Kinematic Fitter for Hydra – Update

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2. The Fitting Procedure
3. Apply 4C Fitter on $p(3.5\mathrm{GeV})p \to pK^+\Lambda$
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Momentum Dependent Uncertainty Estimation – Motivation

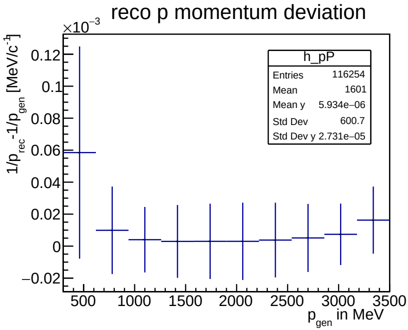
Successful kinematic fitting requires good knowledge of uncertainties

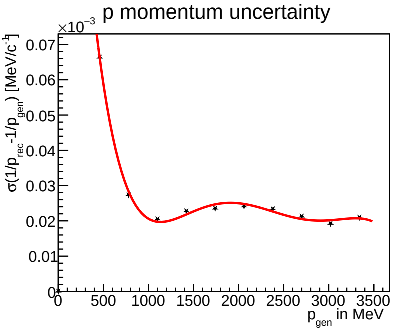
- Covariance matrices not available in Hydra
- Estimate uncertainties from MC data
- Uncertainties dependent on particle type and momentum

Method:

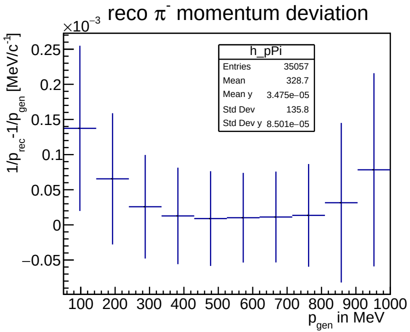
- Calculate difference between generated and reconstructed variables
  $(1/p, \vartheta, \varphi, R, Z)$
- Extract standard deviation for each momentum bin
- Fit polynomial to standard deviations

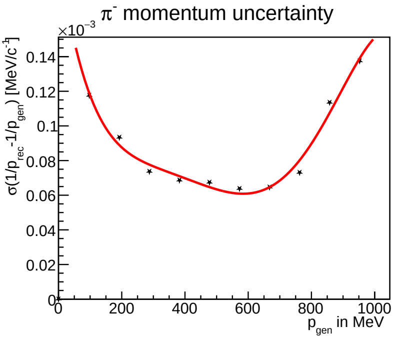
Proton momentum reconstruction uncertainty – Momentum dependent



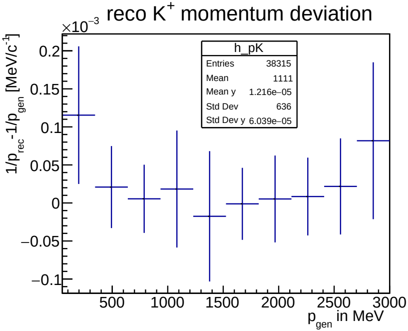


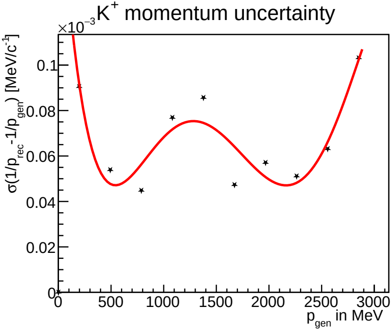
Pion momentum reconstruction uncertainty – Momentum dependent





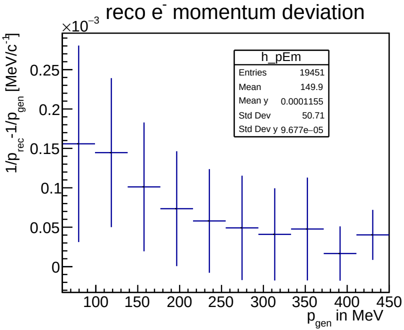
Kaon momentum reconstruction uncertainty – Momentum dependent

