# Consistency of $\Lambda\Lambda$ hypernuclear events

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- Introduction
- $\Lambda\Lambda$ hypernuclei
- Spin dependence in $\Lambda$ hypernuclei
- Consistency of $\Lambda\Lambda$ hypernuclear events

A. Gal, D.J. Millener, Phys. Lett. B 701 (2011) 342

Shell-model predictions for $\Lambda\Lambda$ hypernuclei (Hida event)

# Introduction

# Highlights of Strangeness Nucl. Phys.

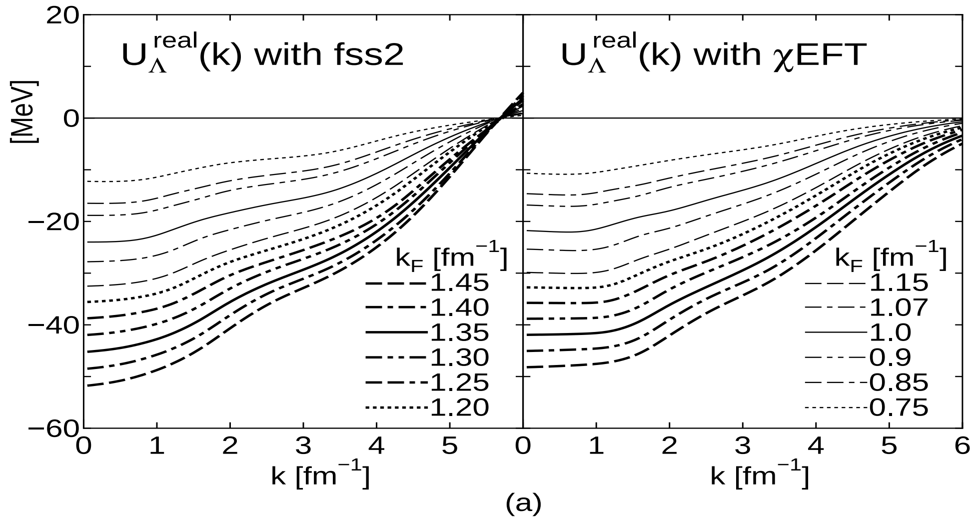
- $\Lambda$ hyperon is bound by 28 MeV in nuclear matter.
  No $\Lambda N$ bound state, binding starts with ${}^{3}_{\Lambda}\mathrm{H}$ $(\Lambda pn)$.

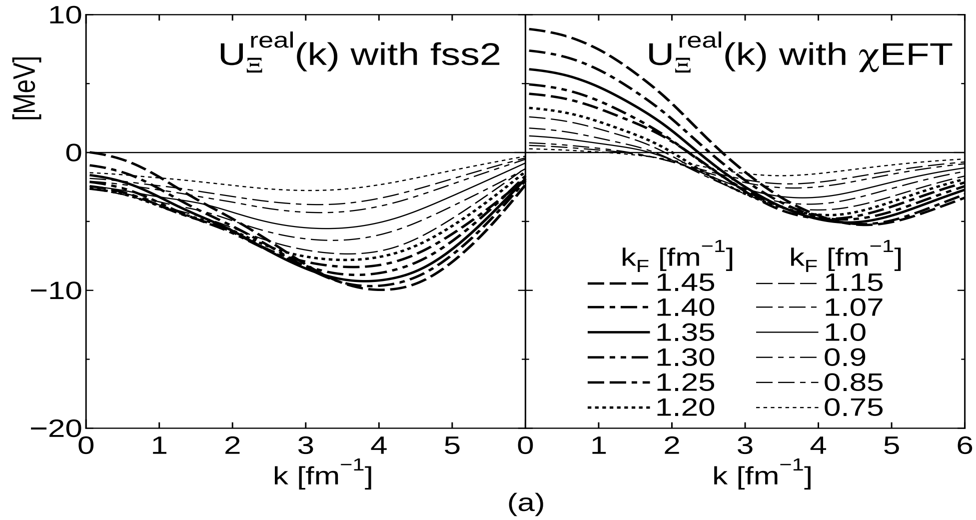
- $\Sigma$ hyperons unbound in nuclei except for ${}^{4}_{\Sigma}\mathrm{He}$.
  $\Sigma N \to \Lambda N$ releases $\sim 80$ MeV, $\Gamma_{\Sigma}(\rho_0) \sim 25$ MeV.

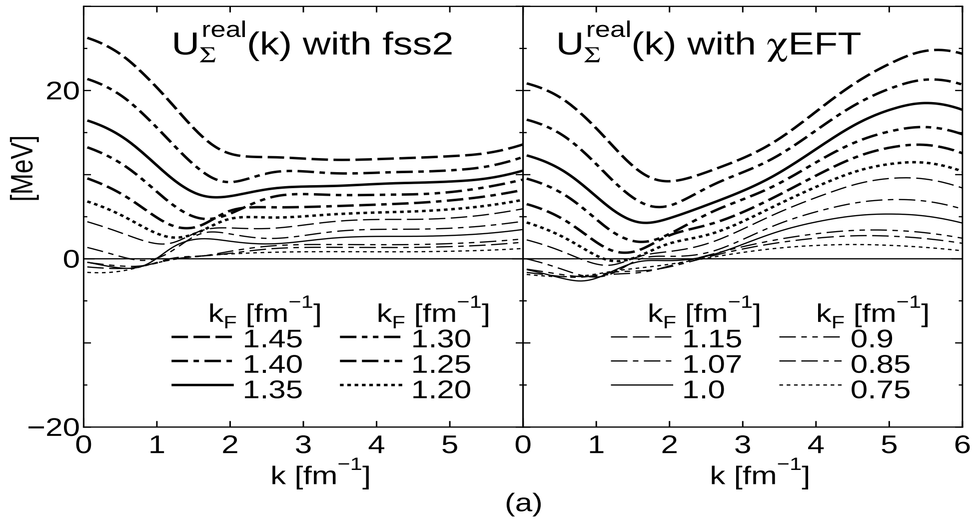
- $\Xi$ hyperons bound by $\sim 15$ MeV in nuclear matter?
  Binding not established yet; $\Xi N \to \Lambda\Lambda$ releases $\sim 25$ MeV.
  $\Xi N \to \Lambda\Lambda$ gets Pauli-blocked as soon as few $\Lambda$s are bound.

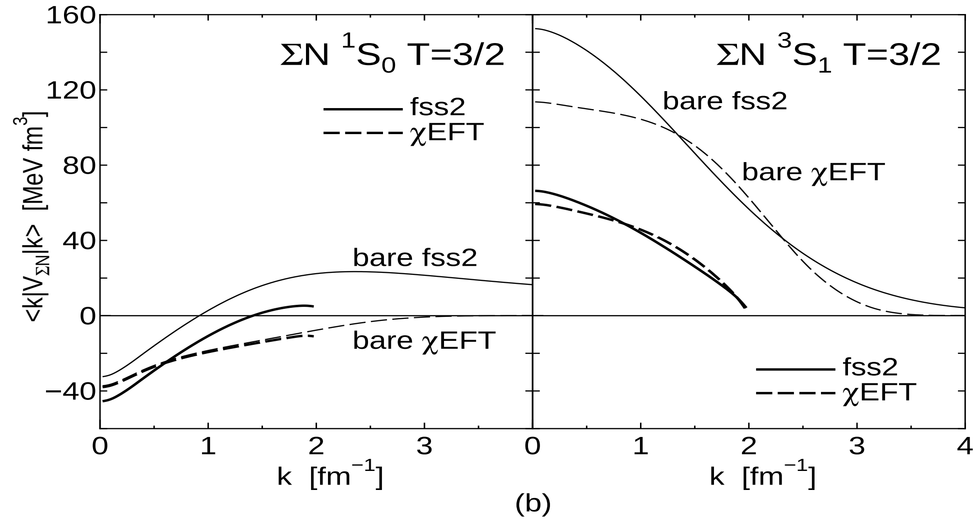
- No $\Lambda\Lambda$, ${}_{\Lambda\Lambda}{}^{3}\mathrm{H}$, but ${}_{\Lambda\Lambda}{}^{5}\mathrm{H}$ likely bound. $S = -2$ $H$ dibaryon?
  Lightest $S = -3, -4$ particle-stable states? case for ${}_{\Lambda\Lambda}{}^{7}_{\Xi}\mathrm{He}$.

