Energy Level Displacement of Excited *np* State of Kaonic Deuterium In Faddeev Equation Approach

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Content

Low–Energy K^-d Scattering

$$K^- + d \to K^- + d$$

Complex S–Wave Scattering Length of K^-d Scattering

$$\tilde{a}^{(0)}_{K^-d} = \frac{m_d}{m_K + m_d} \int d^3x \, |\Phi_d(\vec{r}\,)|^2 \, \hat{A}^{(0)}_{K^-d}(r)$$

Complex P–Wave Scattering Length K^-d Scattering

$$\tilde{a}^{(1)}_{K^-d} = \frac{m_d}{m_K + m_d} \int d^3x \, |\Phi_d(\vec{r}\,)|^2 \, \hat{A}^{(1)}_{K^-d}(r)$$

- R. Machleidt, K. Holinde, Ch. Elster, Phys. Rep. **149**, 1 (1987)

Energy Level Displacements of Kaonic Deuterium, Caused by Strong Low–Energy Interactions

Energy Level Displacement of Excited *ns* State

$$-\epsilon_{ns} + i\,\frac{\Gamma_{ns}}{2} = 2\,\frac{\alpha^3}{n^3}\left(\frac{m_K m_d}{m_K + m_d}\right)^2 \tilde{a}^{(0)}_{K^-d}$$

Energy Level Displacement of Excited *np* State

$$-\epsilon_{np} + i\,\frac{\Gamma_{np}}{2} = 2\,\frac{\alpha^5}{n^3}\left(1 - \frac{1}{n^2}\right)\left(\frac{m_K m_d}{m_K + m_d}\right)^4 \tilde{a}^{(1)}_{K^-d}$$

- S. Deser et al. PR96, 774 (1954); T. L. Trueman, NP26, 57 (1961)
- T. E. O. Ericson and W. Weise, in “Pions and Nuclei”, Clarendon Press, Oxford, 1988; A. N. Ivanov et al., PRA71, 052508 (2005)

Faddeev Equations for S–Wave K^-d Scattering Length: S. S. Kamalov, E. Oset, A. Ramos, NPA 690, 494 (2001)

Faddeev Equations for S–Wave K^-d Scattering Length

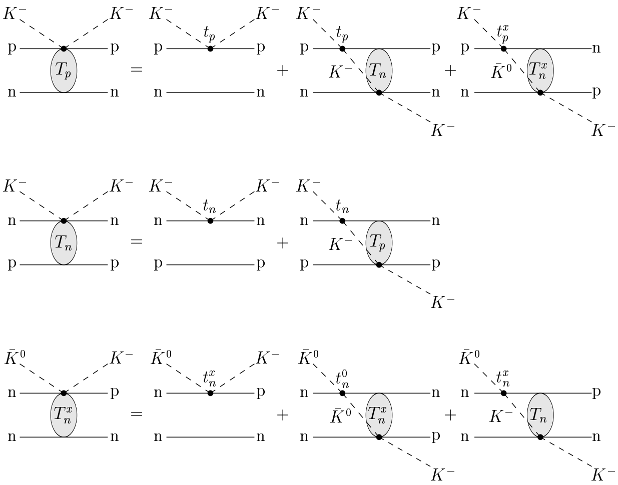
$$T_{Kd}^{(0)} = T_p^{(0)} + T_n^{(0)}$$

$$T_p^{(0)} = t_p^{(0)} + t_p^{(0)} G_0 T_n^{(0)} + t_p^{x(0)} G_0 T_n^{x(0)}$$

$$T_n^{(0)} = t_n^{(0)} + t_n^{(0)} G_0 T_p^{(0)}$$

$$T_n^{x(0)} = t_n^{x(0)} + t_n^{0(0)} G_0 T_n^{x(0)} + t_n^{x(0)} G_0 T_n^{(0)}$$

Faddeev Equations for S–Wave K^-d Scattering Length: S. S. Kamalov, E. Oset, A. Ramos, NPA 690, 494 (2001). Graphical illustration