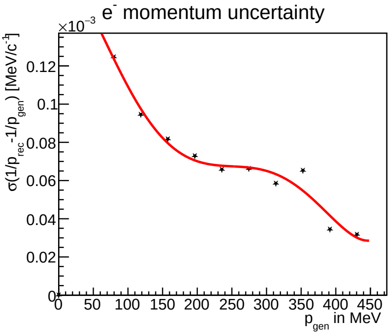




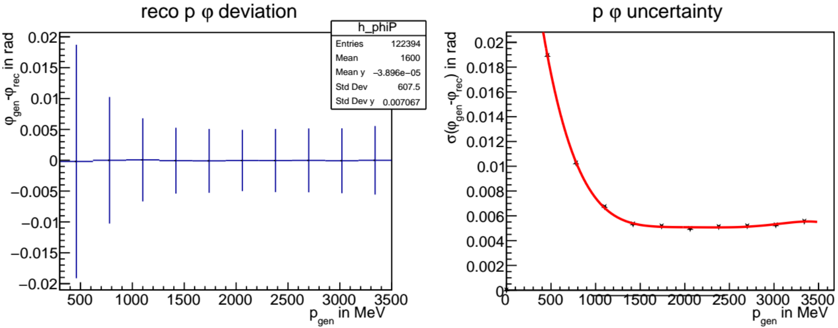
Why are the fluctuations so large?

Electron momentum reconstruction uncertainty – Momentum dependent

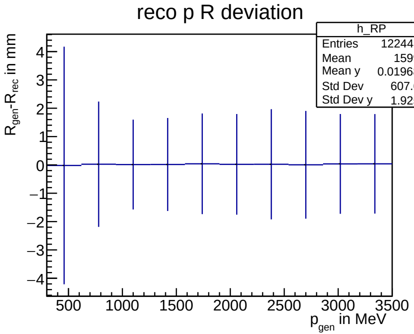


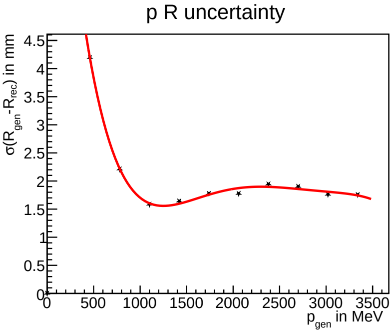


Proton φ reconstruction uncertainty – Momentum dependent

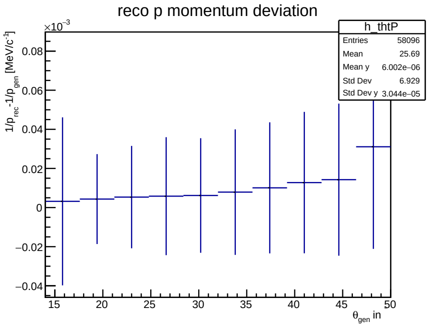


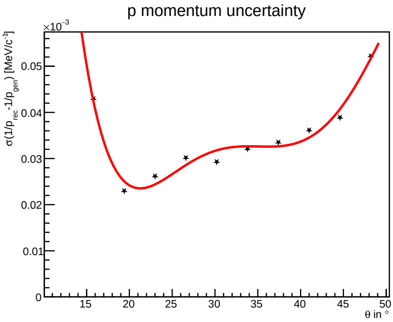
Proton R reconstruction uncertainty – Momentum dependent





Proton momentum reconstruction uncertainty – θ dependent





Take this into account when estimating uncertainties?

Kinematic Fitter for Hydra – Motivation

Possible Appliations for Fitter

- Displaced decay vertices of long-lived particles
- 4C fit of initial four momentum for exclusive analysis
- Fit momentum of missing particle
- Mass constraints
- ...

Kinematic Fitter for Hydra

Suppose there are N measured and M unmeasured variables related by K constraints

$$f_k(\eta_1, \eta_2, ...\eta_N, \xi_1, \xi_2, ...\xi_M) = 0. \quad (1)$$

Goal: Minimize $\chi^2(\eta) = (y - \eta)^T V(y)(y - \eta)$ under those K constraints

Procedure: Introduce Lagrangian Multiplliers λ

$$\chi^2(\eta, \xi, \lambda) = (y - \eta)^T V(y)(y - \eta) + 2\lambda^T f(\eta, \xi) = \mathrm{minimum} \quad (2)$$

y: Measurements, $V(y)$: Covariance matrix of y

The Fitting Procedure – Minimization

χ^2 minimization will result in $N + M + K$ equations:

$$\nabla_\eta \chi^2 = -2V^{-1}(y - \eta) + 2F_\eta^T \lambda = 0$$
$$\nabla_\xi \chi^2 = 2F_\xi^T \lambda = 0$$
$$\nabla_\lambda \chi^2 = 2f(\eta, \xi) = 0$$

where F_η and F_ξ are defined by

$$(F_\eta)_{ij} = \frac{\partial f_i}{\partial \eta_j}, \qquad (F_\xi)_{ij} = \frac{\partial f_i}{\partial \xi_j} \tag{3}$$

$\rightarrow$Solve in iterative procedure

The Fitting Procedure – Iterations

Start from measured values ($\eta^0 = y$) and do fit in iterative procedure. At iteration ν the solution can be calculated by defining

$$r = f^\nu + F^\nu_\eta(\eta^0 - \eta^\nu), \qquad S = F^\nu_\eta V^0 (F^T_\eta)^\nu \tag{4}$$

$$\begin{aligned}
\xi^\nu &= \xi^{\nu-1} - 1 \cdot (F^T_\xi S^{-1} F_\xi)^{-1} F^T_\xi S^{-1} r \\
\Delta\xi &= \xi^\nu - \xi^{\nu-1} \\
\lambda^\nu &= S^{-1} \cdot [r + F_\xi \cdot \Delta\xi] \\
\Delta\eta &= \eta^0 - \eta^\nu \\
\eta^{\nu+1} &= \eta^0 - 1 \cdot V^0 \cdot F^T_\eta \cdot \lambda^\nu \\
V^\nu &= \\
V^0 - 1 \cdot V^0 \cdot & \left[F^T_\eta S^{-1} F_\eta - ((F^T_\eta S^{-1} F_\xi)(F^T_\xi S^{-1} F_\xi)^{-1} (F^T_\eta S^{-1} F_\xi)^T \right] \cdot V^0
\end{aligned}$$

The Fitting Procedure – Stopping Criteria

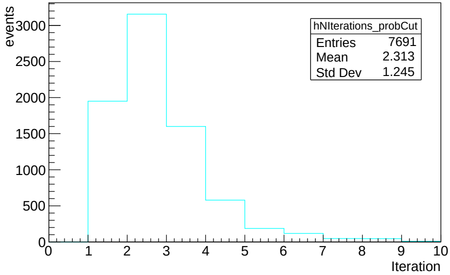
Calculate χ^2 for iteration ν:

$$\chi^2(\eta^\nu, \xi^\nu, \lambda^\nu) = (\eta^0 - \eta^\nu)^T V^0 (\eta^0 - \eta^\nu) + 2(\lambda^\nu)^T f(\eta^\nu, \xi^\nu) \quad (5)$$

Stopping criteria

- Convergence: Difference of this and previous χ^2 is < 1
- Maximum # of iterations $\nu = 10$ reached

Select good fit if converged and $P(\chi^2) > 1\%$



4C Fitter – Constraint Equations

Constraint equations for N particles:

$$f_1 = \sum_{n=1}^{N} \left(\frac{1}{p_n}\right)^{-1} \cdot \sin\vartheta_n \cos\varphi_n - p_{x_{\mathrm{ini}}} = 0$$

$$f_2 = \sum_{n=1}^{N} \left(\frac{1}{p_n}\right)^{-1} \cdot \sin\vartheta_n \sin\varphi_n - p_{y_{\mathrm{ini}}} = 0$$

$$f_3 = \sum_{n=1}^{N} \left(\frac{1}{p_n}\right)^{-1} \cdot \cos\vartheta_n - p_{z_{\mathrm{ini}}} = 0$$

$$f_4 = \sum_{n=1}^{N} \sqrt{\left(\frac{1}{p_n}\right)^{-2} + m_n^2} - E_{\mathrm{ini}} = 0$$

