Neutron spectra from the low-energy $K^- + d$ reaction and the shape of the Λ(1405) resonance

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Motivation

- the Λ(1405) plays a central role in low-enegry Kaon-nuclear physics
- strong and sometimes passionate discussions about its structure
- below the $K^- p$ threshold, experimentally ureachable in two-body reactions with stable particles; only in reactions involving n ≥ 3 particles
- the simplest case :

$$K^- + d \to K^- + d$$

$$\to \overbrace{(\pi + \Sigma)}^{\Lambda(1405)}_{I=0,1} + n$$

$$\to (\pi + \Sigma)_{I=1} + p$$

- a dynamically exact calculation can be performed

What a ***real*** experiment can tell about the Λ(1405) ?

Formulation

Coupled particle channels Faddeev-AGS treatment of the $K^- NN \leftrightarrow \pi\Sigma N$ three-body system.
Only a brief survey, mainly for futher reference, since the formalism was several times presented in front of the majority of the present audience.

AGS transition operators:

$$U_{i1} = (1-\delta_{i1})G_0^{-1} + \sum_{j\neq i} T_j G_0 U_{j1}$$

$i, j = 1,2,3$ pair indices $1 \to (23), 2 \to (13), 3 \to (12)$

$T_j = V_j + V_j G_0 T_j$ Two-particle T operators

The configuration space: particle (α) and isospin (σ_i) labels

$$\alpha = \{1,2,3\} = \{KN_1N_2, \pi\Sigma_1N_2, \pi N_1\Sigma_2\} \qquad \sigma_i = [[t_j t_k]^{I_i} t_i]^I$$

Separable s-wave interactions, isospin conserving

$$V_{\alpha\alpha'}^{I_i} = \left| g_{i\alpha}^{I_i} \right\rangle \lambda_{i\alpha,i\alpha'}^{I_i} \left\langle g_{i\alpha'}^{I_i} \right|,$$

where $i\alpha$ is a particle pair. Thus in the usual way for the T_i we get

$$T_i = \sum_{\alpha\sigma_i,\alpha'\sigma_i'} \left| g_{\alpha\sigma_i} \right\rangle \tau_{\alpha\sigma_i,\alpha'\sigma_i'}^{i} \left\langle g_{\alpha'\sigma_i'} \right| \quad \text{where} \quad \left| g_{\alpha\sigma_i} \right\rangle = \left| g_{i\alpha}^{I_i} \right\rangle$$

Account for K^-, K^0 mass difference:

- for averaged masses the free Green-operator G_0 is diagonal in both particle and isospin indices and we work in pure $I = 1/2$ total isospin state
- for physical masses it becomes non-diagonal in isospin indices $\Rightarrow$ $I = 1/2$ and $I = 3/2$ states will be mixed

The equations for the transition operators take the form:

$$U_{i1} = (1-\delta_{i1})G_0^{-1} + \sum_{j\neq i} \sum_{\alpha\sigma_j,\alpha'\sigma_j'} \left| g_{\alpha\sigma_j} \right\rangle \tau_{\alpha\sigma_j,\alpha'\sigma_j'}^{j} \left\langle X_{\alpha'\sigma_j'}^{j1} \right| \quad \text{with} \quad \left\langle X_{\alpha\sigma_j}^{j1} \right| = \left\langle g_{\alpha\sigma_j} \right| G_0 U_{j1}$$

Finally, after proper antisymmetrization we get the following system of integral equations for the X -s:

$$X^{\Sigma N}(p_\pi) = \int M_{\pi N}(p_\pi, p_N)\left\{\tau_{\pi\Sigma,\pi\Sigma}(p_N)X^{\pi\Sigma}(p_N) + \tau_{\pi\Sigma,KN}(p_N)X^{KN}(p_N)\right\}dp_N$$

$$X^{\pi\Sigma}(p_N) = \int M_{N\pi}(p_N, p_\pi)\tau_{\Sigma N}(p_\pi)X^{\Sigma N}(p_\pi)dp_\pi$$

$$X^{KN}(p_N) = \langle g_{KN} | \Phi_0 \rangle + \int M_{NK}(p_N, p_K)\tau_{NN}(p_K)X^{NN}(p_K)dp_K$$
$$-\int M_{NN}(p_N, p_N)\left\{\tau_{KN,\pi\Sigma}(p_N)X^{\pi\Sigma}(p_N) + \tau_{KN,KN}(p_N)X^{KN}(p_N)\right\}dp_N$$

$$X^{NN}(p_K) = \int M_{KN}(p_K, p_N)\left\{\tau_{KN,\pi\Sigma}(p_N)X^{\pi\Sigma}(p_N) + \tau_{KN,KN}(p_N)X^{KN}(p_N)\right\}dp_N$$

Here we omitted the isospin indices, all quantities are matrices (vectors) in isospin space, the integration variables are the spectator momenta. In the in-homogeneous term $|\Phi_0\rangle = |\varphi_d P_K\rangle$ is the initial state, P_K is the momentum of the incident kaon, $|\varphi_d\rangle$ is the deuteron wave function.

The break-up transition operator can be obtained from the U_{i1} as

$$U_{01} = (U_{11} + U_{21} + U_{31}) / 2$$

and the break-up amplitude reads $A_{BU} = \left\langle \Phi_f \,|U_{01}|\, \Phi_0 \right\rangle$. For the reaction under consideration the properly antisymmetrized final state is

$$\left|\Phi_f\right\rangle = \left|q_{\pi\Sigma}, p_N; \sigma_{\pi\Sigma}\right\rangle = \tfrac{1}{\sqrt{2}}\left(\left|q_{\pi\Sigma_1}, p_{N_2}; \sigma_{\pi\Sigma_1}\right\rangle - \left|q_{\pi\Sigma_2}, p_{N_1}; \sigma_{\pi\Sigma_2}\right\rangle\right).$$

Finally, the break-up amplitude can be expressed in terms of the X -s as

$$A_{BU}(q_{\pi\Sigma}, p_N; \sigma_{\pi\Sigma}) = -\sum_{\sigma'} g^{\sigma}_{\pi\Sigma}(q_{\pi\Sigma})\{\tau^{\sigma,\sigma'}_{\pi\Sigma,KN}(p_N) X^{KN}_{\sigma'}(p_N) + \tau^{\sigma,\sigma'}_{\pi\Sigma,\pi\Sigma}(p_N) X^{\pi\Sigma}_{\sigma'}(p_N)\}$$

$$-\sum_{\sigma'} W_{\sigma,\sigma'} g^{\sigma'}_{\Sigma N}(|\alpha p_N + \beta q_{\pi\Sigma}|)\tau^{\sigma'}_{\Sigma N}(|q_{\pi\Sigma} - \gamma p_N|) X^{\Sigma N}_{\sigma'}(|q_{\pi\Sigma} - \gamma p_N|),$$

where $W_{\sigma,\sigma'}$ is a matrix composed of the isospin recoupling coefficients, while the α, β, γ are mass coefficients of the transformation berween different Jacobi coordinate sets. Here the isospin labels and the summations over them are explicitly indicated.

The on-shell amplitude for a given neutron energy E_n depends on E_n, t and $\sigma_{\pi\Sigma}$:

$$A(E_n, t, \sigma_{\pi\Sigma}) = A_{BU}(q_{\pi\Sigma}, p_N, \sigma_{\pi\Sigma})$$

$$|\, p_N \,| = \sqrt{2E_n \mu_{N,\pi\Sigma}}$$

$$|\, q_{\pi\Sigma} \,| = \sqrt{2(E_{\pi\Sigma N} - E_n)\mu_{\pi\Sigma}}$$

$$\cos(p_N, q_{\pi\Sigma}) = t$$

From experimental point of view it is more transparent to use an amplitude, which corresponds to a definite angle between outgoing particles, e.g. the pion and the neutron:

$$A(E_n, t, \sigma_{\pi\Sigma}) \Rightarrow A(E_n, s, \sigma_{\pi\Sigma}); \quad s = \cos(p_N, p_\pi)$$

For a given neutron energy E_n s and t are in one-to-one correspondence, thus the transition between the amplitudes is straightforward.

These amplitudes correspond to final states with definite $\sigma_{\pi\Sigma}$ isospin quantum numbers

$$\sigma_{\pi\Sigma} = \left\{ \left[[\pi\Sigma]^0 N \right]^{\frac{1}{2}}, \left[[\pi\Sigma]^1 N \right]^{\frac{1}{2}}, \left[[\pi\Sigma]^1 N \right]^{\frac{3}{2}} \right\},$$

but the physically observable ones correspond to definite paricle composition:

$$\sigma_0 = \left\{\pi^+\Sigma^- n, \pi^0\Sigma^0 n, \pi^-\Sigma^+ n\right\},$$

therefore the physical amplitudes read

$$A(E_n, s, \sigma_0) = \sum_{\sigma_{\pi\Sigma}} D_{\sigma_0, \sigma_{\pi\Sigma}} A(E_n, s, \sigma_{\sigma_{\pi\Sigma}})$$

with a suitable transformation matrix $D_{\sigma_0, \sigma_{\pi\Sigma}}$ composed from CG coefficients.

The neutron spectra of different possible processes are then proportional to

$$P(E_n, s, \sigma_0) = | A(E_n, s, \sigma_0) |^2$$

If the angle between π and n is unobserved: $P(E_n, \sigma_0) = \int_{-1}^{1} P(E_n, s, \sigma_0) ds$
and if only the neutron energy is measured:

$$P(E_n) = \sum_{\sigma_0} P(E_n, \sigma_0)$$

Calculation details

separable s-wave interactions

Interactions taken from

N.V. Shevchenko
One- versus two-pole $\bar{K}N - \pi\Sigma$ potential: $K^{-}d$ scattering length
arXiv:1103.4974

These potentials reproduce all known experimental data on the low-energy $\bar{K}N - \pi\Sigma$ system (except, perhaps, the latest SIDDHARTA values of the kaonic hydrogen 1s level shift).

NN potential: two-term separable potential with repulsion, reproduces the deuteron and the singlet- and triplet s-wave phase shifts

ΣN interaction: $I_{\Sigma N} = \frac{1}{2}, \frac{3}{2}, \quad S_{\Sigma N} = 1$, reproducing the scarce experimental data. In $I_{\Sigma N} = \frac{1}{2}$ state two-channel $\Sigma N - \Lambda N$.

πN interaction neglected.

The calculation at present is restricted to $L = 0$, and to incident kaon energies below the deuteron break-up (kaon LAB momenta < 50 MeV/c).

For physical masses $(I = \frac{1}{2}$ and $I = \frac{3}{2}$ mixed) we have 12 unknown functions, while for averaged masses $(I = \frac{1}{2}$ only) – 8.

Numerical method: expansion of the uknown functions on cubic spline basis. Smaller matrices, no problem with logarithmic singularities – they are integrated with known functions.

Results

A by-product: effect of the physical masses on the $K^{-}d$ scattering length (fm):

	averaged	physical
1-pole	-*1.49* + *0.97* i	-*1.52* + *0.98* i
2-pole	-*1.57* + *1.10* i	-*1.60* + *1.12* i

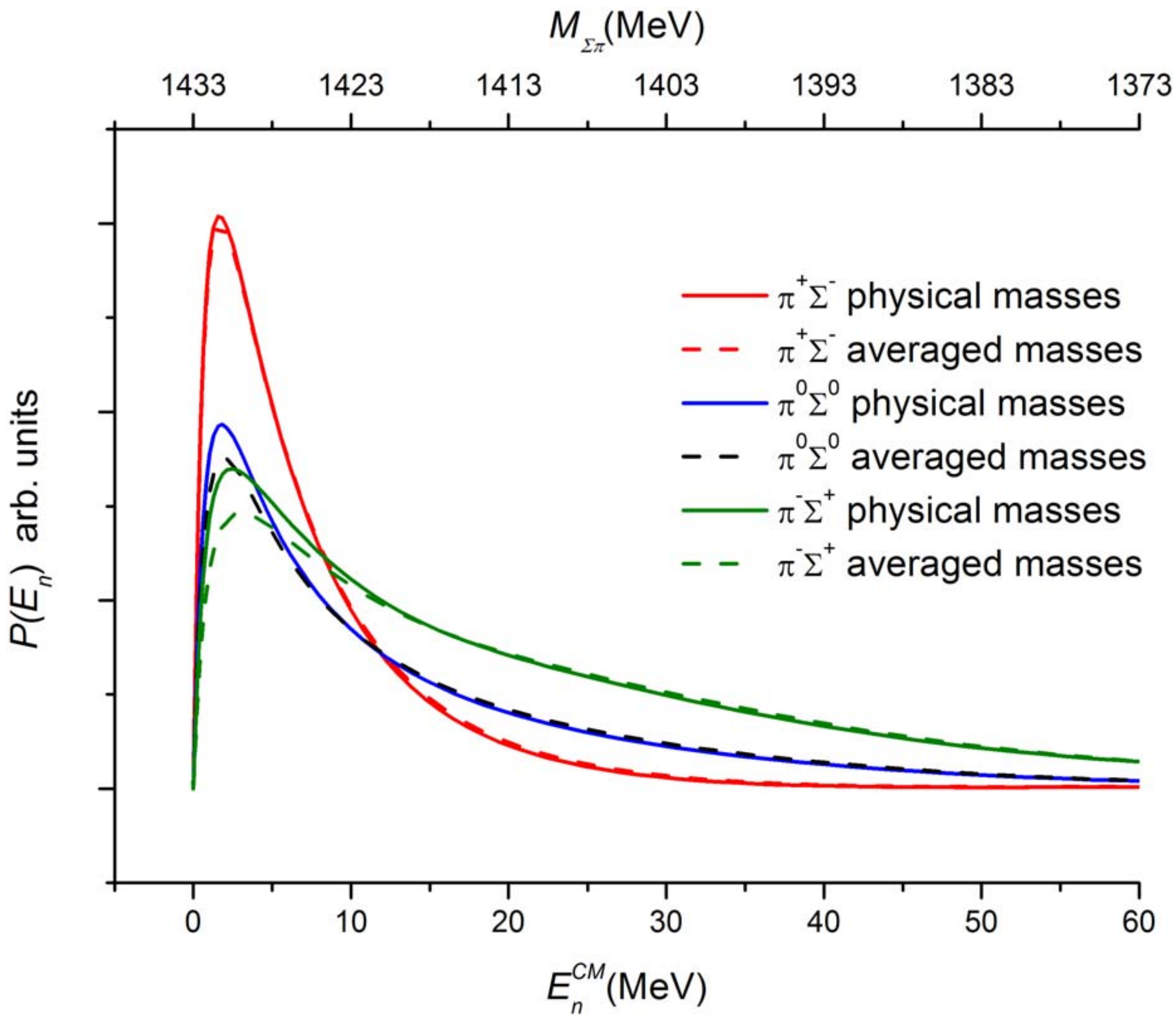
a few percent effect. Now insignificant, may be of some use, if precise $K^{-}d$ atom level shifts will be available.

Similarly small effects in neutron spectra from

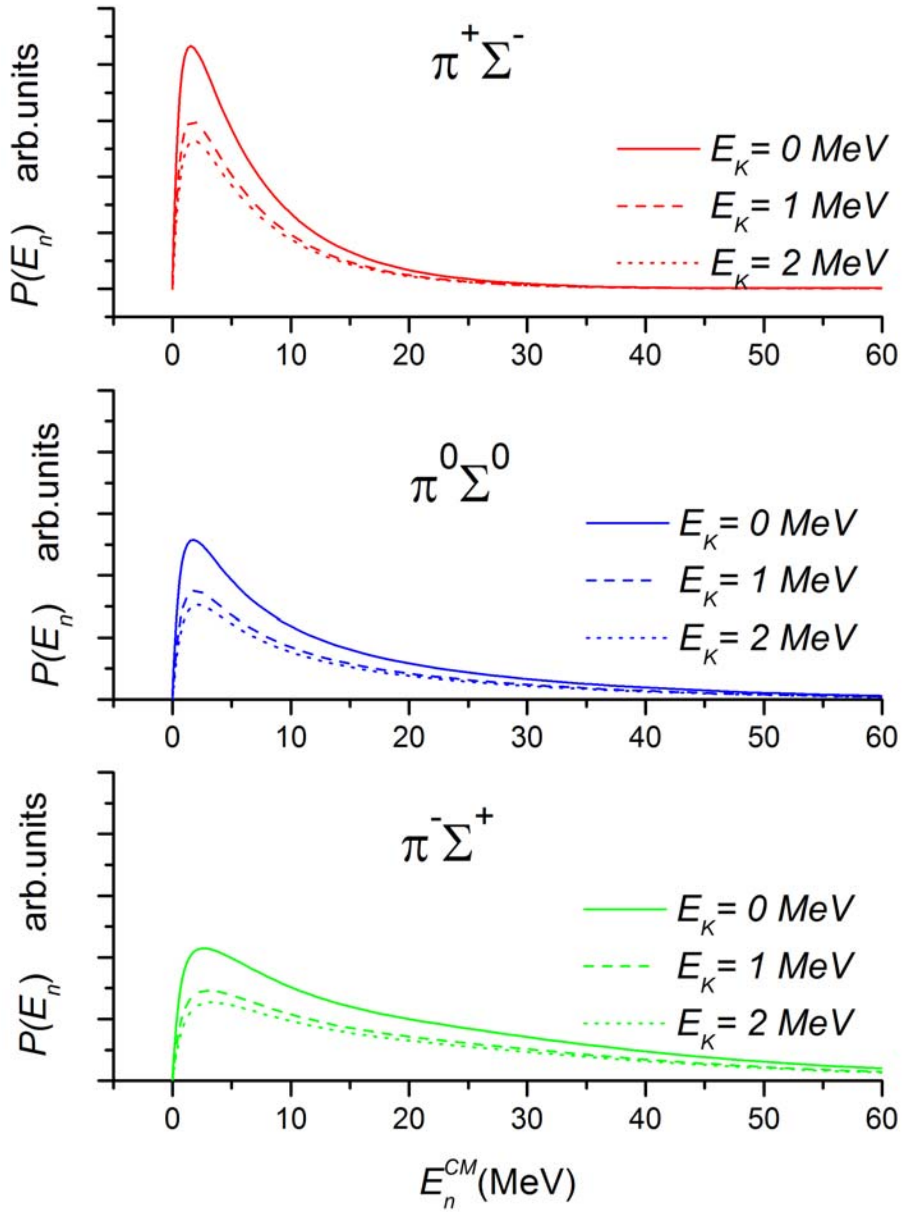
- physical-averaged masses
- incident kaon energy (0,1,2, MeV)
- 1-pole 2-pole versions of $\bar{K}N \leftrightarrow \pi\Sigma$ interactions (for our choice their dominant poles coincide)

The following figures demonstrate the above statement

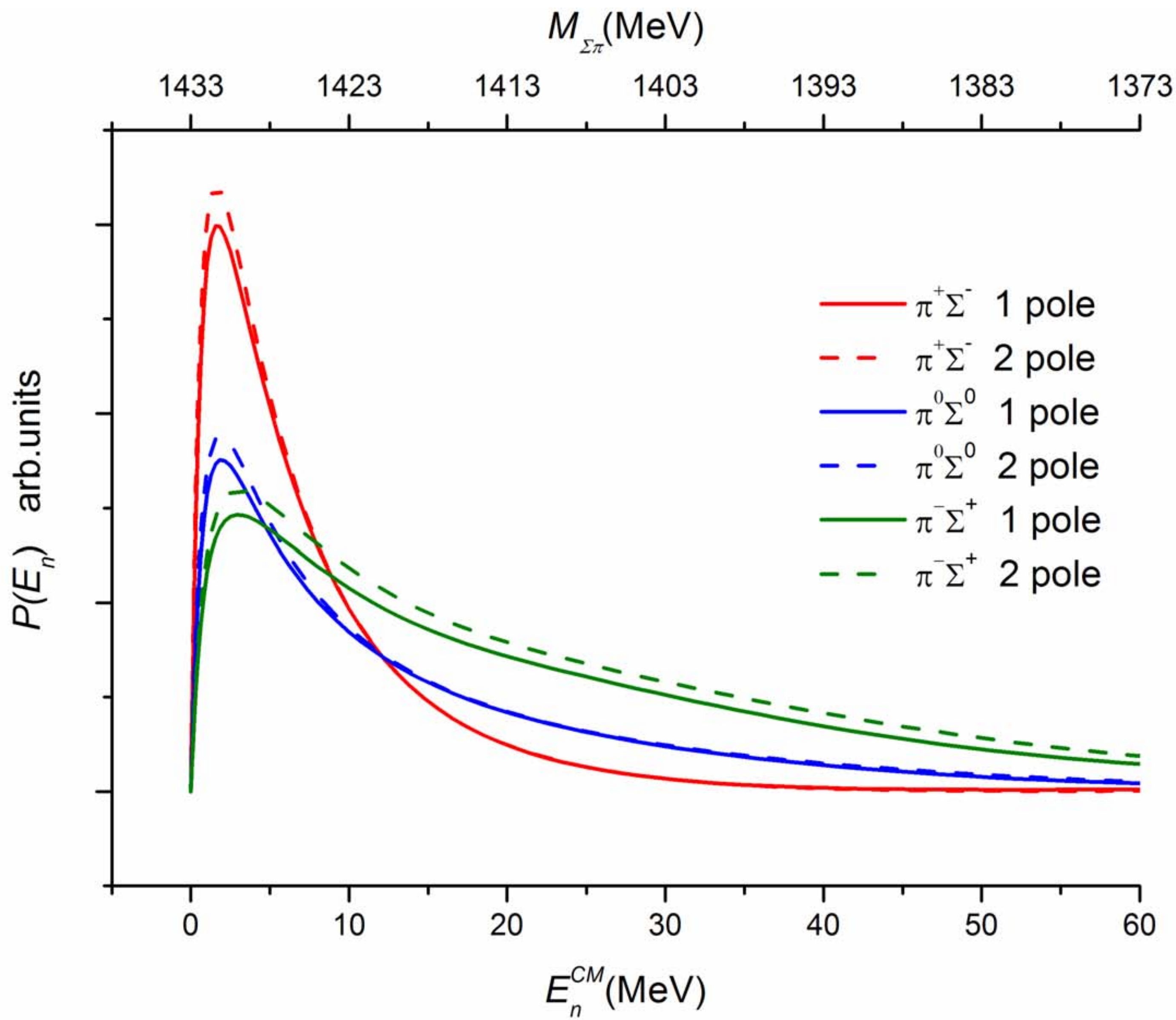
Physical masses



Incident kaon energy



1-pole -2-pole effect



In the angular-integrated spectra there is no signature of the $\Lambda(1405)$ resonance. The $M_{\pi\Sigma}$ region of $\Lambda(1405)$ corresponds to emitted neutron energy of $\sim 20-30\ MeV$. In the considered low-energy $K^- + d$ reaction they are unlikely to be emitted by two kinematical reasons

- quasi-free mechanism (roughly: slow collisions produce mainly slow paricles);
- the momentum distribution of the neutron in the deutron is dominated by the low-energy part.

Esmaili, Akaishi and Yamazaki (EAY) (PRC **83**, 055207) proposed a method to – more or less – eliminate the disturbing kinematical effects in order to reveal the dynamical ones: they suggest to consider the DEViation spectrum:

$$P_{DEV} = \frac{P(E_n)}{P_{nonres}(E_n)}$$

Let's see how this idea can be realized in our case: zero-order iteration of the Faddeev equations - $X \sim$ to inhomogeneous term (page 7):

$$X^{KN}(x_N) = \langle g_{KN} \mid \varphi_d P_K \rangle,$$

all the other X-s are zero. This anzatz is generally called single scattering approximation. The corresponding break-up amplitude is

$$A_{sing}(q_{\pi\Sigma}, p_N; \sigma_{\pi\Sigma}) = g_{\pi\Sigma}(q_{\pi\Sigma})\tau_{\pi\Sigma,KN}(p_N)\langle g_{KN} \mid \varphi_d P_K \rangle = \langle q_{\pi\Sigma}, p_N \mid T_{\pi\Sigma,KN} \mid \varphi_d P_K \rangle,$$

which is the matrix element of the two-body $T_{\pi\Sigma,KN}$ operator between the initial and final states. It contains the two-body *T* operator and the kinematical input: transformation between Jacobi coordinates and the deuteron wave function. This is basically the formula, which EAY used for calculating the transition amplitude from the $K^- d$ atomic state to the $\pi\Sigma n$ continuum.

As for the non-resonant amplitude, they propose to replace $T_{\pi\Sigma,KN}$ by $V_{\pi\Sigma,KN}$ in A_{sing} , that is to use the Born-approximation, which contains all the kinematics.

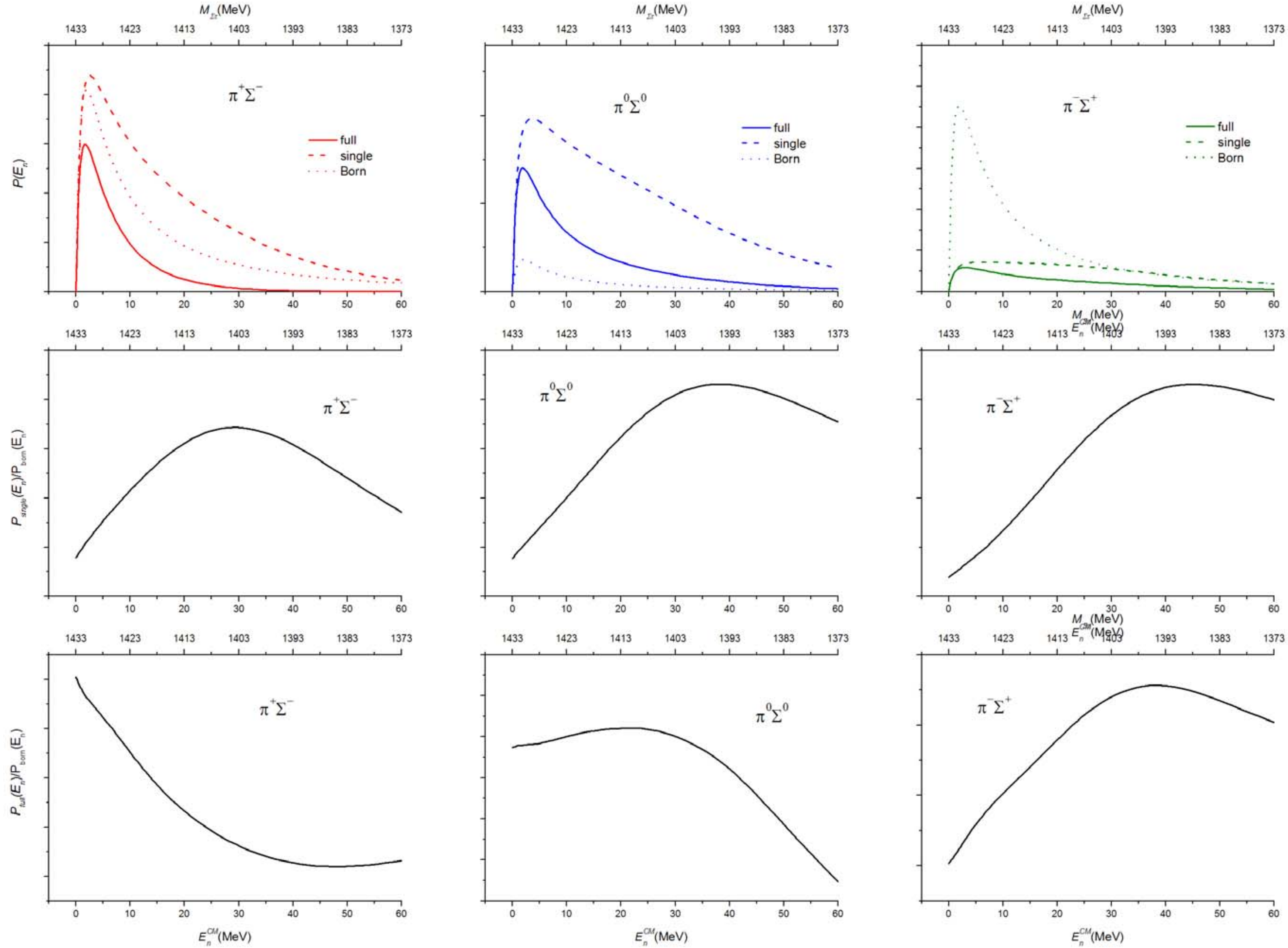
Thus we have three amplitudes with the properties:

A_{full} three-body dynamics + three-body kinematics

A_{sing} two-body dynamics + three-body kinematics

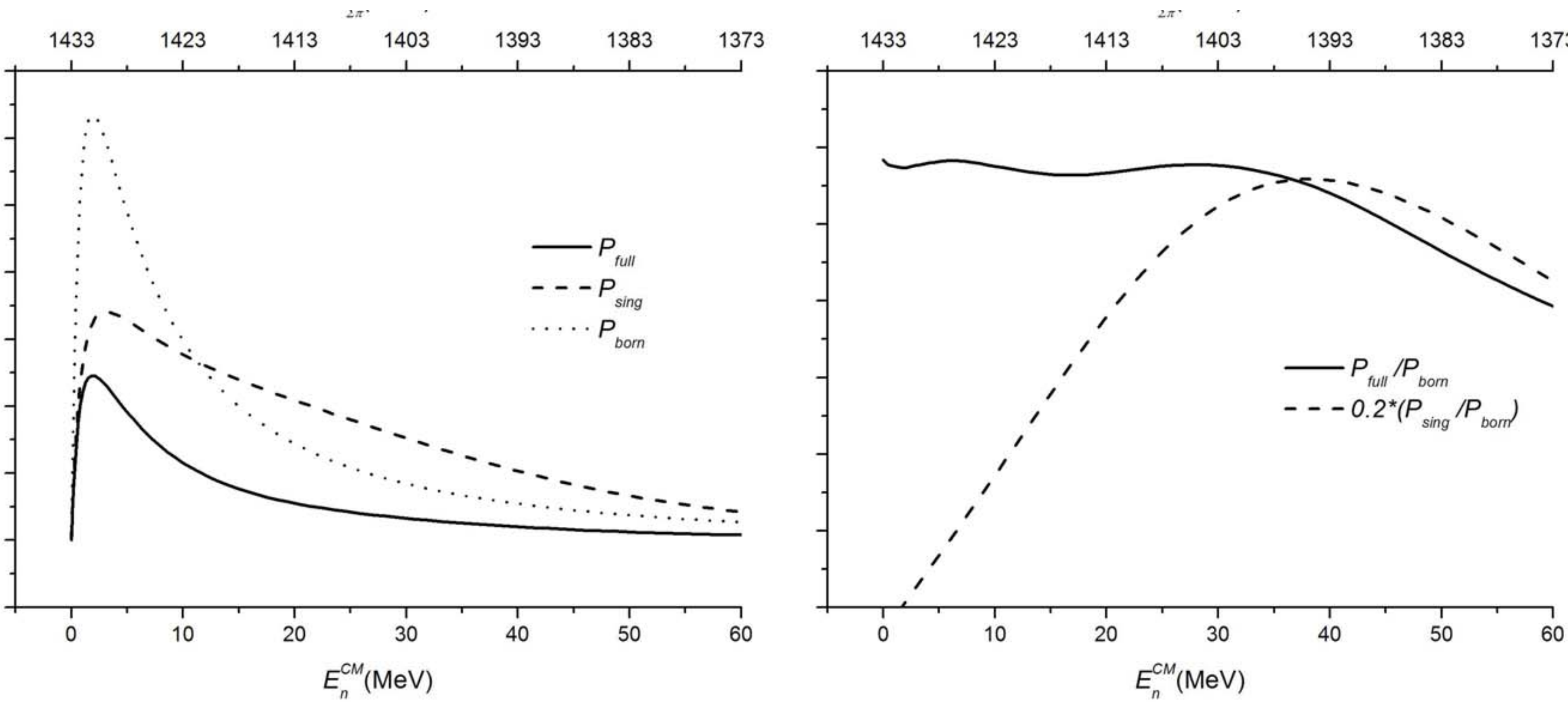
A_{born} three-body kinematics

and we expect that the DEV spectra P_{full} / P_{born} and P_{sing} / P_{born} will display (reveal) three- and two-body dynamics, respectively.