- Strange Hadronic $\{N, \Lambda, \Xi\}$ Matter.



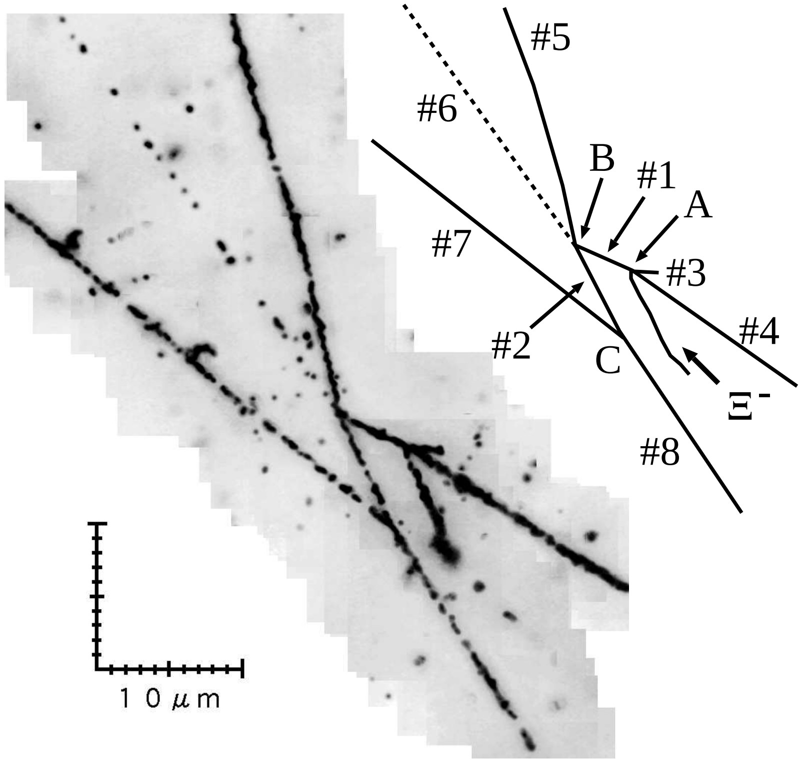




Kohno, PRC **81** (2010) 014003 Nuclear matter hyperon s.p. potentials

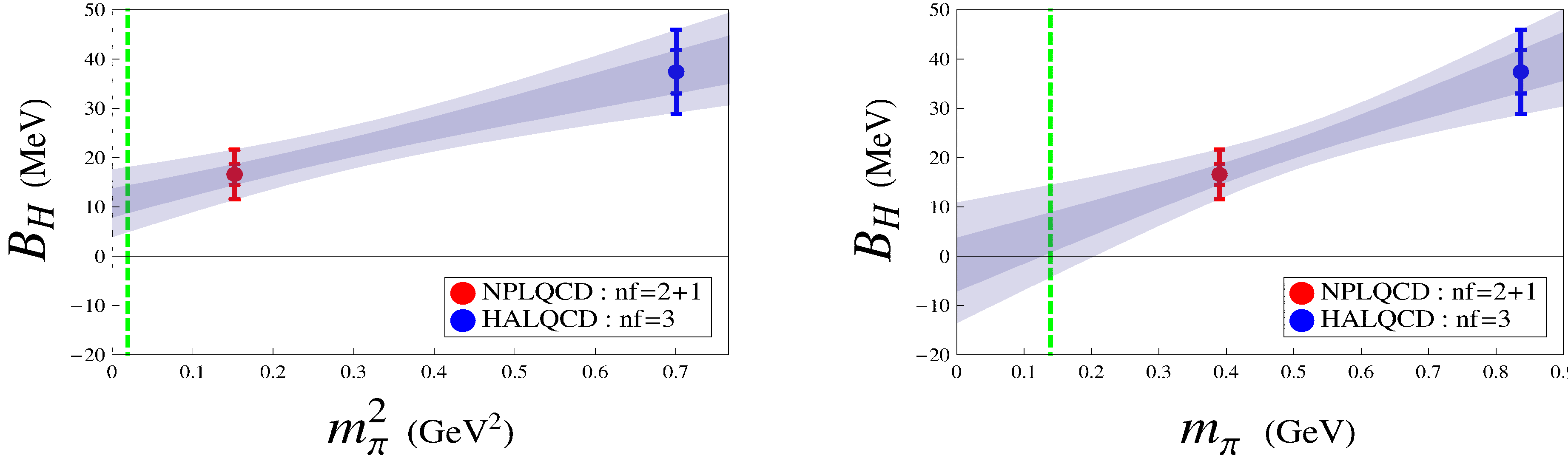
QM fss2, Fujiwara et al., $\chi$ EFT (Polinder et al. 2007)




Nagara event, ${}_{\Lambda\Lambda}^{\ 6}$He, H. Takahashi et al. (KEK-E373) PRL 87 (2001) 212502

$B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{\ 6}\mathrm{He}_{\mathrm{g.s.}}) = 6.91 \pm 0.16$ MeV consistently and unambiguously determined.

- A: $\Xi^-$ capture $\Xi^- + {}^{12}\mathrm{C} \ \rightarrow \ {}_{\Lambda\Lambda}^{\ 6}\mathrm{He} + t + \alpha$
- B: weak decay ${}_{\Lambda\Lambda}^{\ 6}\mathrm{He} \ \rightarrow \ {}_{\Lambda}^{5}\mathrm{He} + p + \pi^-$ (no ${}_{\Lambda\Lambda}^{\ 6}\mathrm{He} \ \rightarrow \ {}^{4}\mathrm{He} + H$)
- C: nonmesonic weak decay of ${}_{\Lambda}^{5}$He to two $Z = 1$ recoils & neutron



