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arXiv:2103.10706

New effective interactions for hypernuclei in density dependent relativistic mean field model

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Date: 2021-04-15

Outline

- Motivation
- Density dependent relativistic mean-field model for hypernuclei & effective ΛN interaction
- Results and discussions
- Summary

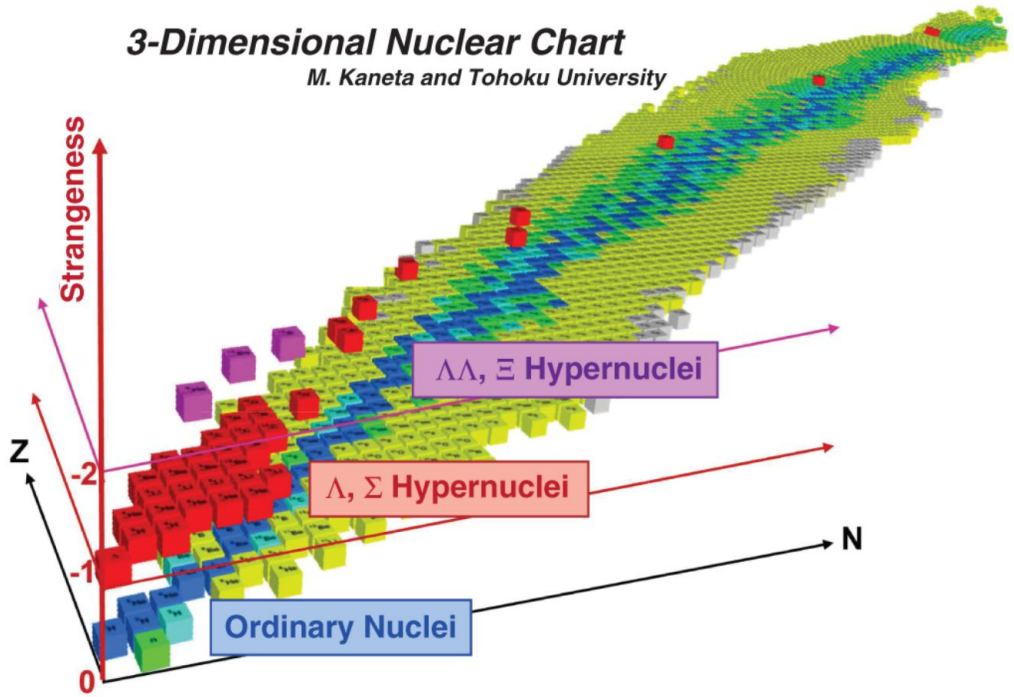
Motivation

The known hypernuclei:

- S=−1: About 40 single Λ hypernuclei (about 20 of them with mass number $A \geq 12$)
- S=−2: A few Ξ hypernuclei and double Λ hypernuclei
- S=+1: Antihypertriton

(Bare or effective) YN and YY interation:

- Lattice QCD
- Chiral effective field theory
- Nijmegen soft-core model
- Juelich hyperon-nucleon model
- Skyrme Hartree Fock model
- Relativistic mean-field model



H. Tamura. Prog. Theor. Exp. Phys. 2012 (2012) 02B012

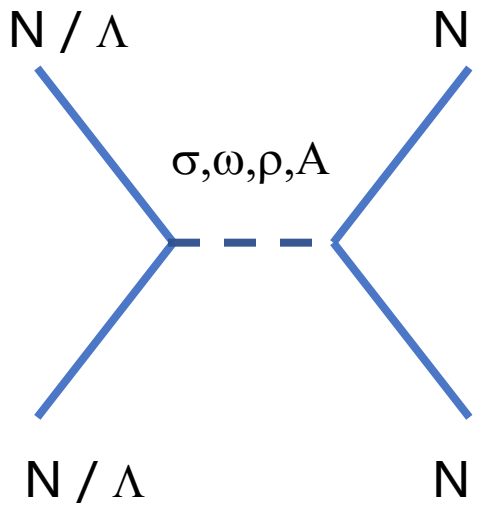
Motivation

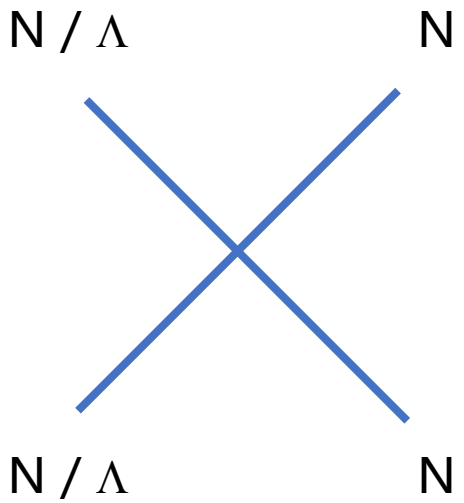
4 classes of effective interactions for RMF model :

Model	NN	ΛN
NL-ME	TM1,TM2,NL1,NL2,NL3,NLSH,PK1,NL-Z,...	NLSH-A, PK1-Y1, et al.
DD-ME	TW99, DD-ME1, DD-ME2, PKDD, DD2, DDME-X, DD-LZ1,...	DD-ME2-a, DD-ME2D-a
NL-PC	PC-PK1, PC-F1, PC-X...	PCY-Si (i=1,2,3,4)
DD-PC	DD-PC1	×

NL: nonlinear coupling
DD: density dependent

ME: meson exchange
PC: point coupling





Motivation

Effective density dependent interaction in MF approach to study hypernuclei and hypernuclear matter

DD hadron field theory:

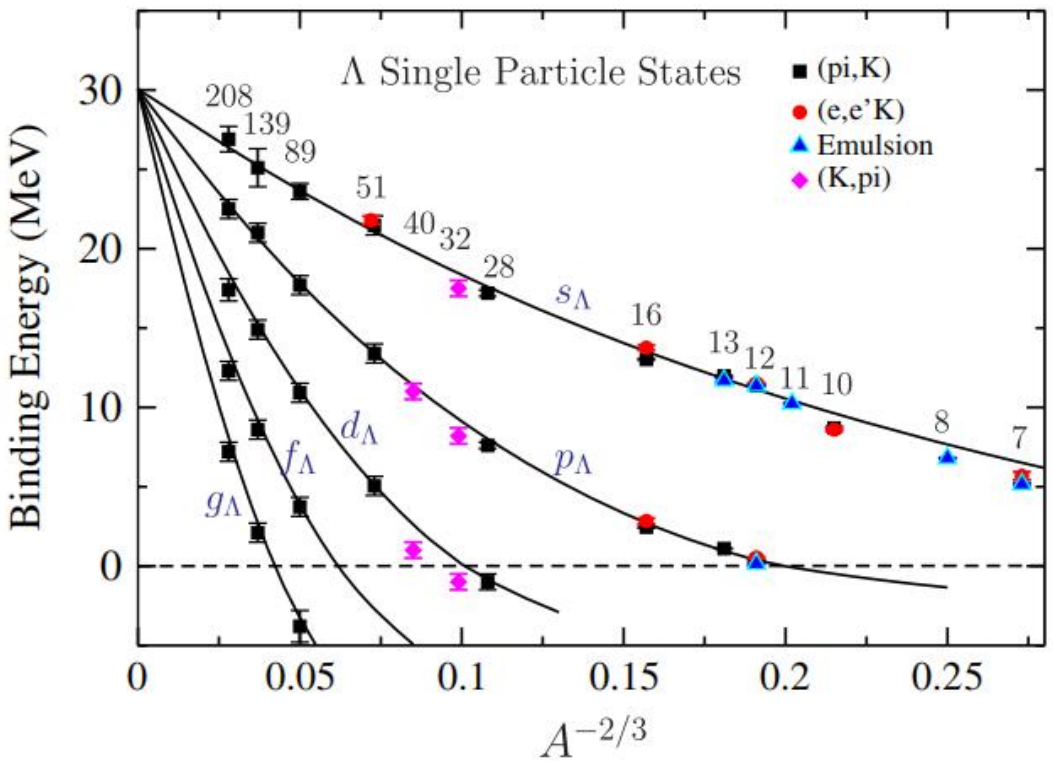
- C.M. Keil, F. Hofmann and H. Lenske. Phys. Rev. C 61 (2000) 064309 (Λ density)
- F. Hofmann, C.M. Keil, H. Lenske. Phys. Rev. C 64 (2001) 025804 (Λ density / total density)

RMF model:

- P. Finelli, N. Kaiser, D. Vretenar and W. Weise. Nucl. Phys. A 831 (2009) 163 (RMF+chiral perturbation theory; total density, B_Λ)
- G. Colucci and A. Sedrakian. Phys. Rev. C 87 (2013) 055806 (total denity, DD-ME2+potential)
- S. Banik, M. Hempel and D. Bandyopadhyay. Astrophys. J. Suppl. Ser. 214 (2014) 22 (total density; DD2+potential at saturation density)
- E.N.E. van Dalen, G. Colucci and A. Sedrakian. Phys. Lett. B 734 (2014) 383 (total density; DD-ME2+mass formula)
- M. Fortin, S.S. Avancini, C. Providencia and I. Vidana. Phys. Rev. C 95 (2017) 065803 (total density / a constant, DD-ME2+B_Λ, $R\omega$ is fixed)
- C. Providencia, M. Fortin, H. Pais and A. Rabhi, Frontiers Astron. Space Sci. 6 (2019) 13 (total density)

- $g_{\sigma\Lambda}$, $g_{\omega\Lambda}$ depend on: Λ density / total density / a constant
- Fitting procedure: (hyper)nuclei are assume to be spherical+ B_Λ from experiment or other prediction / Λ potential

Motivation



A. Gal, E.V. Hungerford and D.J. Millener, Rev. Mod. Phys. 88 (2016) 035004.

Open shell nuclear core
deformation
(Multidimensionally constrained RMF model)

Small spin-orbit splitting
tensor coupling between Λ and ω

In-medium effect
total density dependent coupling constant

Unpaired baryon
equal filling approximation

DDRMF model

Effective Lagrangian:

$$\mathcal{L}_B = \sum_{B=n,p,\Lambda} \bar{\psi}_B \Big(i\gamma_\mu \partial^\mu - M_B - g_{\sigma B}\sigma - g_{\sigma^* B}\sigma^* - g_{\omega B}\gamma_\mu \omega^\mu - g_{\phi B}\gamma_\mu \phi^\mu$$
$$- g_{\rho B}\gamma_\mu \vec{\tau}\cdot\vec{\rho}^\mu - e\gamma_\mu \frac{1-\tau_3}{2} A^\mu + \frac{f_{\omega\Lambda\Lambda}}{4M_\Lambda}\sigma_{\mu\nu}\Omega^{\mu\nu}\Big)\psi_B + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2$$
$$- \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu - \frac{1}{4}\vec{R}^{\mu\nu}\vec{R}_{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^\mu\vec{\rho}_\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$
$$+ \frac{1}{2}\partial_\mu\sigma^*\partial^\mu\sigma^* - \frac{1}{2}m_{\sigma^*}\sigma^{*2} - \frac{1}{4}S^{\mu\nu}S_{\mu\nu} + \frac{1}{2}m_\phi^2\phi^\mu\phi_\mu,$$

Formation for density-dependent coupling constants:

$$g_{mB}(\rho^v) = g_{mB}(\rho_{\rm sat}) f_{mB}(x), \quad x = \rho^v/\rho_{\rm sat},$$

$$f_{mN}(x) = \begin{cases} a_m \dfrac{1 + b_m (x + d_m)^2}{1 + c_m (x + d_m)^2}, & m = \sigma \text{ or } \omega, \\ e^{-a_\rho (x-1)}, & m = \rho, \end{cases}$$

S. Typel and H.H. Wolter. Nucl. Phys. A 656 (1999) 331

Relation between NN and ΛN

$$g_{\sigma\Lambda}(\rho) = R_\sigma g_{\sigma N}(\rho),$$
$$g_{\omega\Lambda}(\rho) = R_\omega g_{\omega N}(\rho),$$
$$g_{\sigma^*\Lambda}(\rho) = R_{\sigma^*} g_{\sigma N}(\rho),$$
$$g_{\phi\Lambda}(\rho) = R_\phi g_{\omega N}(\rho),$$
$$f_{\omega\Lambda\Lambda}(\rho) = R_{\omega\Lambda\Lambda} R_\omega g_{\omega N}(\rho).$$

DDRMF model

Effective Lagrangian:

$$\mathcal{L}_B = \sum_{B=n,p,\Lambda} \bar{\psi}_B \Big(i\gamma_\mu \partial^\mu - M_B - g_{\sigma B}\sigma - g_{\sigma^* B}\sigma^* - g_{\omega B}\gamma_\mu \omega^\mu - g_{\phi B}\gamma_\mu \phi^\mu$$
$$- g_{\rho B}\gamma_\mu \vec{\tau}\cdot\vec{\rho}^\mu - e\gamma_\mu \frac{1-\tau_3}{2} A^\mu + \frac{f_{\omega\Lambda\Lambda}}{4M_\Lambda}\sigma_{\mu\nu}\Omega^{\mu\nu}\Big)\psi_B + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2$$
$$- \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu - \frac{1}{4}\vec{R}^{\mu\nu}\vec{R}_{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^\mu\vec{\rho}_\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$
$$+ \frac{1}{2}\partial_\mu\sigma^*\partial^\mu\sigma^* - \frac{1}{2}m_{\sigma^*}\sigma^{*2} - \frac{1}{4}S^{\mu\nu}S_{\mu\nu} + \frac{1}{2}m_\phi^2\phi^\mu\phi_\mu,$$

Formation for density-dependent coupling constants:

$$g_{mB}(\rho^v) = g_{mB}(\rho_{\rm sat}) f_{mB}(x), \quad x = \rho^v/\rho_{\rm sat},$$

$$f_{mN}(x) = \begin{cases} a_m \dfrac{1 + b_m (x + d_m)^2}{1 + c_m (x + d_m)^2}, & m = \sigma \text{ or } \omega, \\ e^{-a_\rho (x-1)}, & m = \rho, \end{cases}$$

S. Typel and H.H. Wolter. Nucl. Phys. A 656 (1999) 331

Relation between NN and ΛN

$$g_{\sigma\Lambda}(\rho) = R_\sigma g_{\sigma N}(\rho),$$
$$g_{\omega\Lambda}(\rho) = \underset{0}{R_\omega} g_{\omega N}(\rho),$$
$$g_{\sigma^*\Lambda}(\rho) = \underset{0}{R_{\sigma^*}} g_{\sigma N}(\rho),$$
$$g_{\phi\Lambda}(\rho) = R_\phi g_{\omega N}(\rho),$$
$$f_{\omega\Lambda\Lambda}(\rho) = \underset{-1}{R_{\omega\Lambda\Lambda}} R_\omega g_{\omega N}(\rho).$$

Minimization of the least-square deviation

$$\chi^2(\boldsymbol{a}) = \sum_i^N \left(\frac{B_{\Lambda,i}^{\rm exp.} - B_{\Lambda,i}^{\rm cal.}(x_i;\boldsymbol{a})}{\Delta B_{\Lambda,i}^{\rm exp.}} \right)^2,$$

Results: Selections for fitting

NN interaction:
DD-ME2 (G. A. Lalazissis, T. Niksic, D. Vretenar et al. Phys. Rev. C 71 (2005) 024312)
PKDD (W. H. Long, J. Meng, N. Van Giai et al. Phys. Rev. C 69 (2004) 034319)

Experimental data for fitting:

NU	$B_{\Lambda,\mathrm{exp}}$
${}^{12}_{\Lambda}$C	11.36±0.20
${}^{13}_{\Lambda}$C	12.0±0.2
${}^{16}_{\Lambda}$O	13.0±0.2
${}^{28}_{\Lambda}$Si	17.2±0.2
${}^{32}_{\Lambda}$S	17.5±0.5
${}^{40}_{\Lambda}$Ca	18.7±1.1
${}^{51}_{\Lambda}$V	21.5±0.6
${}^{52}_{\Lambda}$V	21.8±0.3
${}^{89}_{\Lambda}$Y	23.6±0.5
${}^{139}_{\Lambda}$La	25.1±1.2
${}^{208}_{\Lambda}$Pb	26.9±0.8

A. Gal, E.V. Hungerford and D.J. Millener. Rev. Mod. Phys. 88 (2016) 035004.
P.H. Pile, S. Bart, et al. Phys. Rev. Lett. 66 (1991) 2585.

