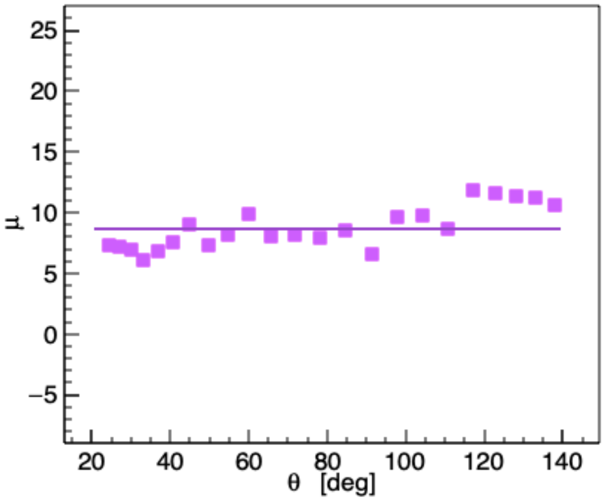


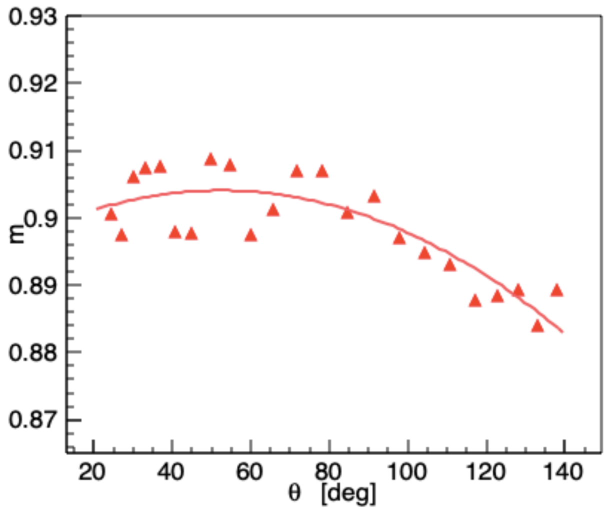




$p_2 =$
$B \exp(-\mu E_\gamma) + m$







Fitted parameters of the lateral development

Fitting Result:

$$\frac{E_{target}}{E_{seed}} = \exp\{-\frac{p_1}{R_M}\xi(r, p_2, p_3, p_4)\} \qquad \xi(r) = r - p_2 r\exp[-\left(\frac{r}{p_3 R_M}\right)^{p_4}] \qquad (R_M = 2.00\ cm)$$

$$p_1(E_\gamma, \theta) = -0.9006 * exp(-3.093 * E_\gamma) + 5.048 * 10^{-5} * (\theta - 88.71)^2 + 3.085$$

$$p_2(E_\gamma, \theta) = 5.546 * 10^{-3} * E_\gamma + 0.9225$$

$$p_3(E_\gamma, \theta) = -8.560 * 10^{-4} * E_\gamma - 1.569 * 10^{-5} * (\theta - 85.84)^2 + 0.7162$$

$$p_4(E_\gamma, \theta) = -2.857 * exp(-1.148 * E_\gamma) + 2.105 * 10^{-4} * (\theta - 80.76)^2 + 4.717$$

Energy dependency

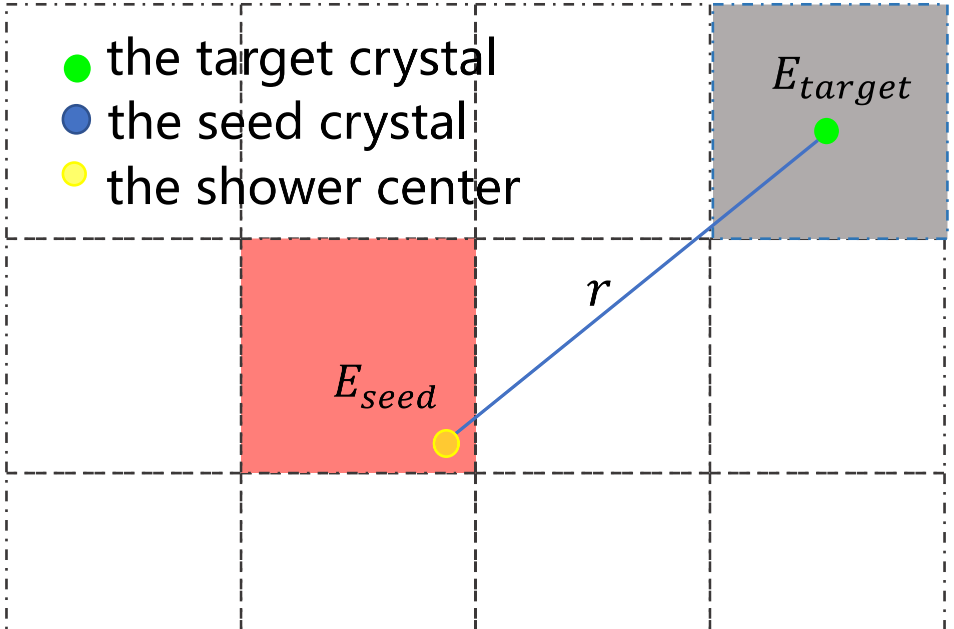
Angle dependency

Seed energy correction

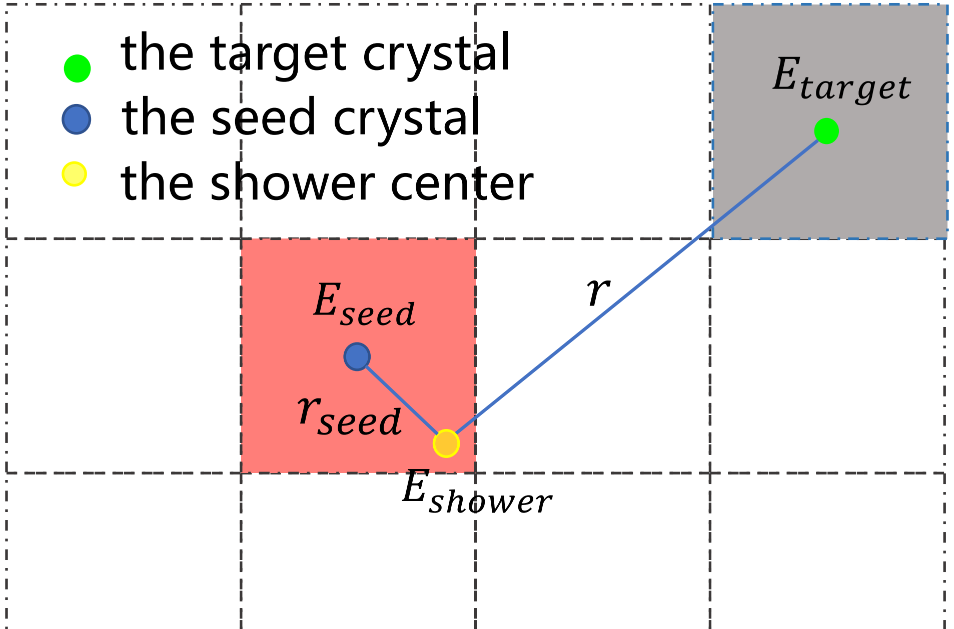
- In the old PandaRoot, the seed energy is used to calculate the $E_{target} = E_{seed} \times f(r)$
- If the shower center does not coincide with the crystal center, E_{seed} needs to be corrected

r or r_{seed}:
the distance from the center of the Bump to the geometric center of the crystal.