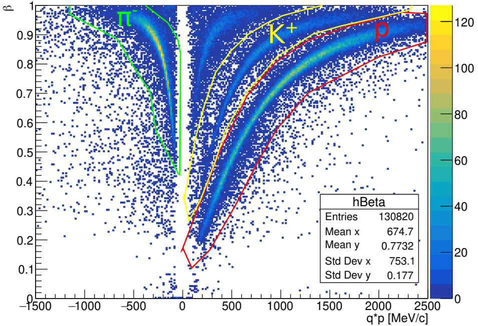
Apply 4C Fitter on $p(3.5\mathrm{GeV})p \to pK^+\Lambda$ Data

Skimmed data set provided by Waleed

- Events contain at least 1 negative and 3 positive tracks
- Initial four momentum: $p_x = 0, p_y = 0, p_z = 4337.96\,\mathrm{MeV}/c$, $E = 5376.54\,\mathrm{MeV}$

PID is assigned by graphical cuts in β vs. $p \times q$ plot

Simulated data



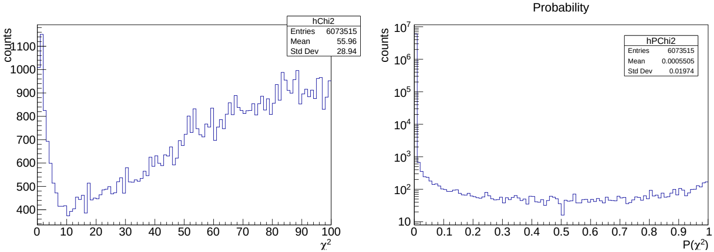
Apply 4C Fitter on Data: $p(3.5\mathrm{GeV})p \to pK^+\Lambda[p\pi^-]$

Modifications:

- HRefitCand inherits from KParticleCand instead of HVirtualCand

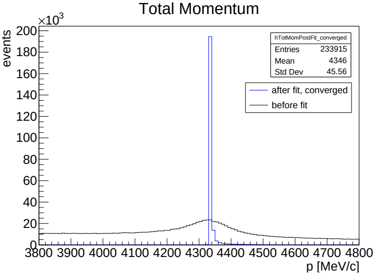
Settings for fitter:

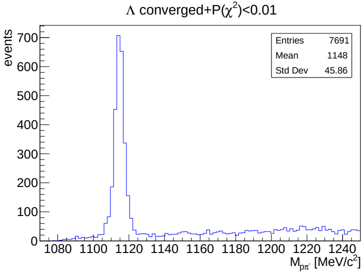
- Learning rate: $lr = 1.$
- Stop iterations when $\Delta\chi^2 < 1$ $\to$converged
- Maximum number of iterations: 10
- Probability cut $P(\chi^2) > 1\%$



Apply 4C Fitter on Data: $p(3.5\mathrm{GeV})p \to pK^+\Lambda[p\pi^-]$

- Total momentum is successfully fitted
- $pp \to pK\Lambda \to pKp\pi^-$ events selected without further selection





Apply Missing Particle Fitter on $p(4.5\mathrm{GeV})p \to pK^+\Sigma^0[\Lambda e^+ e^-]$ Simulation

Reminder

- Dilepton from Σ^0 Dalitz decay has small mass
- Especially positron is difficult to reconstruct

New strategy

- Treat positron as missing particle
- Fit its momentum using kinematic fit with fixed mass
- 4 constraints from energy + momentum conservation
  - 3 unknowns = 1C fit

Missing Particle Fitter – Constraint Equations

Constraint equations for N particles + 1 missing particle:

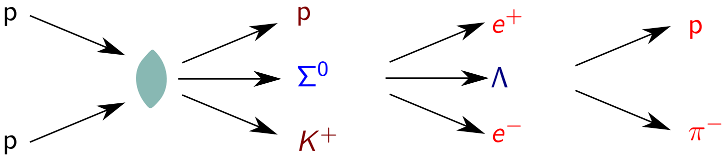
$$f_1 = \sum_{n=1}^{N} \left(\frac{1}{p_n}\right)^{-1} \cdot \sin\vartheta_n \cos\varphi_n + p_{x_{\mathrm{miss}}} - p_{x_{\mathrm{ini}}} = 0$$

$$f_2 = \sum_{n=1}^{N} \left(\frac{1}{p_n}\right)^{-1} \cdot \sin\vartheta_n \sin\varphi_n + p_{y_{\mathrm{miss}}} - p_{y_{\mathrm{ini}}} = 0$$

$$f_3 = \sum_{n=1}^{N} \left(\frac{1}{p_n}\right)^{-1} \cdot \cos\vartheta_n + p_{z_{\mathrm{miss}}} - p_{z_{\mathrm{ini}}} = 0$$

$$f_4 = \sum_{n=1}^{N} \sqrt{\left(\frac{1}{p_n}\right)^{-2} + m_n^2} + \sqrt{p_{\mathrm{miss}}^2 + m_{\mathrm{miss}}^2} - E_{\mathrm{ini}} = 0$$

Data Simulation



Pluto: Event generation

- 200 000 events
- 4.5 GeV beam kinetic energy $\mathrel{\widehat{=}}$ $\sqrt{s} = 3.5\,\mathrm{GeV}$

FwDet HGeant/Hydra

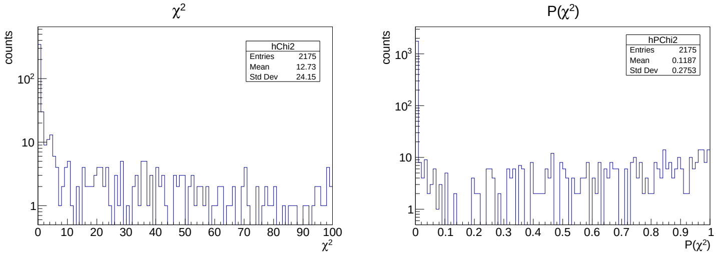
- Λ decay by HGeant $(\mathrm{BR} = 0.639)$
- Simulate detector response

$\Rightarrow$ $\approx$ 127 800 events of charged decay $\Lambda \rightarrow p\pi^-$

Apply Missing Particle Fitter on $p(4.5\mathrm{GeV})p \to pK^+\Sigma^0[\Lambda e^+ e^-]$ Simulation

Settings for fitter:

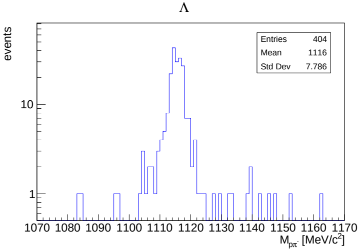
- Initial four momentum: $p_x = 0, p_y = 0, p_z = 5356.72\,\mathrm{MeV}/c$, $E = 6376.54\,\mathrm{MeV}$
- Learning rate: $lr = 1$.
- Stop iterations when $\Delta\chi^2 < 1$ $\to$converged
- Maximum number of iterations: 10
- Probability cut $P(\chi^2) > 1\%$

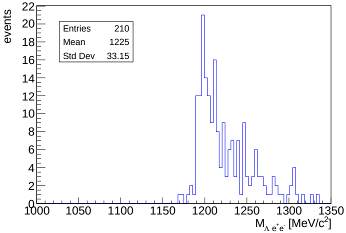


Σ^0 Reconstruction

$\Sigma^0 \to \Lambda e^+ e^-$

- Reconstruct Λ from fitted p and π^-
- Get e^+ momentum from missing particle fit
  - Fit converged, $P(\chi^2) > 1\%$
- Reconstruct Σ^0 from $\Lambda e^+ e^-$
- Λ mass cut: $1100\,\mathrm{MeV} < m_\Lambda < 1130\,\mathrm{MeV}$





Summary and Outlook

Summary

- Momentum dependent uncertainty estimates slightly improve fitting performance
- Fitter can be used with old beam data with slight modifications
- Use missing particle fitter for events where one particle is lost
- Powerful analysis tool

Outlook

- Test displaced vertex fitter for long lived particles (Λ)
- Include Forward Detector
- Integration of fitters in Hydra