${}^{16}_{\Lambda}$N: B_Λ=13.76(16) MeV, CSB

${}^{12}_{\Lambda}$B: B_Λ=11.52(2) MeV, CSB

${}^{13,14}_{\Lambda}$C: only emulsion data

Root mean square (rms) deviation:

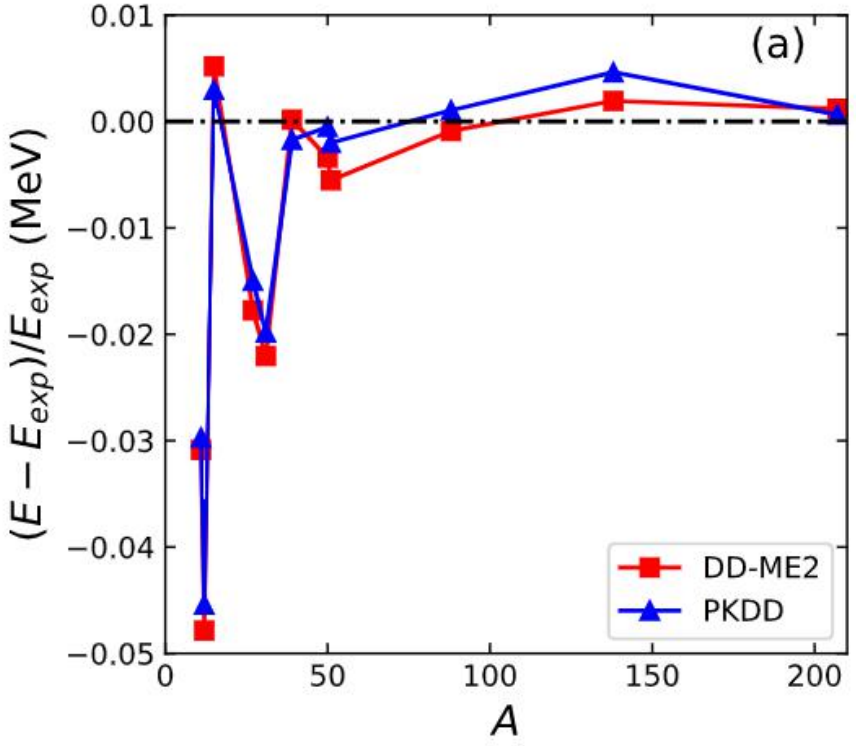
$$\Delta = \sqrt{\frac{1}{N}\sum_{i}^{N}\left(B_{\Lambda,i}^{\mathrm{exp.}} - B_{\Lambda,i}^{\mathrm{cal.}}\right)^2},$$

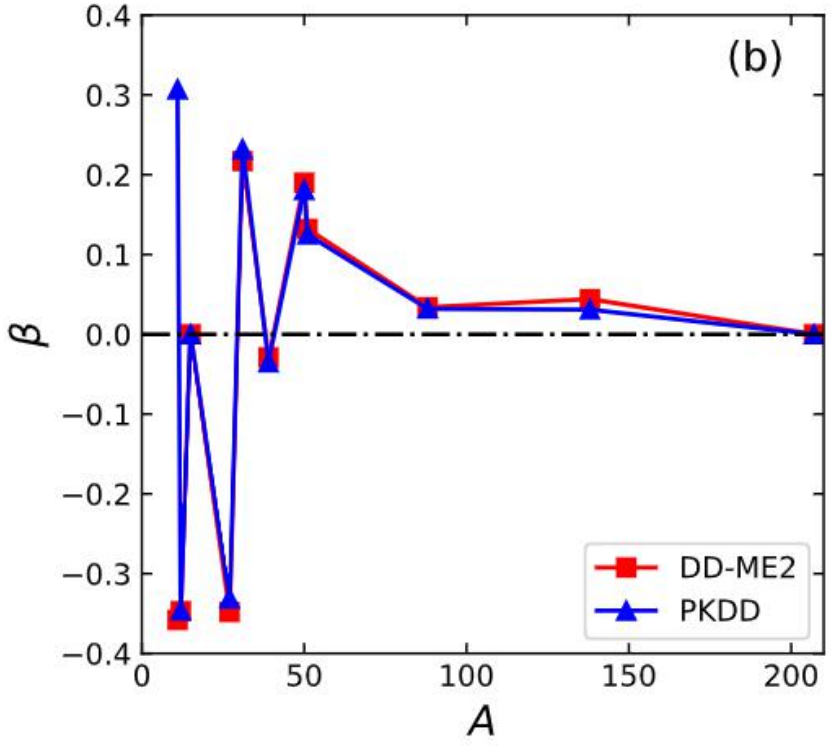
Root of relative square (rrs) deviation:

$$\delta = \sqrt{\frac{1}{N}\sum_{i}^{N}\left(\frac{B_{\Lambda,i}^{\mathrm{exp.}} - B_{\Lambda,i}^{\mathrm{cal.}}}{B_{\Lambda,i}^{\mathrm{exp.}}}\right)^2},$$

Results: Properties of normal core nuclei

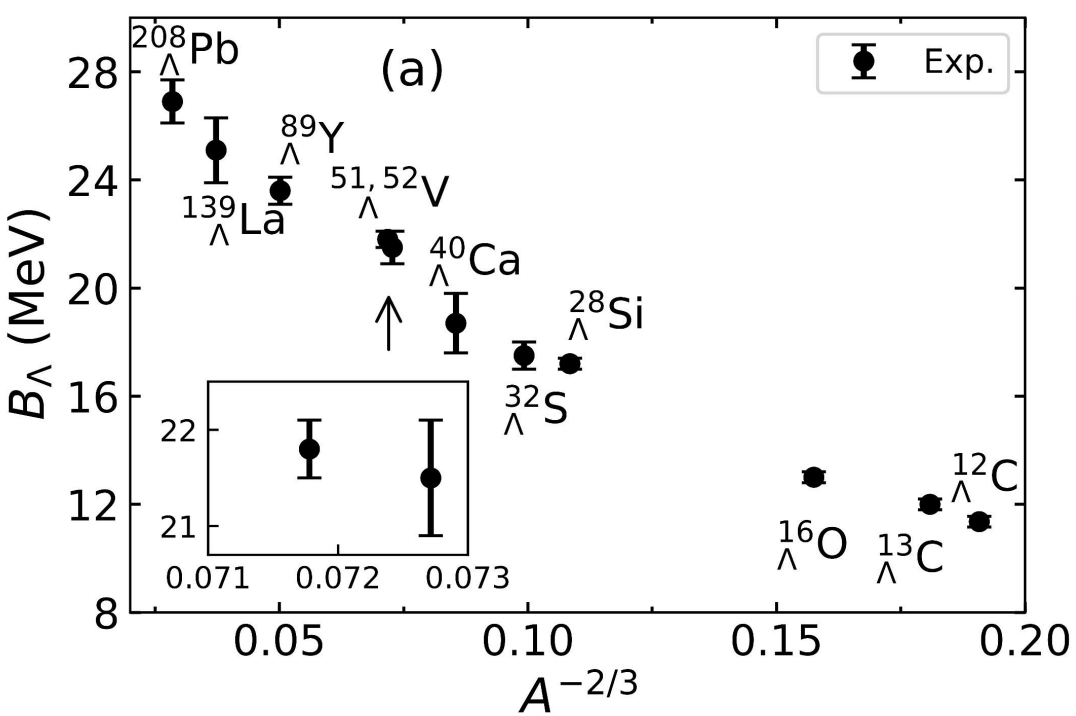
- Nuclear cores: ^{11}C, ^{12}C, ^{15}O, ^{27}Si, ^{31}S, ^{39}Ca, ^{50}V, ^{51}V, ^{88}Y, ^{138}La and ^{207}Pb
- Binding energies calculated with DD-ME2 and PKDD are close to AME2016 (relative deviations are within 5%)
- Axially deformed shape are obtained except ^{15}O and ^{207}Pb

