Influence of Muon Cascade and Muonic Molecule Formation on the μCF Processes Kinetics in Deuterium

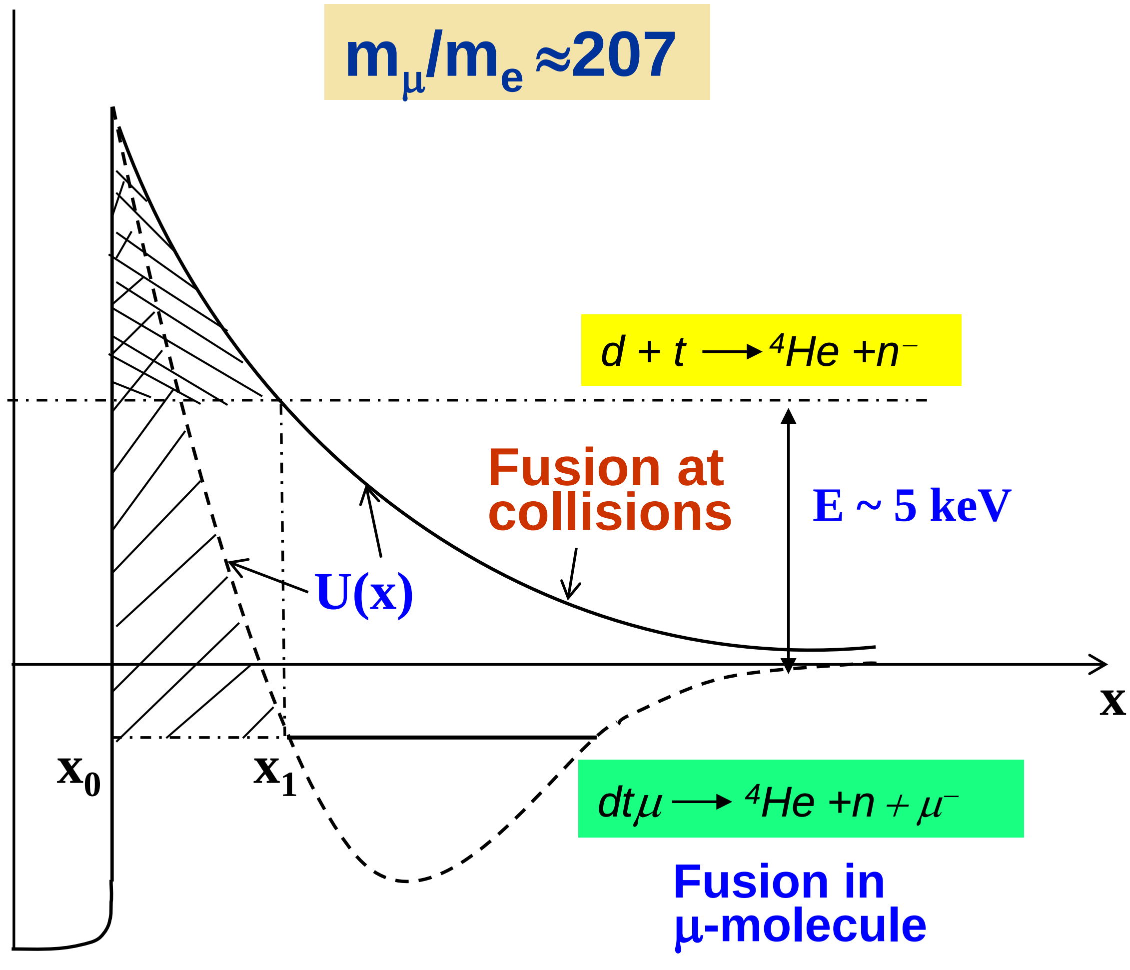
A. Adamczak*) and M.P. Fayfman**)

*) *Institute of Nuclear Physics, Polish Academy of Sciences, Krakow, Poland;*

**) *Research Coordinative Center on the Problem of μCF and Energetic Quantum Systems (MUCATEX), Moscow, Russia.*

The role of muonic molecules in fusion reactions

The μCF idea was suggested by *F.C.Frank (1947), A.D.Sakharov (1948);*
The experimental observation by *L.W. Alvarez' team (1957).*

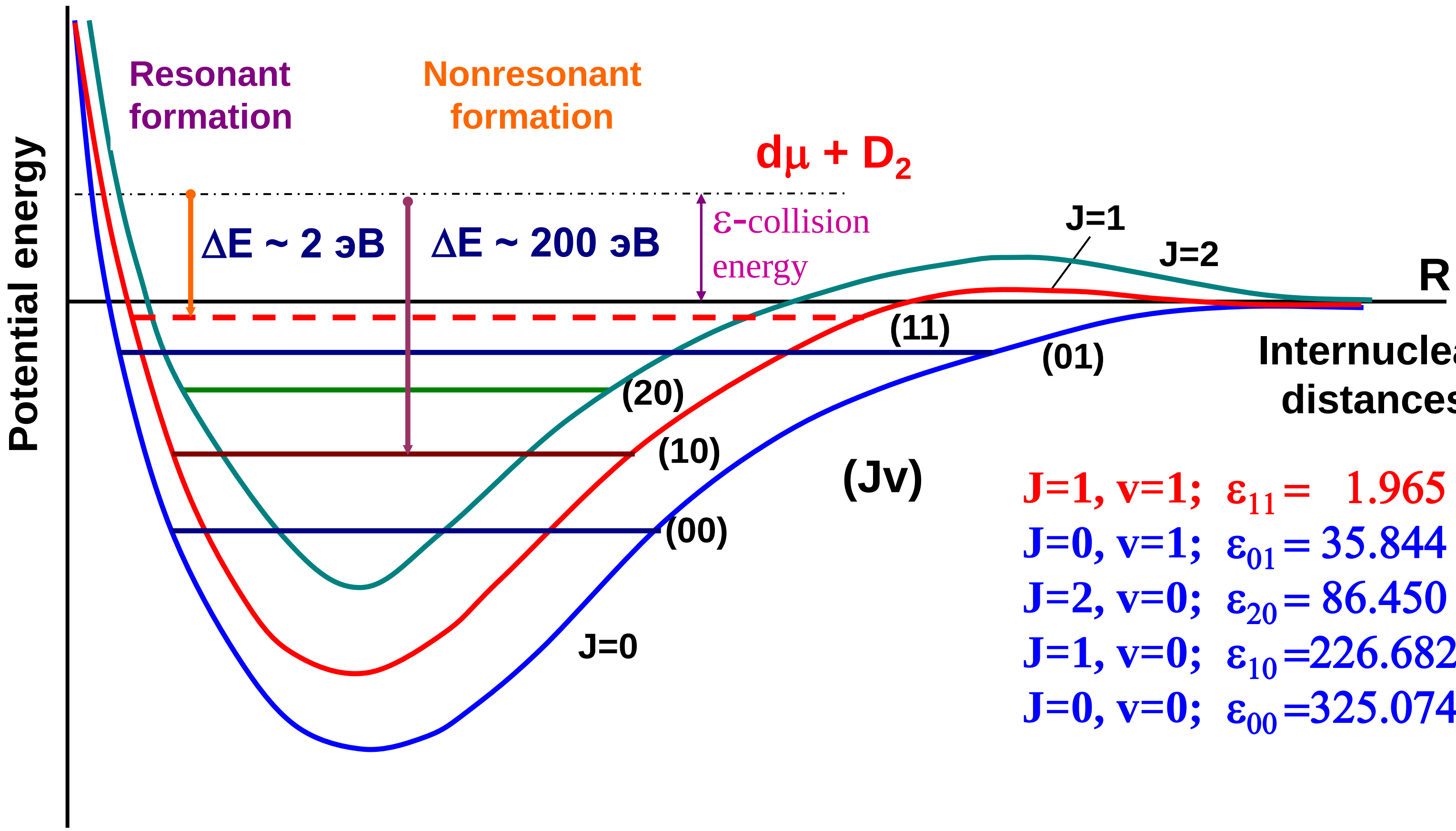


range of nuclear force: $x_0 \approx 10^{-13}$ cm
radius of muonic atom: $x_1 \approx 2{,}5 \cdot 10^{-11}$ cm

Muonic molecule formation in JV-rovibronic state

$$(d\mu)+[D_2]_{K_i\nu_i} \rightarrow [(dd\mu)_{J\nu}de]^{+}_{K_i\nu_i} + e^{-} - \textit{nonresonant formation;}$$

$$(d\mu)+[D_2]_{K_i\nu_i} \rightarrow [(dd\mu)_{J=1,\nu=1}dee]^{*}_{K_f\nu_f} - \textit{resonant formation.}$$