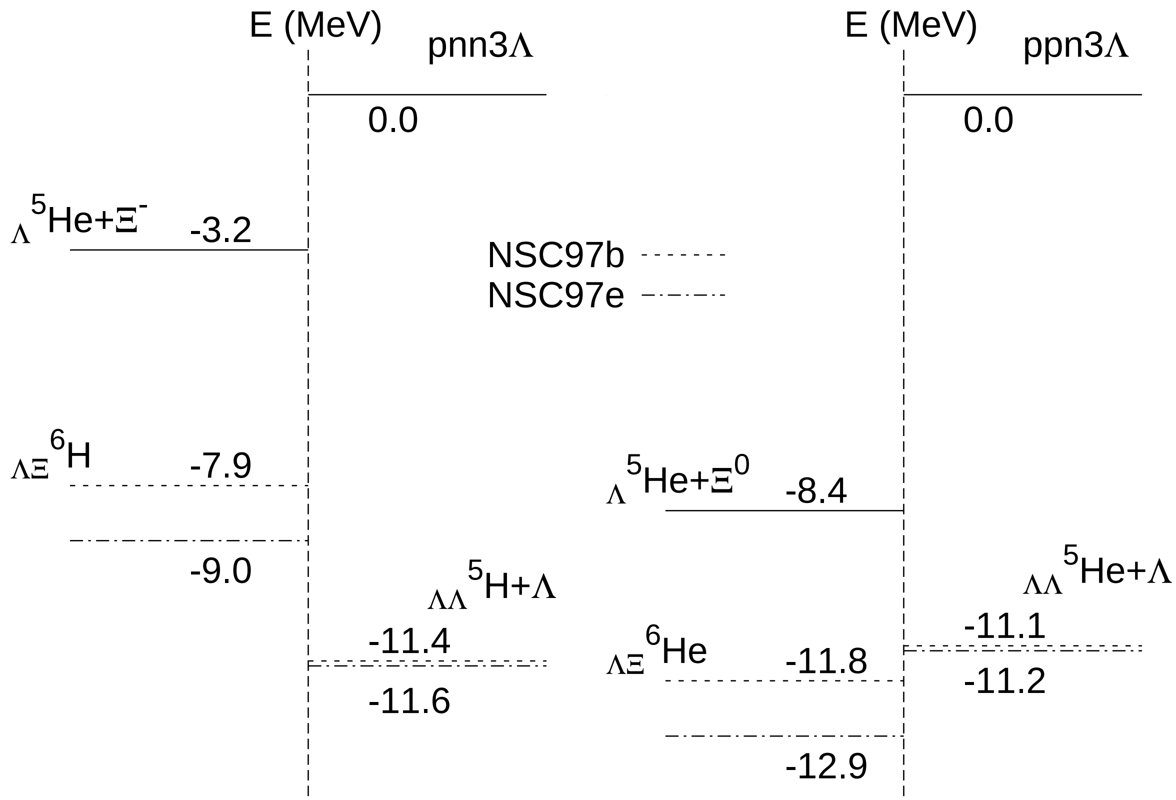

S.R. Beane et al. (NPLQCD Collab.) arXiv:1103.2821 – $H$ dibaryon

LQCD extrapolations: quadratic (left) & linear (right) in $m_\pi$

$H = (uuddss)$ $I = 0$ $J^\pi = 0^+$ approximately **1** SU(3)$_\mathrm{f}$

$$\approx \sqrt{1/8}\ \Lambda\Lambda + \sqrt{1/2}\ N\Xi - \sqrt{3/8}\ \Sigma\Sigma \quad (20\%)$$

- Quark Cluster Model: $H$ could show as a $\Lambda\Lambda$ resonance below $N\Xi$ threshold [Takeuchi+Oka, PRL 66 (1991) 1271; NPA 524 (1991) 649].
- QCD sum rule calculations: $B_H = 40 \pm 70$ MeV; $m_H$ correlated with $m_{nn}$ [Kodama+Oka+Hatsuda, NPA 580 (1994) 445].
- ${}_{\Lambda\Lambda}^{6}$He particle stability rules out $B_{\mathbf{H}} > 7$ MeV.



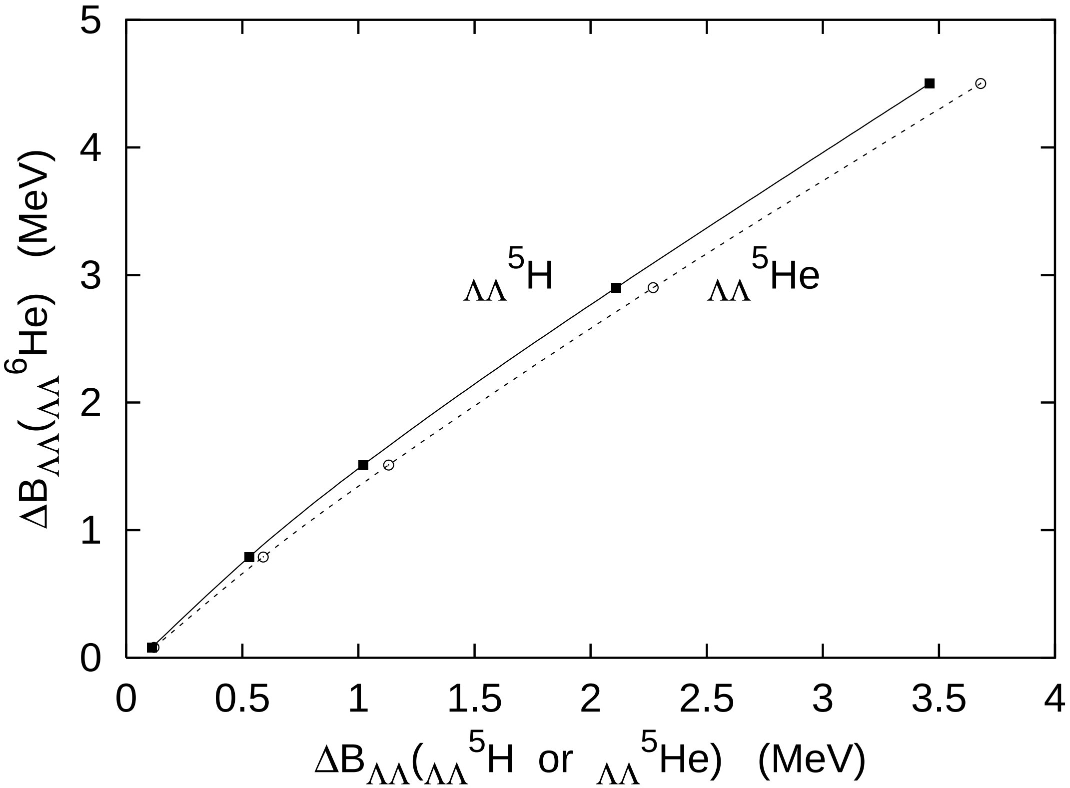

I.N. Filikhin, A. Gal, Phys. Rev. C **65** (2002) 041001(R)

*s*-wave Faddeev calculations for $_{\Lambda\Xi}^{6}$H (left) & $_{\Lambda\Xi}^{6}$He (right)

Onset of $\Xi$ stability to $\Xi N \to \Lambda\Lambda$ requires a bound $_{\Xi}^{5}$He

Nijmegen's ESC04 yields no bound $_{\Xi}^{5}$He; onset at $_{\Lambda\Lambda\Xi}^{7}$He ?