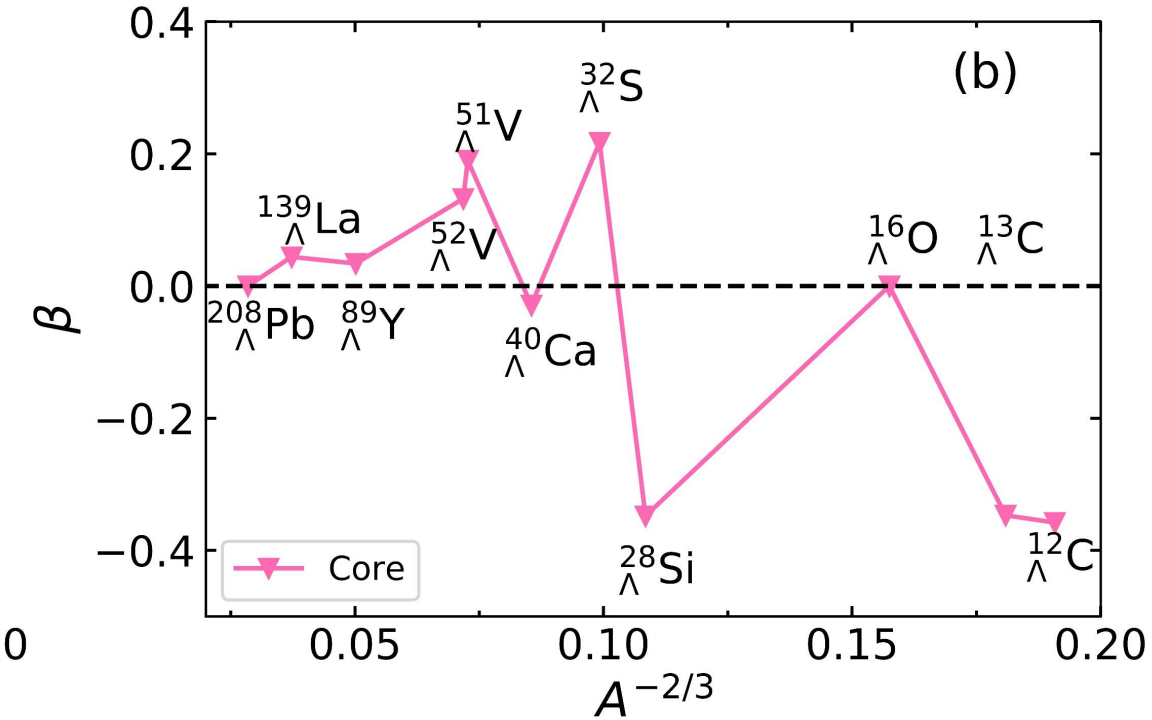




Results: Λ separation energies & deformation

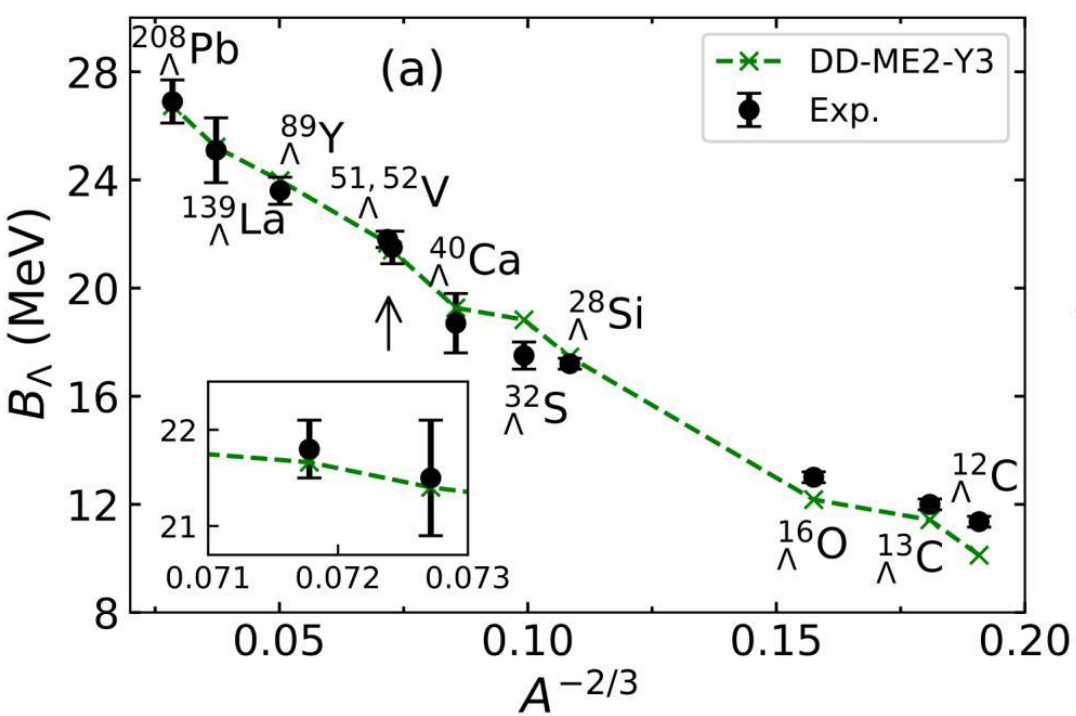
NN interaction: DD-ME2

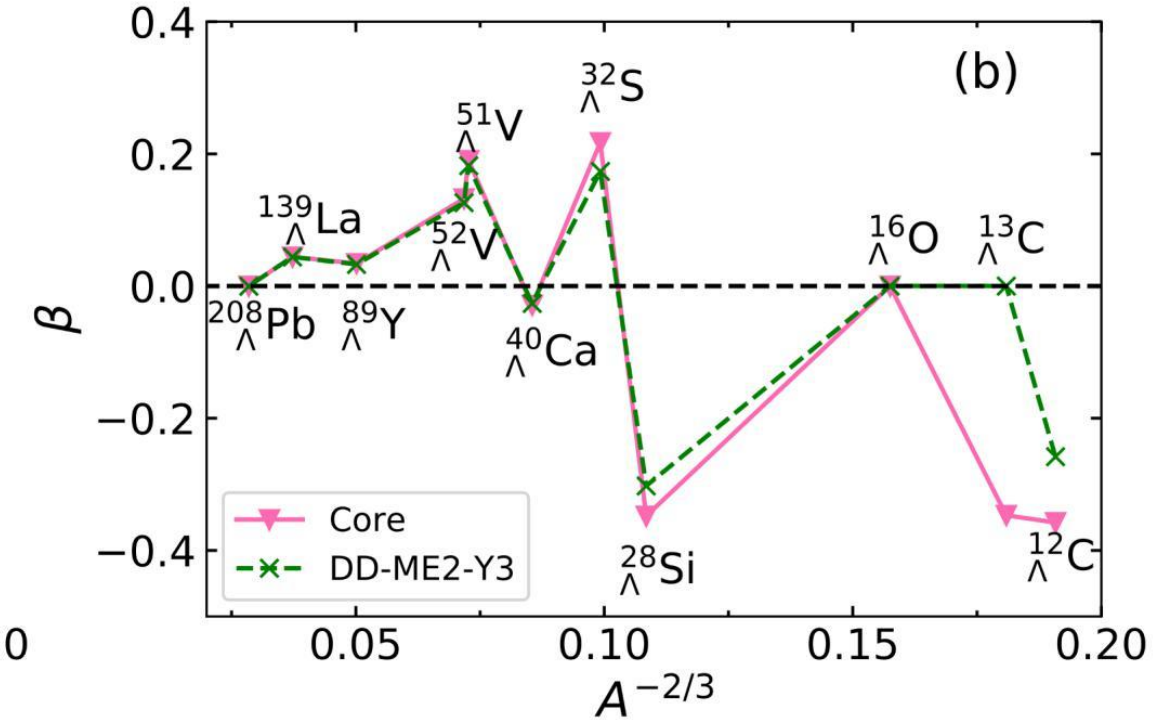




Results: Λ separation energies & deformation

NN interaction: DD-ME2
Reference hypernuclei: $^{208}_{\Lambda}$Pb, $^{139}_{\Lambda}$La, $^{89}_{\Lambda}$Y, $^{51,52}_{\Lambda}$V, $^{40}_{\Lambda}$Ca (Group 3)