J=1, v=1; ε_{11} = 1.965 eV

J=0, v=1; ε_{01} = 35.844 eV

J=2, v=0; ε_{20} = 86.450 eV

J=1, v=0; ε_{10} =226.682 eV

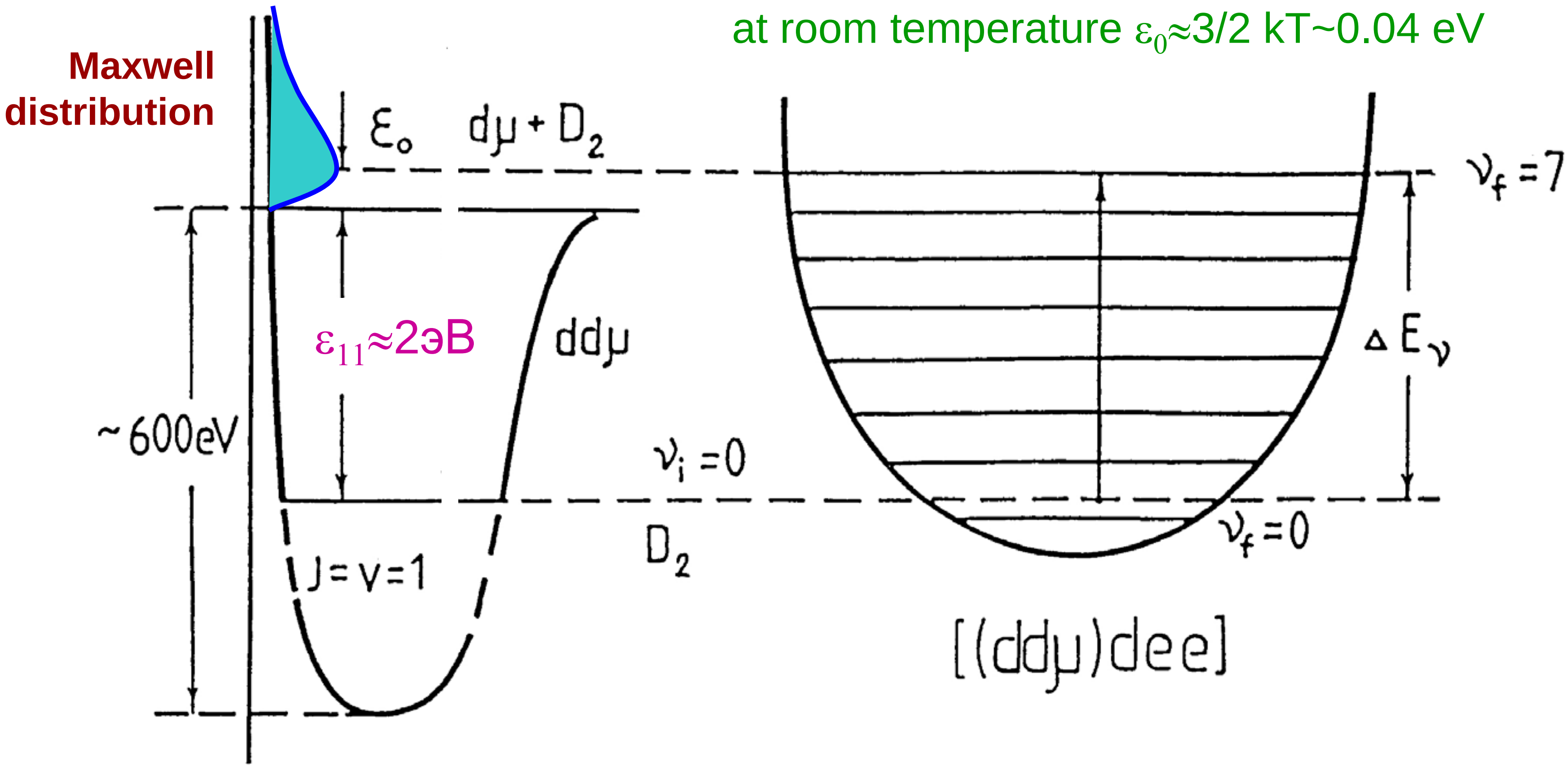
J=0, v=0; ε_{00} =325.074 eV

Resonant ddμ–molecule formation

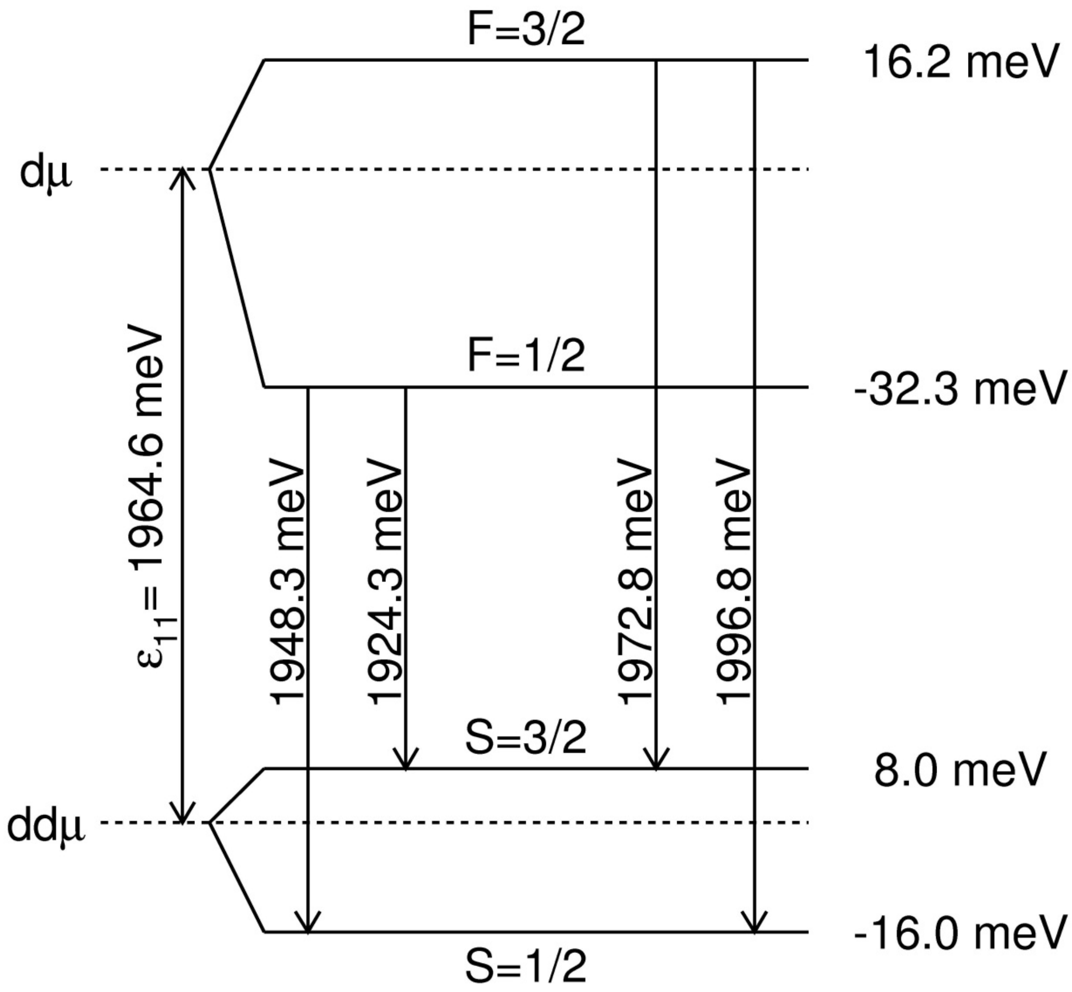
(E.A.Vesman, 1967)

$$\mathbf{d\mu + D_2 \rightarrow \left[\left(dd\mu\right)_{J=v=1} dee\right]^{*}_{vK}}$$

Resonance condiiton: $\boldsymbol{\varepsilon_0 + \left|\varepsilon_{11}\right| = \Delta E_v}$

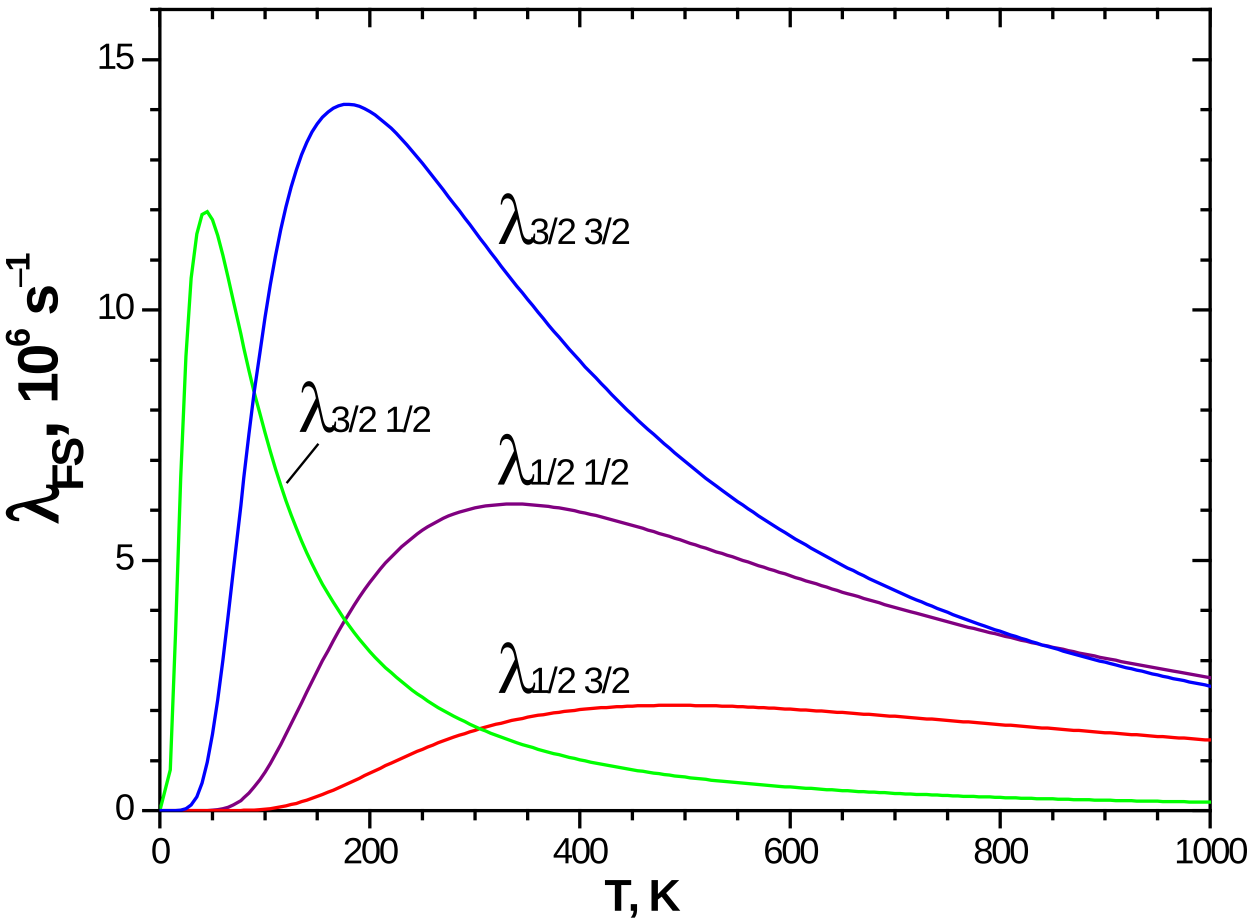


The hyperfine structure of dμ-atom и ddμ-molecule energy levels



$$\Delta\varepsilon^{F}_{d\mu} = \delta\varepsilon^{F=3/2}_{d\mu} - \delta\varepsilon^{F=1/2}_{d\mu} = 48.5\,\mathrm{meV};\qquad \Delta\varepsilon^{S}_{dd\mu} = \delta\varepsilon^{S=3/2}_{dd\mu} - \delta\varepsilon^{S=1/2}_{dd\mu} = 24.0\,\mathrm{meV}$$

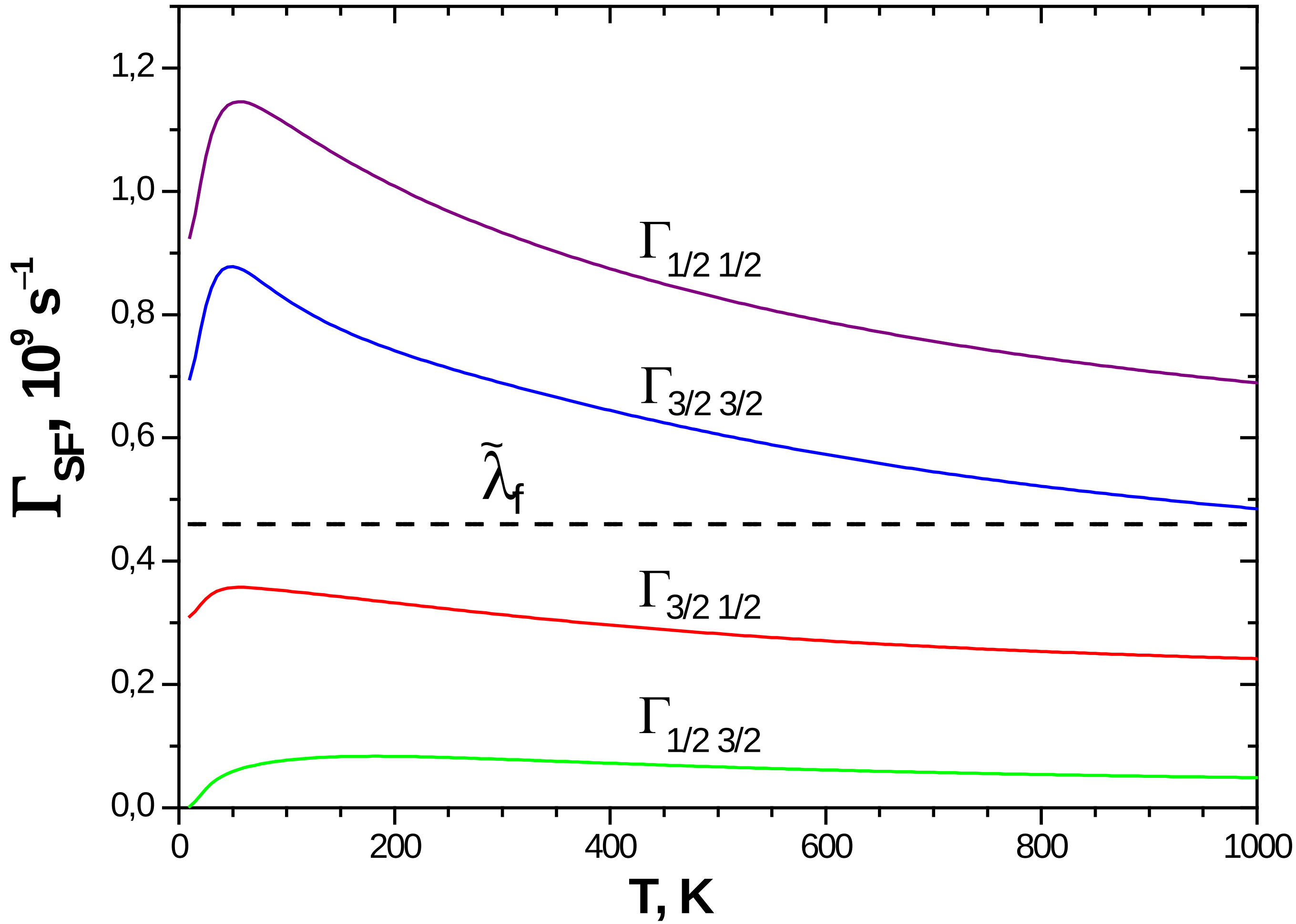
The calculated resonant formation rates of ddμ molecule



$$\lambda_{FS} = \sum_{K_i, K_f} \omega_i(K_i) \lambda^{FS}_{K_i K_f};$$

$$\omega_i(K_i) = \xi(K_i) \frac{2K_i + 1}{Z_i} e^{-\frac{\varepsilon(K_i)}{kT}}, \quad \sum_{K_i} \omega_i(K_i) = 1.$$

The decay rates of resonant formed ddμ -molecule



$$\Gamma_{SF} = \sum_{K_i, K_f} \omega_f(K_f)\Gamma^{SF}_{K_f K_i}, \quad \Gamma_S = \sum_F \Gamma_{SF}.$$

$$\omega_f(K_f) = \frac{2K_f + 1}{Z_f} e^{-\frac{\varepsilon(K_f)}{kT}}, \ \sum_{K_i} \omega_f(K_f) = 1.$$

The effective rates $\tilde{\lambda}_F$ of $dd\mu$ molecule formation

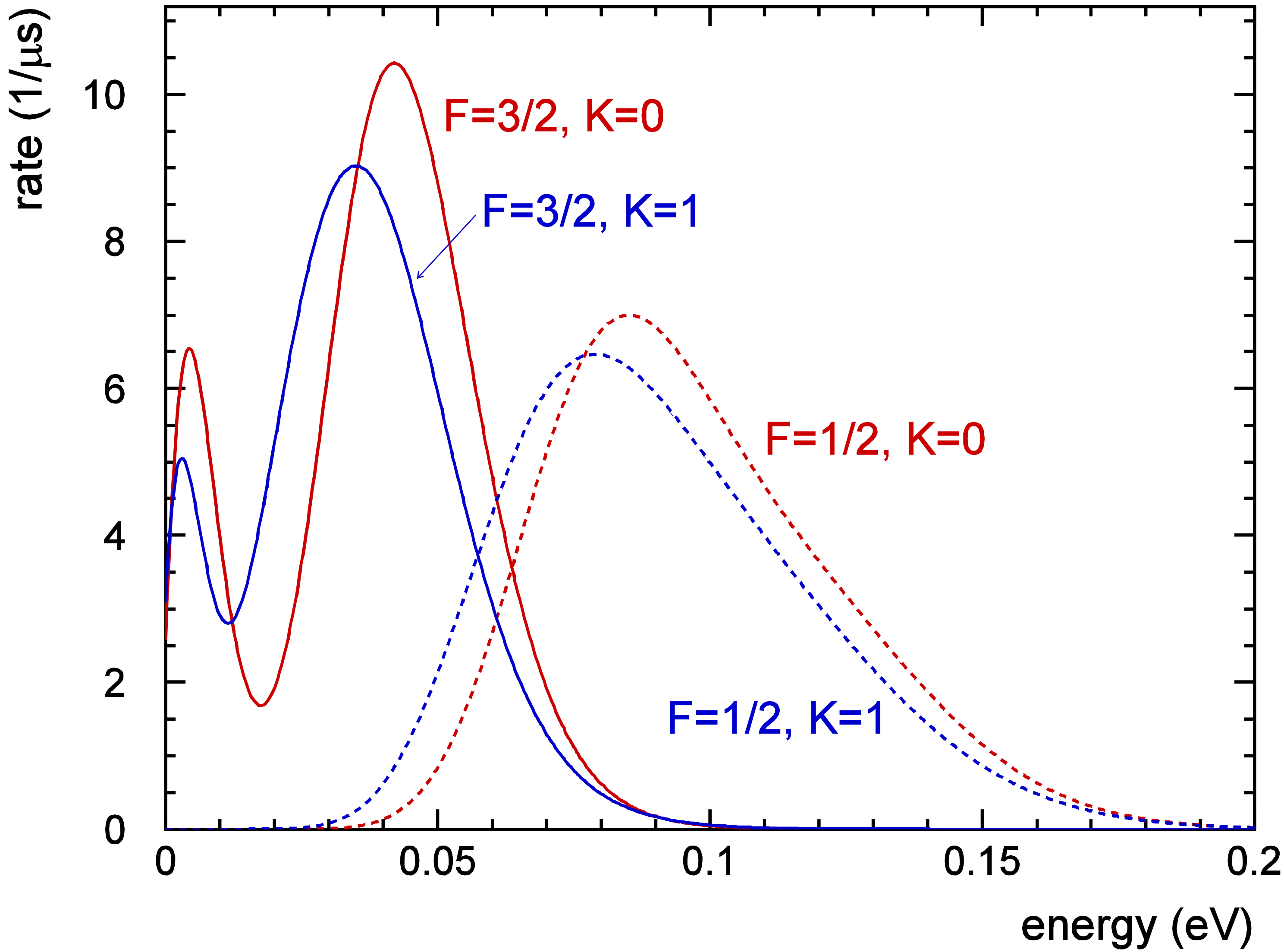
$$\tilde{\lambda}_{\mathbf{F}} = \sum_{\mathbf{S}} \lambda_{\mathbf{FS}} \frac{\tilde{\lambda}_{\mathbf{f}}}{\tilde{\lambda}_{\mathbf{f}} + \Gamma_{\mathbf{S}}},$$

$$\lambda_{FS} = \sum_{K_i, K_f} \omega_i(K_i) \lambda_{K_i K_f}^{FS};$$

$$\Gamma_{SF} = \sum_{K_i, K_f} \omega_f(K_f) \Gamma_{K_f K_i}^{SF},$$

$$\Gamma_S = \sum_F \Gamma_{SF}.$$

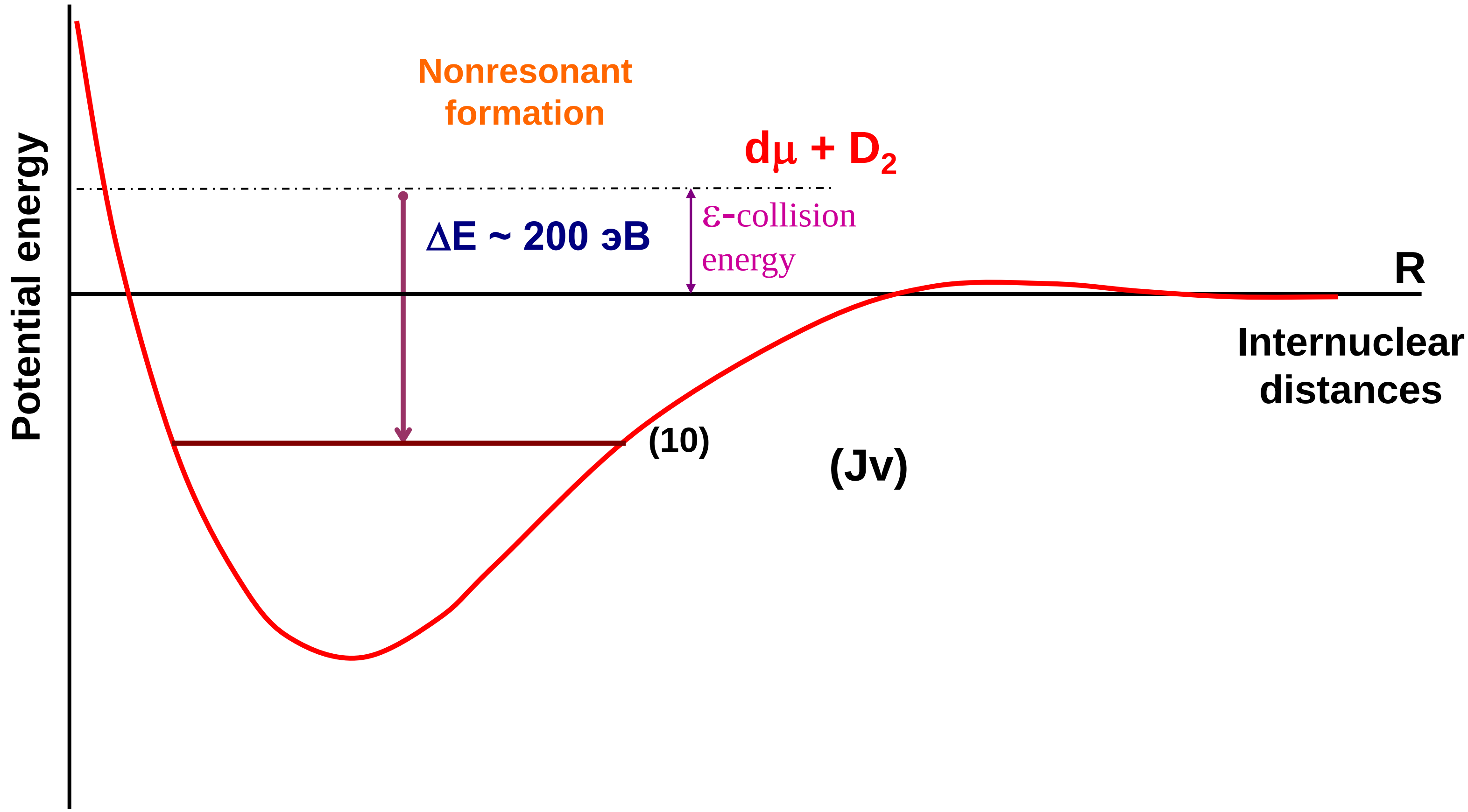
The calculated rates of resonant $dd\mu$ formation in ortho-D_2 (K = 0) and para-D_2 (K = 1) gas at T=40K versus $d\mu$ energy.