# $\Lambda\Lambda$ hypernuclei

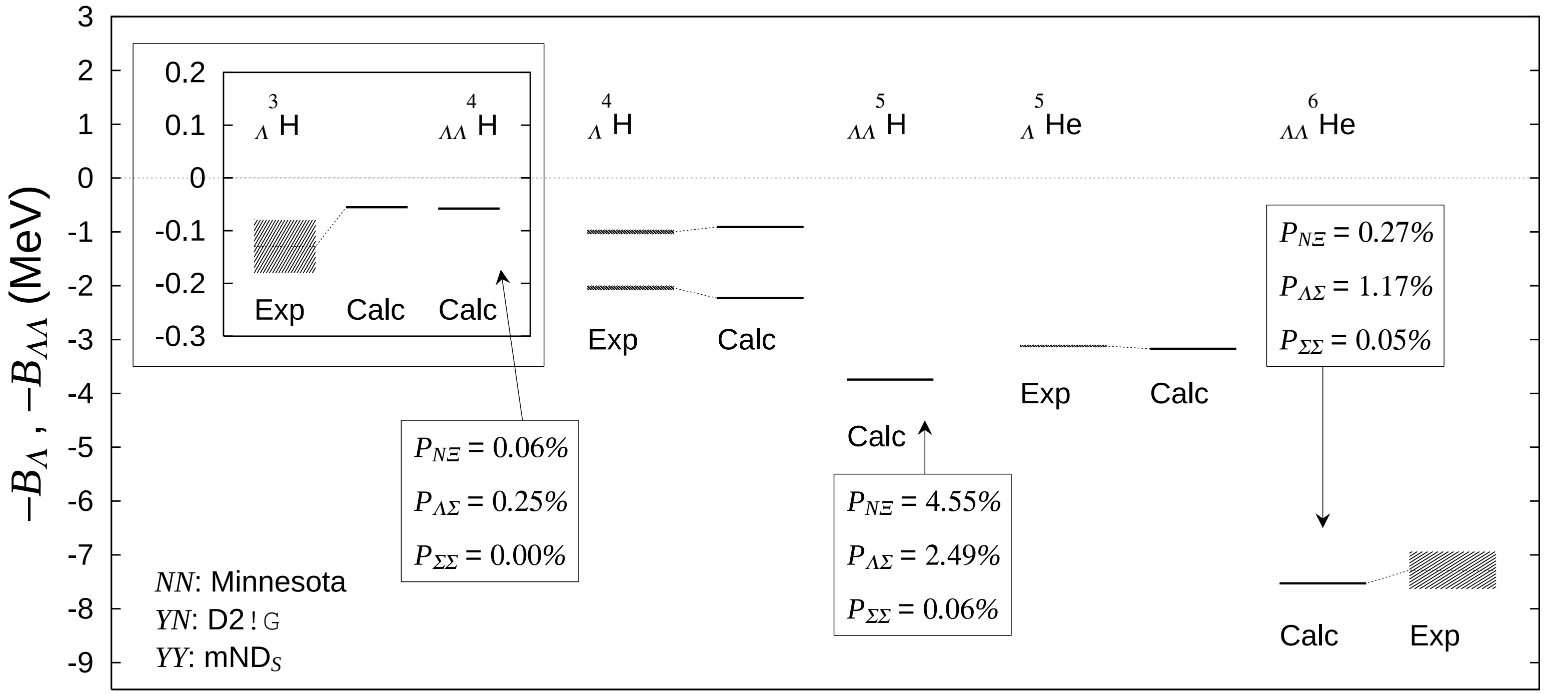



I.N. Filikhin, A. Gal, Nucl. Phys. A **707** (2002) 491

*s*-wave Faddeev calculations of $\Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{6}\mathrm{He})$ vs. $\Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{5}\mathrm{H}, {}_{\Lambda\Lambda}^{5}\mathrm{He})$

$\Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{6}\mathrm{He}) \equiv \mathrm{B}_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{6}\mathrm{He}) - 2\mathrm{B}_{\Lambda}({}_{\Lambda}^{5}\mathrm{He})$

$\Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{6}\mathrm{He}) \approx 1$ MeV implies that ${}_{\Lambda\Lambda}^{5}\mathrm{H}$ & ${}_{\Lambda\Lambda}^{5}\mathrm{He}$ are also bound



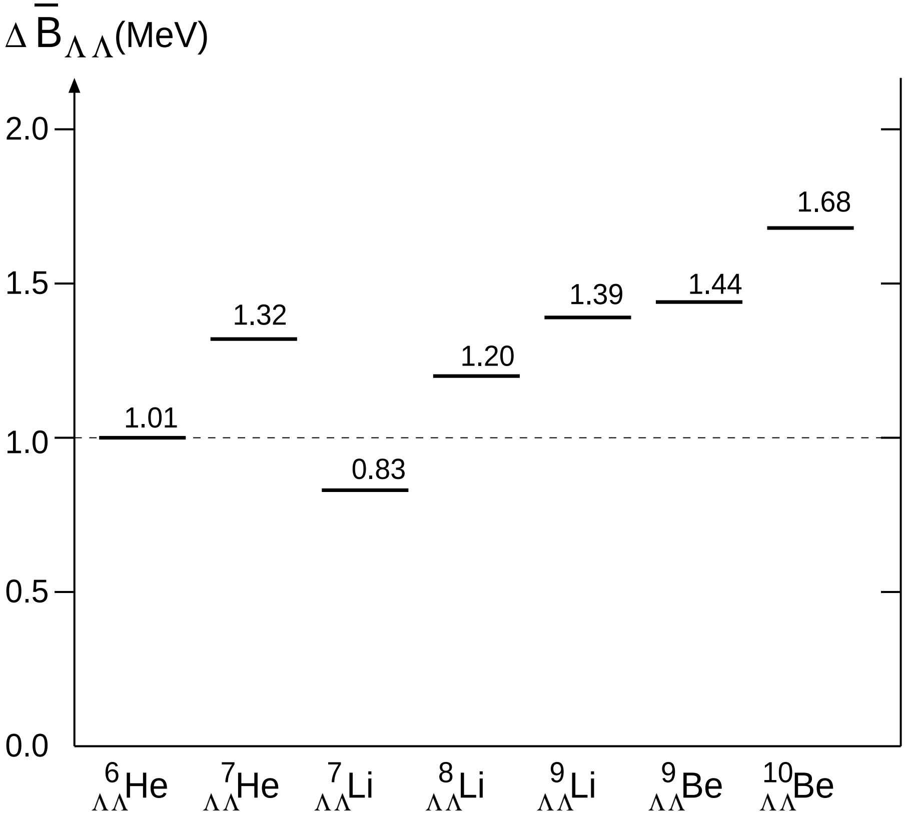

H. Nemura, S. Shinmura, Y. Akaishi, K.S. Myint, PRL **94** (2005) 202502

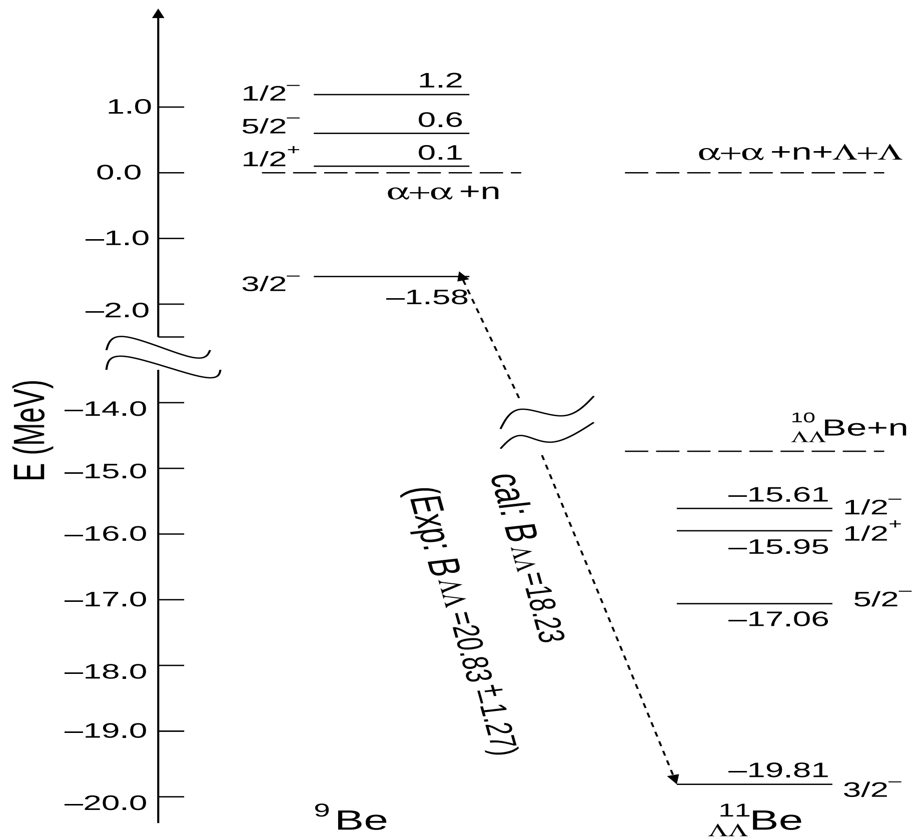
Calculated $\Lambda$ & $\Lambda\Lambda$ separation energies of $s$-shell hypernuclei