Faddeev Equations for S–Wave K^-d Scattering Length: S. S. Kamalov, E. Oset, A. Ramos, NPA 690, 494 (2004). Fixed Centre Approximation

$$\hat{A}^{(0)}_{Kd}(r) = \hat{A}^{(0)}_p(r) + \hat{A}^{(0)}_n(r)$$

$$\hat{A}^{(0)}_p(r) = \hat{a}^{(0)}_p + \hat{a}^{(0)}_p \frac{1}{r}\hat{A}^{(0)}_n(r) - \hat{a}^{(0)}_x \frac{1}{r}\hat{A}^{x(0)}_n(r)$$

$$\hat{A}^{(0)}_n(r) = \hat{a}^{(0)}_n + \hat{a}^{(0)}_n \frac{1}{r}\hat{A}^{(0)}_p(r)$$

$$\hat{A}^{x(0)}_n(r) = \hat{a}^{(0)}_x - \hat{a}^{0(0)}_n \frac{1}{r}\hat{A}^{x(0)}_n(r) + \hat{a}^{(0)}_x \frac{1}{r}\hat{A}^{(0)}_n(r)$$

$$\hat{a}^{(0)}_p = \left(1 + \frac{m_K}{m_N}\right)\tilde{a}_{K^-p}(K^-p)\,,\ \hat{a}^{(0)}_n = \left(1 + \frac{m_K}{m_N}\right)\tilde{a}_{K^-n}(K^-n)$$

$$\hat{a}^{(0)}_x = \left(1 + \frac{m_K}{m_N}\right)\tilde{a}_{K^-p}(\bar{K}^0 n)\,,\ \hat{a}^{0(0)}_n = \left(1 + \frac{m_K}{m_N}\right)\tilde{a}_{\bar{K}^0 n}(\bar{K}^0 n)$$

Faddeev Equations for P–Wave K^-d Scattering Length

$$T^{(1)}_{Kd} = T^{(1)}_p + T^{(1)}_n$$

$$T^{(1)}_p = t^{(1)}_p + t^{(1)}_p G_0 T^{(0)}_n + t^{(0)}_p G_0 T^{(1)}_n + t^{x(1)}_p G_0 T^{x(0)}_n + t^{x(0)}_p G_0 T^{x(1)}_n$$

$$T^{(1)}_n = t^{(1)}_n + t^{(1)}_n G_0 T^{(0)}_p + t^{(0)}_n G_0 T^{(1)}_p$$

$$T^{x(1)}_n = t^{x(1)}_n + t^{0(1)}_n G_0 T^{x(0)}_n + t^{0(0)}_n G_0 T^{x(1)}_n + t^{x(1)}_n G_0 T^{(0)}_n + t^{x(0)}_n G_0 T^{(1)}_n$$

Solution of Faddeev Equations for P–Wave K^-d Scattering Length: M. Faber *et al.* arXiv:1012.3933 [nucl-th]. Fixed Centre Approximation

$$T^{(1)}_{Kd} = T^{(1)}_p + T^{(1)}_n \to A^{(1)}_{Kd}(r) = A^{(1)}_p(r) + A^{(1)}_n(r)$$

$$\hat{A}^{(1)}_p(r) = \hat{a}^{(1)}_p + \frac{1}{6}\,\hat{a}^{(1)}_p\frac{1}{r}\hat{A}^{(0)}_n(r) + \frac{1}{6}\,\hat{a}^{(0)}_p\frac{1}{r}\hat{A}^{(1)}_n(r)$$

$$-\frac{1}{6}\,\hat{a}^{(1)}_x\frac{1}{r}\hat{A}^{x(0)}_n(r) - \frac{1}{6}\,\hat{a}^{(0)}_x\frac{1}{r}\hat{A}^{x(1)}_n(r)$$

$$\hat{A}^{(1)}_n(r) = \hat{a}^{(1)}_n + \frac{1}{6}\,\hat{a}^{(1)}_n\frac{1}{r}\hat{A}^{(0)}_p(r) + \frac{1}{6}\,\hat{a}^{(0)}_n\frac{1}{r}\hat{A}^{(1)}_p(r)$$

$$\hat{A}^{x(1)}_n(r) = \hat{a}^{(1)}_x - \frac{1}{6}\,\hat{a}^{0(1)}_n\frac{1}{r}\hat{A}^{x(0)}_n(r) - \frac{1}{6}\,\hat{a}^{0(0)}_n\frac{1}{r}\hat{A}^{x(1)}_n(r)$$

$$+\frac{1}{6}\,\hat{a}^{(1)}_x\frac{1}{r}\hat{A}^{(0)}_n(r) + \frac{1}{6}\,\hat{a}^{(0)}_x\frac{1}{r}\hat{A}^{(1)}_n(r)$$

Solution of Faddeev Equations for P–Wave K^-d Scattering Length: M. Faber *et al.* arXiv:1012.3933 [nucl-th]. Fixed Centre Approximation

$$\hat{A}_p^{(1)}(r)\left(1+\frac{1}{6}\frac{\hat{a}_n^{0(0)}}{r}-\frac{1}{36}\frac{\hat{a}_n^{(0)}\hat{a}_p^{(0)}}{r^2}-\frac{1}{216}\frac{\hat{a}_n^{(0)}(\hat{a}_p^{(0)}\hat{a}_n^{0(0)}-(\hat{a}_x^{(0)})^2)}{r^3}\right)=$$

$$=\hat{a}_p^{(1)}+\frac{1}{6}\frac{\hat{a}_p^{(1)}\hat{a}_n^{0(0)}}{r}+\frac{1}{6}\frac{\hat{a}_n^{(1)}\hat{a}_p^{(0)}}{r}-\frac{1}{6}\frac{\hat{a}_x^{(1)}\hat{a}_x^{(0)}}{r}$$

$$+\frac{1}{36}\frac{\hat{a}_n^{(1)}(\hat{a}_p^{(0)}\hat{a}_n^{0(0)}-(\hat{a}_x^{(0)})^2)}{r^2}+\frac{1}{6}\frac{\hat{a}_p^{(1)}}{r}\hat{A}_n^{(0)}(r)-\frac{1}{6}\frac{\hat{a}_x^{(1)}}{r}\hat{A}_n^{x(0)}(r)$$

$$+\frac{1}{36}\frac{\hat{a}_p^{(1)}\hat{a}_n^{0(0)}}{r^2}\hat{A}_n^{(0)}(r)+\frac{1}{36}\frac{\hat{a}_n^{(1)}\hat{a}_p^{(0)}}{r^2}\hat{A}_p^{(0)}(r)$$

$$+\frac{1}{36}\frac{\hat{a}_n^{0(1)}\hat{a}_x^{(0)}}{r^2}\hat{A}_n^{x(0)}(r)-\frac{1}{36}\frac{\hat{a}_x^{(1)}\hat{a}_n^{0(0)}}{r^2}\hat{A}_n^{x(0)}(r)-\frac{1}{36}\frac{\hat{a}_x^{(1)}\hat{a}_x^{(0)}}{r^2}\hat{A}_n^{(0)}(r)$$

$$+\frac{1}{216}\frac{\hat{a}_n^{(1)}(\hat{a}_p^{(0)}\hat{a}_n^{0(0)}-(\hat{a}_x^{(0)})^2)}{r^3}\hat{A}_p^{(0)}(r)$$

Solution of Faddeev Equations for P–Wave K^-d Scattering Length: M. Faber *et al.* arXiv:1012.3933 [nucl-th]. Fixed Centre Approximation

$$\hat{A}_n^{(1)}(r) = \hat{a}_n^{(1)} + \frac{1}{6}\frac{\hat{a}_n^{(1)}}{r}\hat{A}_p^{(0)}(r) + \frac{1}{6}\frac{\hat{a}_n^{(0)}}{r}\hat{A}_p^{(1)}(r)$$

Solution of Faddeev Equations for P–Wave K^-d Scattering Length: M. Faber *et al.* arXiv:1012.3933 [nucl-th]. Effective Strong Low–Energy $\bar{K}N$ Interaction

$$\mathcal{L}_{\rm int}(x) = \mathcal{L}^{(0)}_{\rm int}(x) + \mathcal{L}^{(1)}_{\rm int}(x) =$$

$$= 4\pi[\hat{a}^{(0)}_p K^{-\dagger}(x)K^-(x)\bar{p}(x)p(x) + \hat{a}^{(0)}_x \bar{K}^{0\dagger}(x)K^-(x)\bar{n}(x)p(x)]$$

$$+4\pi[\hat{a}^{(0)}_n K^{-\dagger}(x)K^-(x)\bar{n}(x)n(x) + \hat{a}^{(0)}_x K^{-\dagger}(x)\bar{K}^0(x)\bar{p}(x)n(x)]$$

$$+4\pi[\hat{a}^{0(0)}_n \bar{K}^{0\dagger}(x)\bar{K}^0(x)\bar{n}(x)n(x)]$$

$$+12\pi[\hat{a}^{(1)}_p \bigtriangledown K^{-\dagger}(x)\cdot\bigtriangledown K^-(x)\bar{p}(x)p(x) + \hat{a}^{(1)}_x \bigtriangledown \bar{K}^{0\dagger}(x)\cdot\bigtriangledown K^-(x)\bar{n}(x)p(x)]$$

$$+12\pi[\hat{a}^{(1)}_n \bigtriangledown K^{-\dagger}(x)\cdot\bigtriangledown K^-(x)\bar{n}(x)n(x) + \hat{a}^{(1)}_x \bigtriangledown K^{-\dagger}(x)\cdot\bigtriangledown \bar{K}^0(x)\bar{p}(x)n(x)]$$

$$+12\pi[\hat{a}^{0(1)}_n \bigtriangledown \bar{K}^{0\dagger}(x)\cdot\bigtriangledown \bar{K}^0(x)\bar{n}(x)n(x)]$$

Solution of Faddeev Equations for P–Wave K^-d Scattering Length: M. Faber *et al.* arXiv:1012.3933 [nucl-th]. Wave Function of The Deuteron

Wave Function of The Deuteron

$$|d(-\vec{k},\lambda)\rangle =$$

$$= \frac{\sqrt{2E_d(\vec{k}\,)}}{(2\pi)^3}\int \frac{d^3k_p}{\sqrt{2E_N(\vec{k}_p)}}\frac{d^3k_n}{\sqrt{2E_N(\vec{k}_n)}}\,\delta^{(3)}(\vec{k}+\vec{k}_p+\vec{k}_n)$$

$$\times\,\tilde{\Phi}_d\left(\frac{\vec{k}_p-\vec{k}_n}{2}\right)[a^\dagger_p(\vec{k}_p,\sigma_p)a^\dagger_n(\vec{k}_n,\sigma_n)]_{\sigma_p+\sigma_n=\lambda}|0\rangle$$

Solution of Faddeev Equations for P–Wave K^-d Scattering Length: M. Faber *et al.* arXiv:1012.3933 [nucl-th]. Fixed Centre Approximation

Complex P–wave Scattering Length

in Single (Impulse) and Double Scattering Approximation

$$\tilde{a}^{(1)}_{K^-d} = \frac{m_d}{m_K + m_d}$$

$$\times\Big(\hat{a}^{(1)}_p + \hat{a}^{(1)}_n + \frac{1}{3}\,(\hat{a}^{(0)}_p \hat{a}^{(1)}_n + \hat{a}^{(0)}_n \hat{a}^{(1)}_p - \hat{a}^{(0)}_x \hat{a}^{(1)}_x) \int \frac{d^3x}{r}\,|\Phi_d(\vec{r}\,)|^2\Big) =$$

$$= -\,0.262 + i\,0.548\,\mathrm{fm}^3$$

Solution of Faddeev Equations for P–Wave K^-d Scattering Length: M. Faber *et al.* arXiv:1012.3933 [nucl-th]. Fixed Centre Approximation