Old PandaRoot version



Updated version



Seed energy correction

In the new update, E_{target} can be calculated by the lateral development f(r):

$$E_{target} = E_{shower} \times f(r)$$

E_{shower}, which is not available in the reconstruction algorithm, can be related to the E_{seed} :

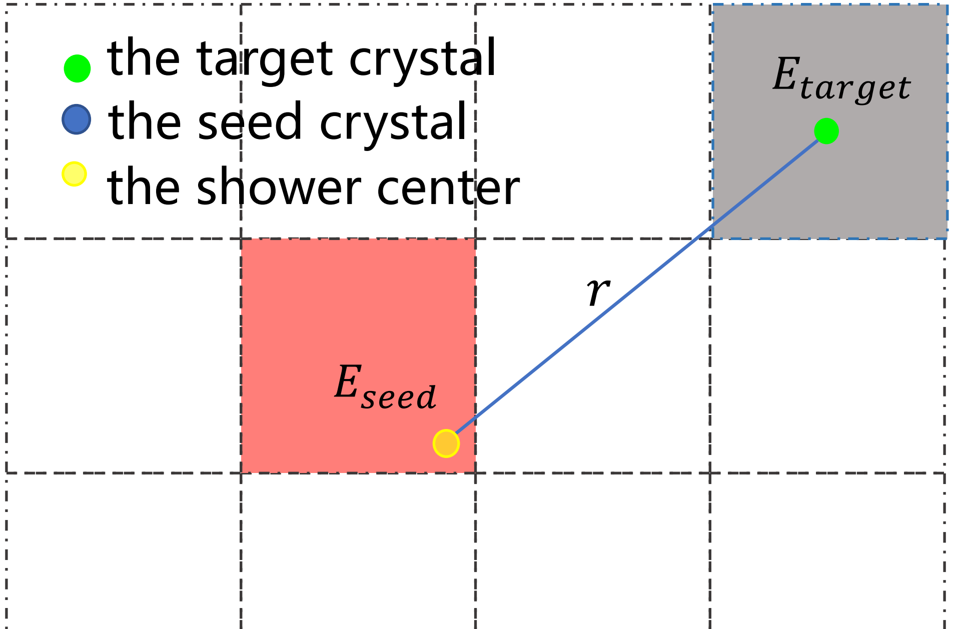
$$E_{shower} = \frac{E_{seed}}{f(r_{seed})}$$

In the end, E_{target} can be calculated as ($\frac{1}{f(r_{seed})}$ as the correction factor) :

$$E_{target} = \frac{E_{seed}}{f(r_{seed})} \times f(r)$$

r or r_{seed}: the distance from the center of the Bump to the geometric center of the crystal.

Old PandaRoot version



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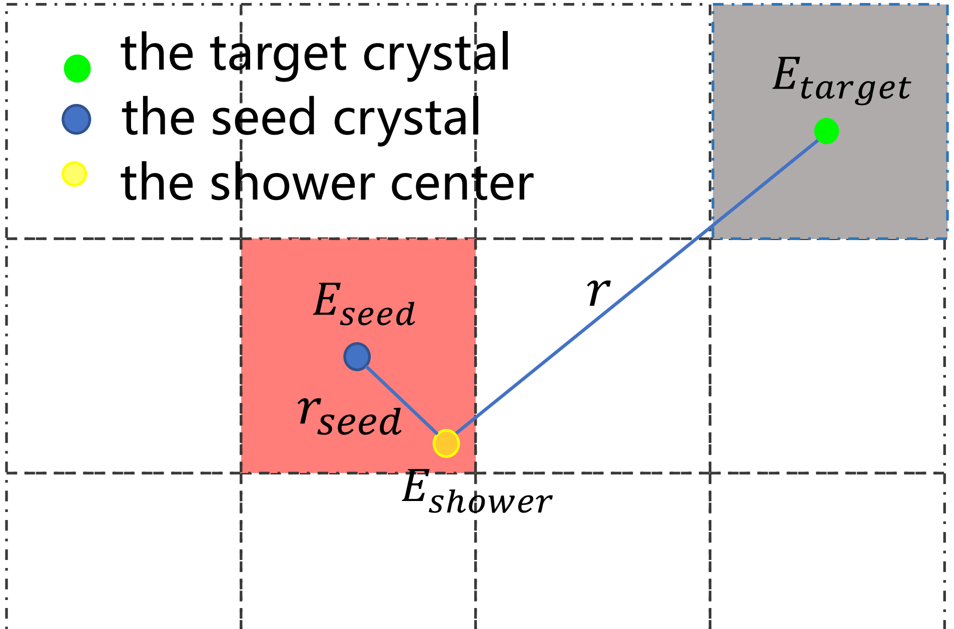
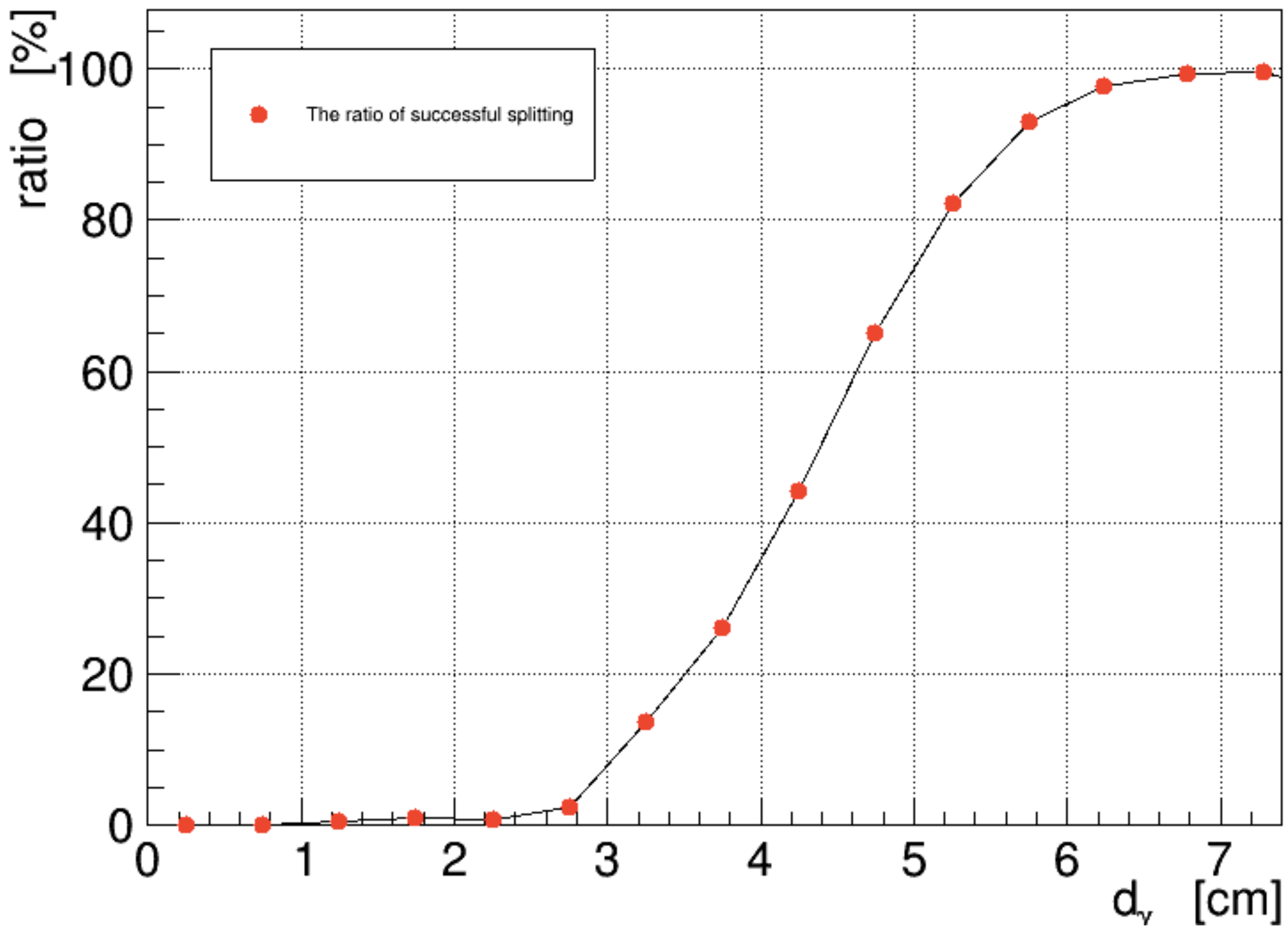