$\Lambda N - \Sigma N$ and $\Lambda\Lambda - \Xi N$ mixings are important

$^{6}_{\Lambda\Lambda}$He: the only uniquely determined $\Lambda\Lambda$ hypernucleus

$^{4}_{\Lambda\Lambda}$H unlikely bound [Filikhin-Gal, PRL 89 (2002) 172502]




Hiyama et al. NPA **754** (2005) 103c (left) PRL **104** (2010) 212502 (right)

ΛΛ binding energies in 3-, 4- & 5-body cluster models

Have cluster models reached their physics & computational limits?

Species accepted so far beyond $^{6}_{\Lambda\Lambda}$He: $^{10}_{\Lambda\Lambda}$Be & $^{13}_{\Lambda\Lambda}$B

# Ambiguities in identifying $\Lambda\Lambda$ emulsion events

- $\Lambda\Lambda$ hypernuclei often formed in, or decay to, excited states. Unseen $\gamma$ energy should be added or subtracted accordingly.

- Demachiyanagi (KEK-E373): $\Xi^- + {}^{12}\mathrm{C} \rightarrow {}^{10}_{\Lambda\Lambda}\mathrm{Be}^*(2^+) + t$.
  From production: $B_{\Lambda\Lambda} = 11.90 \pm 0.13 \rightarrow 14.94 \pm 0.13$ MeV.

- Danysz et al. (1963): ${}^{10}_{\Lambda\Lambda}\mathrm{Be} \rightarrow {}^{9}_{\Lambda}\mathrm{Be}^*(5/2^+, 3/2^+) + p + \pi^-$.
  From decay: $B_{\Lambda\Lambda} = 17.7 \pm 0.4 \rightarrow 14.7 \pm 0.4$ MeV.

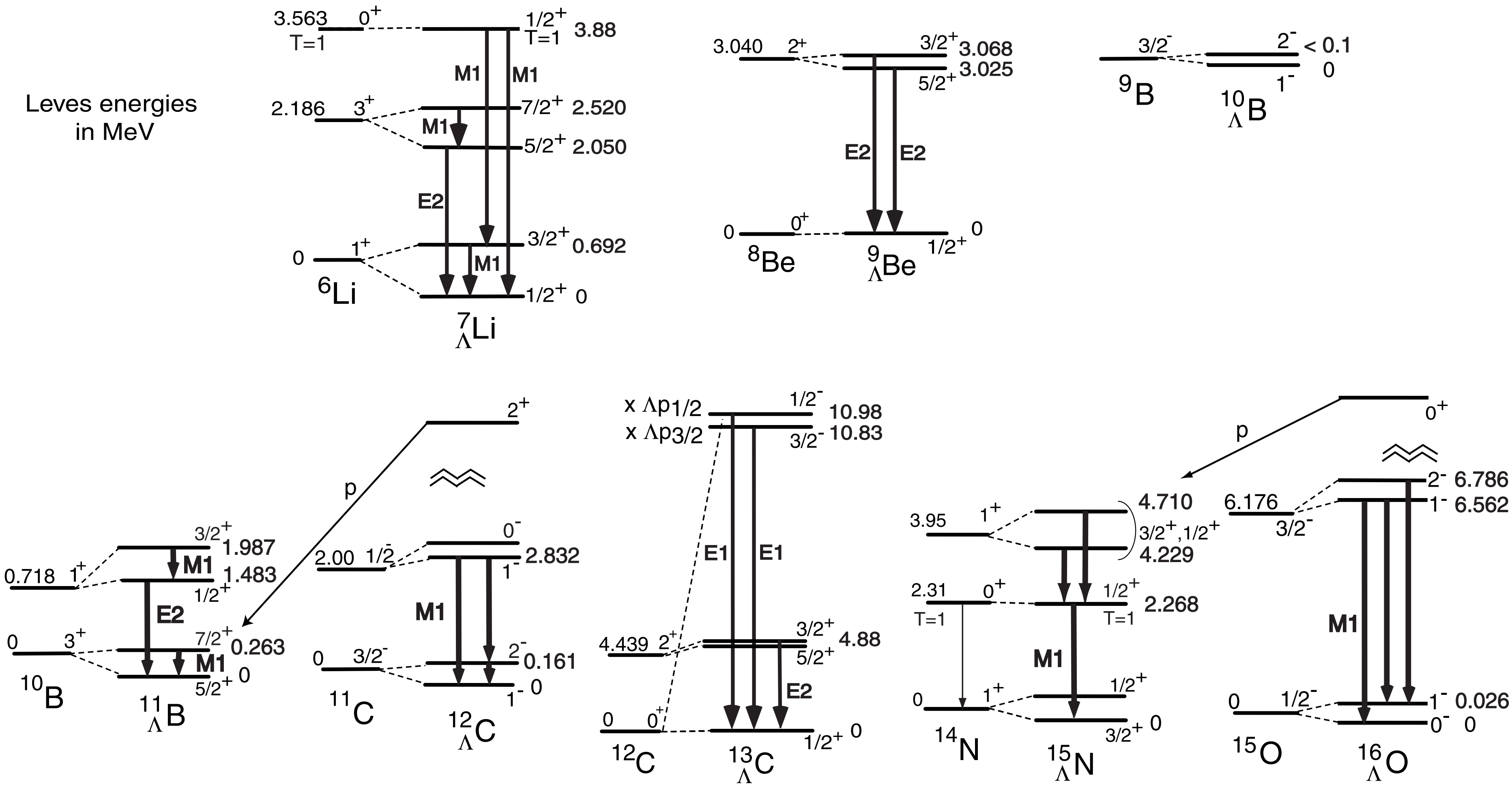
- KEK-E176: $\Xi^- + {}^{14}\mathrm{N} \rightarrow {}^{13}_{\Lambda\Lambda}\mathrm{B} + p + n$, followed by ${}^{13}_{\Lambda\Lambda}\mathrm{B} \rightarrow {}^{13}_{\Lambda}\mathrm{C}^*(5/2^+, 3/2^+) + \pi^-$, with $E_{\mathrm{x}} \approx 4.8$ MeV.
  Both production & decay consistent with $B_{\Lambda\Lambda} = 23.4 \pm 0.7$ MeV.
  Other, less likely interpretations are not ruled out,
  S. Aoki et al., NPA 828 (2009) 191.