The nonresonant formation of muonic molecules

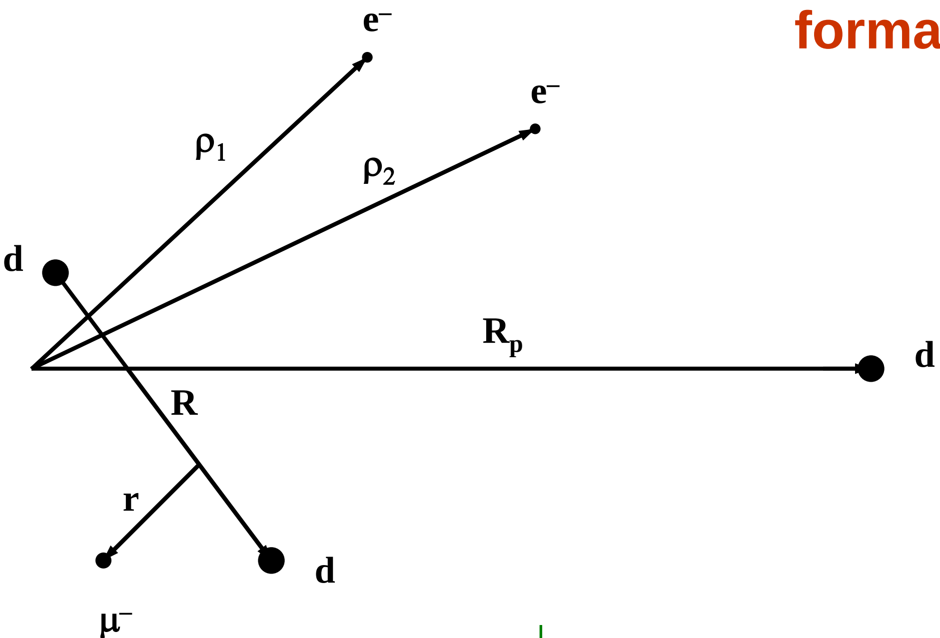
(Ya.B. Zel'dovich, 1954)

$$(d\mu) + D_2 \rightarrow [(dd\mu)_{Jv}de]^+ + e^-$$



The calculation of nonresonant formation rates of muonic molecule

(M.Faifman, 1989)



$$(\mathbf{d\mu}) + \mathbf{D_2} \to [(\mathbf{dd\mu})_{\mathbf{J\nu}}\mathbf{de}]^+ + \mathbf{e}^-$$

$$d\lambda = 2\pi\beta N_0 \left|t_{fi}\right|^2 \delta(E_f - E_i)\frac{d\vec{q}}{(2\pi)^3};$$

$$E_i = \varepsilon_{col} + E_I^e; \quad E_f = \varepsilon_{J\nu} + \frac{q^2}{2m_e};$$

$$\left|t_{fi}\right|^2 = \sum_{M_J^{(f)}} \left| \int d\vec{R}\,d\vec{r}\,d\vec{\rho}_1 d\vec{\rho}_2 \Psi_{mes}^{(f)*}(\vec{r},\vec{R})\psi_{el}^{(f)*}(\vec{\rho}_{1,}\vec{\rho}_2)\hat{V}_{\text{int}}\psi_{el}^{(i)}(\vec{\rho}_{1,}\vec{\rho}_2)\Psi_{mes}^{(i)}(\vec{r},\vec{R}) \right|^2$$

$$\psi^{(\mathrm{i})}(\vec{\rho}_1,\vec{\rho}_2) = \phi^{(i)}(\vec{\rho}_1)\phi^{(i)}(\vec{\rho}_2)$$

$$\psi^{(\mathrm{f})}(\vec{\rho}_1,\vec{\rho}_2) = \frac{1}{2}[\phi^{(i)}(\vec{\rho}_1)\phi^{(f)}(\vec{\rho}_2) + \phi^{(i)}(\vec{\rho}_2)\phi^{(f)}(\vec{\rho}_1)]$$

$$\phi^{(i)}(\vec{\rho}) = \left[\frac{Z_{eff}^3}{2\pi(1+\Delta)}\right]^{\frac{1}{2}} (e^{-Z_{eff}\rho} + e^{-Z_{eff}\left|\vec{\rho}-\vec{R}_p\right|})$$

$$\phi^{(f)}(\vec{\rho}) = \sum_l i^l(2l+1)e^{-i\sigma_l}\frac{1}{q}R_{ql}(\rho)P_l(\cos\Theta)$$

$$V_{\mathrm{int}} = \sum_{j=a,b,\mu} z_j \left[\frac{1}{\rho_1} - \frac{1}{R^e_{1j}} + \frac{1}{\rho_2} - \frac{1}{R^e_{2j}} \right]; \quad \mathrm{R}^{\mathrm{e}}_{\mathrm{ij}} = \left| \mathbf{R}^{\mathrm{e}}_{\mathrm{i}} - \mathbf{R}^{\mathrm{e}}_{\mathrm{j}} \right|; \quad z_{\mathrm{a}} = z_{\mathrm{b}} = 1, z_{\mathrm{m}} = -1.$$

$$V^{(0)}_{\mathrm{int}} = \frac{4\pi}{3} Q \delta(\rho), \qquad V^{(1)}_{\mathrm{int}} = -2 \frac{\vec{d}\vec{\rho}}{\rho^3},$$

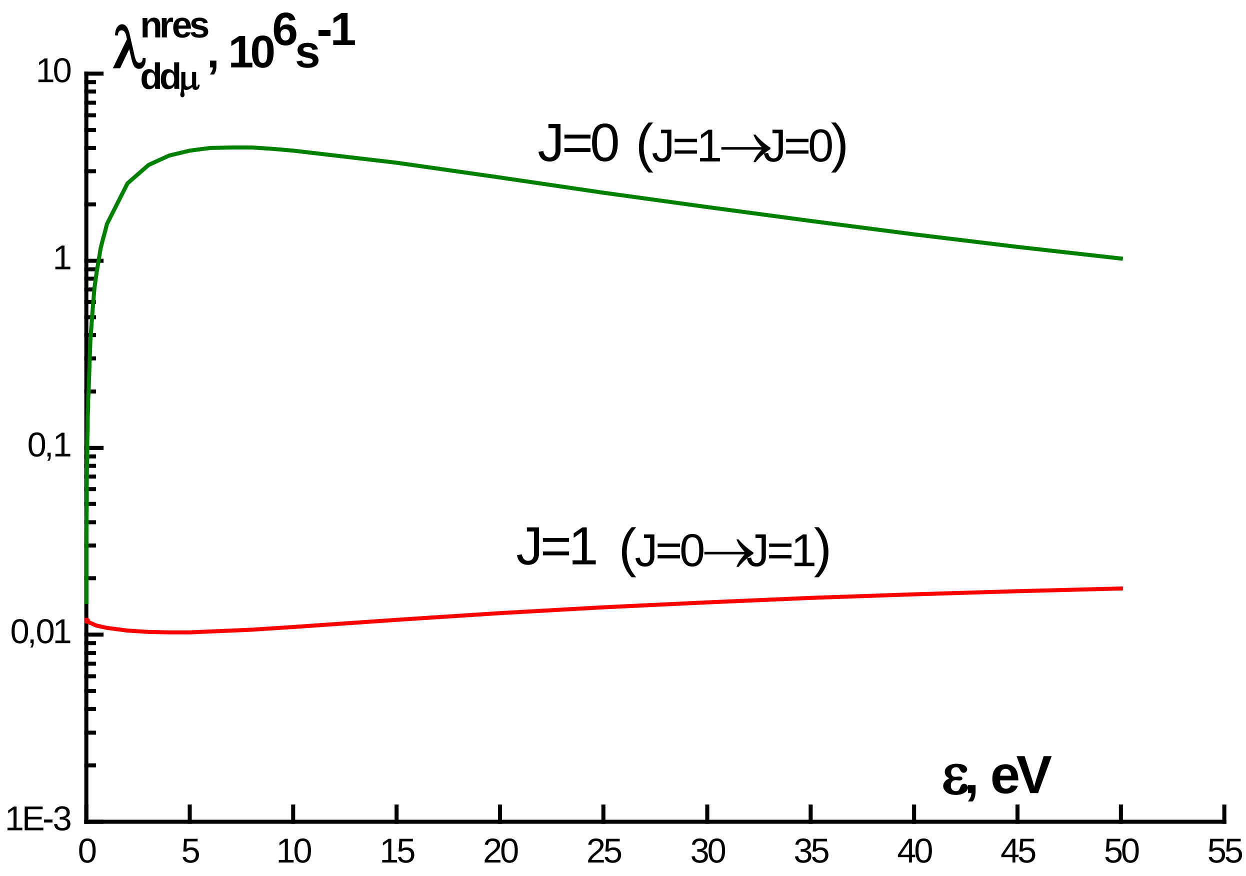
$$Q = \sum_{j=a,b,\mu} z_j R_j^2 \qquad \vec{d} = \sum_{j=a,b,\mu} z_j \vec{R}_j$$

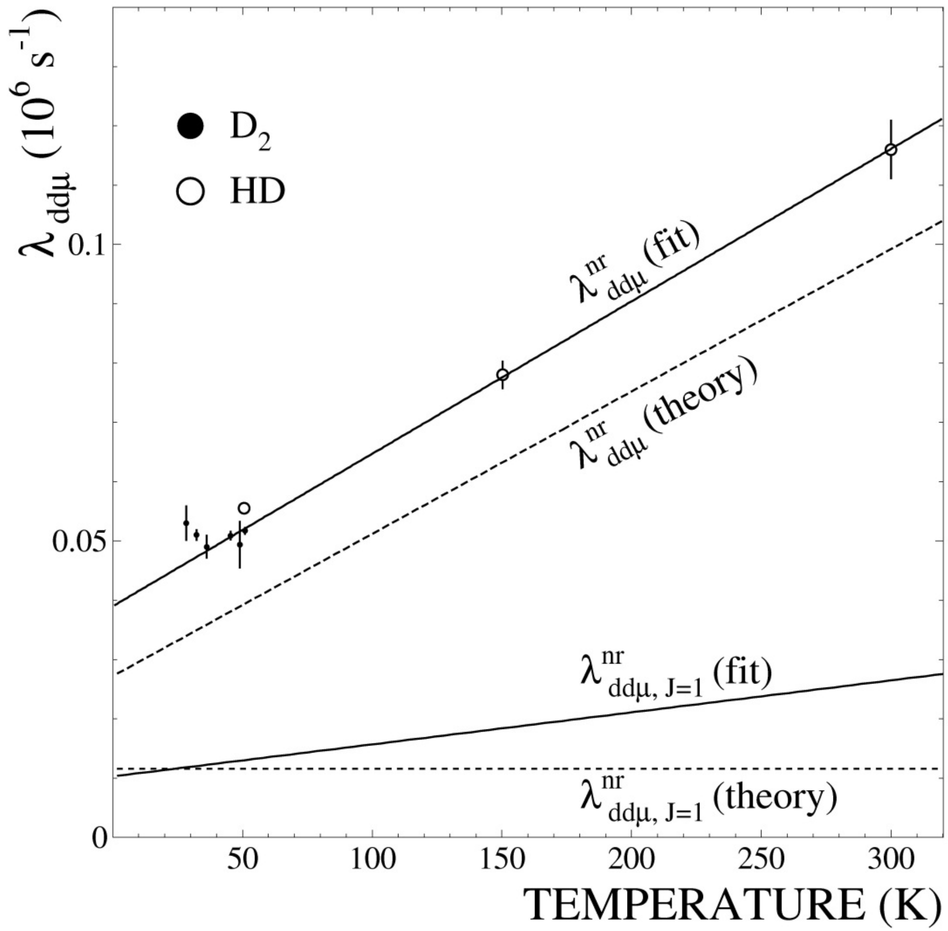
The rates of nonresonant formation

$$\lambda(E0) = \frac{8\pi}{9} \beta q N_0 I_0^2(q) \left| \left\langle Q \right\rangle \right|^2 - the \;\; monopole \;\; transition;$$

$$\lambda(E1) = \frac{4}{3\pi} \beta q N_0 I_1^2(q) \left| \left\langle \vec{d} \right\rangle \right|^2 - the \;\; dipole \;\; transition.$$