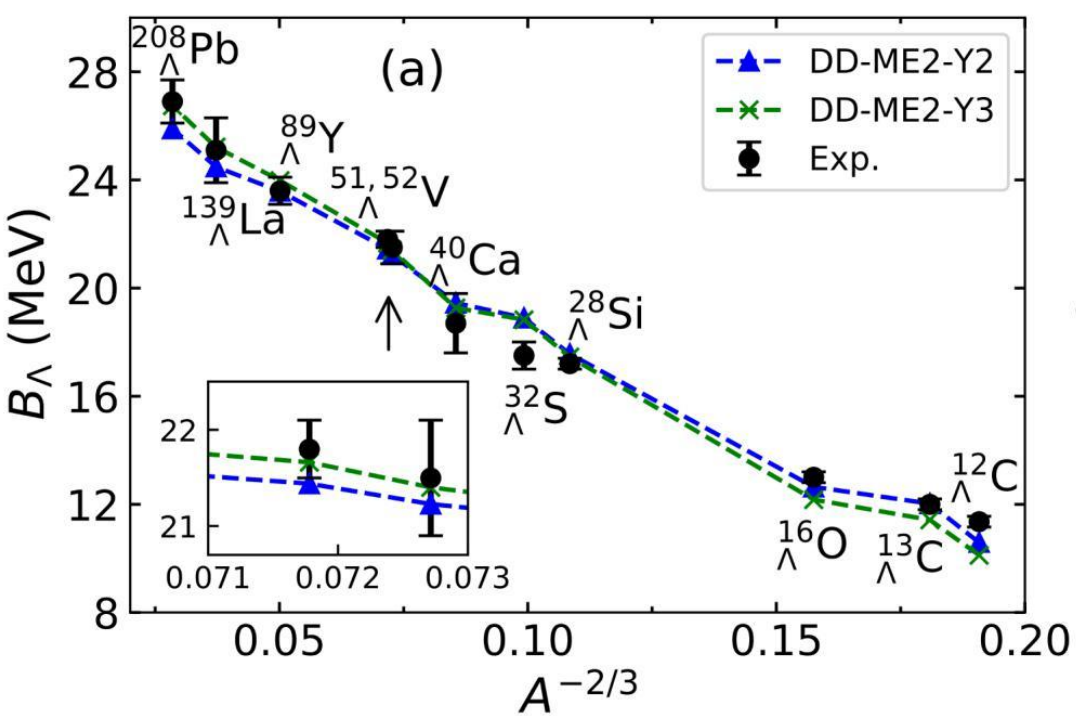


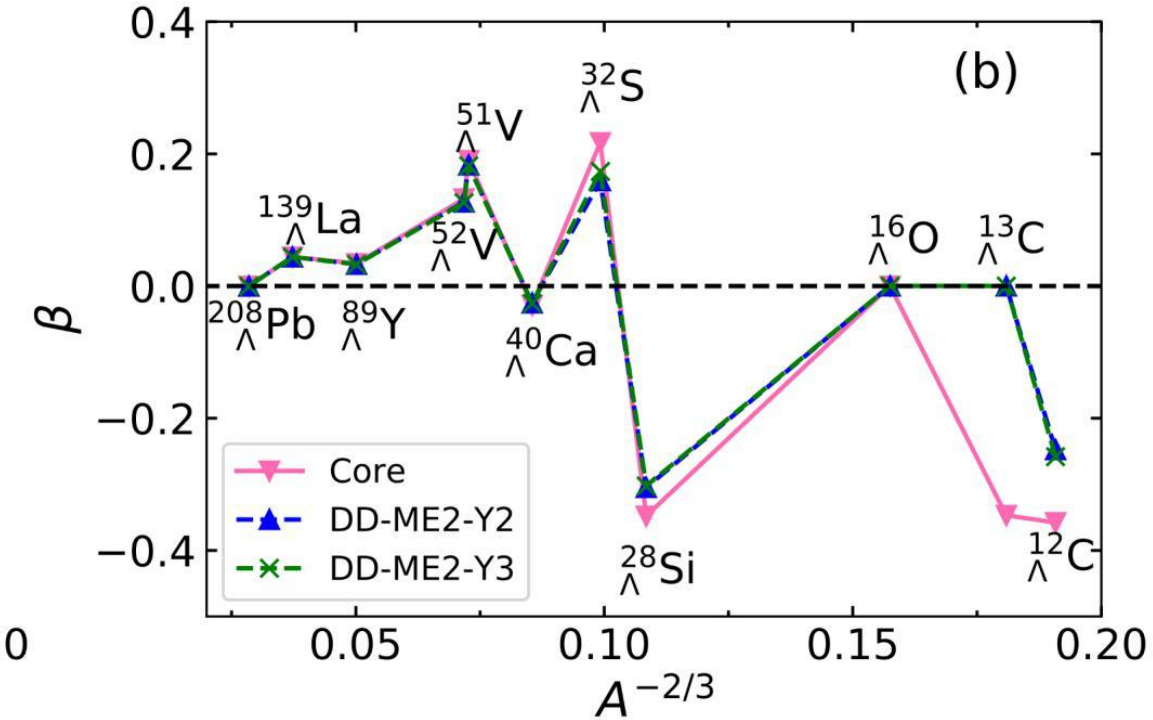
R_σ	R_ω	$\bar{\chi}^2$	$\bar{\chi}^2_{\rm all}$	Δ	δ	Hypernuclei	Effective interactions
0.577	0.611	0.185	6.690	0.668	4.817	Group 3	DD-ME2-Y3

Results: Λ separation energies & deformation

NN interaction: DD-ME2

Reference hypernuclei: $^{208}_{\Lambda}$Pb, $^{139}_{\Lambda}$La, $^{89}_{\Lambda}$Y, $^{51,52}_{\Lambda}$V, $^{40}_{\Lambda}$Ca, $^{32}_{\Lambda}$S, $^{28}_{\Lambda}$Si, $^{16}_{\Lambda}$O (Group 2)