Splitting efficiency



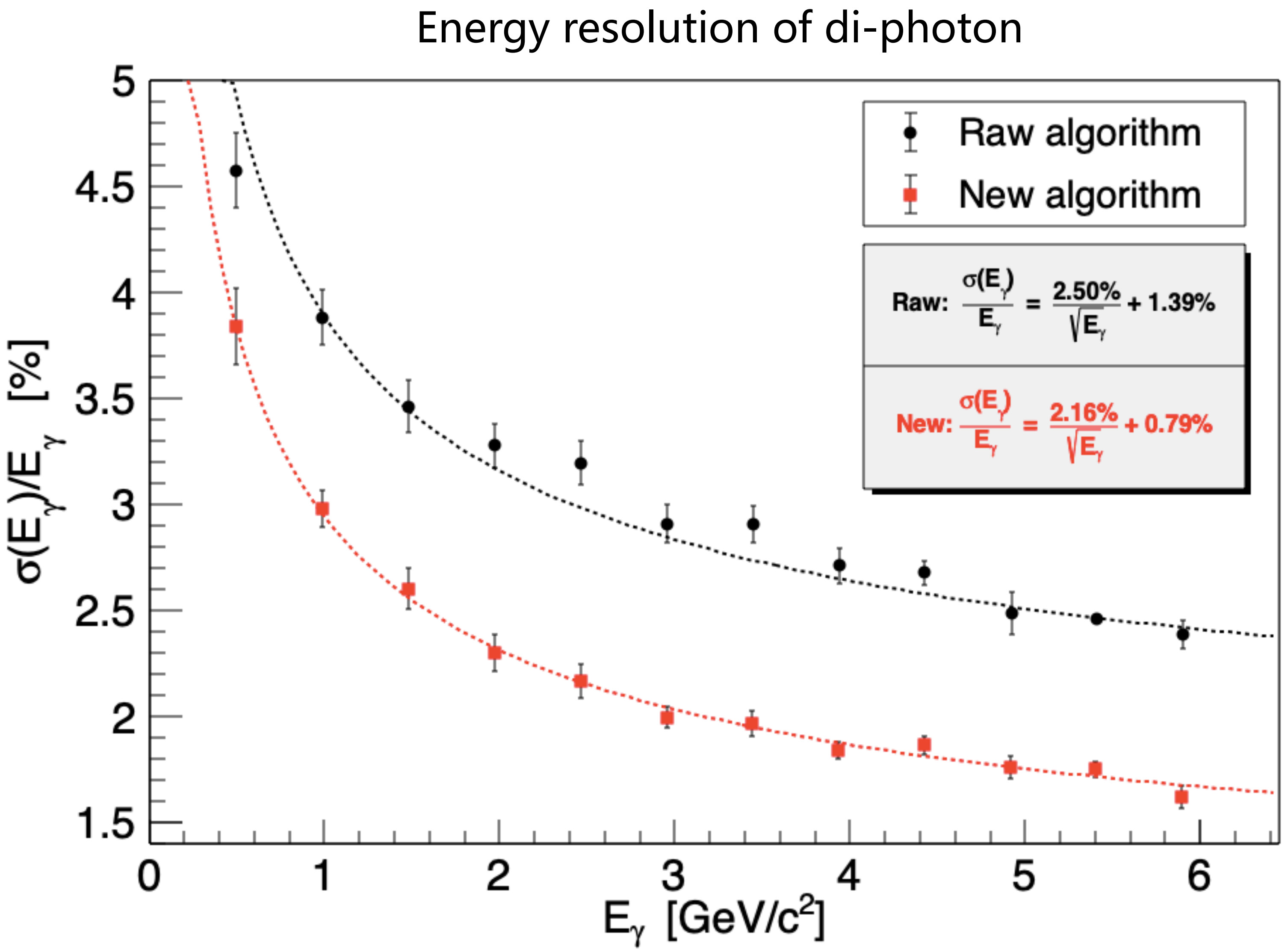
d_γ: The distance between two shower centers

$$ratio = \frac{N_{splitting}}{N_{total}} \times 100\ (\%)$$

Control sample:

- di-photon (0 ~ 6GeV)
- Events 10000
- Geant4
- Generator: Box
- Phi(0, 360)
- Theta(22, 140)

Energy resolution (di-photon)



- The angle between two photons < 6.75 (deg)

Range of simulated samples:

- Energy

0.5~6 GeV

- Theta

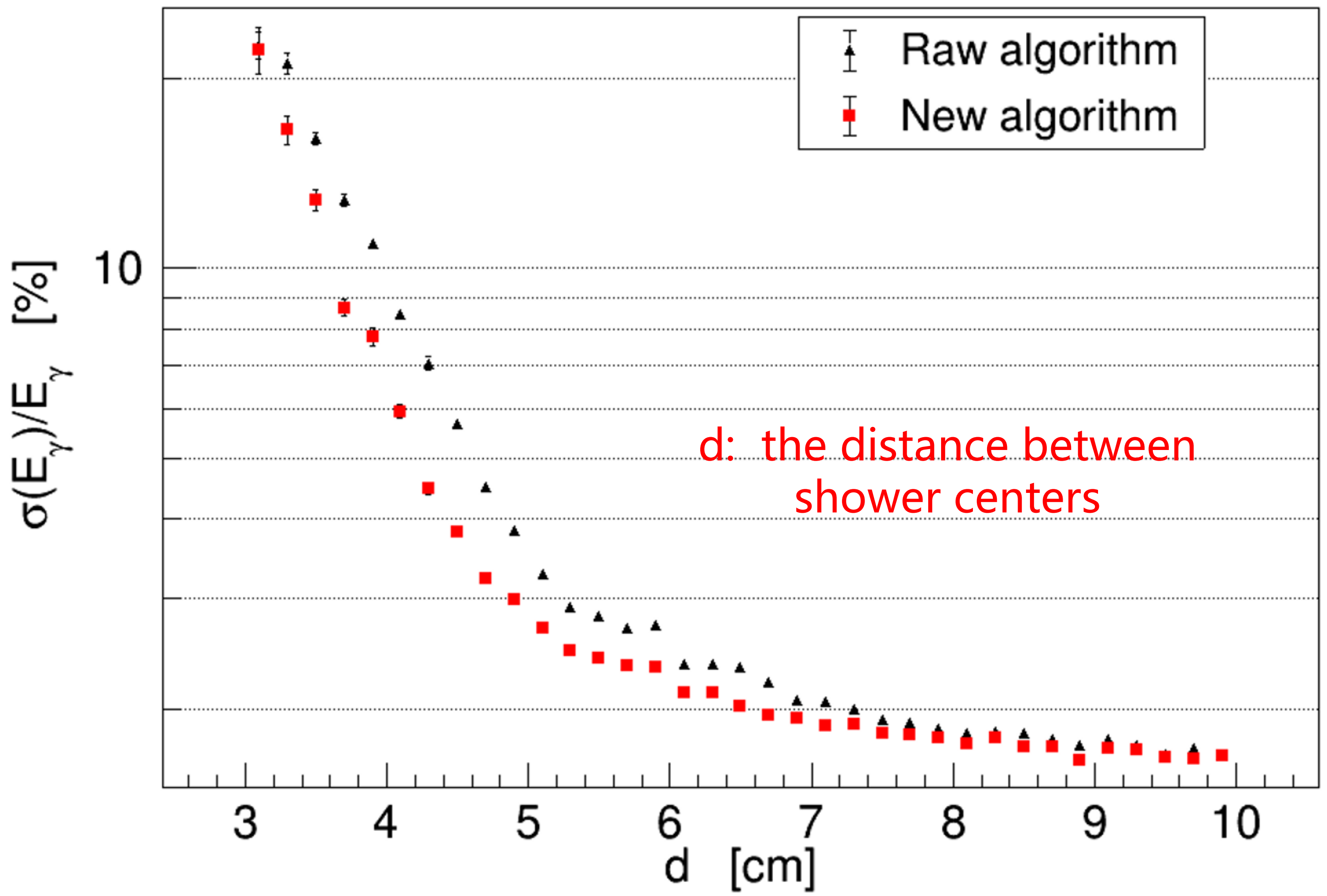
67.7938 ~ 73.8062 {deg}

- Phi

0.625 ~ 7.375 {deg}

Energy resolution (di-photon)

Energy resolution of di-photon



Range of simulated samples:

- Energy

0.5~6 GeV

- Theta

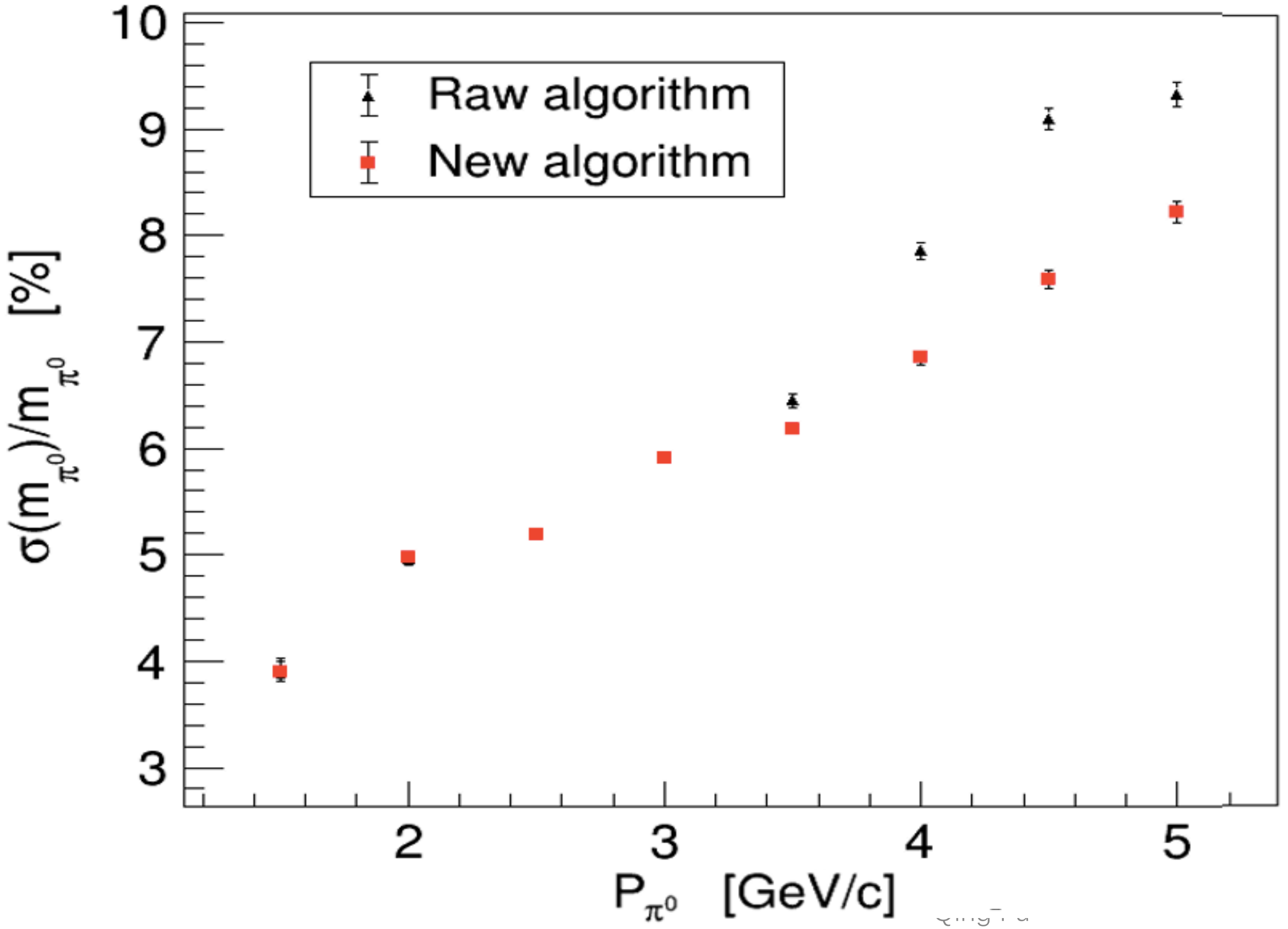
Range12: 70.8088 ~ 72.8652

- Phi

Square area calculated according to theta

Energy resolution (pi0)

pi0 mass resolution



Range of simulated samples:

- Energy

0.5~5 GeV

- Theta

22~140 (deg)

- Phi

0~360 (deg)

Summary

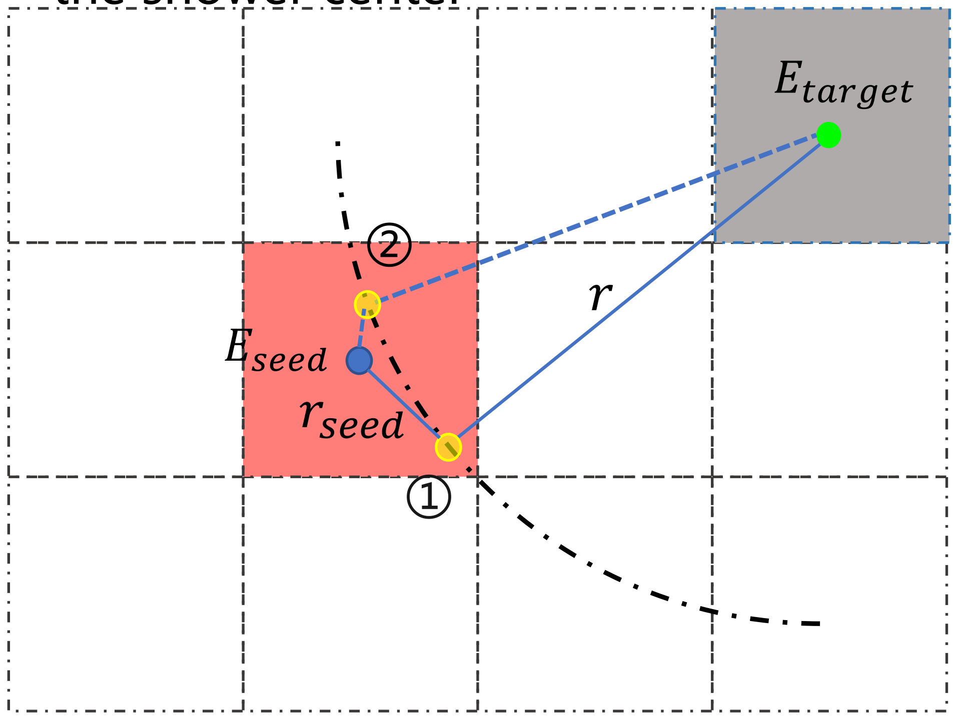
- The lateral development of the cluster using Geant4 simulation is measured
  - Lateral development with the crystal dimension is considered
  - Energy and angle dependent is considered
- Seed energy is corrected while applying the lateral development in cluster-splitting
- Several checks are done in reconstruction, including
  - Splitting efficiency
  - Energy resolution for di-photon samples
  - Mass resolution for pi0 samples
- Improvements are seen with the new algorithm. Will finalizing the code.

Backup

The seed energy dependency

Consider two cases where the photon hits the seed at different positions:

- the target crystal
- the seed crystal
- the shower center



$$E_{target} = E_{seed} \exp(-2.5\, r/R_M)$$

$case1$:	r	E_{target}	E_{seed}
	‖	‖	X
$case2$:	r	E_{target}	E_{seed}

- For the same r, $\frac{E_{target}}{E_{seed}}$ depends on r_{seed}.

The detector geometry dependency

According to the definition of $f(r)$:

$$f(r) = p_0 \exp[-\frac{p_1}{R_M}\xi(r)] \qquad \xi(r) = r - p_2 r \exp[-\left(\frac{r}{p_3 R_M}\right)^{p_4}] \qquad (R_M = 2.00\ cm)$$

$$f(r)/f(r_{seed}) = p_0 \exp[-\frac{p_1}{R_M}\xi(r)]/\, p_0 \exp[-\frac{p_1}{R_M}\xi(r_{seed})] = \exp\{-\frac{p_1}{R_M}[\xi(r) - \xi(r_{seed})]\}$$

$$\frac{E_{target}}{E_{seed}} = \exp\{-\frac{p_1}{R_M}[\xi(r, p_2, p_3, p_4) - \xi(r_{seed}, p_2, p_3, p_4)]\}$$

$$Raw: \quad \frac{E_{target}}{E_{seed}} = \exp(-\frac{\varepsilon}{R_M} r)$$

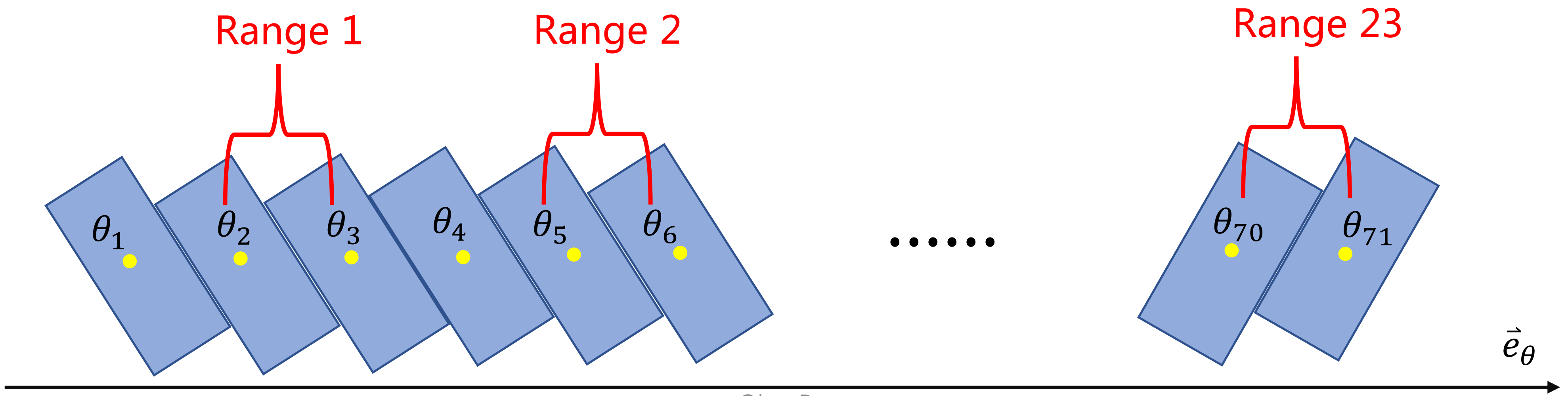
Control sample

$< 1GeV$

- Gamma (0.2, 0.3, 0.4…0.9 GeV)
- Events 10000
- Geant4
- Generator: Box
- Phi(0, 360)
- Theta(Range1, Range2,… ,Range23)