The rates of nonresonant ddμ formation in deeply bound (J=0 and J=1) states





$$\boldsymbol{\lambda}^{\mathbf{nres}}_{dd\mu}(\mathrm{T}) = \boldsymbol{\lambda}^{\mathbf{nres}}_{\mathbf{J=0}}(\mathrm{T}) + \boldsymbol{\lambda}^{\mathbf{nres}}_{\mathbf{J=1}}(\mathrm{T});$$

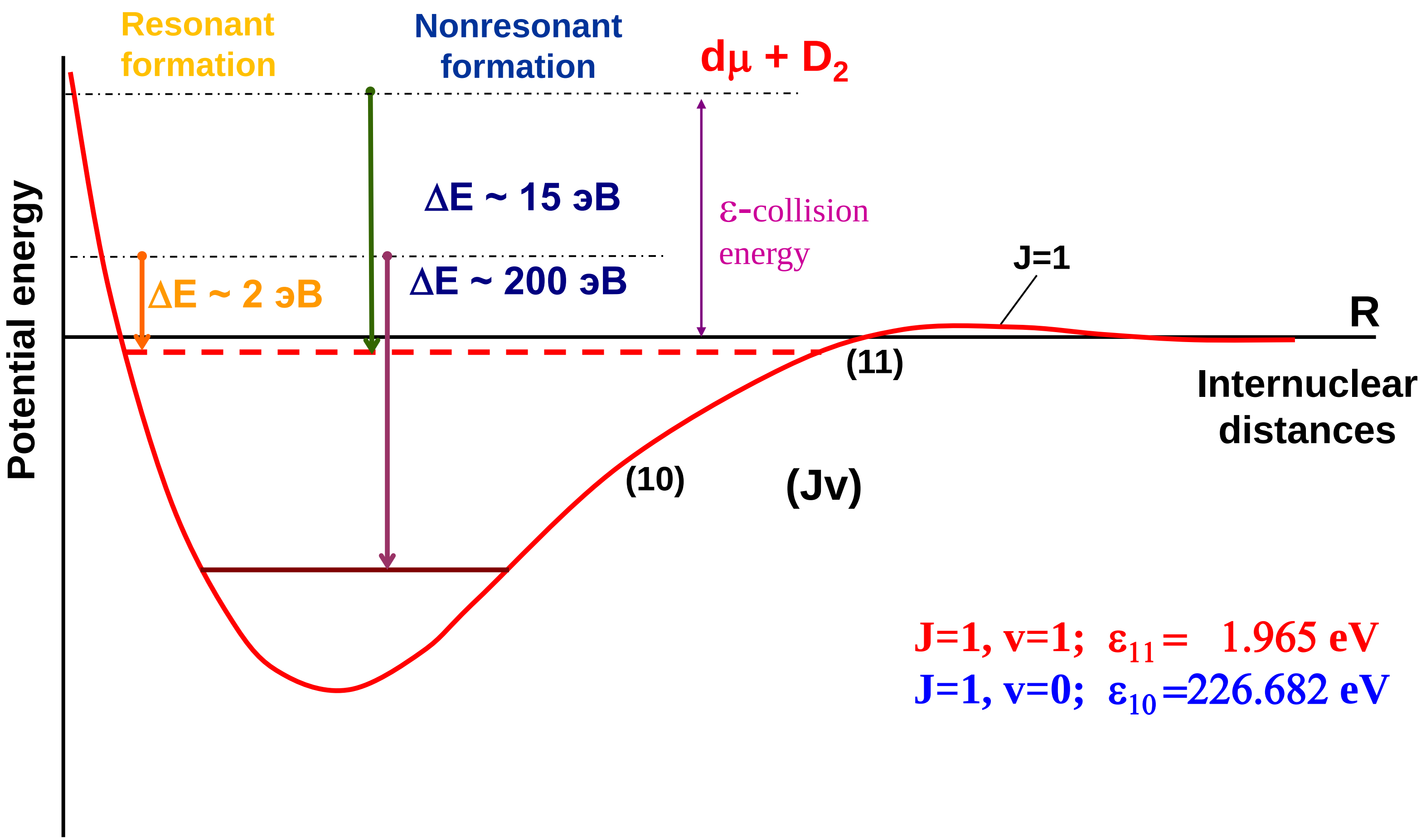
$\boldsymbol{\lambda}^{\mathbf{nres}}_{dd\mu}(\mathrm{T}) = [2.57(17) \cdot 10^{-4}\ \mathbf{T} + 0.039(1)] \cdot 10^{6}\mathrm{s}^{-1}$ – PSI-PNPI-IMEP-LBNL-TUM *experiment*

D.V.Balin et al., Phys.El.Part. and Nucl., **42** (2) 361 (2011);

$\boldsymbol{\lambda}^{\mathbf{nres}}_{dd\mu}(\mathrm{T}) = [2.4 \cdot 10^{-4}\,\mathbf{T} + 0.027] \cdot 10^{6}\mathrm{s}^{-1}$ – *theory* (M. P. Faifman, MCF 4, 341, 1989)

Muonic molecule formation

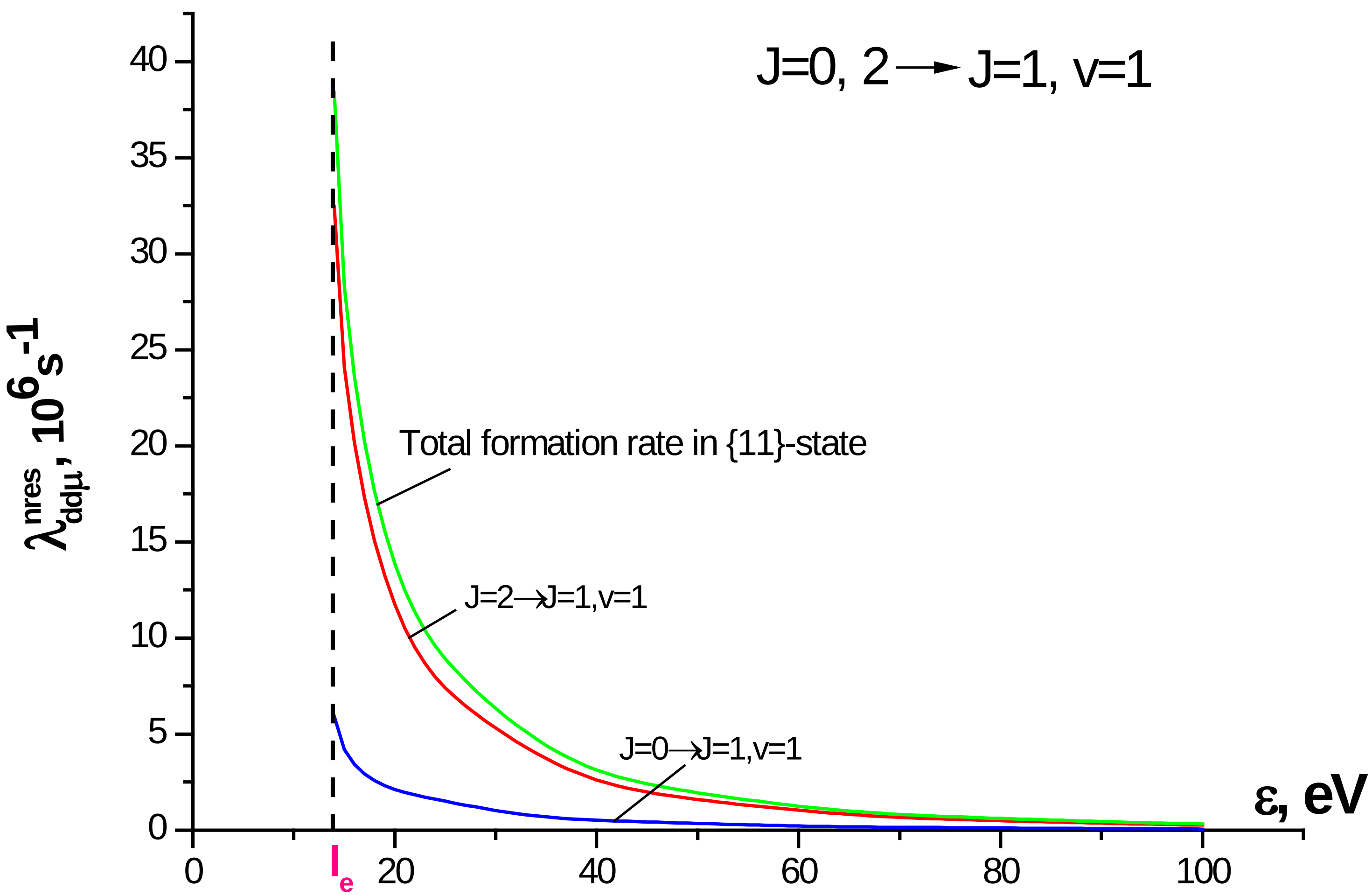
$$(d\mu) + D_2 \rightarrow [(dd\mu)_{J\nu}de]^{+} + e^{-} \; - \; \textit{nonresonant formation.}$$