# Shell-model approach

- Account for the $\Lambda$-nuclear interactions of each $\Lambda$ using $\overline{B}_\Lambda({}^{\mathrm{A}-1}_{\Lambda}\mathrm{Z})$, the $(2J+1)$-averaged $B_\Lambda$ in the ${}^{\mathrm{A}-1}_{\Lambda}\mathrm{Z}$ g.s. doublet, as appropriate to a spin-zero configuration $(1s_\Lambda)^2$ in ${}^{\mathrm{A}}_{\Lambda\Lambda}\mathrm{Z}$.
- Identify $< V_{\Lambda\Lambda} >_{\mathrm{SM}}$ with
  $B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) - 2B_\Lambda({}^{5}_{\Lambda}\mathrm{He}) = 0.67 \pm 0.17$ MeV.
  Note $< V_{\Lambda\Lambda} >_{\mathrm{CM}} = B_{\Lambda\Lambda}(V_{\Lambda\Lambda} \neq 0) - B_{\Lambda\Lambda}(V_{\Lambda\Lambda} = 0)$,
  $\Rightarrow 0.54\ ({}^{6}_{\Lambda\Lambda}\mathrm{He}),\ 0.53\ ({}^{10}_{\Lambda\Lambda}\mathrm{Be}),\ 0.56\ ({}^{11}_{\Lambda\Lambda}\mathrm{Be})$ MeV.
- Use $B^{\mathrm{SM}}_{\Lambda\Lambda}({}^{\mathrm{A}}_{\Lambda\Lambda}\mathrm{Z}) = 2\overline{B}_\Lambda({}^{\mathrm{A}-1}_{\Lambda}\mathrm{Z}) + < V_{\Lambda\Lambda} >_{\mathrm{SM}}$.
- Input $\overline{B}_\Lambda({}^{\mathrm{A}-1}_{\Lambda}\mathrm{Z})$ requires spin dependence analysis.
- Modify where nuclear core is unbound (${}^{8}\mathrm{Be}$).

# Spin dependence in $\Lambda$ hypernuclei:

# shell-model approach




# Level schemes of $\Lambda$ hypernuclei from recent $\gamma$-ray measurements

H. Tamura et al., Nucl. Phys. A 835 (2010) 3 [HYP-X]

# p-shell $\Lambda$ hypernuclei

$$V_{\Lambda N} = V_0(r) + V_\sigma(r)\, s_N \cdot s_\Lambda + V_{LS}(r)\, l_{N\Lambda} \cdot (s_\Lambda + s_N) + V_{ALS}(r)\, l_{N\Lambda} \cdot (s_\Lambda - s_N) + V_T(r)\, S_{12}$$

For $p_N s_Y$ : $\quad V_{\Lambda N} = \bar{V} + \Delta\, s_N \cdot s_\Lambda + S_\Lambda\, l_N \cdot s_\Lambda + S_N\, l_N \cdot s_N + T\, S_{12}$

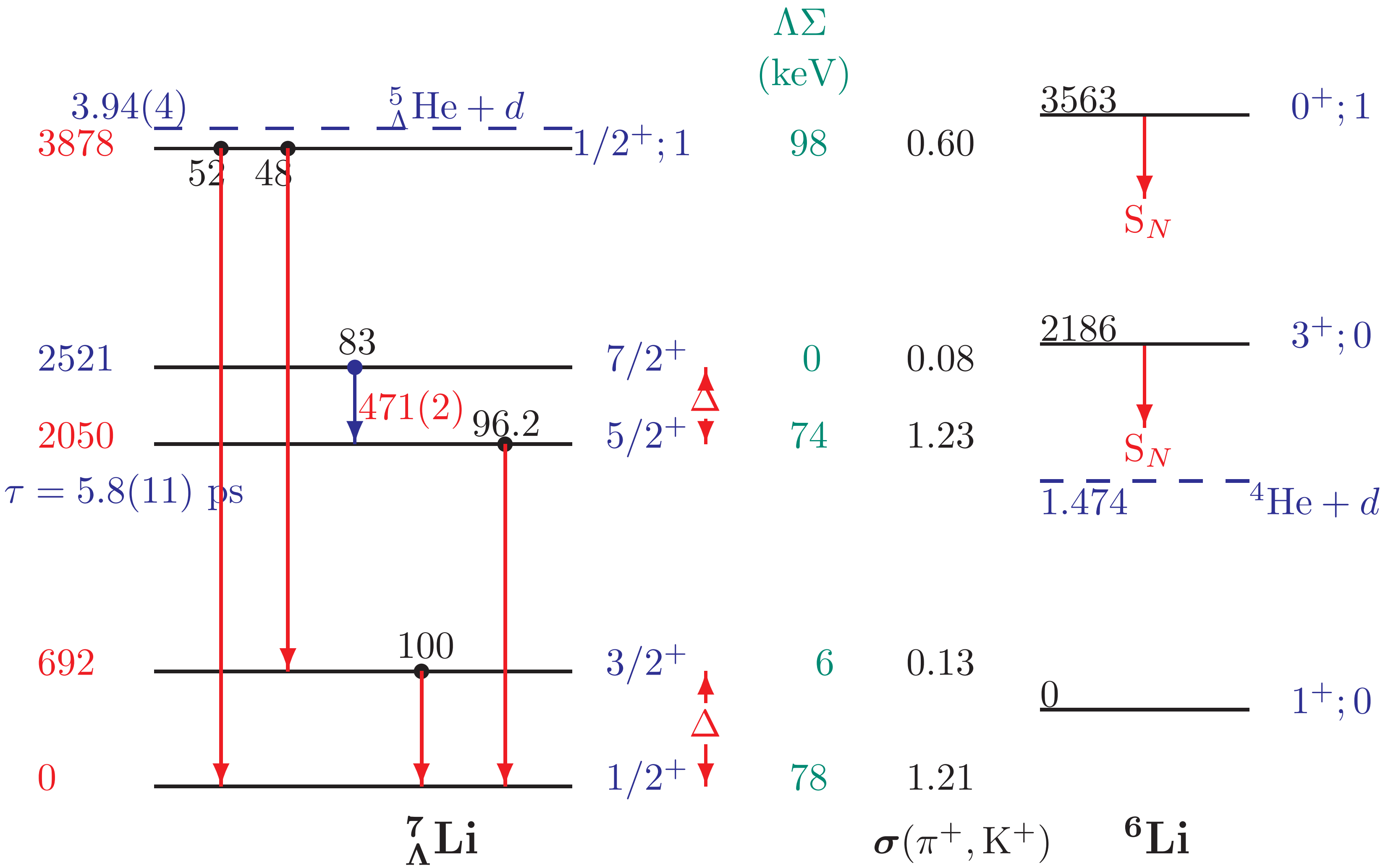
R.H Dalitz, A. Gal, Ann. Phys. 116 (1978) 167