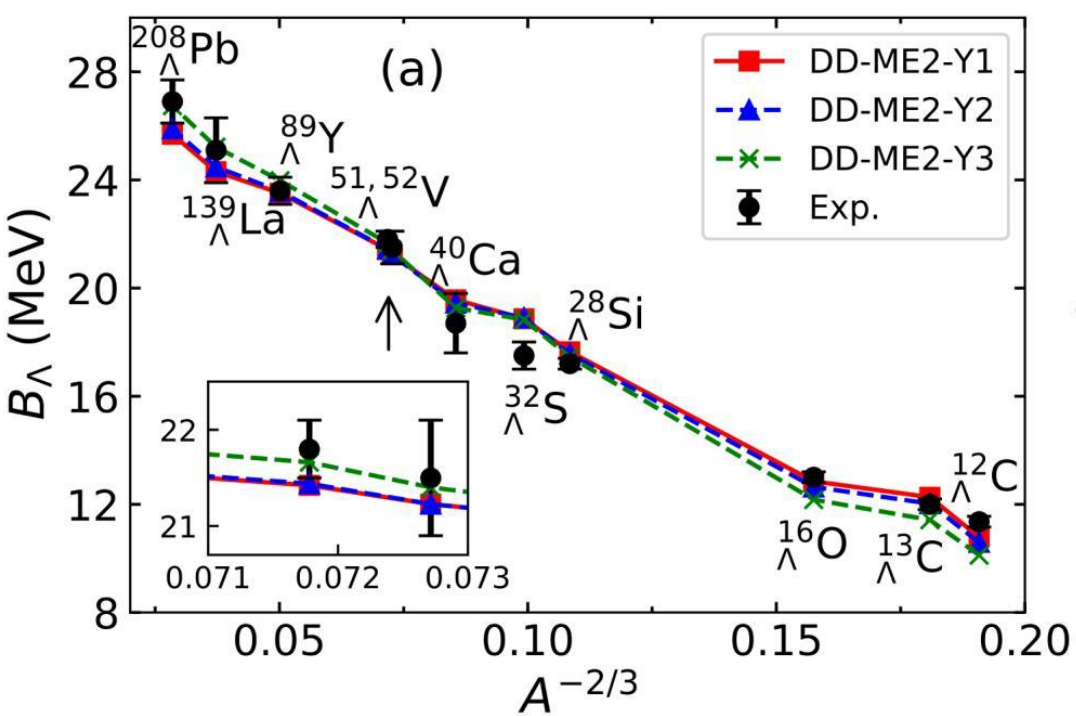


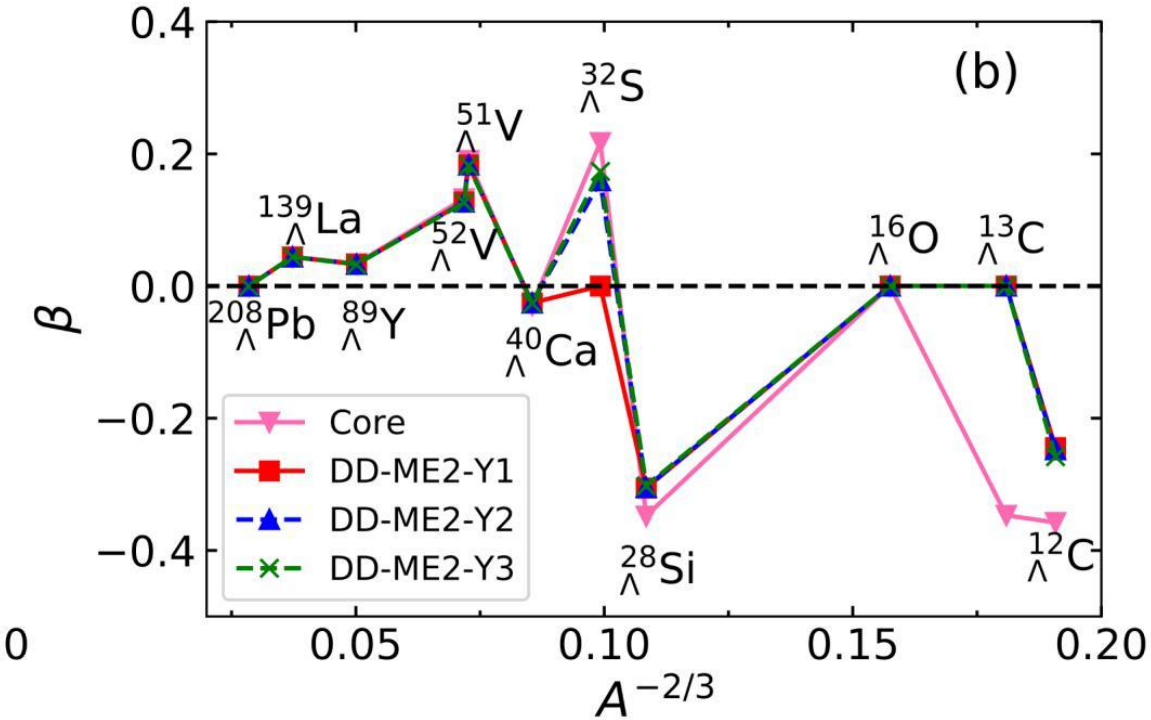
R_σ	R_ω	$\bar{\chi}^2$	$\bar{\chi}^2_{\rm all}$	Δ	δ	Hypernuclei	Effective interactions
0.417	0.415	1.867	3.009	0.672	3.840	Group 2	DD-ME2-Y2
0.577	0.611	0.185	6.690	0.668	4.817	Group 3	DD-ME2-Y3

Results: Λ separation energies & deformation

NN interaction: DD-ME2

Reference hypernuclei: $^{208}_{\Lambda}$Pb, $^{139}_{\Lambda}$La, $^{89}_{\Lambda}$Y, $^{51,52}_{\Lambda}$V, $^{40}_{\Lambda}$Ca, $^{32}_{\Lambda}$S, $^{28}_{\Lambda}$Si, $^{16}_{\Lambda}$O, $^{13}_{\Lambda}$C, $^{12}_{\Lambda}$C (Group 1)

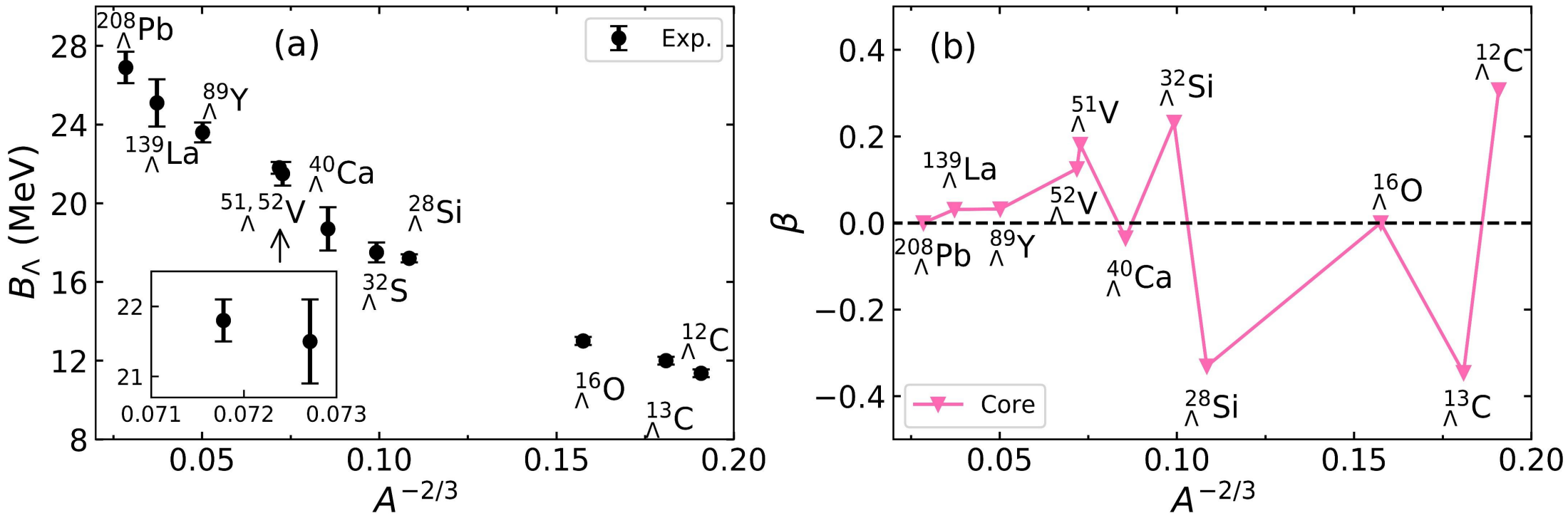




R_σ	R_ω	$\bar{\chi}^2$	$\bar{\chi}^2_{\rm all}$	Δ	δ	Hypernuclei	Effective interactions
0.366	0.352	2.543	2.543	0.711	3.759	Group 1	DD-ME2-Y1
0.417	0.415	1.867	3.009	0.672	3.840	Group 2	DD-ME2-Y2
0.577	0.611	0.185	6.690	0.668	4.817	Group 3	DD-ME2-Y3

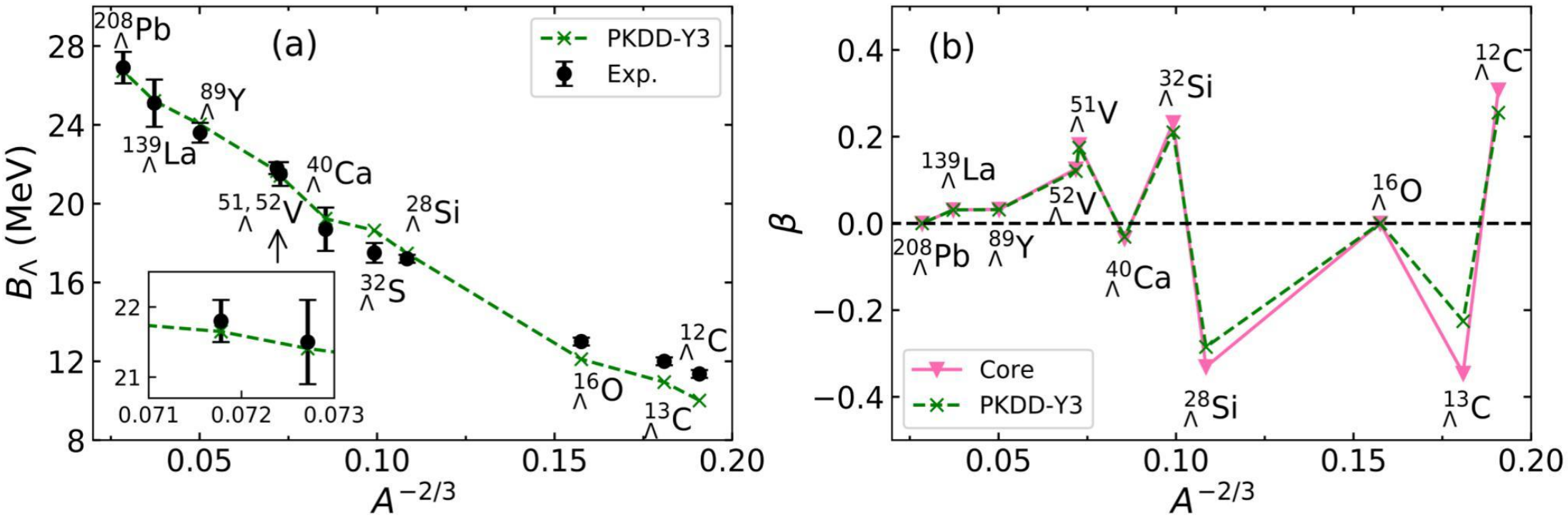
Results: Λ separation energies & deformation

NN interaction: PKDD



Results: Λ separation energies & deformation

NN interaction: PKDD
Reference hypernuclei: $^{208}_{\Lambda}$Pb, $^{139}_{\Lambda}$La, $^{89}_{\Lambda}$Y, $^{51,52}_{\Lambda}$V, $^{40}_{\Lambda}$Ca (Group 3)

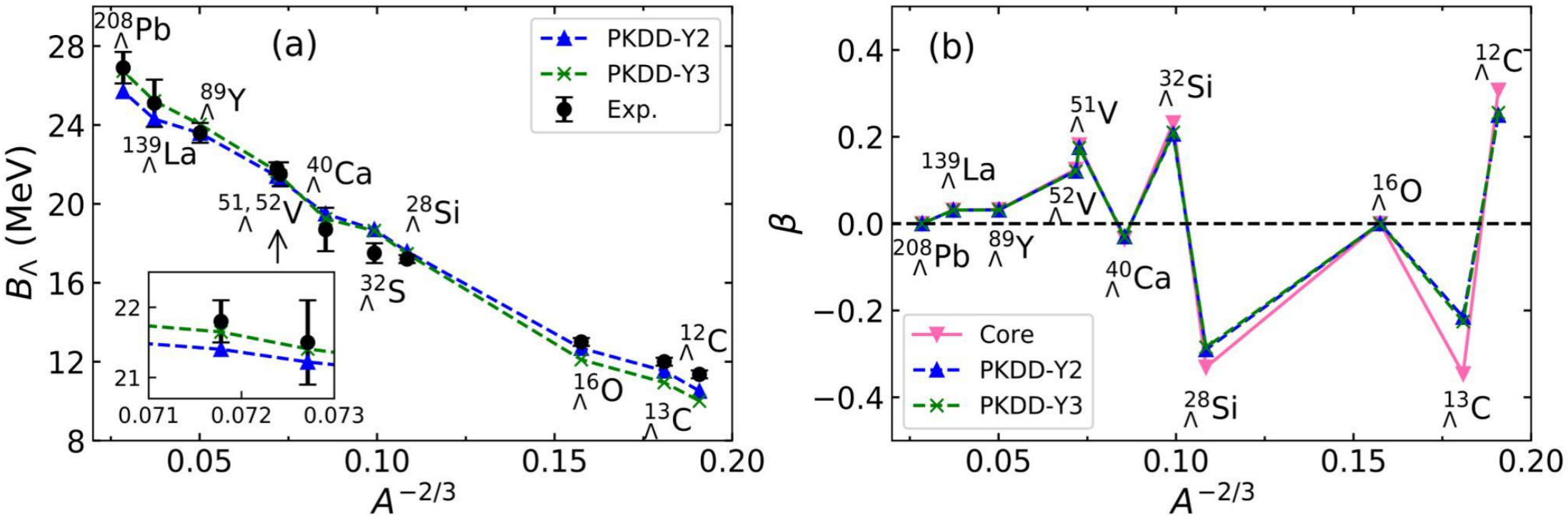


R_σ	R_ω	$\bar{\chi}^2$	$\bar{\chi}^2_{\rm all}$	Δ	δ	Hypernuclei	Effective interactions
0.659	0.712	0.211	9.233	0.716	5.419	Group 3	PKDD-Y3

Results: Λ separation energies & deformation

NN interaction: PKDD

Reference hypernuclei: $^{208}_{\Lambda}$Pb, $^{139}_{\Lambda}$La, $^{89}_{\Lambda}$Y, $^{51,52}_{\Lambda}$V, $^{40}_{\Lambda}$Ca, $^{32}_{\Lambda}$S, $^{28}_{\Lambda}$Si, $^{16}_{\Lambda}$O (Group 2)

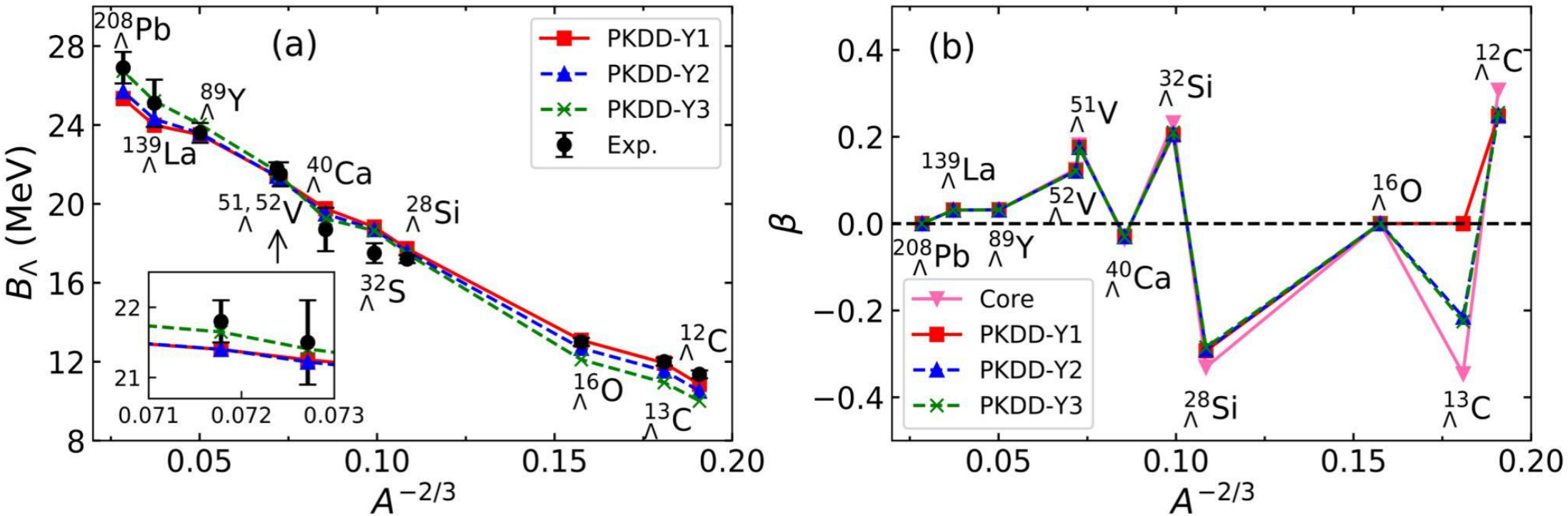


R_σ	R_ω	$\bar{\chi}^2$	$\bar{\chi}^2_{\rm all}$	Δ	δ	Hypernuclei	Effective interactions
0.465	0.472	1.889	3.666	0.712	4.060	Group 2	PKDD-Y2
0.659	0.712	0.211	9.233	0.716	5.419	Group 3	PKDD-Y3

Results: Λ separation energies & deformation

NN interaction: PKDD

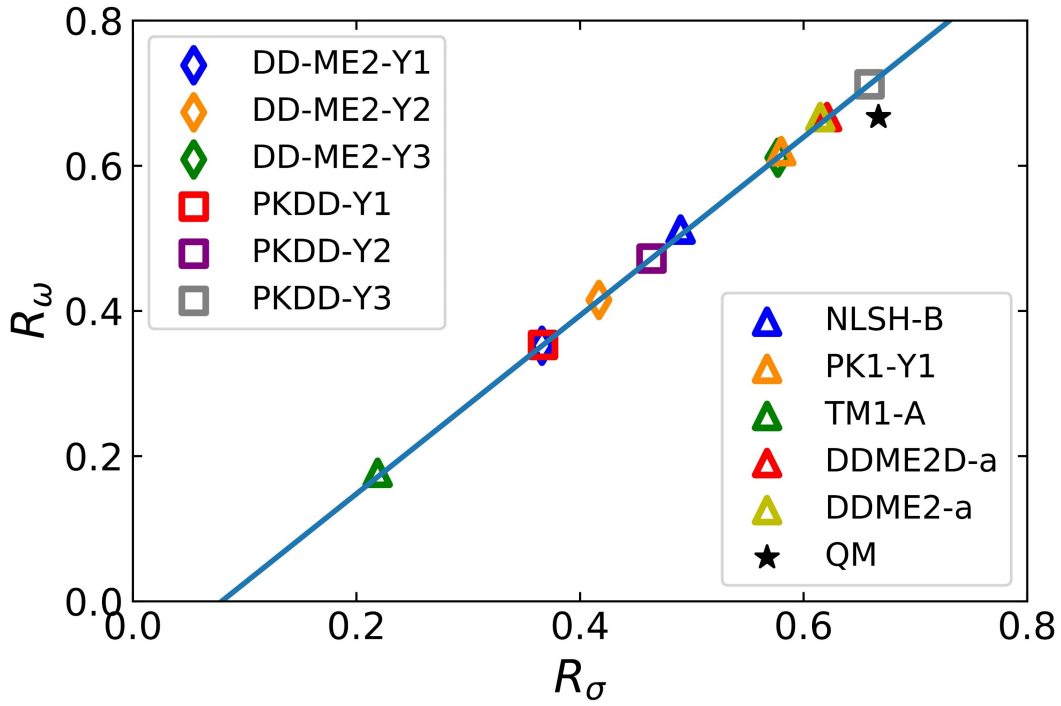
Reference hypernuclei: $^{208}_{\Lambda}$Pb, $^{139}_{\Lambda}$La, $^{89}_{\Lambda}$Y, $^{51,52}_{\Lambda}$V, $^{40}_{\Lambda}$Ca, $^{32}_{\Lambda}$S, $^{28}_{\Lambda}$Si, $^{16}_{\Lambda}$O, $^{13}_{\Lambda}$C, $^{12}_{\Lambda}$C (Group 1)