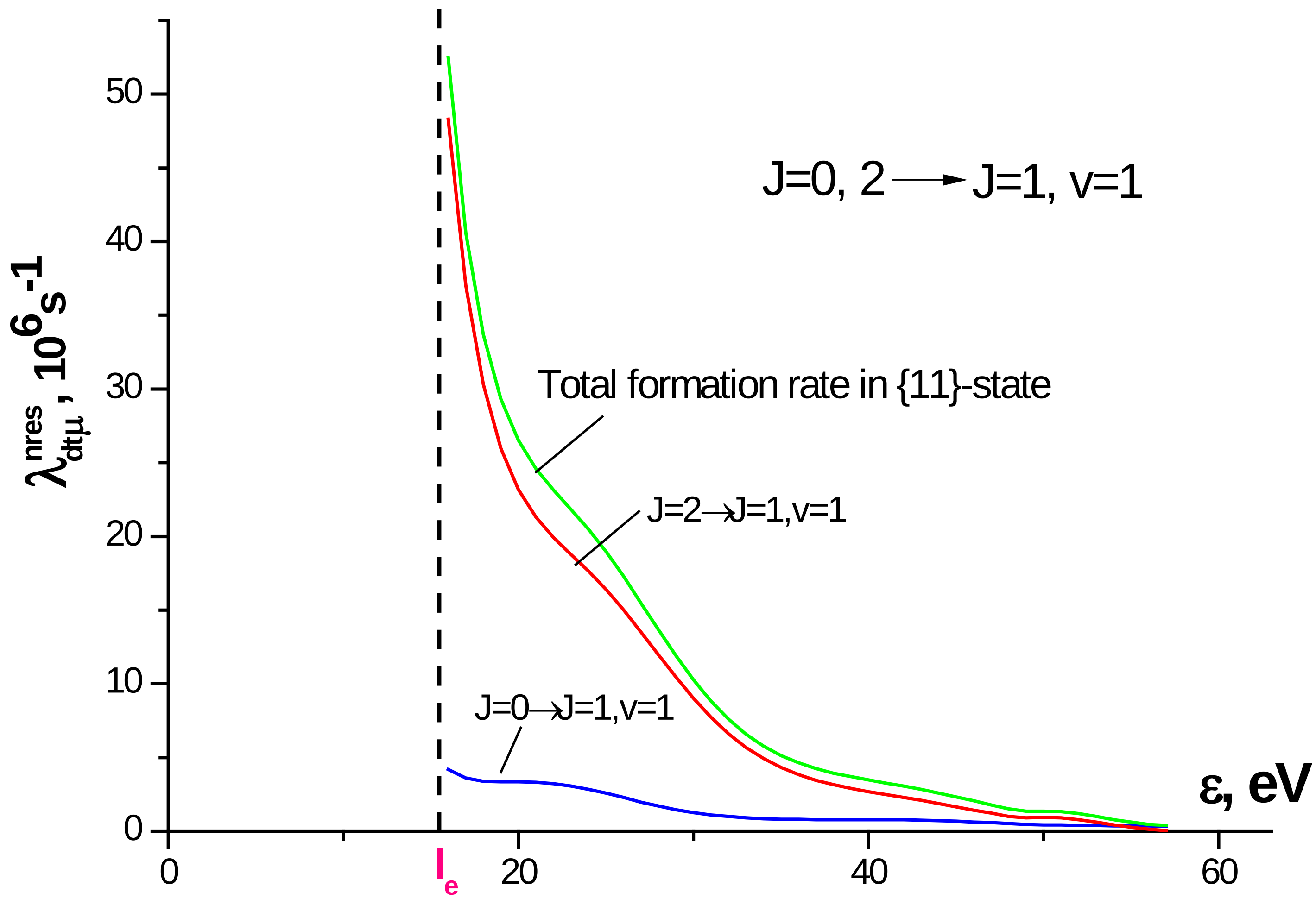
J=1, v=1; ε_{11} = 1.965 eV

J=1, v=0; ε_{10} =226.682 eV

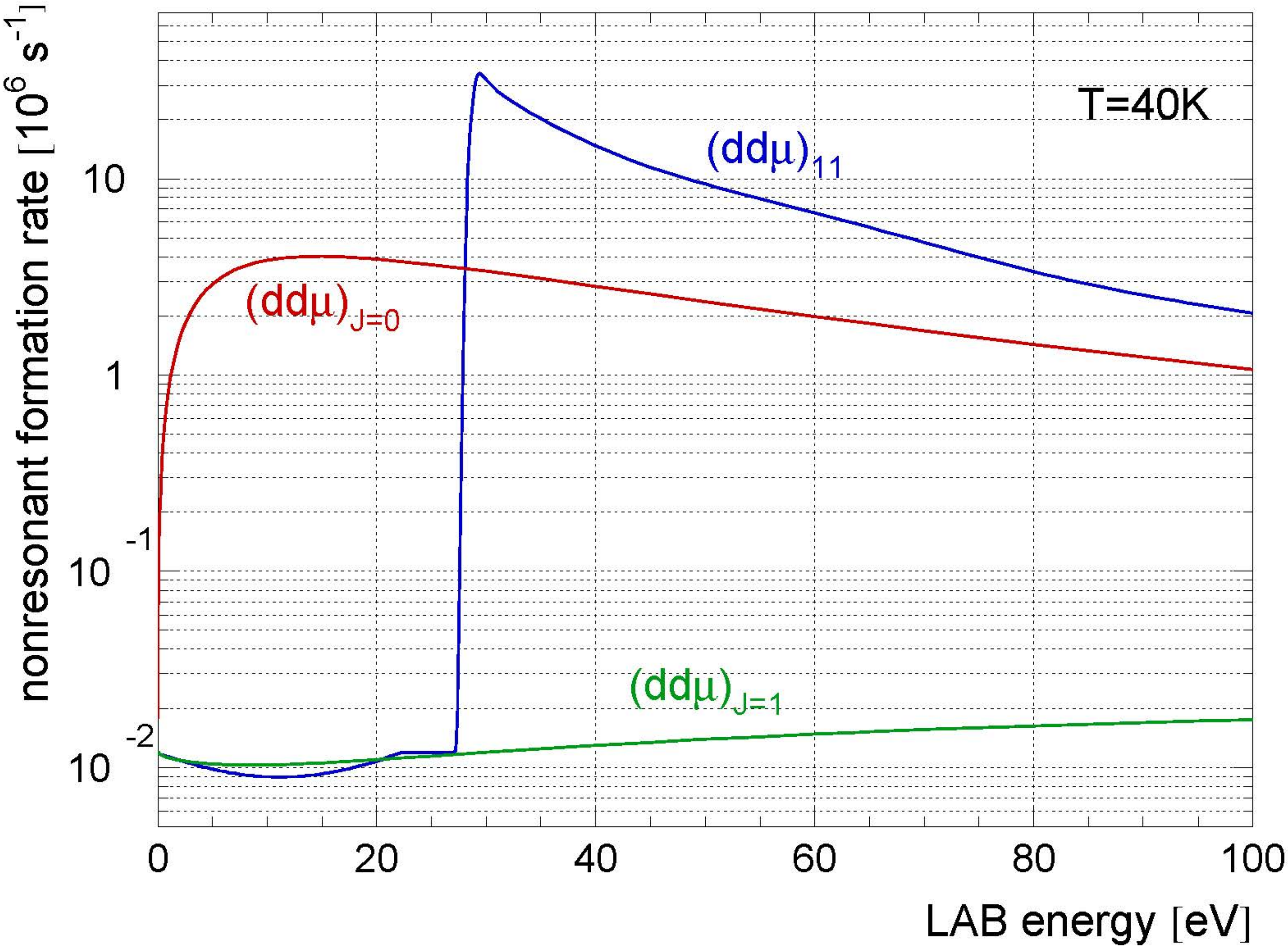
$$(d\mu) + D_2 \rightarrow [(dd\mu)_{11} de]^+ + e^-$$



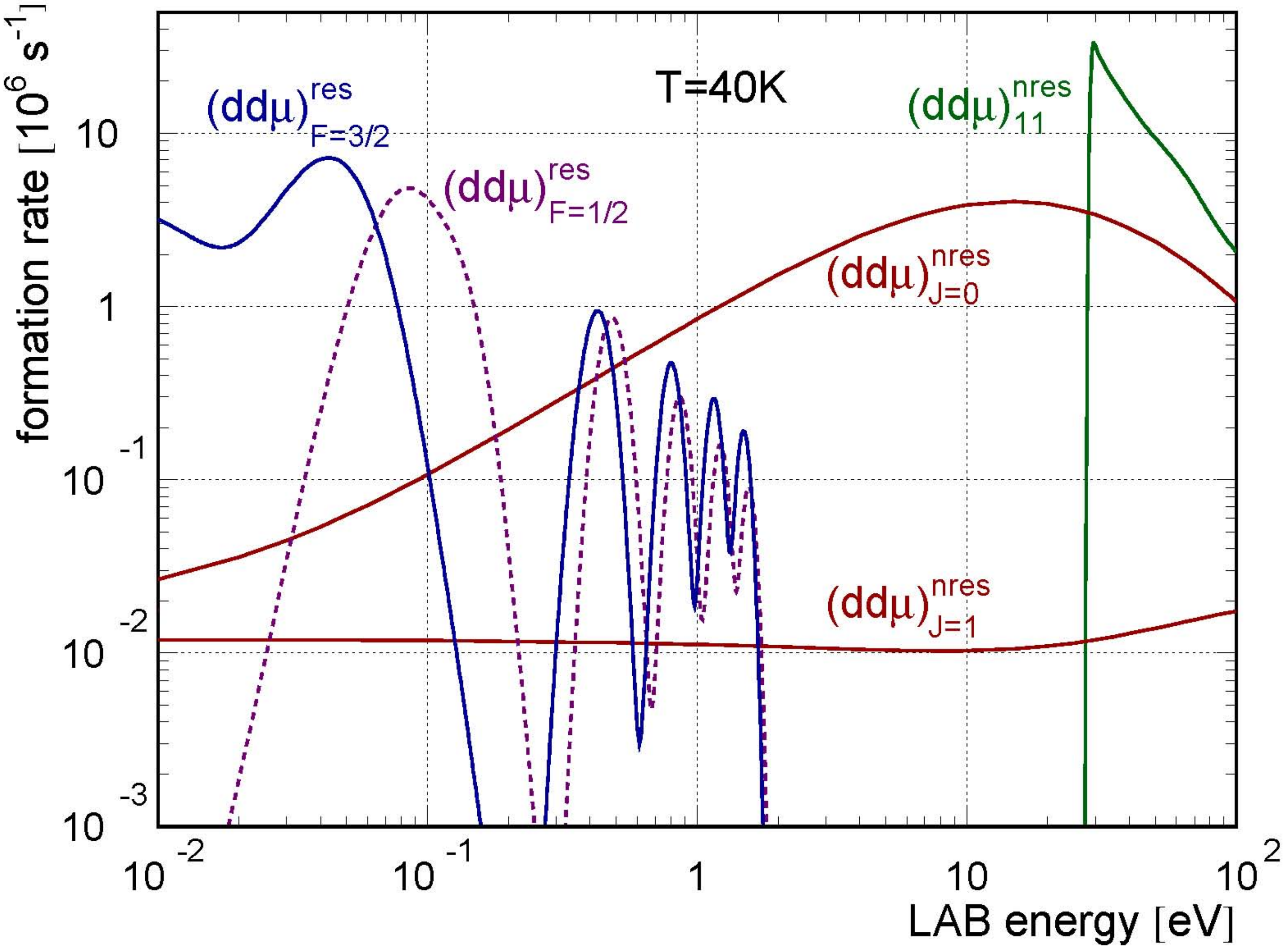
$$(t\mu) + D_2 \rightarrow [(dt\mu)_{11} de]^+ + e^-$$