$\geq 1GeV$

- Gamma (1, 1.5, 2, 2.5…6 GeV)
- Events 10000
- Geant4
- Generator: Box
- Phi(0, 360)
- Theta(Range1, Range2,… ,Range23)



Control sample

Phi: 0~360

Theta(deg):

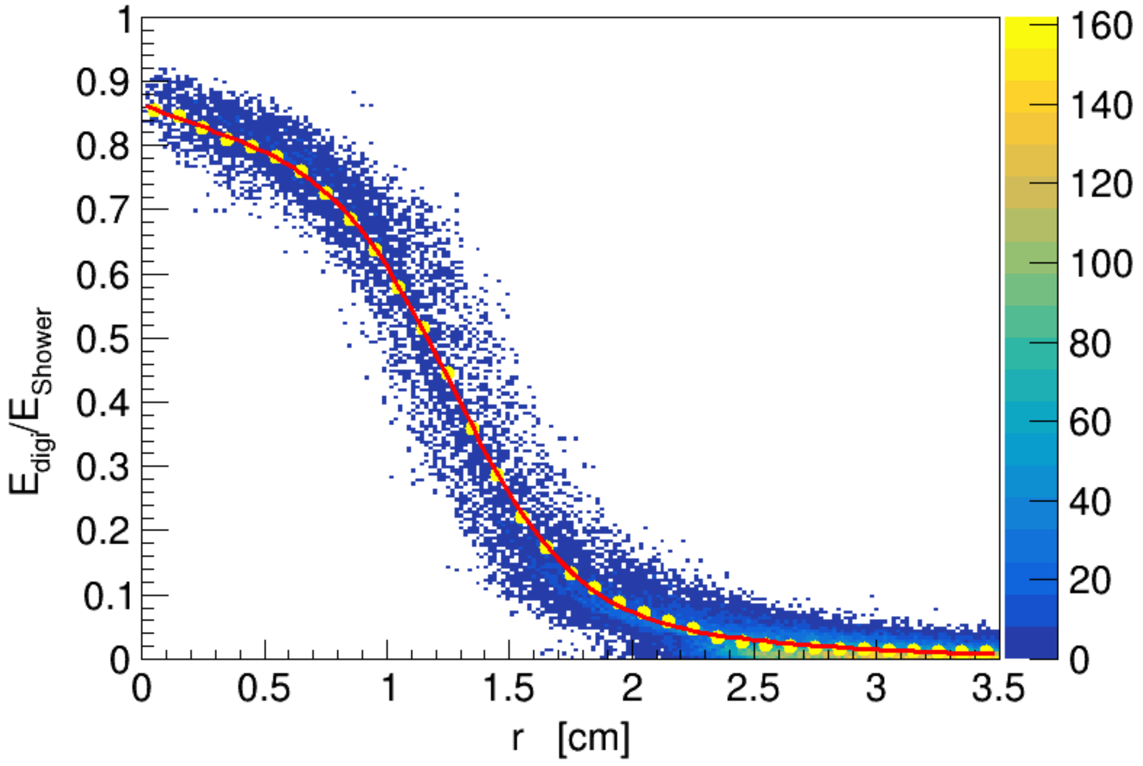
Range1: 23.8514 ~ 24.6978
Range2: 26.4557 ~ 27.3781
Range3: 29.4579 ~ 30.4916
Range4: 32.6536 ~ 33.7759
Range5: 36.1172 ~ 37.3507
Range6: 39.9051 ~ 41.2390
Range7: 44.2385 ~ 45.7355
Range8: 48.8451 ~ 50.4459
Range9: 53.7548 ~ 55.4790
Range10: 59.0059 ~ 60.8229
Range11: 64.7855 ~ 66.7591
Range12: 70.8088 ~ 72.8652
Range13: 77.0506 ~ 79.1942
Range14: 83.4997 ~ 85.6749
Range15: 90.2068 ~ 92.4062
Range16: 96.8200 ~ 99.0099
Range17: 103.361 ~ 105.534
Range18: 109.793 ~ 111.893
Range19: 116.067 ~ 118.019
Range20: 121.838 ~ 123.686
Range21: 127.273 ~ 129.033
Range22: 132.400 ~ 134.031
Range23: 137.230 ~ 138.679

Fitting results

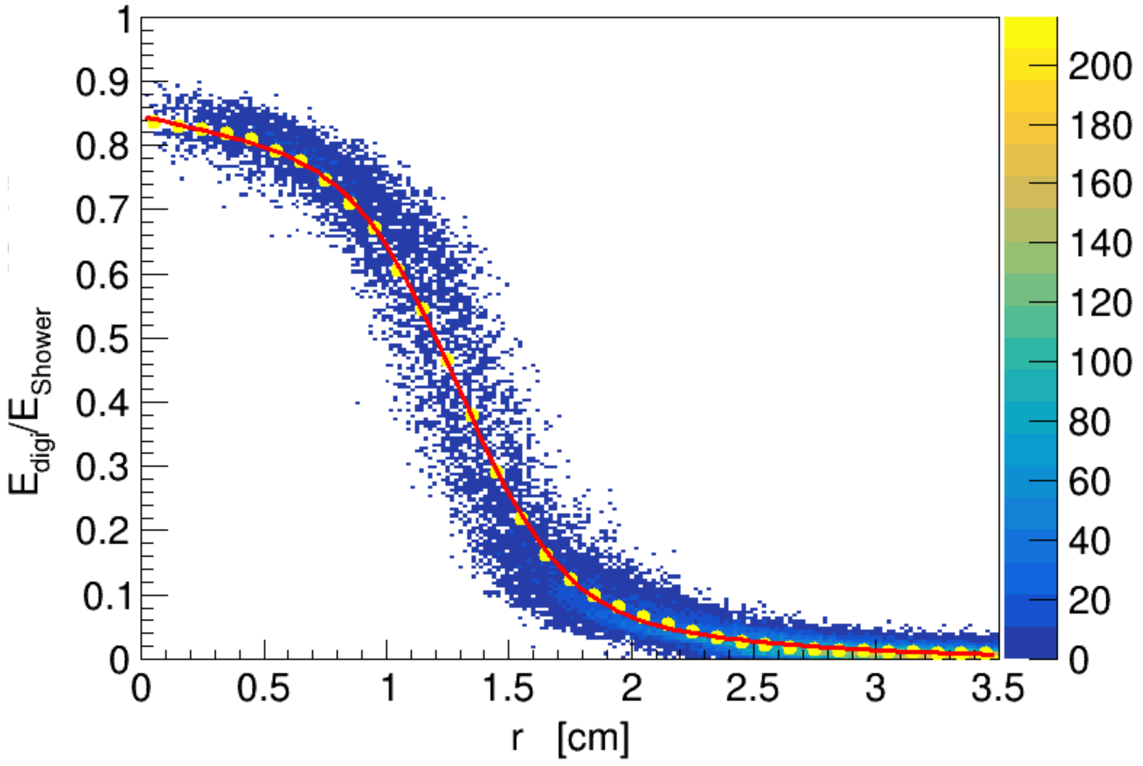
Fitting function:

$$f(r) = p_0 \exp\left[-\frac{p_1}{R_M}\xi(r, p_2, p_3, p_4)\right], \quad \xi(r, p_2, p_3, p_4) = r - p_2 r \exp[-\left(\frac{r}{p_3 R_M}\right)^{p_4}]$$

Range12; 0.5 GeV



Range12; 1 GeV

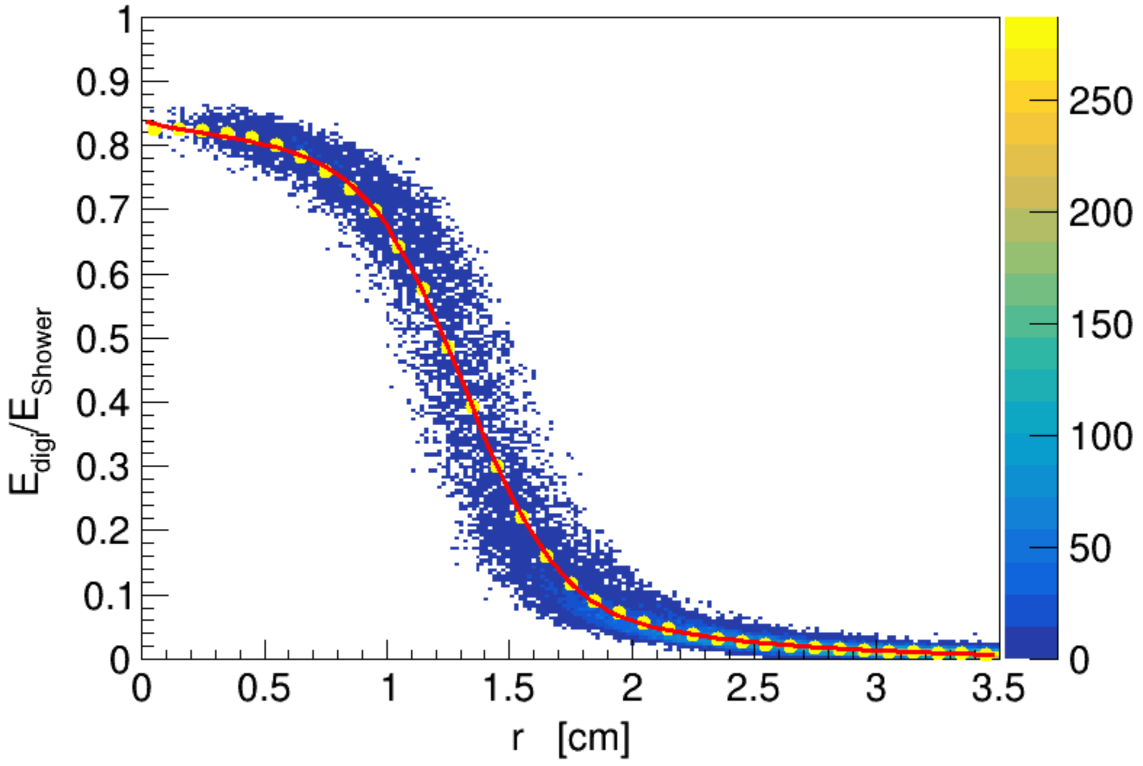


Fitting results

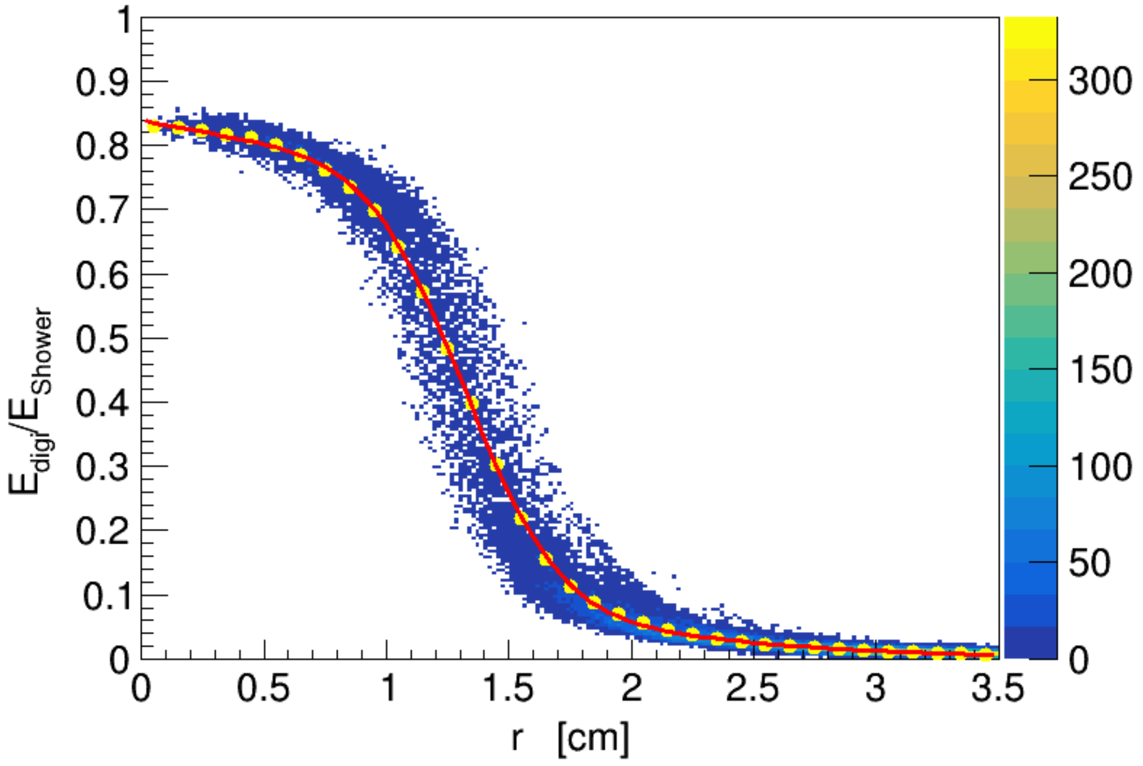
Fitting function:

$$f(r) = p_0 \exp\left[-\frac{p_1}{R_M}\xi(r, p_2, p_3, p_4)\right], \quad \xi(r, p_2, p_3, p_4) = r - p_2 r \exp[-\left(\frac{r}{p_3 R_M}\right)^{p_4}]$$

