Hyperfine spectroscopy of simple systems in external magnetic and electric fields

D.Bakalov
Institute for nuclear research
Bulgarian Academy of sciences

Simple systems?

- “Simple” enough to allow for high accuracy theoretical description
- Accessible to high precision spectroscopy experiments

Simple systems?

- Long-lived,
- dominant electromagnetic interactions

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- 2-body:

Ordinary: hydrogen-like atoms and ions

Exotic: positronium, muonium, μ^-p

Simple systems?

- Long-lived,
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- 2-body:

  Ordinary: hydrogen-like atoms and ions

  Exotic: positronium, muonium, $\mu^- p$
- 3-body: molecular ions $H2^+$,HD^+,

  exotic: antiprotonic helium, $p\mu^- p$

Precision HD+ spectroscopy

- High accuracy goals (Duesseldorf team):

  10^{-10} for the determination of the electron-to-proton and electron-to-deuteron mass ratio

  10^{-16} for testing the time variability of fundamental constants

Precision HD+ spectroscopy

- Theoretical uncertainties: below 10 kHz

This requires

- Higher orders relativistic and QED effects
- Hyperfine structure

but also

Corrections due to external magnetic and electric fields (constant or oscillating)

External fields

- Magnetic field of the Earth (~0.5G)
- Incompletely shielded fields (?)

but also:

- Trap fields
- Bias fields
- …

Hyperfine structure of HD^+

- Classification:

  L: orbital momentum

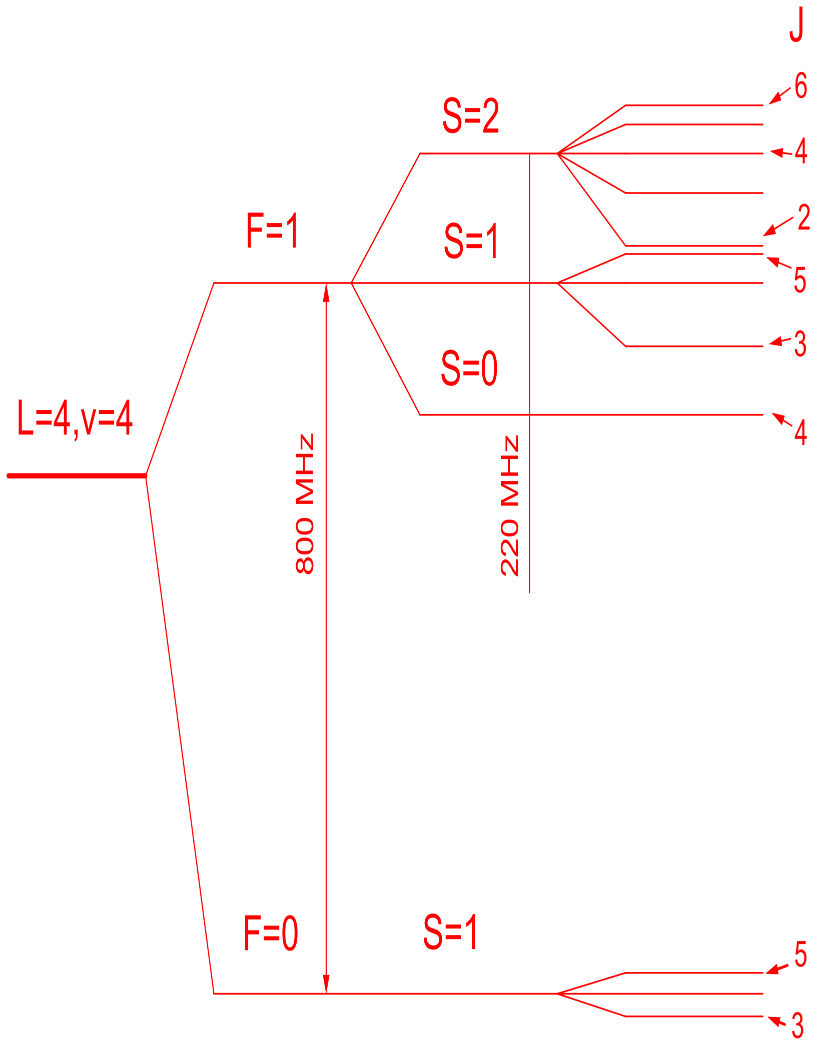
  V: vibrational q.n.

  $F = S_p + S_e$

  $S = F + S_d$

  $J = S + L$

  (J_z)



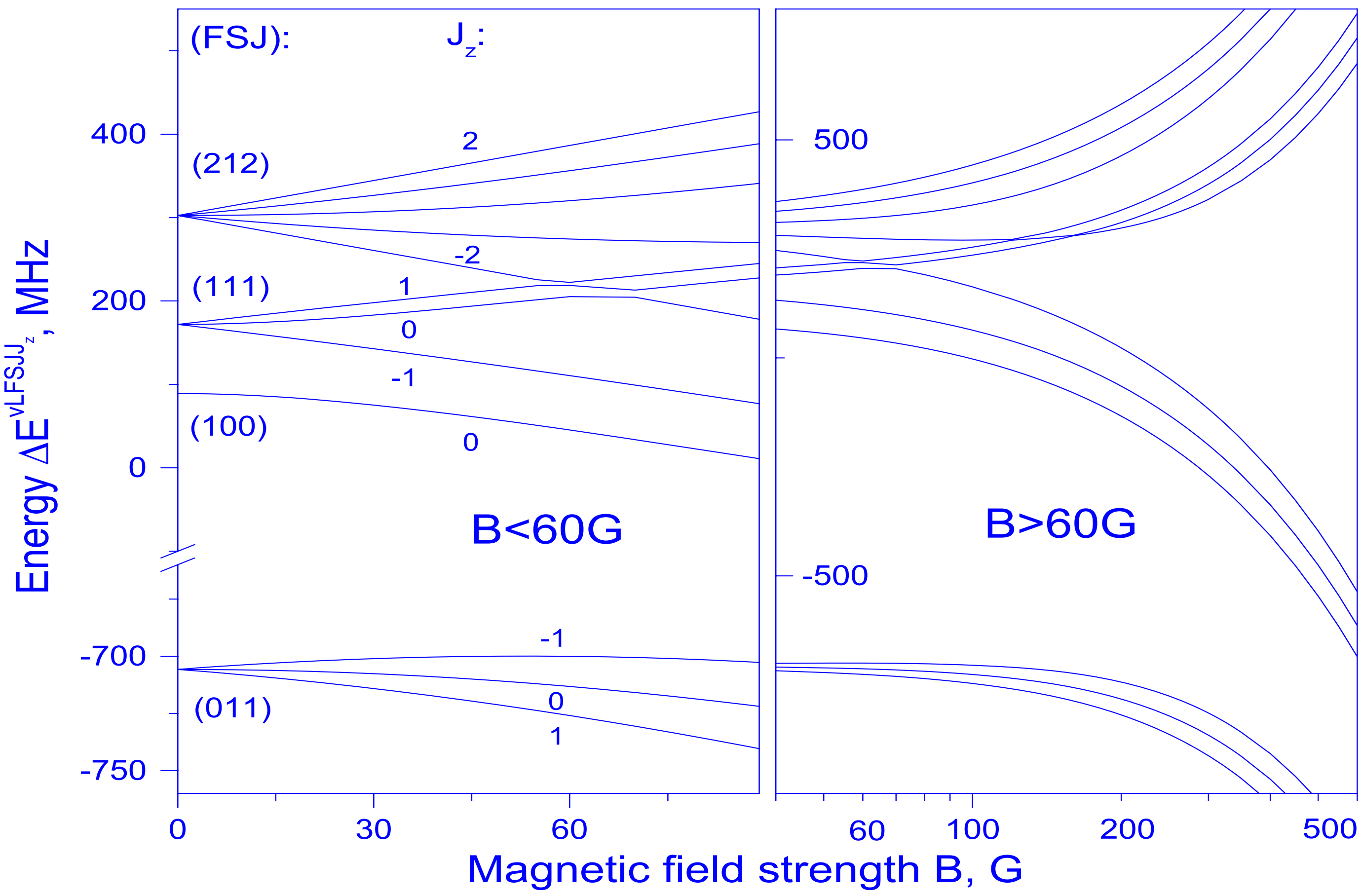
Effective Hamiltonian

$$H^{hfs} = E_1(S_p.S_e)+E_2(S_d.S_e)+E_3(S_d.S_p)+$$
$$+E_4(S_d.L)+E_5(S_p.L)+E_6(S_e.L)+\ldots$$

$$H=H^{hfs}+H^{mag}$$

$$H^{mag} =$$
$$E_{10}(L.B)+E_{11}(S_p.B)+E_{12}(S_d.B)+E_{13}(S_e.B)$$

Zeeman splitting (L=0,v=0)



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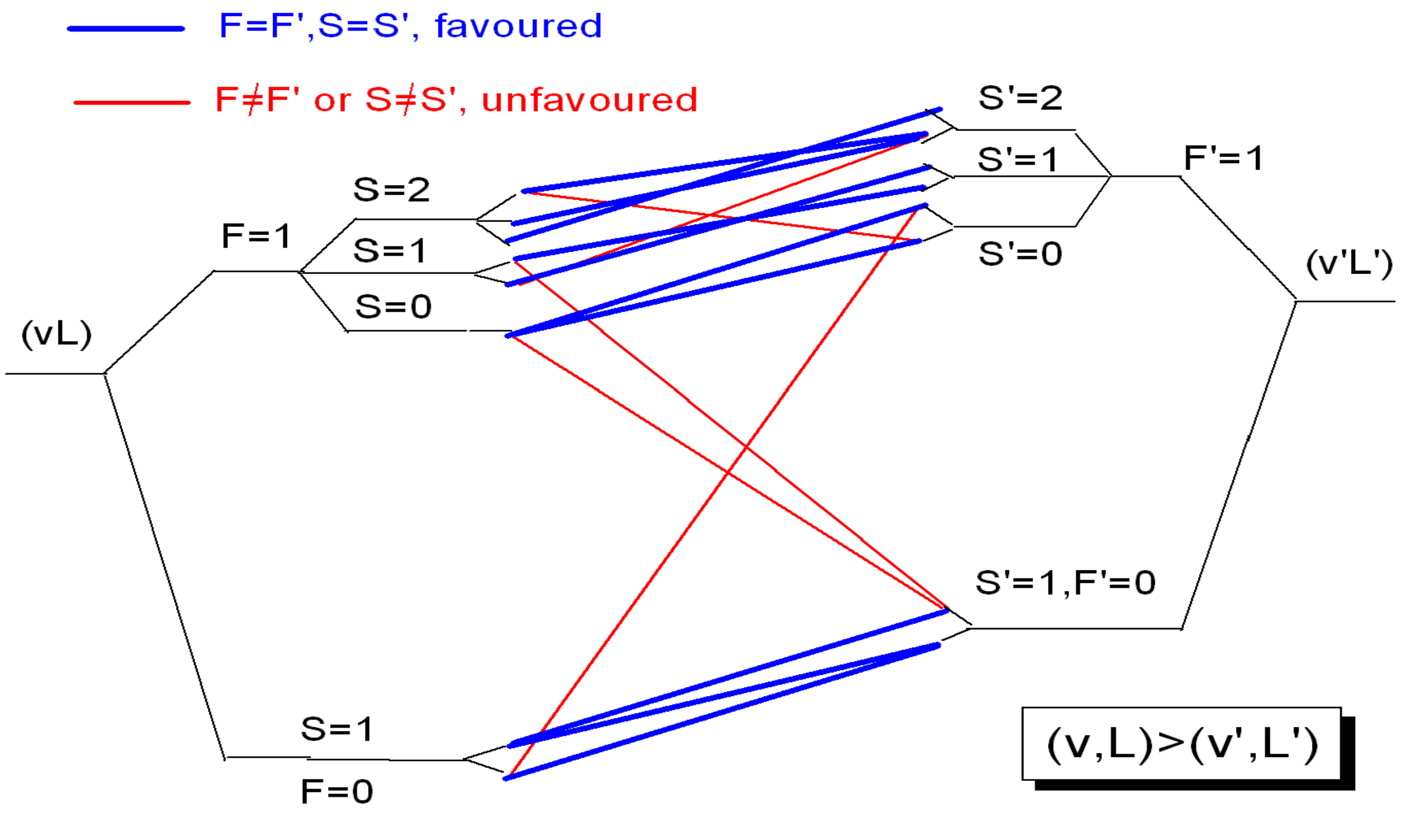
E1 transition spectra



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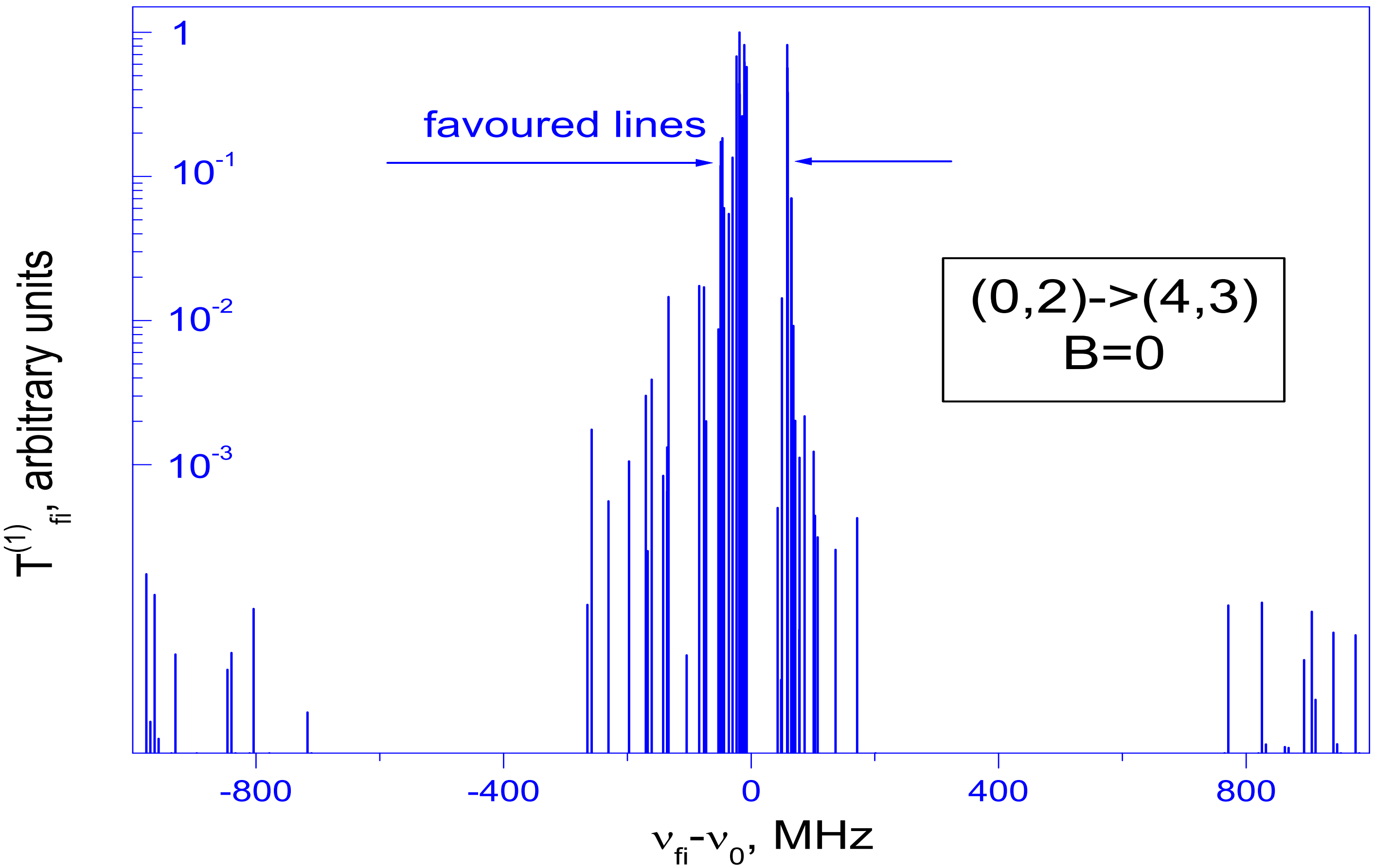
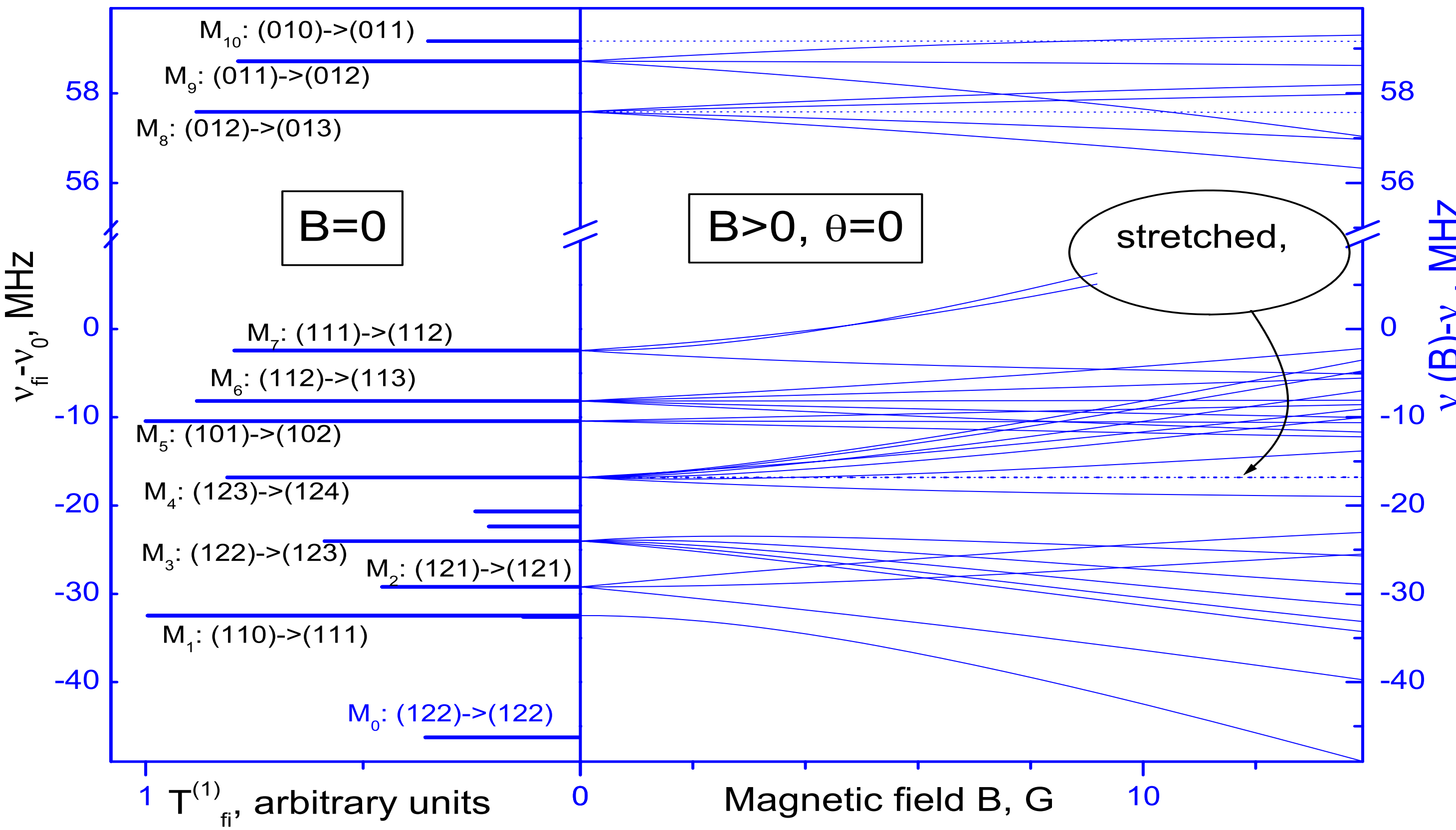
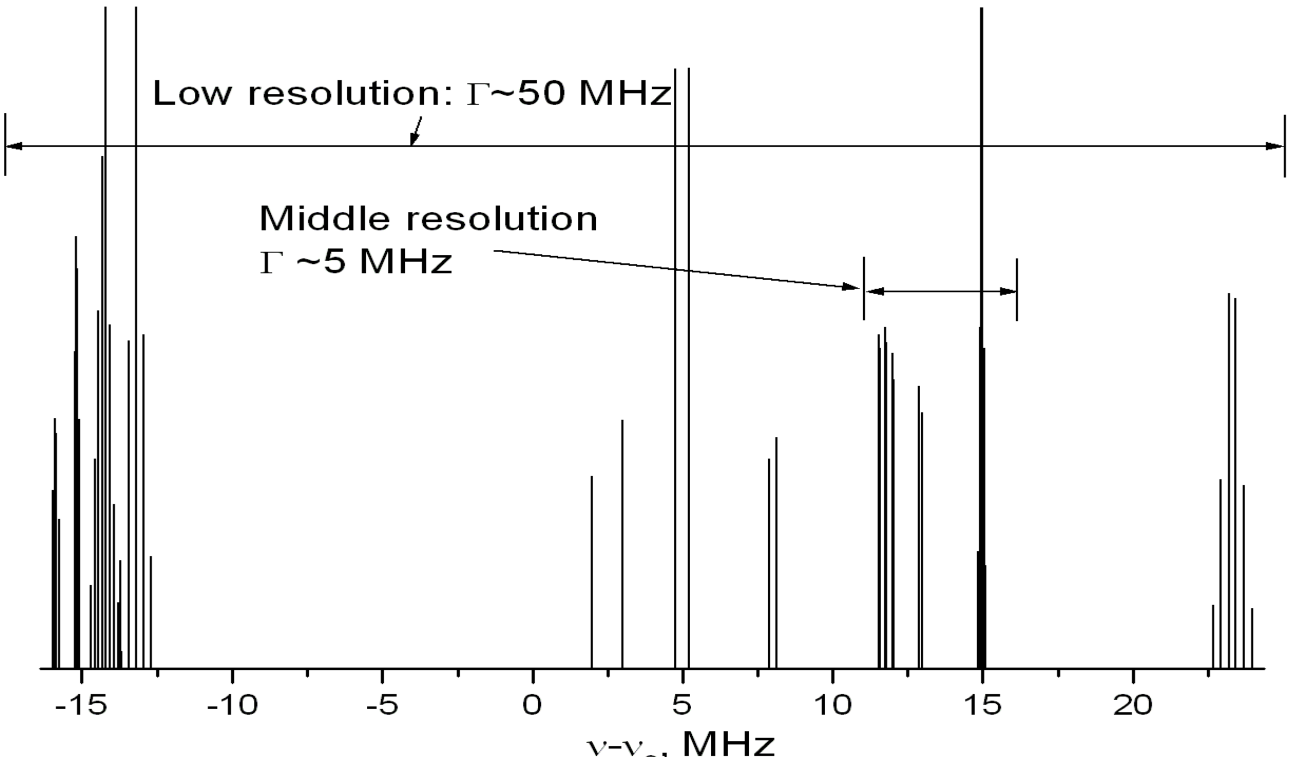
HFS of transition spectra



Zeeman structure of (0,1)→(4,2)



Observable Zeeman effects



Observable Zeeman effects

Dependence on the spectral resolution.

Low resolution >50 MHz: line broadening

Middle resolution: also shift of the h.f. lines:

(F,S,J)	$(1,2,3)$		$(1,2,1)$		$(1,1,2)$		$(0,1,3)$		$(0,1,2)$		$(0,1,1)$	
$\bar{\nu}-\nu_0$, MHz	-23.481		-14.257		-6.902		58.251		58.861		59.426	
$\theta =$	0	$\pi/2$	0	$\pi/2$	0	$\pi/2$	0	$\pi/2$	0	$\pi/2$	0	$\pi/2$
$\delta\bar{\nu}(0.5\,\mathrm{G})$, kHz	5.1	5.1	-1.5	-3.0	-1.4	-2.7	-0.1	0.1	-0.2	-0.1	0.0	-0.1
$\delta\bar{\nu}(1.0\,\mathrm{G})$, kHz	16.9	17.0	-5.1	-9.8	-4.8	-8.7	-0.3	0.4	-0.5	-0.1	0.0	-0.5

High resolution spectroscopy

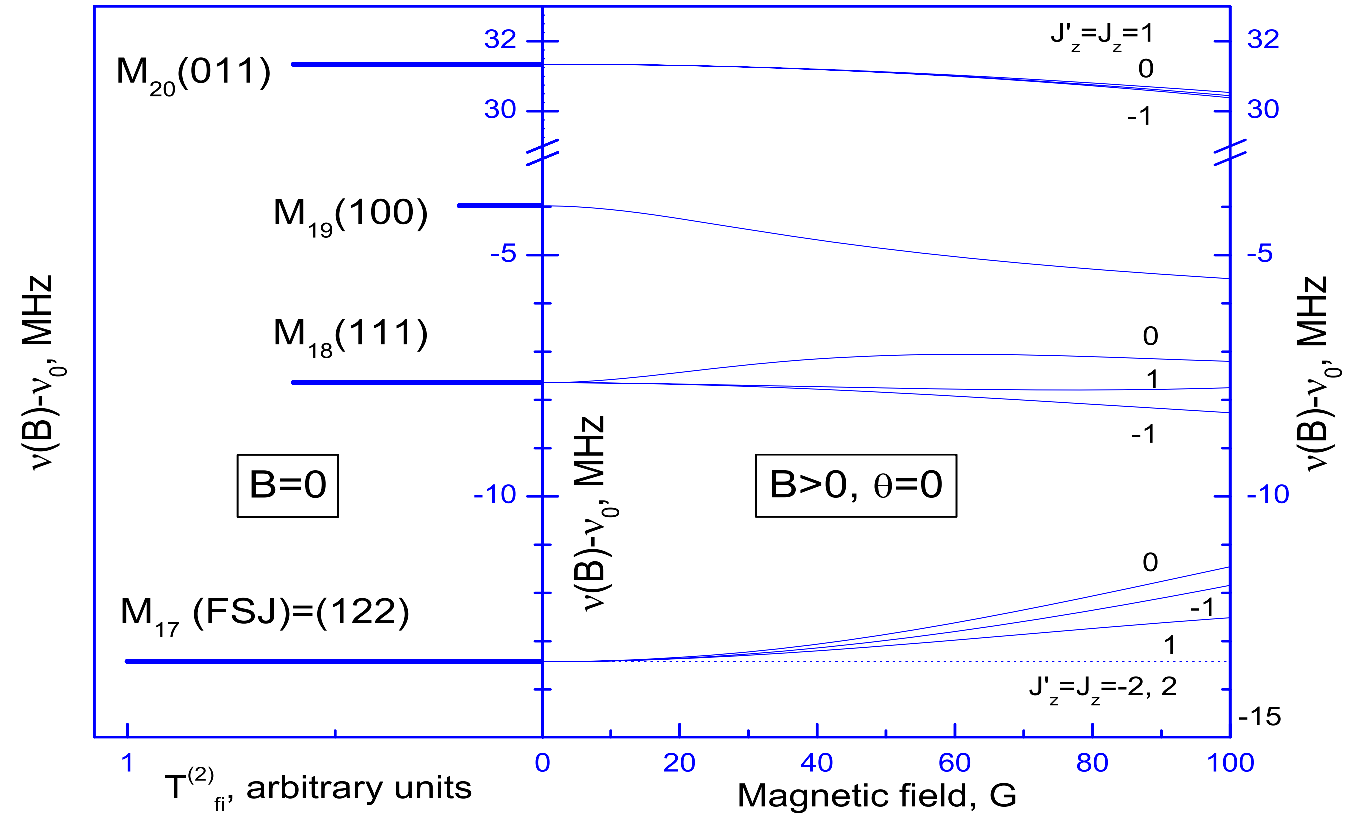
- Individual Zeeman components resolved
- Search for transitions, insensitive to B

$$f(B)-f(0) = t.J_z.B+(q+r.J_z^2).B^2$$

1. Transition between stretched lines: no quadratic dependence on B
2. More appropriate choices:

   Zeeman shift below 40 Hz/G^2

2-gamma transition (00)→(20)



Stark effect in HD^+

$H = H^{hfs} + H^{mag} + H^{el}$

$H^{el} = -E.d + q.Q + \ldots$

The E.d term – in second order of P.T.

The q.Q term – in first order of P.T.

Polarizabilities of HD^+

Static Stark effect in HD^+

- In absence of magnetic fields and HFS:

$$\Delta f_i = f_i(E) - f_i(0) = -\frac{1}{2}\alpha_i^{(D)}\mathbf{E}^2 + \frac{1}{2}\alpha^{(Q)} \cdot \mathbf{q}$$

$$\mathbf{E} = -\nabla V; \quad \mathbf{q}_{mn} = \nabla_m \nabla_n V$$

$$\alpha^{(D)} = -2\sum_k \frac{|\langle k|\mathbf{d}|i\rangle|^2}{f_i(0) - f_k(0)} \quad \alpha_{mn}^{(Q)} = \langle i|\mathbf{Q}_{nm}|i\rangle$$

Convergence: v'=v, L'=L±1: ~99%; el. excitations: ~1%

Moss et al, 2002, Koelemeij, 2011, Bakalov et al., 2011

Static Stark effect in HD^+

- Dipole polarizabilities $a^{(D)}_{vLM}$:

  increase fast with v

  decrease slowly with L

  Typically $0.3\text{-}3\times10^{-8}$ kHz/(V/m)^2

  and up to 10^{-6} kHz/(V/m)^2 for v>6
- Quadrupole polarizabilities $a^{(Q)}_{vLM}$:

  same behavior with v and L

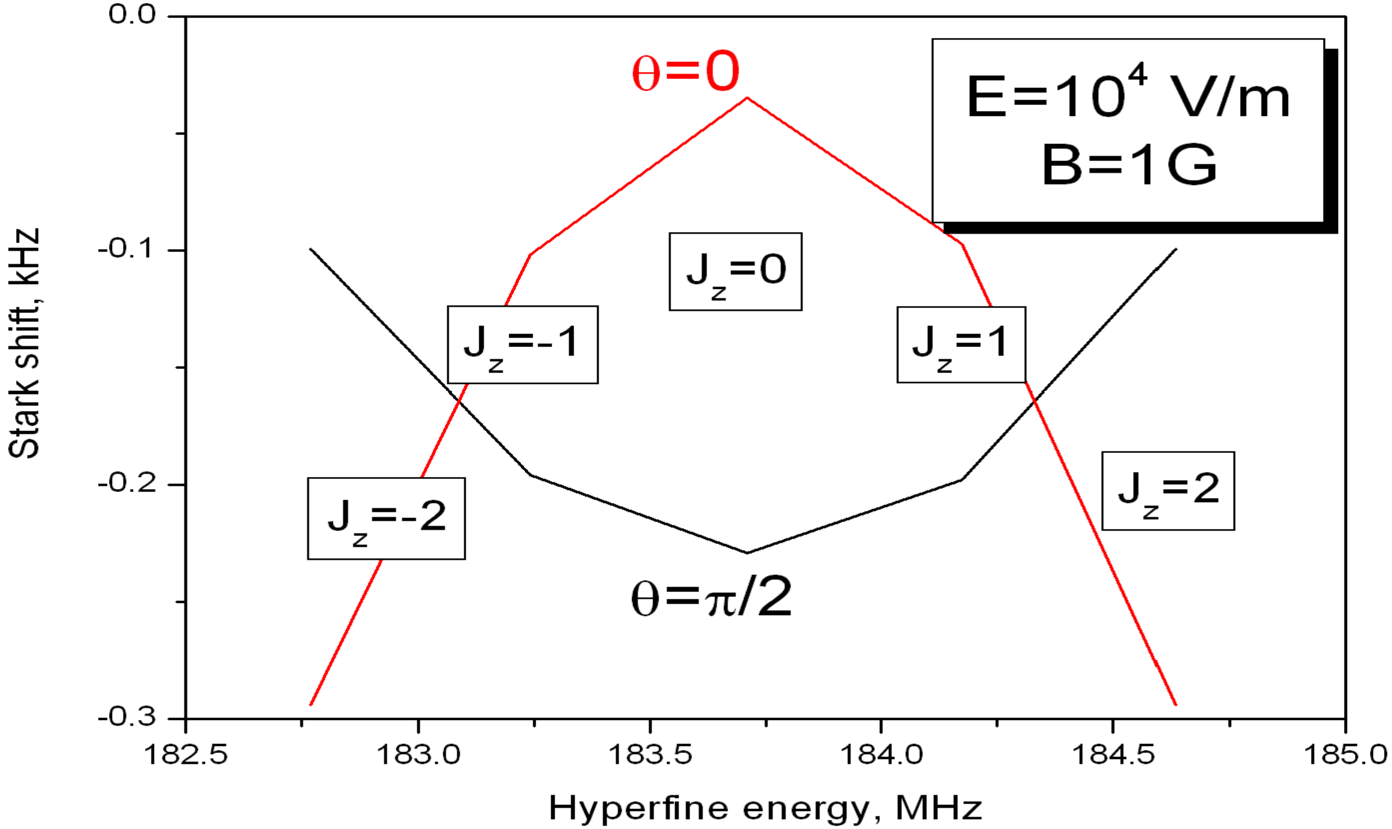
  Typically: 1-2 orders of magnitude smaller

Stark and Zeeman effects with HFS

$$f_i(E) - f_i(0) = -\mathrm{E}^2 \sum_k (\sin^2\theta (A_{-1}^{(ki)2} + A_1^{(ki)2}) + \cos^2\theta A_0^{(ki)2}) / (f_i(0) - f_k(0))$$

$$A_q^{(ki)} = \langle k || \mathbf{d} | i \rangle \sum_{FSJ'J} \sqrt{2J+1} \begin{Bmatrix} J' & 1 & J \\ L & S & L' \end{Bmatrix} (-1)^{J'+L'+S} \beta^k_{FSJ'} \beta^i_{FSJ} C^{J'J'_z}_{JJ_z,1q}$$

Stark + Zeeman effects



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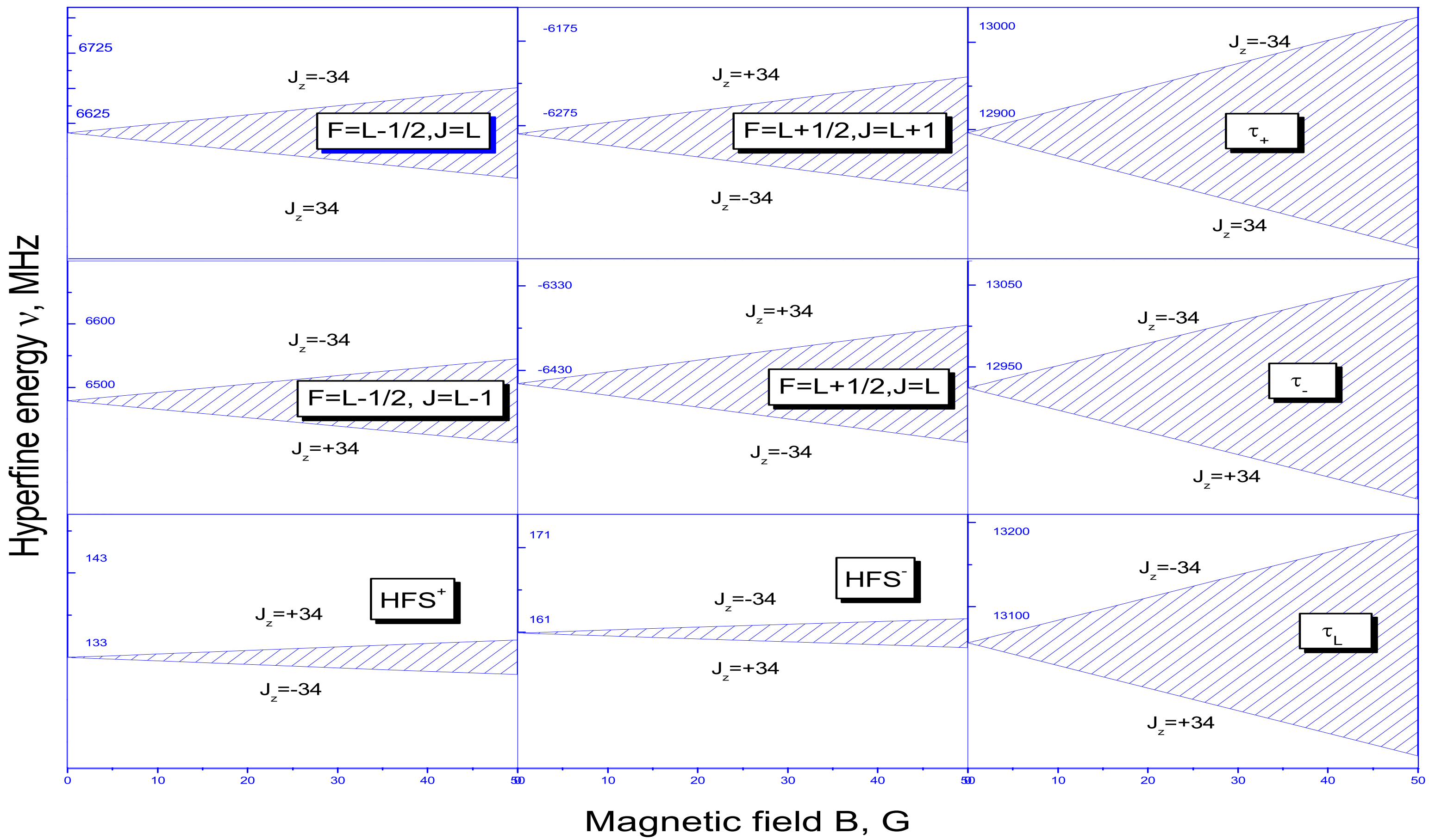
Stark + Zeeman effects

- The Stark shift depends on $|J_z|$, while the Zeeman depends on J_z:
- The Stark shift of the weighed mean of a h.f. multiplet differs from the simple form.
- Deformation of the hyperfine structure

Effects of up to a few kHz for E~10^8 V/m

Zeeman shift in antiprotonic He



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Zeeman shift in antiprotonic He

- The response of antiprotonic He levels to external magnetic fields – weaker, but

  $J_z >> 1$
- Quadratic effects – need to be studied,

  expected to be stronger in ^3He
- Possible shift of the weighed mean of the multiplets of unresolved Zeeman lines.

Next steps to do

- Dynamic Stark (and Zeeman) effects incl. Black Body Radiation shift