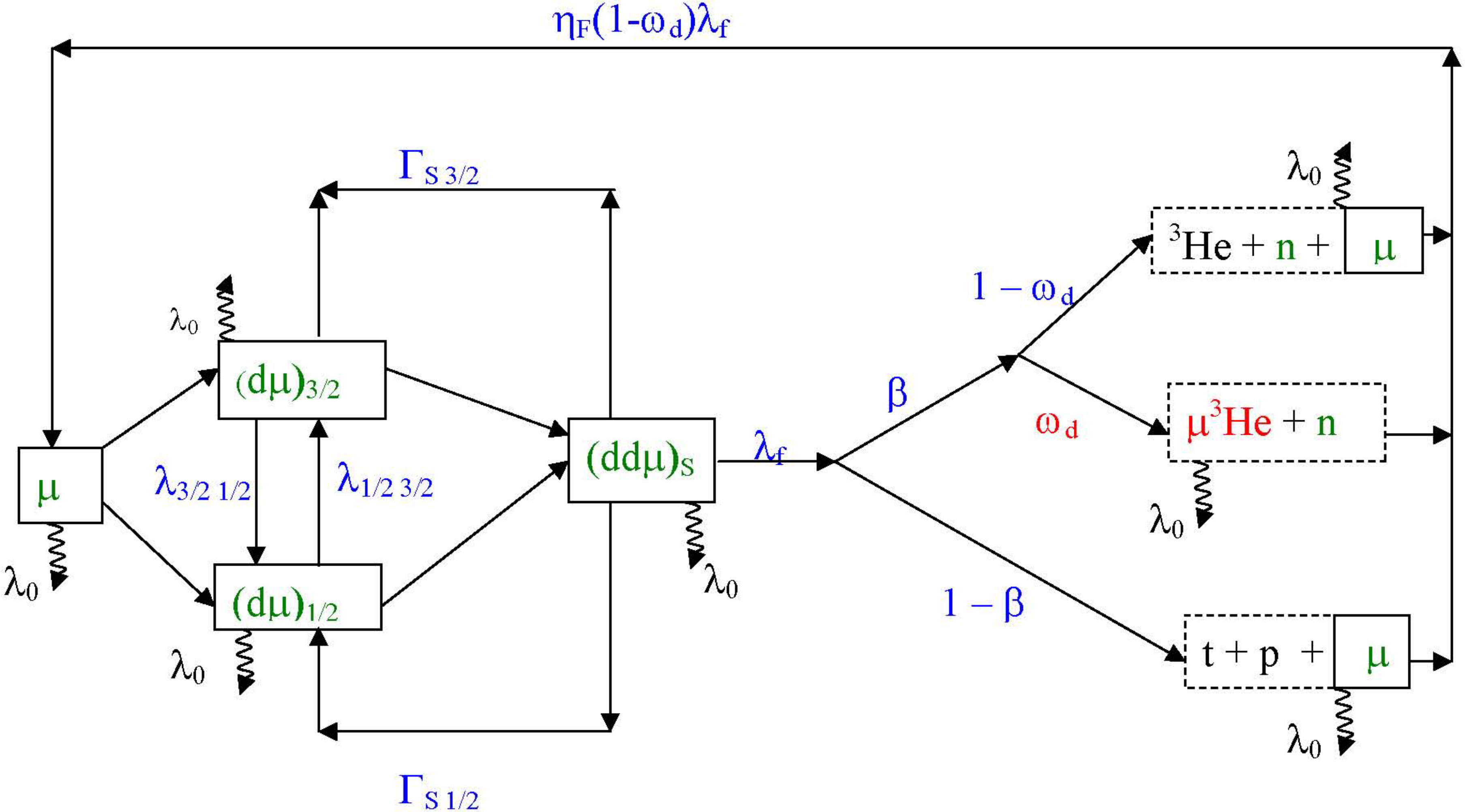
The comparison of nonresonant $dd\mu$ formation rates in deeply (J=0 and J=1) and weakly (J=v=1) bound states



The calculated dependences of resonant and nonresonant ddμ formation rates versus dμ energy



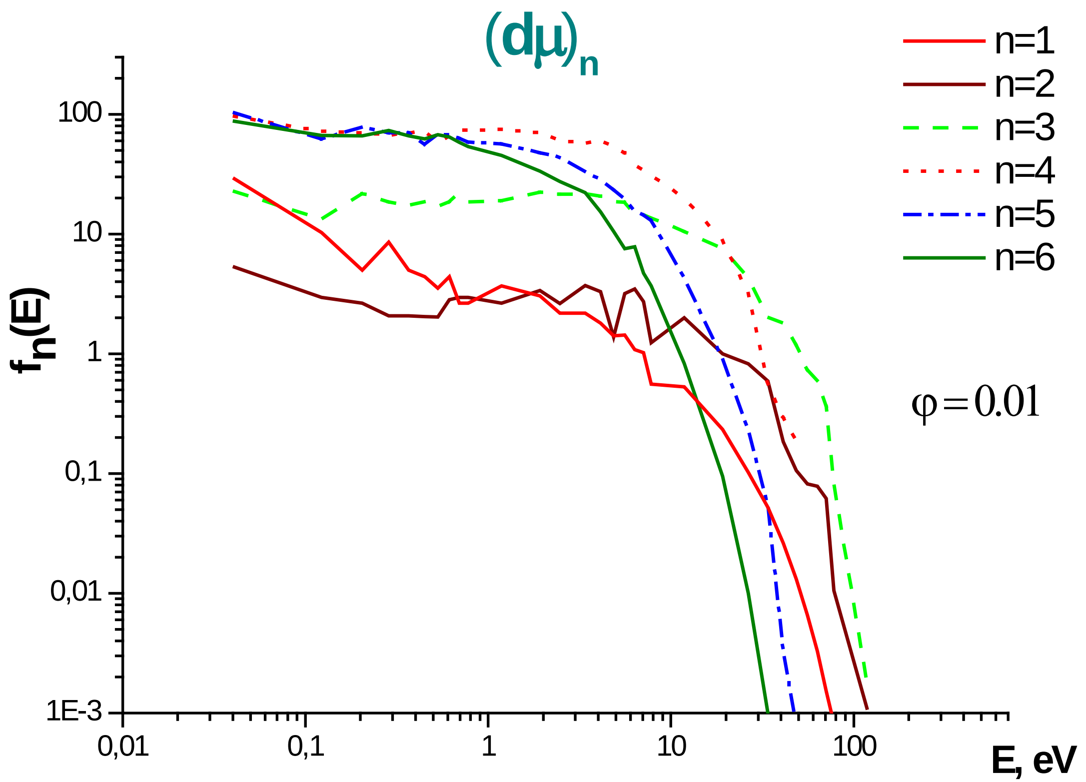
The scheme of μCF processes in pure deuterium



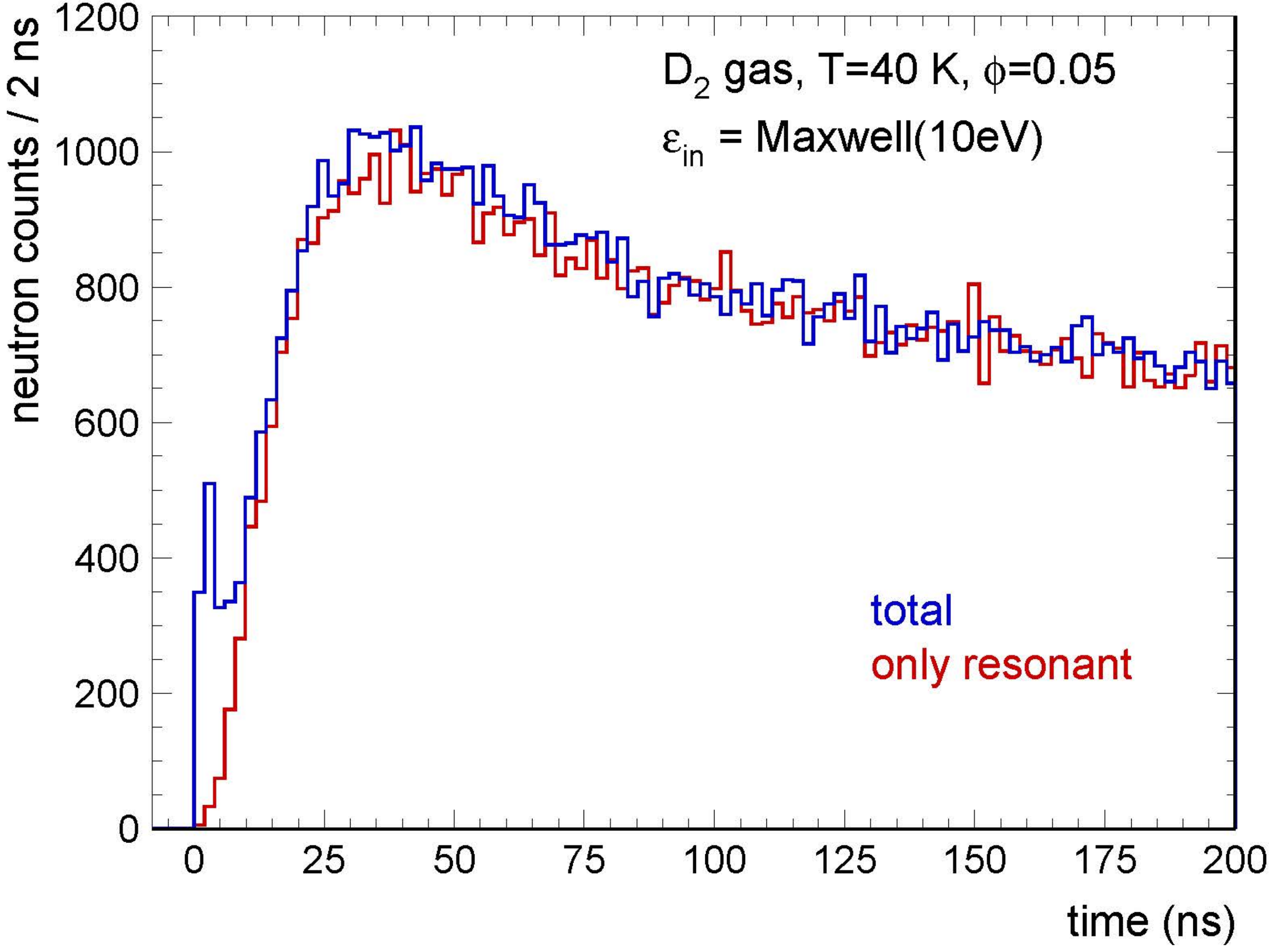
The main processes and their rates of μCF cycle in D_2

Processes	Rates
Muon decay: $\mu^- \rightarrow \nu_\mu + e^- + \tilde{\nu}_e$	$\lambda_0 = 0.46 \cdot 10^6\ s^{-1}$
μ-atom formation/cascade	$\lambda_a \sim 10^{12}\ s^{-1}$
Spin-flip	$\lambda_{3/2\ 1/2} \sim 3 \cdot 10^7\ s^{-1}$
μ-molecule formation	$\lambda_{dd\mu} \sim 3 \cdot 10^6\ s^{-1}$
Fusion	$\lambda_f \sim 4 \cdot 10^8\ s^{-1}$
Sticking	$\omega_s \approx 12\%$