Range12; 3 GeV

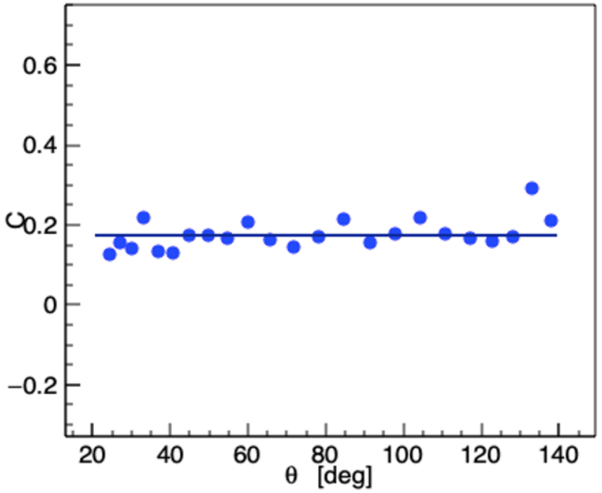


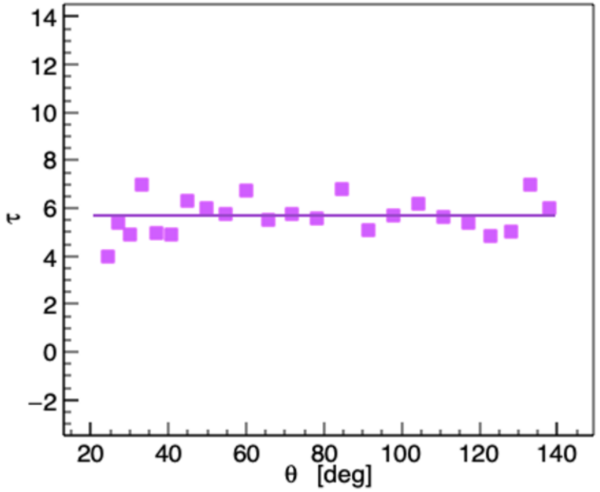
Range12; 6 GeV

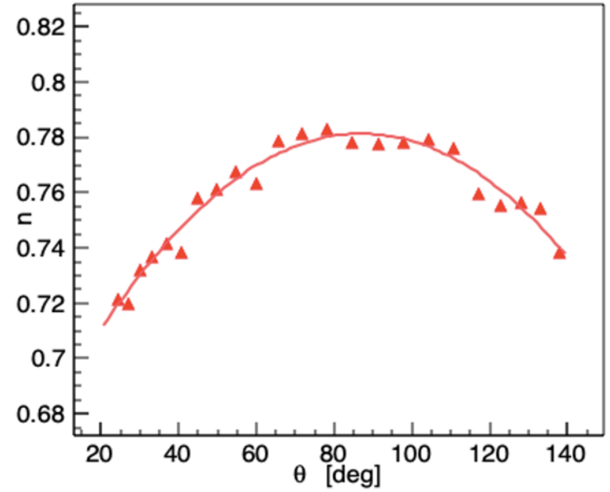


Angle dependency of parameters

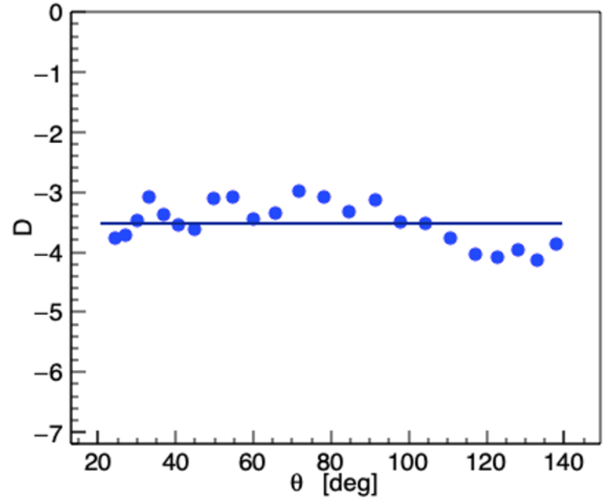
$$p_3 = C\exp(-\tau E_\gamma) + n$$

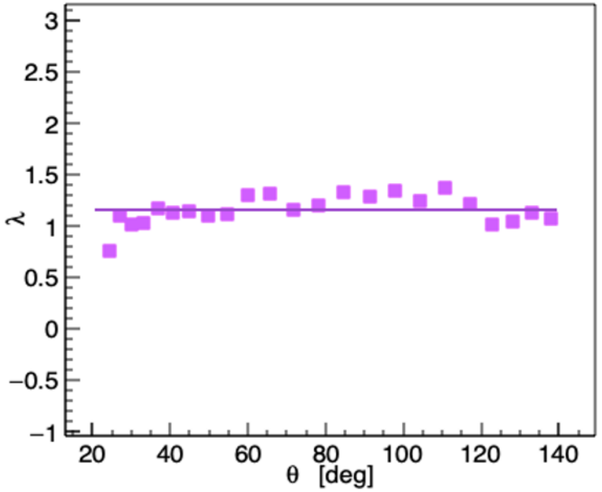


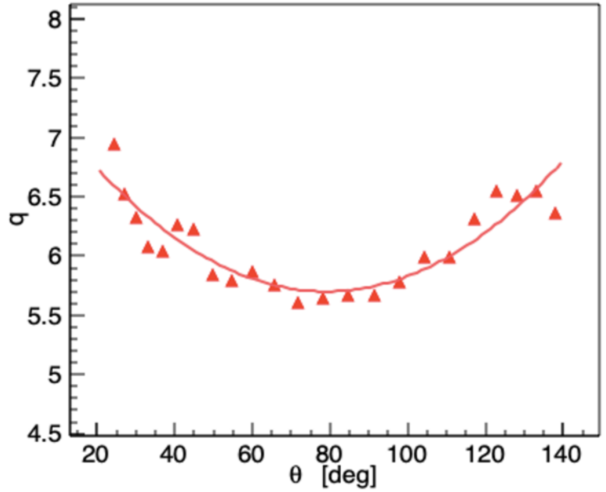




$$p_4 = D\exp(-\lambda E_\gamma) + q$$







Energy resolution (pi0)

The angle between the two photons produced by the decay of pi0 changes with its momentum: