THEORY of KAONIC DEUTERIUM

in view of SIDDHARTA

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- **Intro II: Effective Field Theory for hadronic atoms**
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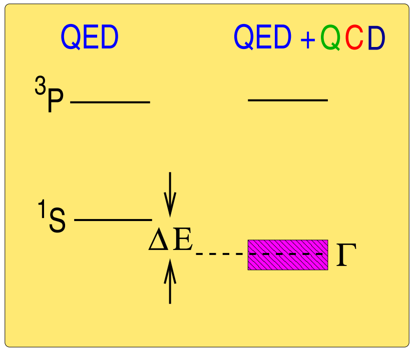
Introduction

INTRODUCTORY REMARKS

- Hadronic atoms are bound by the static Coulomb force (QED)
- Many species: $\pi^+\pi^-$, $\pi^\pm K^\mp$, $\pi^- p$, $\pi^- d$, $K^- p$, $K^- d$, ...
- Bohr radii $\gg$ typical scale of strong interactions
- Small average momenta $\Rightarrow$ non-relativistic approach
- Observable effects of QCD
  - ⋆ energy shift ΔE from the Coulomb value
  - ⋆ deacy width Γ



$\Rightarrow$ access to scattering at zero energy! = S-wave scattering lengths

- These scattering lengths are very sensitive to the chiral & isospin symmetry breaking in QCD

Weinberg, Gasser, Leutwyler, . . .

- can be analyzed systematically & consistently in the framework of low-energy Effective Field Theory (including virtual photons)

EFFECTIVE FIELD THEORY for HADRONIC ATOMS

- Three step procedure utilizing *nested* effective field theories

- Step 1: *construction*

  Construct non-relativistic effective Lagrangian (complex couplings)
  & solve Coulomb problem exactly, corrections in perturbation theory

- Step 2: *matching*

  relate complex couplings of $\mathcal{L}_{\rm eff}$ to QCD parameters, e.g. scattering lengths
  & express complex energy shift in terms of QCD parameters

- Step 3: *extraction*

  extract scattering length(s) from the measured complex energy shift

  $\Rightarrow$ most precise way of determining hadron-hadron scattering lengths

Comprehensive review: Gasser, Lyubovitskij, Rusetsky, Phys. Rept. **456** (2008) 167

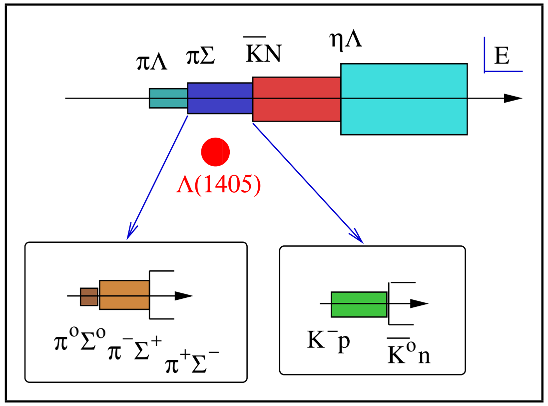
Analysis of kaonic hydrogen

UGM, Raha, Rusetsky, Eur. Phys. J. C **35** (2004) 349 [hep-ph/0402261]

FEATURES OF KAONIC HYDROGEN

- Strong ($K^-p \to \pi^0\Lambda, \pi^\pm\Sigma^\mp, \ldots$) and weaker electromagnetic ($K^-p \to \gamma\Lambda, \gamma\Sigma^0, \ldots$) decays

  $\to$ complicated (interesting) analytical structure



- Average momentum $\langle p^2\rangle = \alpha\,\mu \simeq 2$ MeV

  $\to$ highly non-relativistic

- Bohr radius $r_B = 1/(\alpha\,\mu) \simeq 100$ fm

- Binding energy $E_{1s} = \frac{1}{2}\,\alpha^2\,\mu + \ldots \simeq 8$ keV

- Width $\Gamma_{1s} \simeq 250$ eV $\ll E_{1s}$

- $\Delta = m_n + M_{K^0} - m_p + M_{K^-} > 0 \Rightarrow$ unitary cusp

- Isospin breaking, small parameter $\delta \sim \alpha \sim (m_d - m_u)$

$$\Delta E = \underbrace{\delta^3}_{\text{LO}} + \underbrace{\delta^4}_{\text{NLO}} + \ldots$$

ENERGY SHIFT in KAONIC HYDROGEN

a) recoil corrections

b) transvere photon exchange

c) finite size corrections

d) vacuum polarisation

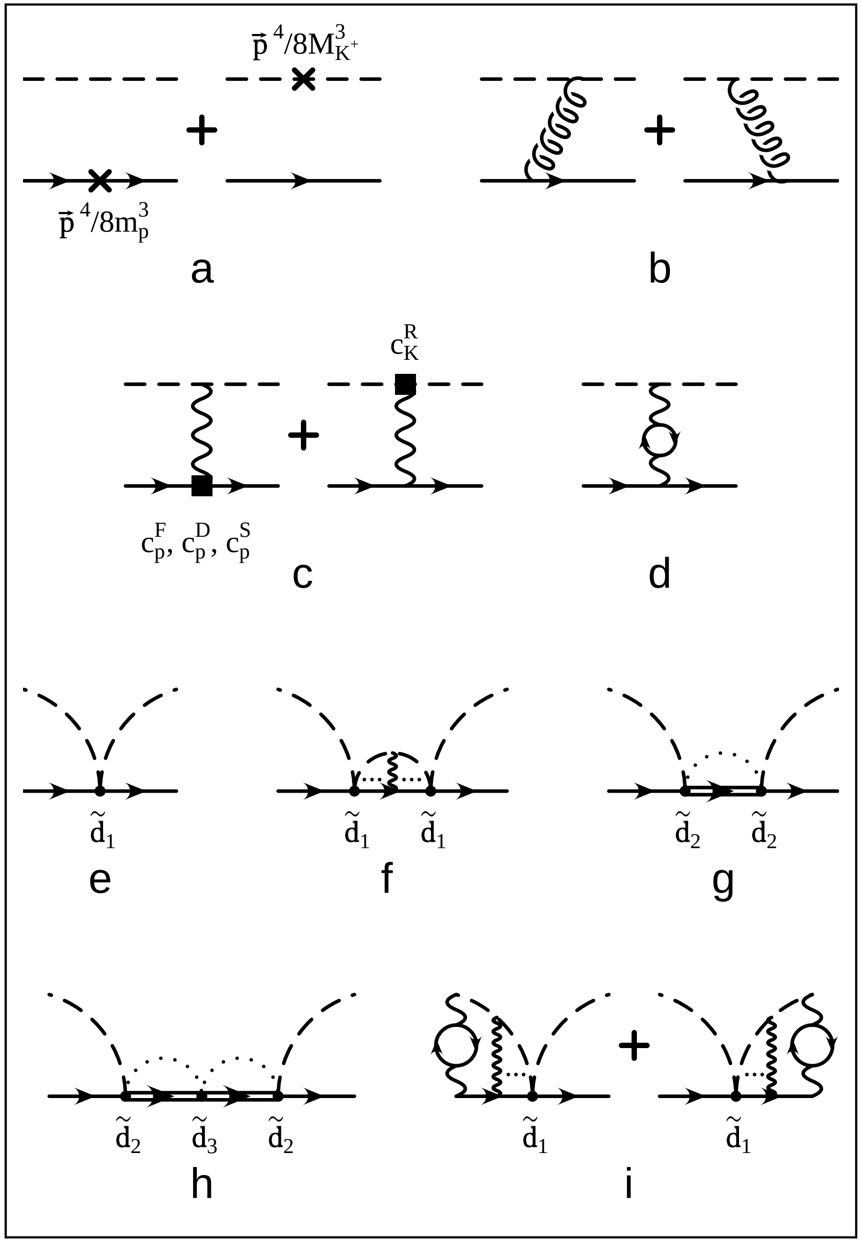
e) leading $K^- p$ interaction

f) $K^- p$ interaction w/ Coulomb ladders

g) leading $\bar{K}^0 n$ intermediate state

h) iterated $\bar{K}^0 n$ intermediate state

i) Coulomb ladders in the $K^- p$ interaction



COMPLEX ENERGY SHIFT in KAONIC HYDROGEN

UGM, Raha, Rusetsky, Eur. Phys. J. C **35** (2004) 349 [arXiv:hep-ph/0402261]

$$\Delta E_{1s} - \frac{i}{2}\Gamma_{1s} = -2\alpha^3\mu_c^2\, a_p \left(1 - 2\alpha\mu_c(\ln\alpha - 1)\, a_p + \ldots\right)$$

$$a_p = \frac{(a_0 + a_1)/2 + q_0 a_0 a_1}{1 + q_0(a_0 + a_1)/2} \qquad q_0 = \sqrt{2\mu_0\Delta}$$

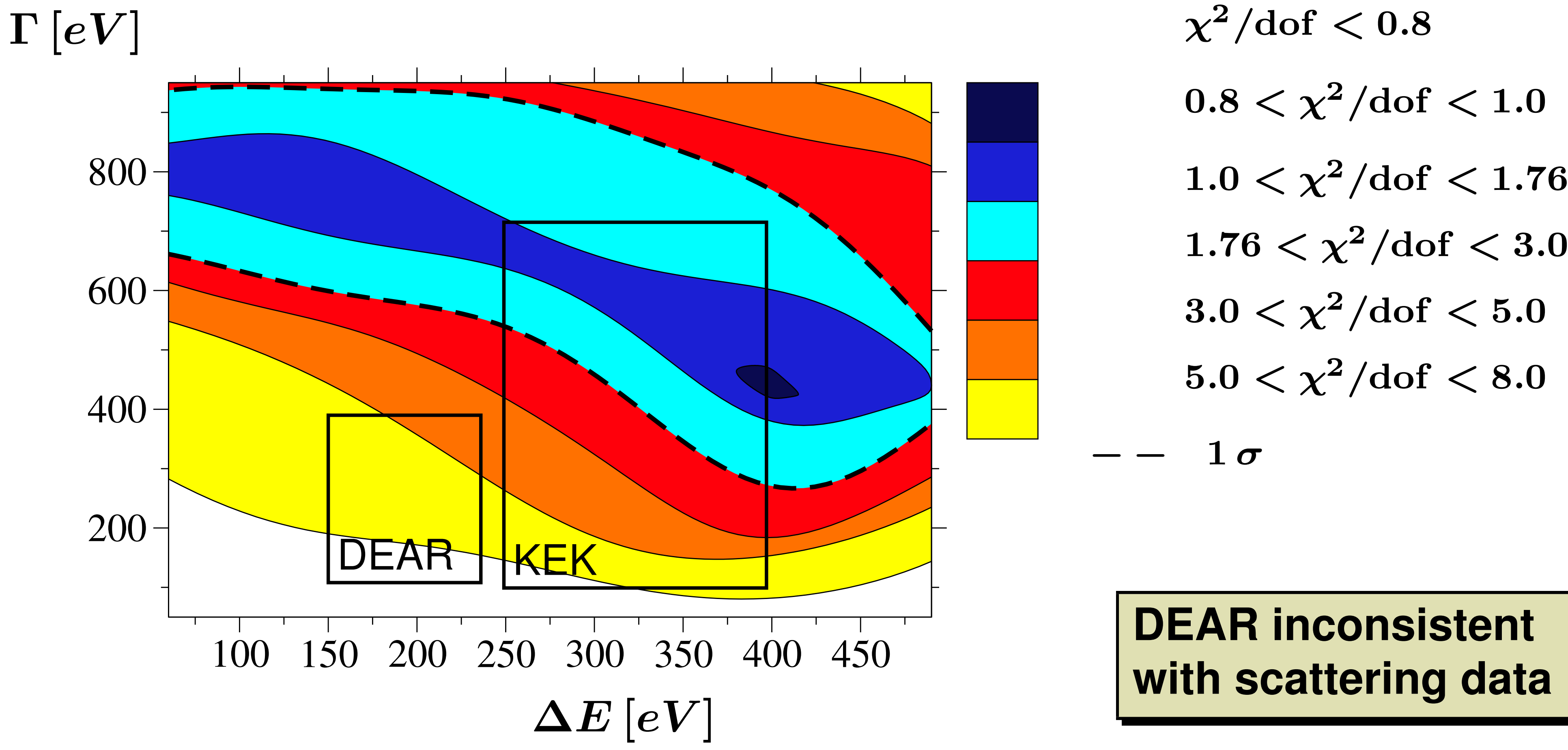
- $\mathcal{O}(\sqrt{\delta})$ and $\mathcal{O}(\delta\ln\delta)$ terms:
  - ⋆ Parameter-free, in terms of a_0 and a_1
  - ⋆ Numerically by far dominant ⇒ **unitary cusp** $(K^- p \to \bar{K}^0 n)$
- Much smaller effects from further isospin violation in T_{KN} and vacuum polarization: $\delta_n^{\rm vac} \simeq 1\%$