Triple Scattering Contribution

$$(\tilde{a}^{(1)}_{K^-d})_{\rm tr.sc.} = \frac{m_d}{m_K + m_d}$$

$$\times \frac{1}{36}\Big[\hat{a}^{(1)}_p\Big(7\hat{a}^{(0)}_p\hat{a}^{(0)}_n + (\hat{a}^{(0)}_n)^2 - (\hat{a}^{0(0)}_n)^2\Big)$$

$$+\hat{a}^{(1)}_n\Big(7(\hat{a}^{(0)}_p\hat{a}^{(0)}_n - (\hat{a}^{(0)}_x)^2) + \hat{a}^{(0)}_p(\hat{a}^{(0)}_n + \hat{a}^{0(0)}_n) - 2\hat{a}^{(0)}_n\hat{a}^{(0)}_x\Big)$$

$$+\hat{a}^{(1)}_x\hat{a}^{(0)}_x)\Big(\hat{a}^{0(0)}_n - \hat{a}^{(0)}_n\Big) + \hat{a}^{0(1)}_n(\hat{a}^{(0)}_x)^2\Big]\int\frac{d^3x}{r^2}\,|\Phi_d(\vec{r}\,)|^2 =$$

$$= -0.015 - i\,0.023\,{\rm fm}^3$$

Chiral $SU(3) \times SU(3)$ Dynamics and $SU(3)$ Coupled–Channel Approach: arXiv:1012.3933 [nucl-th] $M^{-1} = M_0^{-1} - G$ for $\bar{K}N \to PB$

$$\mathcal{L}_{\chi D}(x) = \langle \bar{B}(x) i\gamma^{\mu}[s_{\mu}(x), B(x)]\rangle - g_A\,(1-\alpha_D)\langle \bar{B}(x)\gamma^{\mu}[p_{\mu}(x), B(x)]\rangle$$

$$+\alpha_D\,\langle \bar{B}(x)\gamma^{\mu}\{p_{\mu}(x), B(x)\}\rangle + \frac{1}{4}\,b_D\langle \bar{B}(x)\{\chi_{+}(x), B(x)\}\rangle$$

$$+\frac{1}{4}\,b_F\langle \bar{B}(x)[\chi_{+}(x), B(x)]\rangle + \frac{1}{4}\,b_0\langle \bar{B}(x)\langle \chi_{+}(x)\rangle B(x)\rangle$$

$$+\frac{1}{2}\,d_1\langle \bar{B}(x)\{p_{\mu}(x), [p^{\mu}(x), B(x)]\}\rangle + \frac{1}{2}\,d_2\langle \bar{B}(x)[p_{\mu}(x), [p^{\mu}(x), B(x)]]\rangle$$

$$+\frac{1}{2}\,d_3\langle \bar{B}(x)p_{\mu}(x)\rangle\langle p^{\mu}(x)B(x)\rangle + \frac{1}{2}\,d_4\langle \bar{B}(x)\langle p_{\mu}(x)p^{\mu}(x)\rangle B(x)\rangle$$

$$+g_{\Lambda^*}\,\bar{\Lambda}^*(x)\gamma^{\mu}\gamma^5\langle p_{\mu}(x)B(x)\rangle + \sqrt{2}\,g_{\Delta}\bar{D}^{abc}_{\mu}(x)\Theta^{\mu\nu}\gamma^5(p_{\nu}(x))^d_a B^e_b(x)\,\varepsilon_{cde} + \ldots$$

$$\epsilon_{1s}^{(\rm exp)} = 283(37)\,\mathrm{eV} \qquad \Gamma_{1s}^{(\rm exp)} = 541(92)\,\mathrm{eV}$$