D.J. Millener, A. Gal, C.B. Dover, R.H. Dalitz, PRC 31 (1985) 499

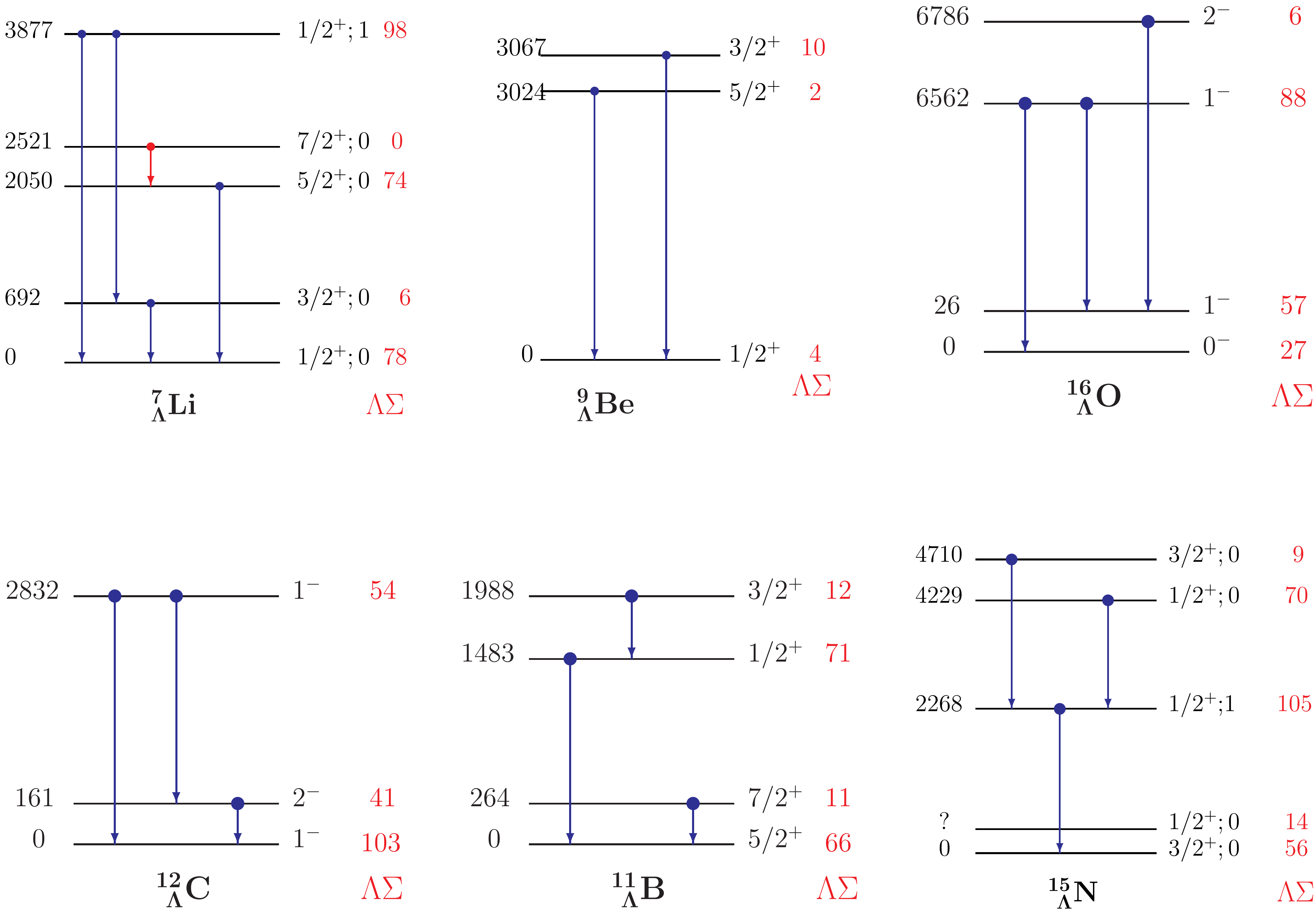
| (MeV) | $\bar{V}$ | $\Delta$ | $S_\Lambda$ | $S_N$ | $T$ |
|---|---|---|---|---|---|
| $N\Lambda$-$N\Lambda$ $\quad A = 7-$ ? | (-1.32) | 0.430 | $-0.015$ | $-0.390$ | 0.030 |
| $A = 11 - 16$ | (-1.32) | 0.330 | $-0.015$ | $-0.350$ | 0.024 |
| $N\Lambda$-$N\Sigma$ | 1.45 | 3.04 | $-0.085$ | $-0.085$ | 0.157 |

D.J. Millener, Nucl. Phys. A 804 (2008) 84

# $^{7}_{\Lambda}$Li $\gamma$ rays – Hyperball, KEK E419 and BNL E930




H. Tamura et al., PRL 84 (2000) 5963 K. Tanida et al., PRL 86 (2001) 1982


3877
2521
2050
692
0
1/2+; 1
7/2+; 0
5/2+; 0
3/2+; 0
1/2+; 0
98
0
74
6
78
7ΛLi
ΛΣ
3067
3024
0
3/2+
5/2+
1/2+
10
2
4
ΛΣ
9ΛBe
6786
6562
26
0
2−
1−
1−
0−
6
88
57
27
16ΛO
ΛΣ
2832
161
0
1−
2−
1−
54
41
103
12ΛC
ΛΣ
1988
1483
264
0
3/2+
1/2+
7/2+
5/2+
12
71
11
66
11ΛB
ΛΣ
4710
4229
2268
?
0
3/2+; 0
1/2+; 0
1/2+;1
1/2+; 0
3/2+; 0
9
70
105
14
56
15ΛN
ΛΣ

# Doublet spacings in $p$-shell hypernuclei (in keV)

| | $J_u^\pi$ | $J_l^\pi$ | $\Lambda\Sigma$ | $\Delta$ | $S_\Lambda$ | $S_N$ | $T$ | $\Delta E^{\rm th}$ | $\Delta E^{\rm exp}$ |
|---|---|---|---|---|---|---|---|---|---|
| ${}^{7}_{\Lambda}$Li | $3/2^+$ | $1/2^+$ | 72 | 628 | $-1$ | $-4$ | $-9$ | 693 | 692 |
| ${}^{7}_{\Lambda}$Li | $7/2^+$ | $5/2^+$ | 74 | 557 | $-32$ | $-8$ | $-71$ | 494 | 471 |
| ${}^{8}_{\Lambda}$Li | $2^-$ | $1^-$ | 151 | 396 | $-14$ | $-16$ | $-24$ | 450 | (442) |
| ${}^{9}_{\Lambda}$Be | $3/2^+$ | $5/2^+$ | $-8$ | $-14$ | 37 | 0 | 28 | 44 | 43 |
| ${}^{11}_{\Lambda}$B | $7/2^+$ | $5/2^+$ | 56 | 339 | $-37$ | $-10$ | $-80$ | 267 | 264 |
| ${}^{11}_{\Lambda}$B | $3/2^+$ | $1/2^+$ | 61 | 424 | $-3$ | $-44$ | $-10$ | 475 | 505 |
| ${}^{12}_{\Lambda}$C | $2^-$ | $1^-$ | 61 | 175 | $-22$ | $-13$ | $-42$ | 153 | 161 |
| ${}^{15}_{\Lambda}$N | $3/2_2^+$ | $1/2_2^+$ | 65 | 451 | $-2$ | $-16$ | $-10$ | 507 | 481 |
| ${}^{16}_{\Lambda}$O | $1^-$ | $0^-$ | $-33$ | $-123$ | $-20$ | 1 | 188 | 23 | 26 |
| ${}^{16}_{\Lambda}$O | $2^-$ | $1_2^-$ | 92 | 207 | $-21$ | 1 | $-41$ | 248 | 224 |

# $\Lambda N$ interaction matrix elements in Nijmegen $BB$ models

- G-Matrix elements from $N\Lambda$-$N\Sigma$ calculation fitted with sums of Gaussians, Yukawas, OBEP forms, ...
- $p_N s_\Lambda$ matrix elements (MeV) calculated using Woods-Saxon wave functions.

| | | $p$-shell | | | | | $s$-shell | |
|---|---|---|---|---|---|---|---|---|
| | | $\overline{V}$ | $\Delta$ | $S_\Lambda$ | $S_N$ | $T$ | $\overline{V}_s$ | $\Delta_s$ |
| fit-DJM | ${}^7_\Lambda$Li | −1.142 | 0.438 | −0.008 | −0.414 | 0.031 | −1.387 | 0.497 |
| | ${}^{16}_{\Lambda}$O | −1.161 | 0.441 | −0.007 | −0.401 | 0.030 | | |
| NSC97f | ${}^7_\Lambda$Li | −1.086 | 0.421 | −0.149 | −0.238 | 0.055 | −1.725 | 0.775 |
| ESC04a | ${}^7_\Lambda$Li | −1.287 | 0.381 | −0.108 | −0.236 | 0.013 | −1.577 | 0.850 |
| ESC08a | ${}^7_\Lambda$Li | −1.221 | 0.146 | −0.074 | −0.241 | 0.055 | −1.796 | 0.650 |

- Fitted matrix elements are roughly constant with A - same YNG interaction, WS wells have $R=r_0A^{1/3}$, but rms radii of $p$-shell nuclei are roughly constant.