D. Eiras and J. Soto, Phys. Lett. **B 491** (2000) 101 [hep-ph/0005066]

THE DEAR MYSTERY

- Analyse scattering data in chiral unitary approach @ NLO: scan 14 parameters by $\sim$ 10000 MC fits

Borasoy, Nißler, UGM, Phys. Rev. **C 74** (2006) 055201



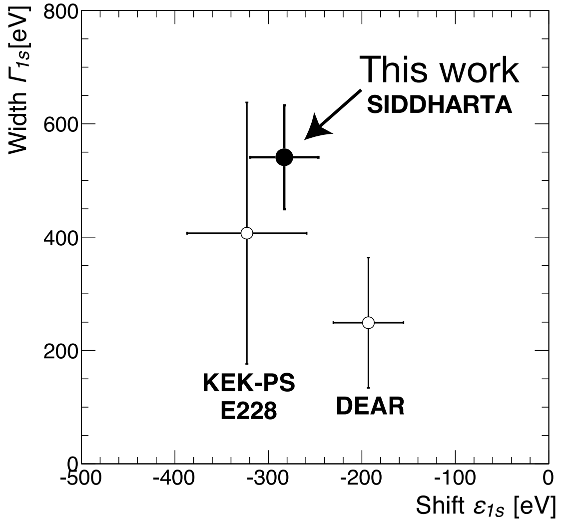
DEAR inconsistent with scattering data

THE SIDDHARTA RESOLUTION

- SIDDHARTA 2011:

$$\Delta E_{1s} = (-283 \pm 36 \pm 6)\ \mathrm{eV}\,, \quad \Gamma_{1s} = (541 \pm 89 \pm 22)\ \mathrm{eV}$$

a_p [fm]	Experiment
$-0.82 + i\,0.64$	KpX [1]
$-0.48 + i\,0.35$	DEAR [2]
$-0.66 + i\,0.81$	SIDDHARTA [3]
$-0.85 + i\,0.78$	Average SIDDHARTA [3] & scattering data [4]



[1] M. Iwasaki *et al.*, Phys. Rev. Lett. **78** (1997) 3067.