M. Bazzi *et al.* (SIDDHARTA), arXiv: 1105.3090 [nucl–ex]

Numerical Results: M. Faber *et al.* arXiv:1012.3933 [nucl-th]

Complex S– and P–Wave Scattering Lengths
of K^-d Scattering

$$\tilde{a}^{(0)}_{K^-d} = -\,1.273 + i\,2.435\,\mathrm{fm}$$

$$\tilde{a}^{(1)}_{K^-d} = -\,0.352 + i\,0.432\,\mathrm{fm}^3$$

Energy Level Displacements of Kaonic Deuterium

$$\epsilon_{1s} = 0.766\,\mathrm{keV} \qquad \Gamma_{1s} = 2.933\,\mathrm{keV}$$

$$\epsilon_{2p} = 4.158\,\mathrm{meV} \qquad \Gamma_{2p} = 10.203\,\mathrm{meV}$$

Yield of X–Rays of K_α emission line

$$Y_{K^-p} = 1.80\,\%\,, \qquad \Gamma_{1p} = 1.979\,\mathrm{meV}$$

$$Y^{(\mathrm{exp})}_{K^-p} = 1.5(5)\,\% \quad (\mathrm{KEK})\,\mathrm{PRC58},\,2366\,(1998)$$

$$Y_{K^-d} = 0.27\,\%\,, \qquad \Gamma_{2p} = 10.203\,\mathrm{meV}$$

Comparison With Results, Obtained by W. Weise *et al.*

- B. Borasoy, R. Nißler, W. Weise, EPJA 25, 79 (2005):
  S–Wave $\bar{K}N$ Scattering Lengths
- W. Weise, R. Härtle, NPA 804, 173 (2008):
  P–Wave $\bar{K}N$ Scattering Lengths

Complex S– and P–Wave Scattering Lengths
of K^-d Scattering

$$\tilde{a}^{(0)}_{K^-d} = -1.951 + i\,0.996\,\mathrm{fm}$$

$$\tilde{a}^{(1)}_{K^-d} = -0.174 + i\,0.113\,\mathrm{fm}^3$$

Energy Level Displacements of Kaonic Deuterium

$$\epsilon_{1s} = 1.175\,\mathrm{keV} \qquad \Gamma_{1s} = 1.200\,\mathrm{keV}$$

$$\epsilon_{2p} = 2.053\,\mathrm{meV} \qquad \Gamma_{2p} = 2.675\,\mathrm{meV}$$

Yield of X–Rays of K_α emission line

$$Y_{K^-d} = 1.9\,\% \qquad \Gamma_{2p} = 2.675\,\mathrm{meV}$$

Accuracy of Solution is Approximately 15 %

Assumption

- We assume that the accuracy of our solution of the Faddeev equation in the fixed centre approximation for the complex P–wave scattering length of $K^- d$ scattering is of about 15 %

Ground of Assumption

- A. Gal, arXiv: nucl–th/0607067: Accuracy of S–wave scattering length of $K^- d$ scattering as a solution of Faddeev equations in the fixed centre approximation is $(10 - 25)\,\%$
- V. Baru, E. Epelbaum, A. Rusetsky, EPJA 42, 111 (2009): Nucleon recoil corrections to the double-scattering contribution to S–wave scattering length of $K^- d$ scattering is $(10 - 15)\,\%$

Summary

- We have proposed the solution of the Faddeev coupled–channel equations in fixed centre approximation for the P–wave scattering length of K^-d scattering within chiral $SU(3) \times SU(3)$ dynamics and $SU(3)$ coupled channel approach for low–energy $\bar{K}N$ scattering
- We have calculated the energy level displacements for the ground 1s and the excited 2p states of kaonic deuterium and the yield of X–rays of K_α emission line
- The obtained results can be used for the planning of the experiments on the measurements of the energy level displacement of the ground state of kaonic deuterium and for SIDDHARTA Collaboration, measuring currently the energy level displacement of the ground state of kaonic deuterium

Thank You for Attention