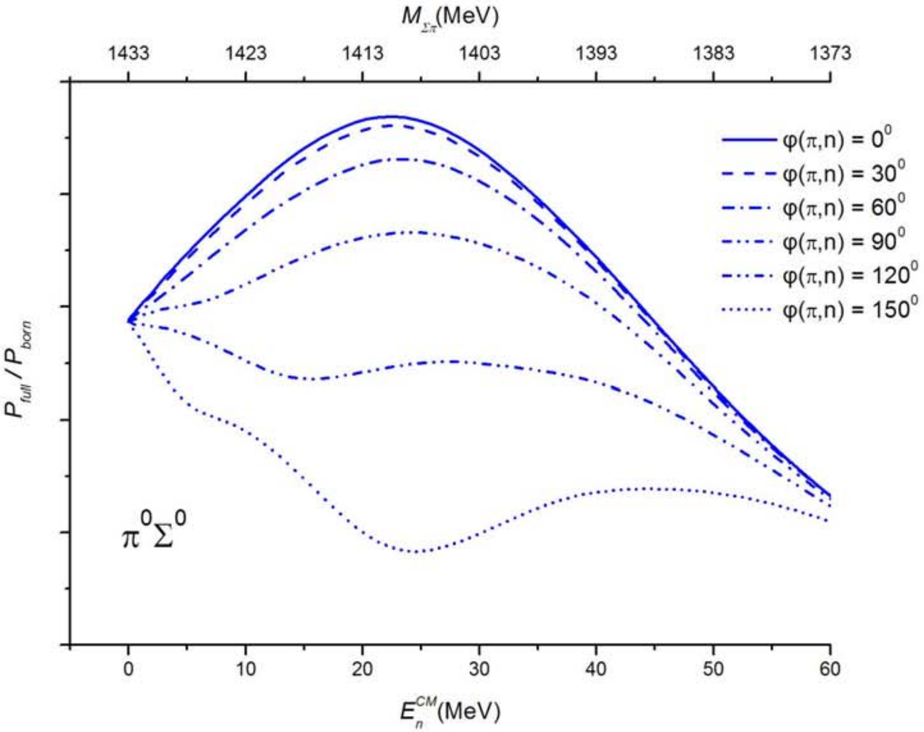
M_Σπ(MeV)
1433 1423 1413 1403 1393 1383 1373
P(E_n)
π+Σ−
full
single
Born
π0Σ0
full
single
Born
π−Σ+
full
single
Born
0 10 20 30 40 50 60
P_single(E_n)/P_born(E_n)
π+Σ−
π0Σ0
π−Σ+
P_full(E_n)/P_born(E_n)
π+Σ−
π0Σ0
π−Σ+
E_n^CM(MeV)

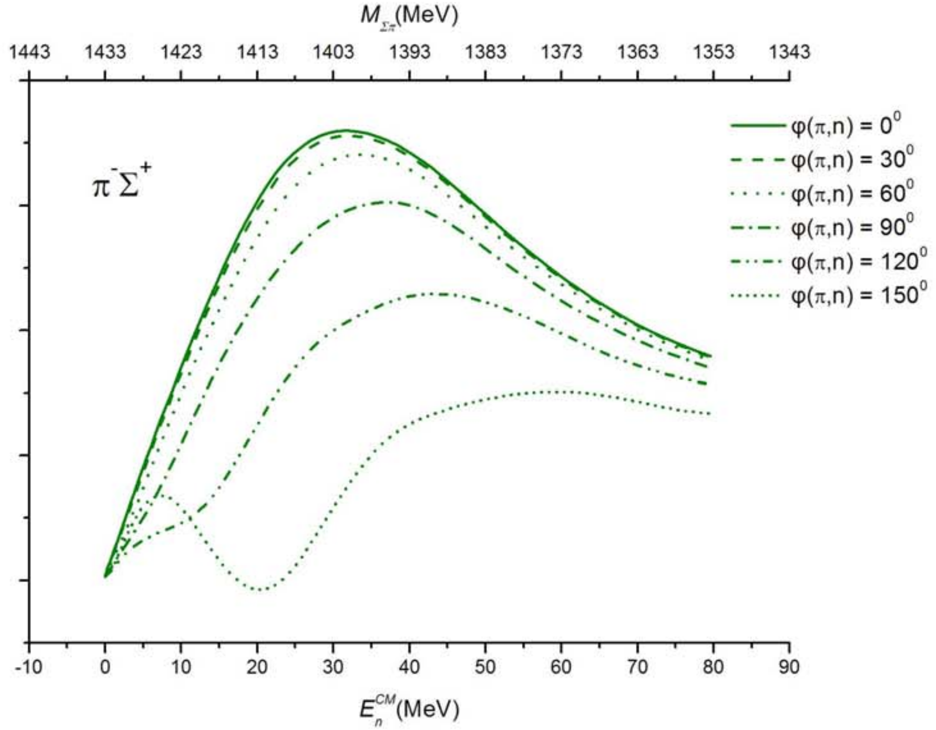
If we sum up these spectra of different $\pi\Sigma$ charge states, we get the full neutron spectrum :



While the trace of the 1405 resonace is clearly seen on the P_{sing} / P_{born} curve, its signature on the full curve is much less apparent.

For certain charge combinations and angles between π and n the corresponding DEV spectra show a more pronounced trace of the Λ(1405) resonance:





Conclusions

- no sign of the Λ(1405) resonance in any (differential, total or summed over $\pi\Sigma$ charge states) direct neutron spectra from the low-energy $K^- + d$ reaction (probably the same is true for K^- capture from an atomic orbital);

- kinematical effects mask the dynamical ones;

- using the DEV spectrum method some resonance structure can be revealed, however, it is not straightforward to relate the obtained (calculated or deduced from experiment) maxima to the pole position of Λ(1405);

- calculations (or experiments) for higher incident antikaon energies might improve the situation.