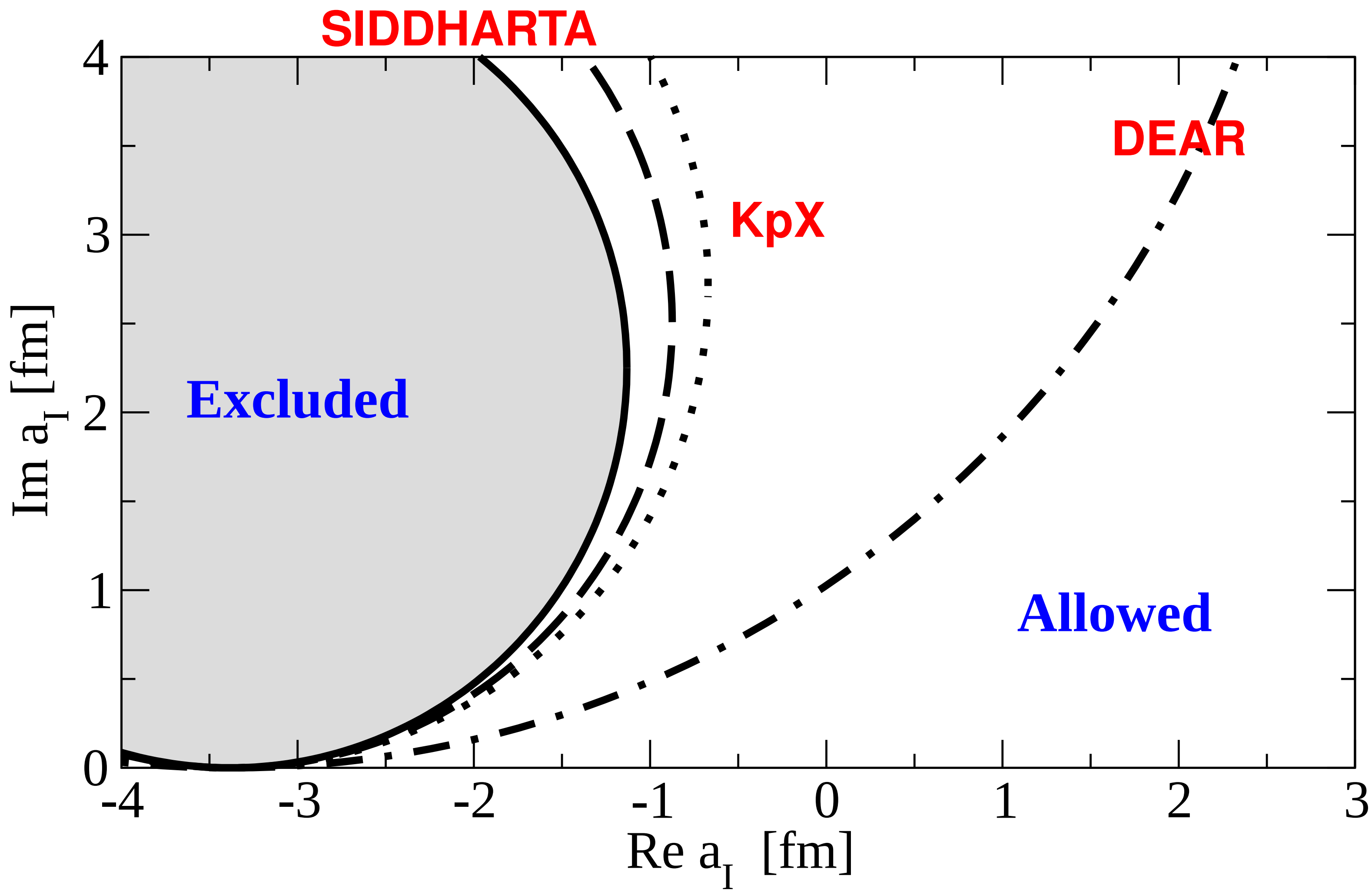
[2] G. Beer *et al.* [DEAR Collaboration], Phys. Rev. Lett. **94** (2005) 212302.

[3] M. Bazzi *et al.*, [arXiv:1105.3090 [nucl-ex]].

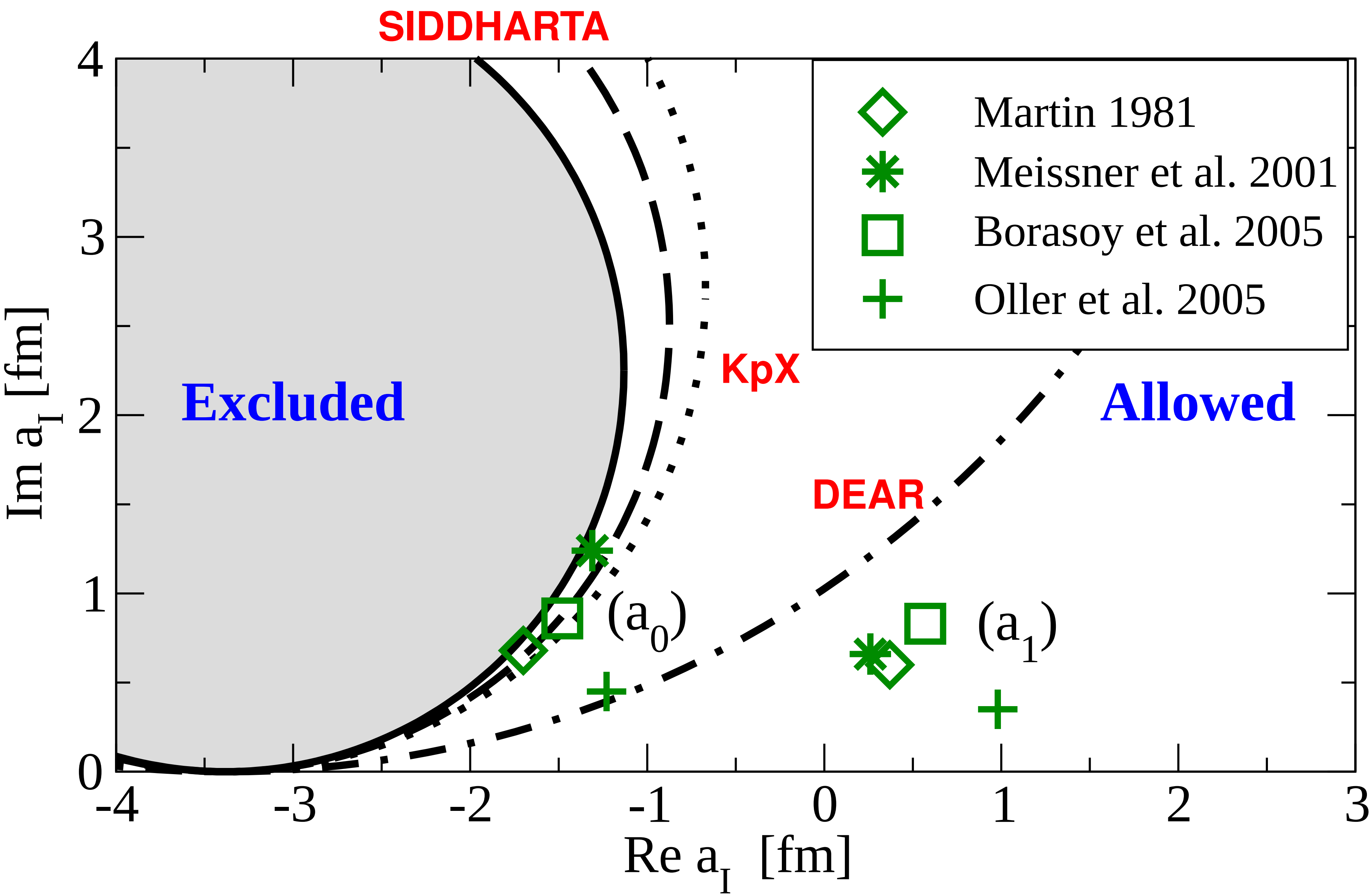
[4] B. Borasoy, UGM, R. Nissler, Phys. Rev. **C74** (2006) 055201.