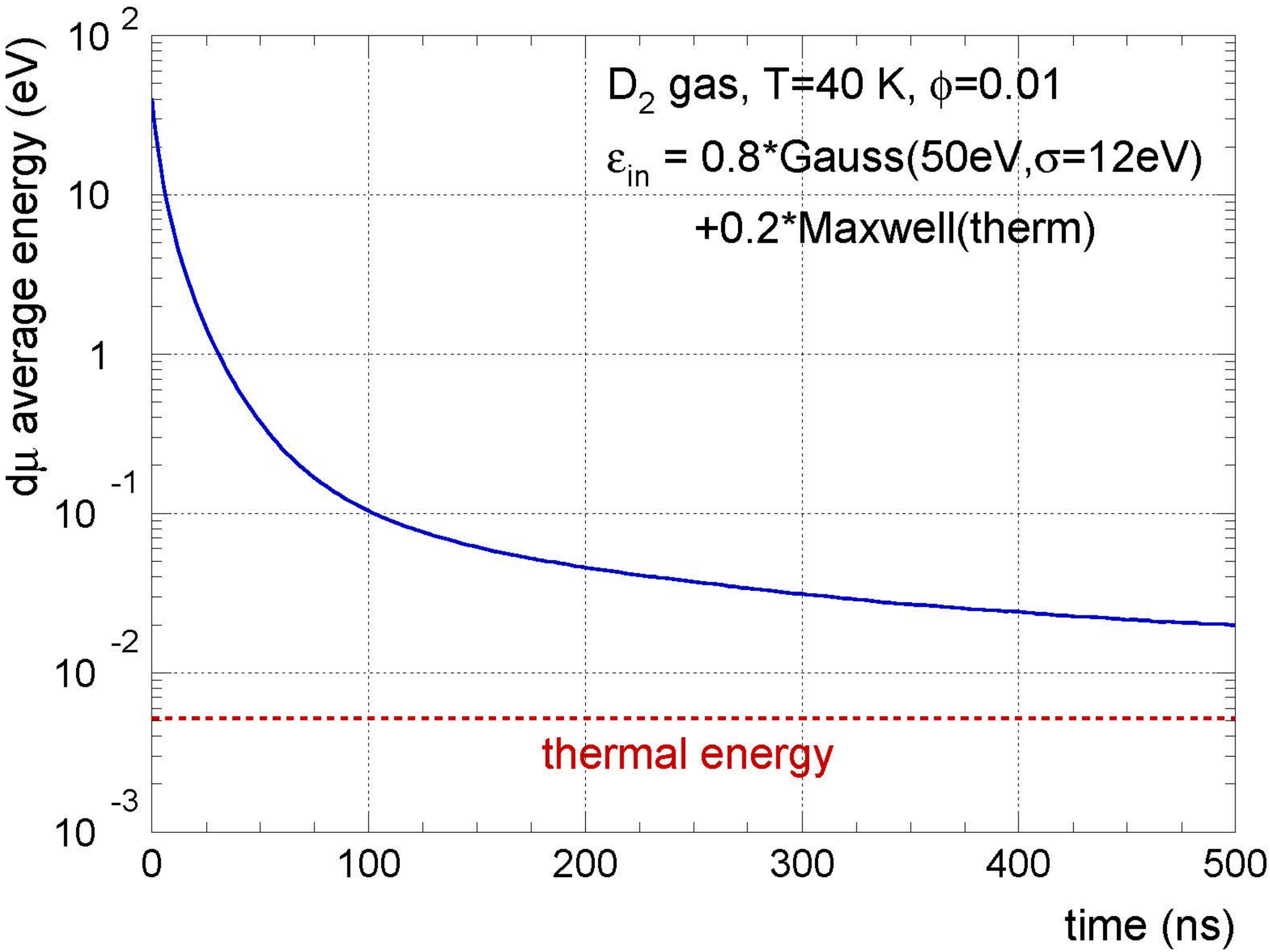
The distributions over the kinetic energies of dμ atoms in different exited states at density φ=0.01 LHD.



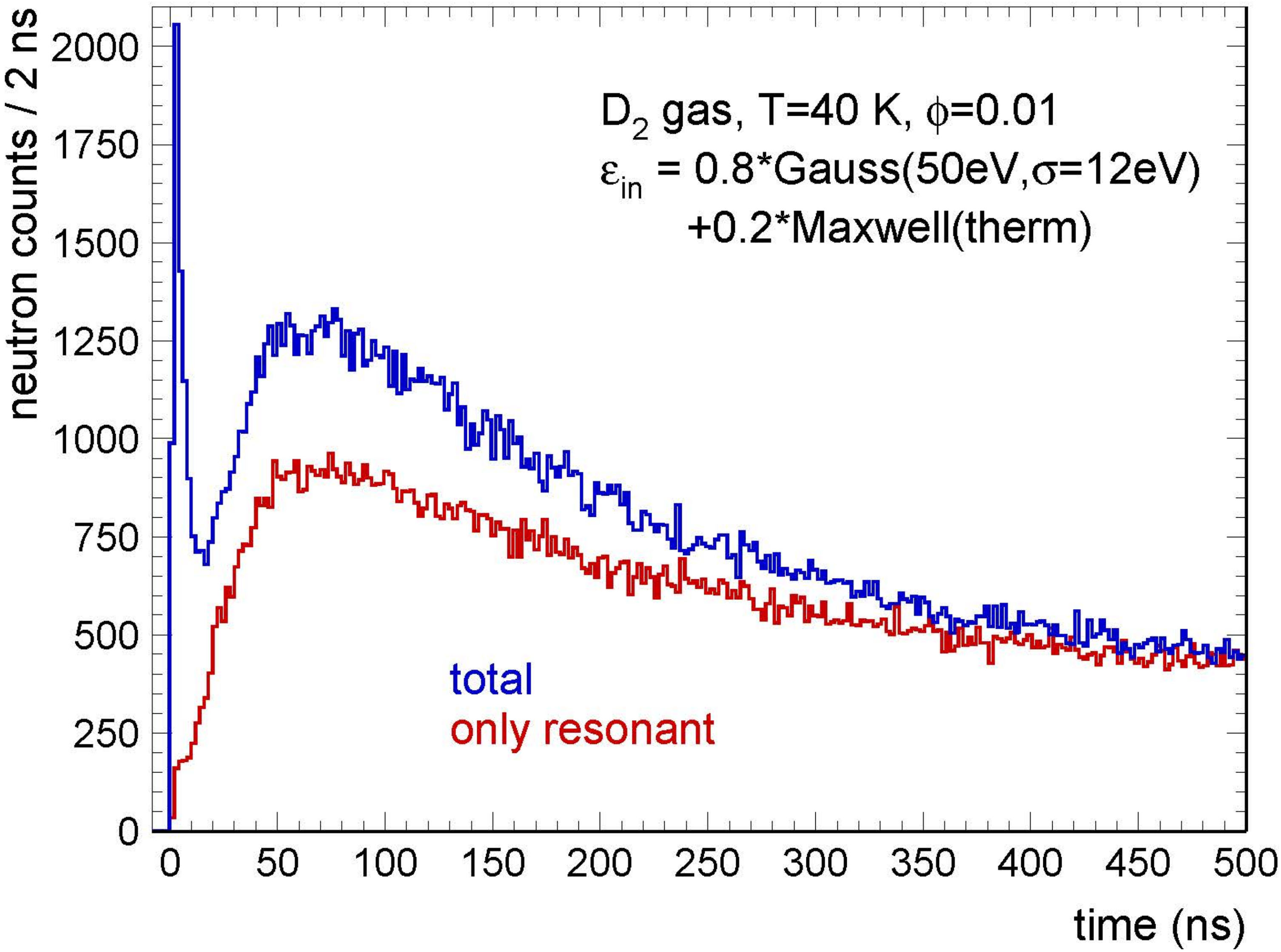
The time spectrum of neutrons from *dd*-fusion in D2 gas

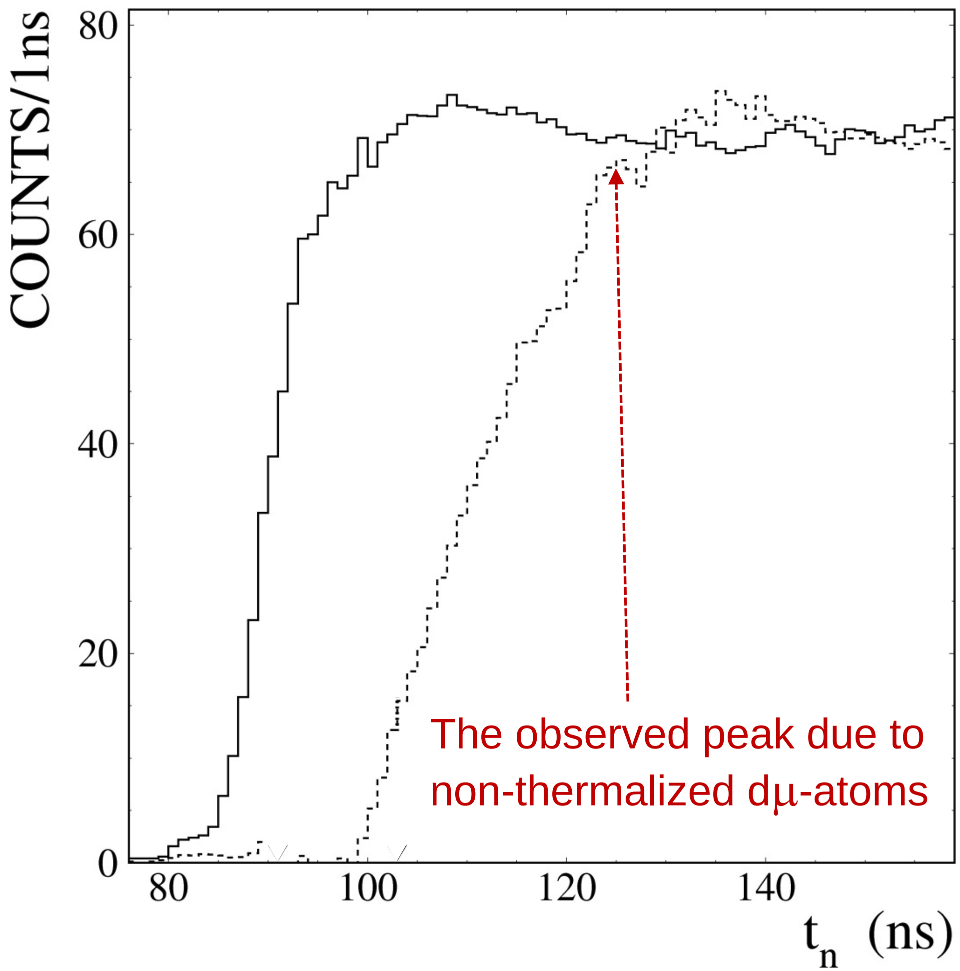


Mean kinetic energy of *dμ* atoms in D2 gas



The peak due to nonresonant *ddμ* formation in the above-threshold energy region





Initial parts of time distributions of electrons from μ-decay (solid line) and neutrons from *dd* fusion (dashed line) measured by the neutron detectors.

D.V.Balin et al., Phys.El.Part. And Nucl., 42 (2) 361 (2011).

Summary

•The non-resonant formation of *ddμ* and *dtμ* molecules in the weakly bound state was considered in the above-threshold energy region, and the calculated rates are about one order of magnitude higher than previously expected;

•The role of the non-resonant *ddμ* formation by non-thermalized dμ-atoms for kinetics in D_2 gas has been revealed as the directly observed in experiments peak in the neutron time spectra at very short initial times;

•The detailed comparison of the observed and calculated neutron spectra at times before the complete thermalization of *dμ* atoms provides an opportunity to test the cascade theory.