$p_N s_\Lambda$ $\Lambda$-$\Sigma$ coupling parameters from Nijmegen baryon-baryon potentials.

| Source | Interaction | $\bar{V}'$ | $\Delta'$ | $S'_\Lambda$ | $S'_N$ | $T'$ |
|---|---|---|---|---|---|---|
| Akaishi (s-shell) | NSC97e/f | 1.45 | 3.04 | −0.09 | −0.09 | 0.16 |
| Yamamoto | NSC97f | 0.96 | 3.62 | −0.07 | −0.07 | 0.31 |
| Halderson | NSC97e | 0.75 | 3.51 | −0.45 | −0.24 | 0.31 |
| Halderson | NSC97f | 1.10 | 3.73 | −0.45 | −0.23 | 0.30 |
| Halderson ∗ | ESC04a | −2.30 | −2.59 | −0.17 | −0.17 | 0.23 |
| Halderson | ESC08a | 1.05 | 4.71 | −0.07 | 0.02 | 0.32 |

∗ D. Halderson, Phys. Rev. C 77, 034304 (2008).

- ${}^4_\Lambda$H/${}^4_\Lambda$He $0^+$ $\bar{V}'_s + 3/4\ \Delta'_s$
- ${}^4_\Lambda$H/${}^4_\Lambda$He $1^+$ $\bar{V}'_s - 1/4\ \Delta'_s$
- Effective central interaction from second-order tensor in ESC04 has a peculiar radial behavior (see Halderson)

# $\Lambda$-$\Sigma$ & spin-dependent contributions to $B_\Lambda$(g.s.)

| emul. (keV) | ${}^{8}_{\Lambda}$Li $1^-$ | ${}^{9}_{\Lambda}$(Li-B) $3/2^+$ | ${}^{9}_{\Lambda}$Be $1/2^+$ | ${}^{10}_{\Lambda}$(Be-B) $1^-$ | ${}^{11}_{\Lambda}$B $5/2^+$ | ${}^{11}_{\Lambda}$Be $1/2^+$ | ${}^{12}_{\Lambda}$B $1^-$ |
|---|---|---|---|---|---|---|---|
| $\Lambda$-$\Sigma$ | 160 | 183 | 4 | 35 | 66 | 99 | 103 |
| $\Delta$ | 288 | 350 | 0 | 125 | 203 | 2 | 108 |
| $S_\Lambda$ | $-6$ | $-10$ | 0 | $-13$ | $-20$ | 0 | $-14$ |
| $S_N$ | 192 | 434 | 207 | 386 | 652 | 540 | 704 |
| $T$ | $-9$ | $-6$ | 0 | $-15$ | $-43$ | 0 | $-29$ |
| sum | 625 | 952 | 211 | 518 | 858 | 641 | 869 |
| $B_\Lambda^{\mathrm{exp}}$(g.s.) | 6.80 | 8.44 | 6.71 | 8.94 | 10.24 | | 11.37 |
| (MeV) | 0.03 | 0.10 | 0.04 | 0.11 | 0.05 | | 0.06 |
| $\overline{V}$ | $-1.02$ | $-1.09$ | $-0.84$ | $-1.06$ | $-1.04$ | | $-1.05$ |

$$B_\Lambda^{\mathrm{exp}}(\mathrm{g.s.}) = [B_\Lambda^{\mathrm{exp}}({}^{5}_{\Lambda}\mathrm{He}) = 3.12 \pm 0.02\ \mathrm{MeV}] - (A-5)\overline{V} + \text{‘sum’}$$

# Consistency of $\Lambda\Lambda$ events

## $B_{\Lambda\Lambda}$: theory vs. KEK experiments

| event | ${}^{A}_{\Lambda\Lambda}$Z | $B^{\rm exp}_{\Lambda\Lambda}$ | $B^{\rm CM}_{\Lambda\Lambda}$ | $B^{\rm SM}_{\Lambda\Lambda}$ |
|---|---|---|---|---|
| E373-Nagara | ${}^{6}_{\Lambda\Lambda}$He | $6.91 \pm 0.16$ | $6.91 \pm 0.16$ | $6.91 \pm 0.16$ |
| E373-DemYan | ${}^{10}_{\Lambda\Lambda}$Be | $14.94 \pm 0.13$ | $14.74 \pm 0.16$ | $14.97 \pm 0.22$ † |
| E176-G2 | ${}^{11}_{\Lambda\Lambda}$Be | $17.53 \pm 0.71$ | $18.23 \pm 0.16$ | $18.40 \pm 0.28$ |
| E373-Hida | ${}^{11}_{\Lambda\Lambda}$Be | $20.83 \pm 1.27$ | $18.23 \pm 0.16$ | $18.40 \pm 0.28$ |
| E373-Hida | ${}^{12}_{\Lambda\Lambda}$Be | $22.48 \pm 1.21$ | – | $20.72 \pm 0.20$ |
| E176-E2 | ${}^{12}_{\Lambda\Lambda}$B | $20.02 \pm 0.78$ | – | $20.85 \pm 0.20$ |
| E176-E4 | ${}^{13}_{\Lambda\Lambda}$B | $23.4 \pm 0.7$ | – | $23.21 \pm 0.21$ |

† $B^{\rm SM}_{\Lambda\Lambda}({}^{10}_{\Lambda\Lambda}{\rm Be}) = 2\,\overline{B}_{\Lambda}({}^{9}_{\Lambda}{\rm Be}) + 4\,[\overline{V}({}^{9}_{\Lambda}{\rm Be}) - \overline{V}_{\rm average}] + < V_{\Lambda\Lambda} >_{\rm SM}$.

- All E176 entries refer to a single event, see NPA 828 (2009) 191.
- Hida-event [K. Nakazawa, H. Takahashi, PTPS 185 (2010) 335] interpretations are dubious.
- $B^{\rm SM}_{\Lambda\Lambda} \approx B^{\rm CM}_{\Lambda\Lambda}$, but SM spans a wider $A$ range.

# Conclusions

- Shell model works well for light $\Lambda\Lambda$ hypernuclei with accuracy bounded by 0.3 MeV, including 0.16 MeV uncertainty from ${}_{\Lambda\Lambda}^{6}$He.
- No sound SM or CM interpretation for Hida event.
- Need more data for systematics and for studying possible continuum effects from $H$ dibaryon.
- Next exp. phase @J-PARC (E07) with $K^-$ beam, and @FAIR (PANDA) with antiprotons.