ALLOWED REGIONS for a_0 & a_1

- universal circle: $$a_0 + a_1 + \frac{2q_0}{1 - q_0 a_p}\, a_0 a_1 - \frac{2a_p}{1 - q_0 a_p} = 0\,, \quad \mathrm{Im}\, a_I > 0$$

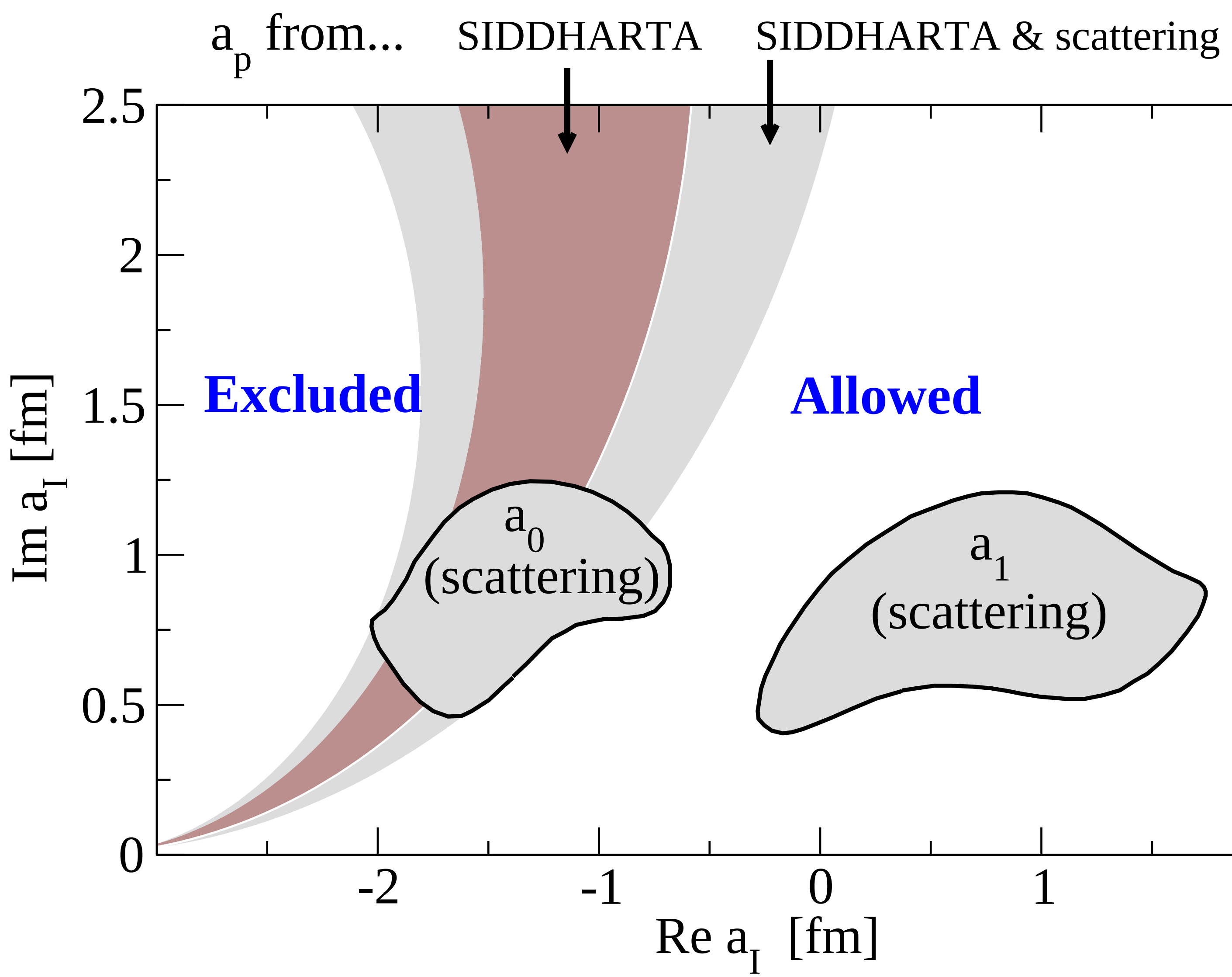


ALLOWED REGIONS for a_0 & a_1 cont'd



ALLOWED REGIONS: ERROR ANALYSIS

- combining experimental and theoretical errors



Analysis of kaonic deuterium

UGM, Raha, Rusetsky, Eur. Phys. J. **C 47** (2006) 473 [nucl-th/0603029]

Döring, UGM, arXiv:1108.5912 [nucl-th]

WHY KAONIC DEUTERIUM?

- Further info on a_{KN}: $a_{Kd} = \frac{1}{2}(a_0 + 3a_1) + \text{double scattering} + \ldots$

- Expected characteristics: $E^d_{1s} \simeq \frac{1}{2}\mu_{Kd}\alpha^2 = 10.4$ keV

  $\Gamma^d_{1s} = 1/\tau \simeq 1.2$ keV

- Sensitivity to deuteron structure → convergence of multiple scattering series?

  Baru, Epelbaum, Rusetsky, Eur. Phys. J. **A 42** (2009) 111

- Experimental terra incognita: → SIDDHARTA(2) @ DAΦNE

- Tackle the **inverse** problem here:

  Assume synthetic data for the complex Kd scattering length

  → what constraints does this put on a_0 & a_1 ?

THEORETICAL FRAMEWORK

- Resummation of the multiple scattering series in the static limit (FCA)

Kamalow, Oset, Ramos, Nucl. Phys. **A690** (2001) 494
Bahaoui, Fayard, Mizutani, Saghai, Phys. Rev. **C 68** (2003) 064001

$$\left(1+\frac{M_K}{m_d}\right)A_{Kd} = \int_0^\infty dr\,\left(u^2(r)+w^2(r)\right)\hat{a}_{Kd}(r)$$

$$\hat{a}_{Kd}(r) = \frac{a_p + a_n + (2a_p a_n - b_x^2)/r - 2b_x^2 a_n/r^2}{1 - a_p a_n/r^2 + b_x^2 a_n/r^3} + \delta\hat{a}_{Kd}$$

with

- $u(r), w(r)$ = deuteron S-, D-wave function
- $\delta\hat{a}_{Kd} \sim$ 3-body LEC (small, cf. K^-–absorption)

see detailed discussion in UGM, Raha. Rusetsky, EPJ **C 41** (2005) 213

- $b_x^2 = a_x^2/(1+a_u/r)$
- $a_{p,n,x,u}$ = threshold scattering amplitudes for $K^-p \to K^-p$,

$$K^-n \to K^-n,\, K^-p \to \bar{K}^0 n,\, \bar{K}^0 n \to \bar{K}^0 n$$

- note: large IV corrections (unitary cusp) in the individual amplitudes

RESULTS

- Isospin-breaking effects in A_{Kd} come out very small (despite the large IV corrections in the elementary amplitudes)

- Use synthetic data for A_{Kd} based on calcs in the literature together w/ the kaonic hydrogen results

  $\Rightarrow$ constraints on $\bar{K}N$ scattering lengths

  $\Rightarrow$ consistent with extraction of $\bar{K}^0 d$ scatt. length from $pp \to d\bar{K}^0 K^+$

  Sibirtsev, Büscher, Grishina, Hanhart, Kondryatchuk, Krewald, M., PLB **601** (2004) 132

- what values can A_{Kd} take so that solutions for a_0 and a_1 exist?

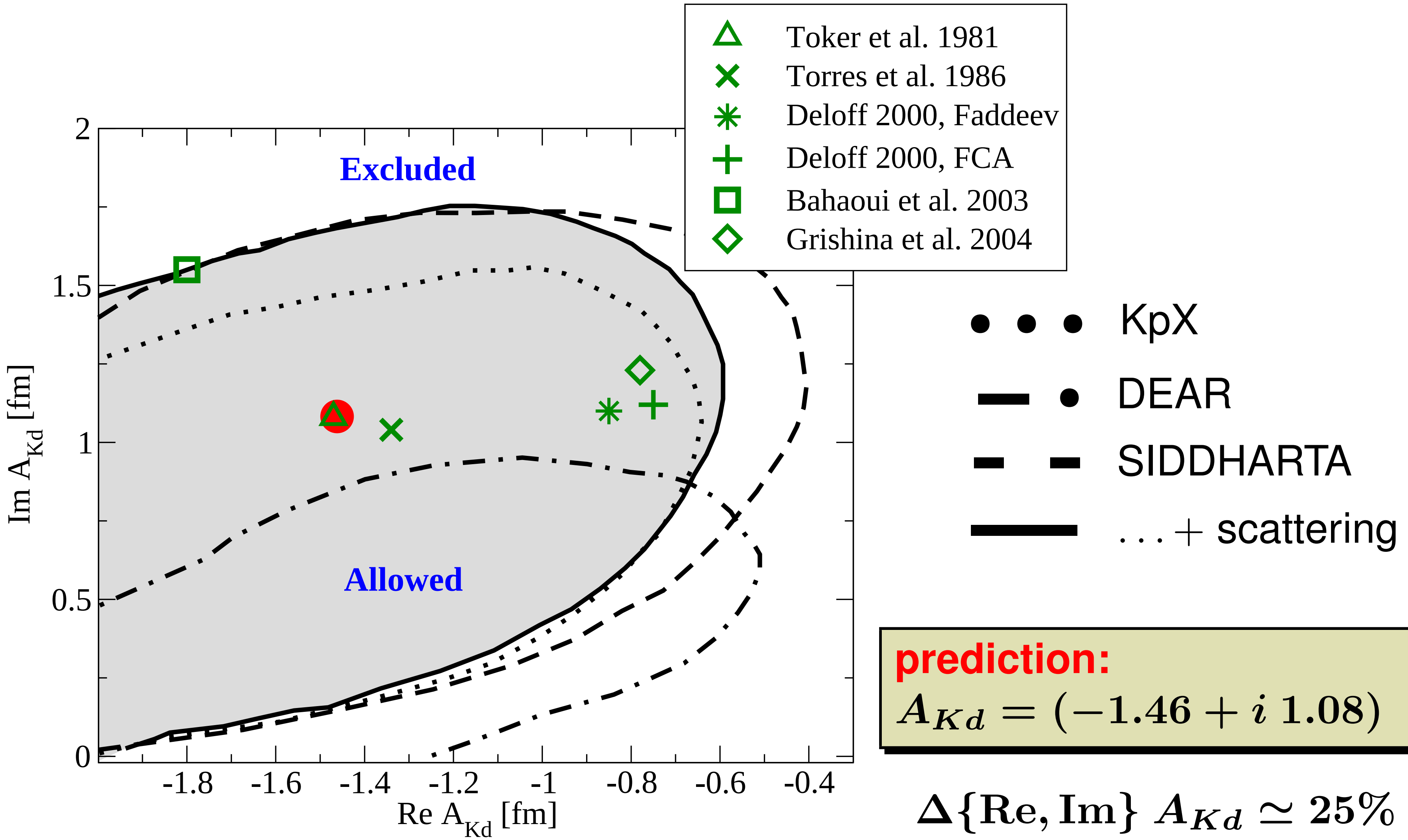
$\Rightarrow$ scan the interval $-2\ \mathrm{fm} \leq \mathrm{Re}\ A_{Kd} \leq 0$ and $0 \leq \mathrm{Im}\ A_{Kd} \leq 2\ \mathrm{fm}$

and impose constraint from SIDDHARTA (or DEAR or KpX) $\quad \to$ fig.

- make a prediction for A_{Kd}

KAON–DEUTERON SCATTERING LENGTH

- Area where solutions exist for a_0 and a_1 consistent with kaonic hydrogen data



prediction:
$A_{Kd} = (-1.46 + i\,1.08)$ fm

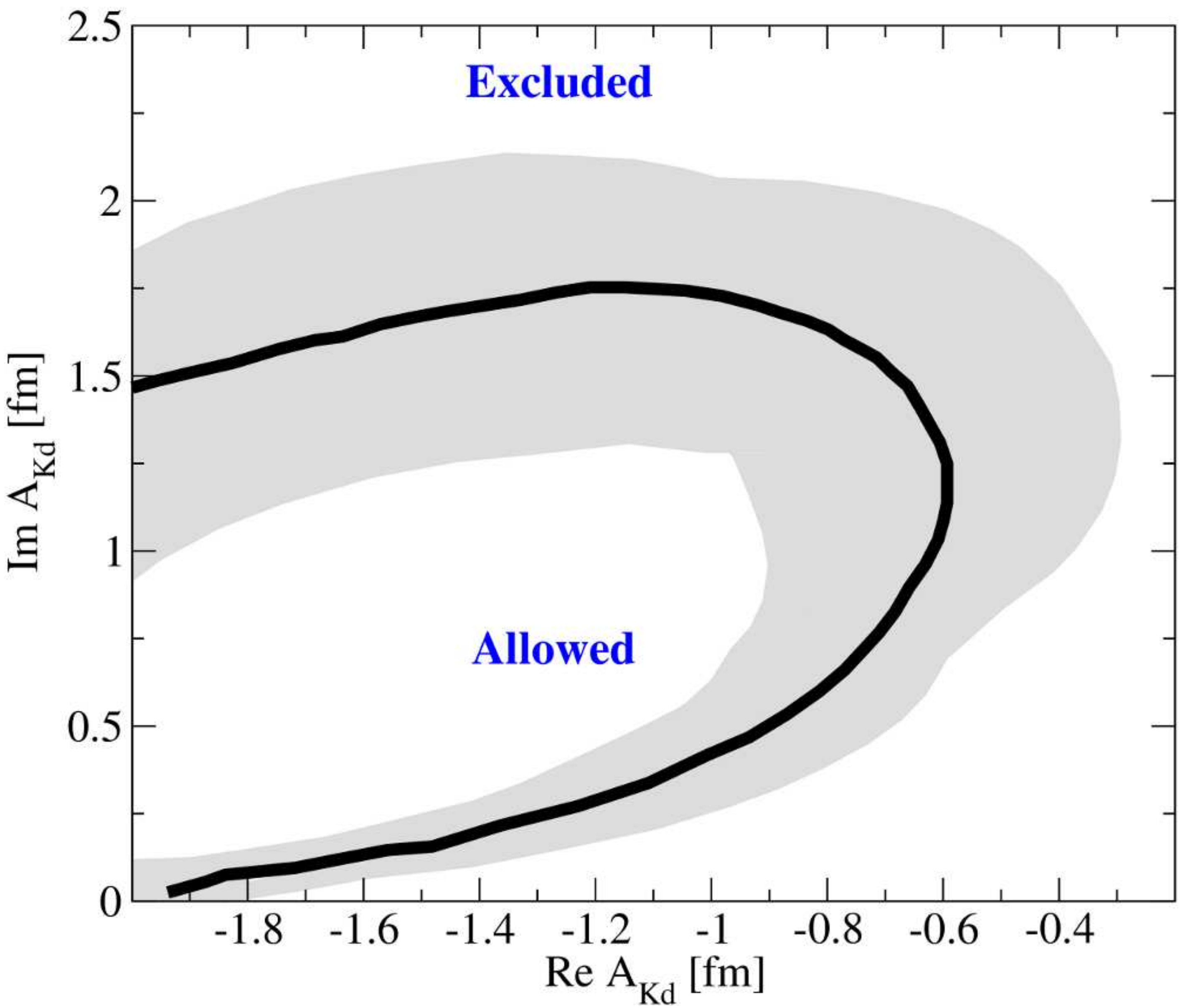
$\Delta\{\mathrm{Re},\mathrm{Im}\}\, A_{Kd} \simeq 25\%$

SUMMARY & OUTLOOK

- Hadronic atoms can be systematically analyzed in Effective Field Theory
- New SIDDHARTA kaonic hydrogen data are consistent with scattering data
- Kaonic deuterium poses further stringent constraints → SIDDHARTA2
- Prediction based on the scattering data only: $A_{Kd} = (-1.46 + i\,1.08)\ \mathrm{fm}$
- Much remains to be done, e.g. recoil corrections to the multiple scattering series Baru, Epelbaum, Rusetsky, EPJ **A42** (2009) 111
- Beautiful interplay of EXP and TH → much remains to be learned

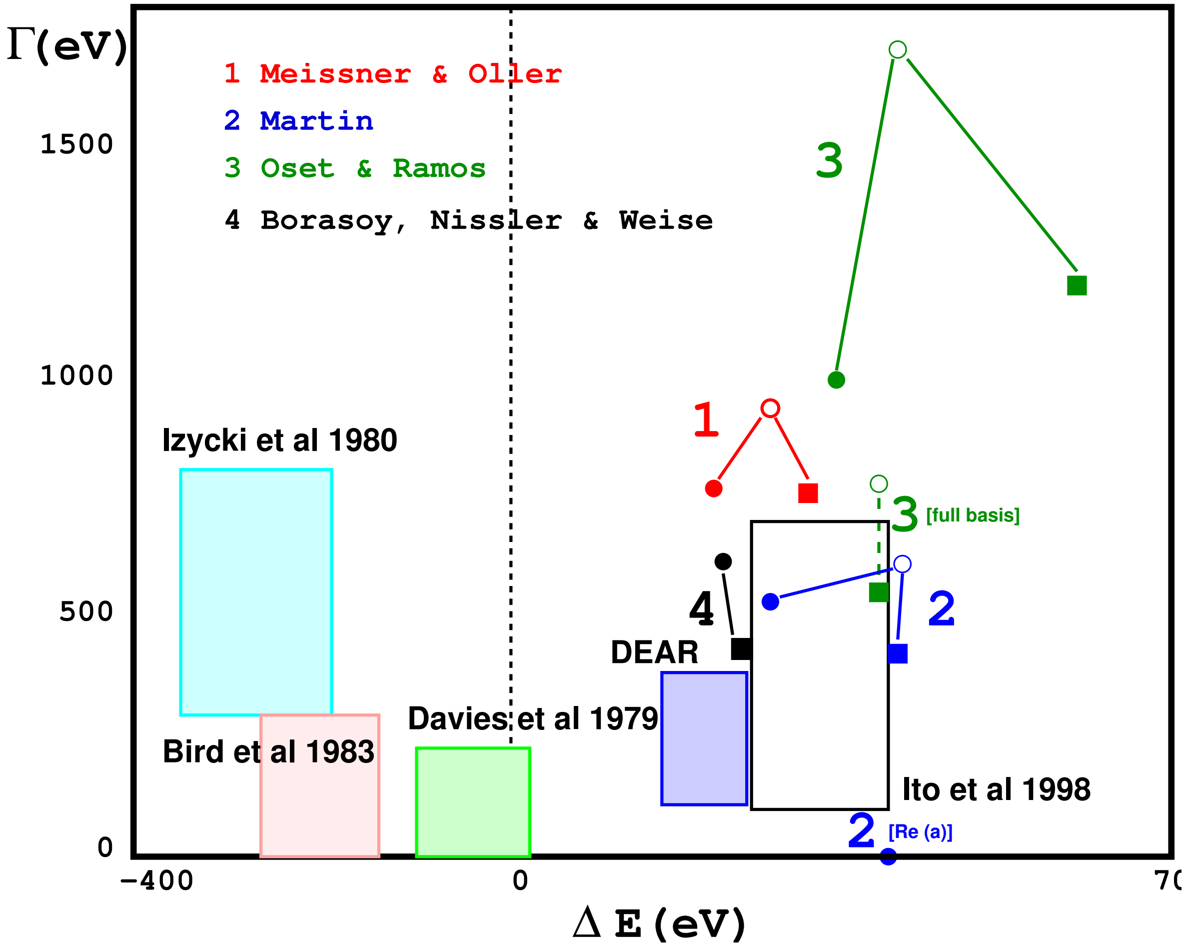
SPARES

ALLOWED VALUES for A_{Kd}: UNCERTAINTIES



ENERGY SHIFT and WIDTH in KAONIC HYDROGEN

- compare existing bound state data and predictions based on scattering data



- ● leading Deser formula
- ○ incl. unitarity cusp
- ■ full IV corrections

MO, Phys. Lett. B500 (2001) 263

M, Nucl. Phys. B179 (1981) 33

OR, Nucl. Phys. A 635 (1998) 99

BNW, Phys. Rev. Lett. 94 (2005) 213401

⇒ Recent DEAR data apparently not consistent with the (older) scattering data