R_σ	R_ω	$\bar{\chi}^2$	$\bar{\chi}^2_{\rm all}$	Δ	δ	Hypernuclei	Effective interactions
0.367	0.353	2.580	2.580	0.816	4.018	Group 1	PKDD-Y1
0.465	0.472	1.889	3.666	0.712	4.060	Group 2	PKDD-Y2
0.659	0.712	0.211	9.233	0.716	5.419	Group 3	PKDD-Y3

Results: Parameter correlation

Rσ and Rω are correlated —> Λ potential
B_Λ of light hypernuclei —> small Rσ —> mass dependent? Structure effect?



$$R_\omega = 1.228R_\sigma - 0.097$$

$$-U_B \approx g_{\sigma B}\sigma + g_{\omega B}\omega,$$

$$R_m = g_{m\Lambda}/g_{mN},\ m = \sigma \text{ or } \omega.$$

$$R_\omega \approx \frac{-U_\Lambda - R_\sigma g_{\sigma N}\sigma}{-U_N - g_{\sigma N}\sigma}.$$

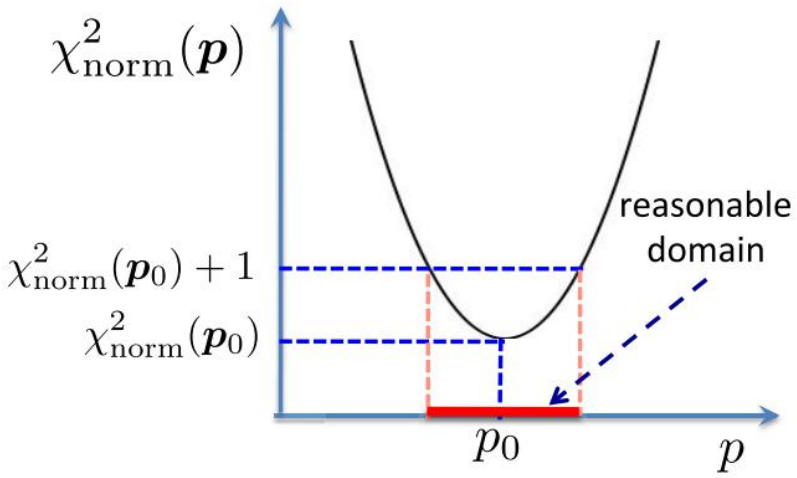
Results: Theoretical uncertainty & parameter correlation

$$\chi^2(\mathbf{p}) = \sum_{i=1}^{N_{type}} \sum_{j=1}^{n_i} \left(\frac{O_{i,j}(\mathbf{p}) - O_{i,j}^{exp}}{\Delta O_{i,j}} \right)^2$$

$$s = \frac{\chi^2(\mathbf{p}_0)}{N_{data} - N_{par}}$$

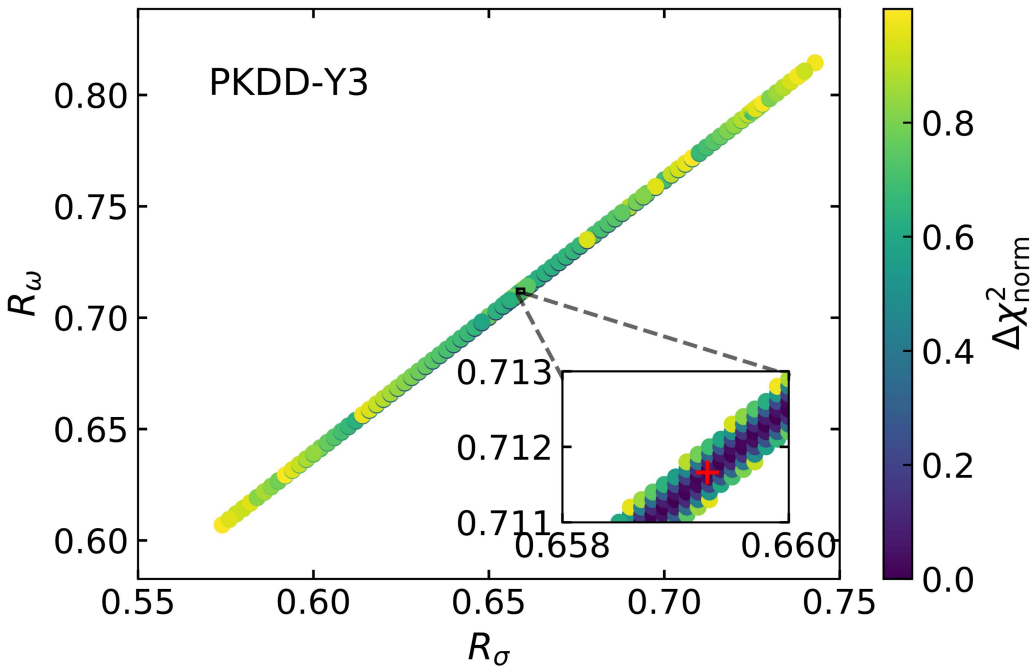
$$\chi^2_{norm}(\mathbf{p}) = \frac{1}{s}\chi^2(\mathbf{p})$$

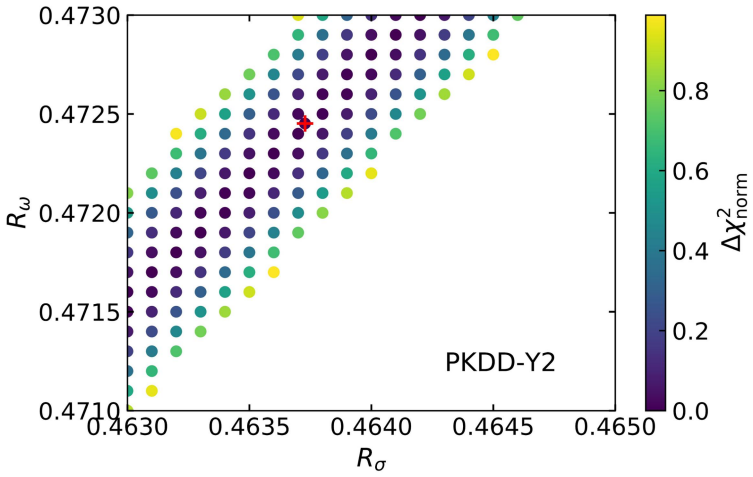
$$\chi^2_{norm}(\mathbf{p}) \leq \chi^2_{norm}(\mathbf{p}_0) + \Delta\chi^2_{max}$$

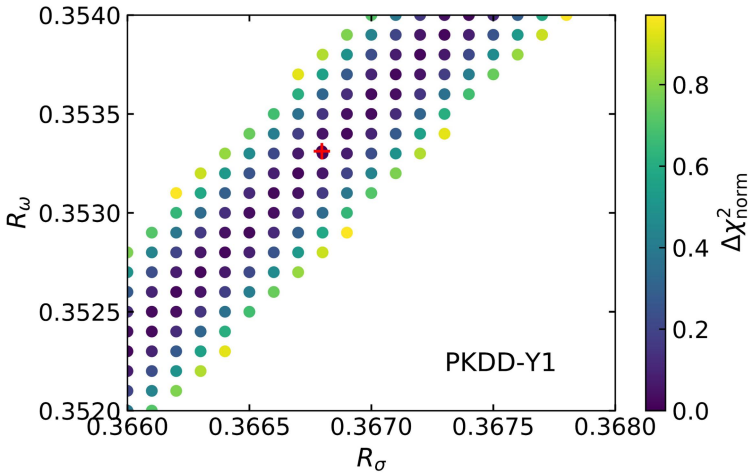


J. Dobaczewski, W. Nazarewicz and P-G. Reinhard.
J. Phys. G: Nucl. Part. Phys. 41 (2014) 074001.

Results: Theoretical uncertainty & parameter correlation







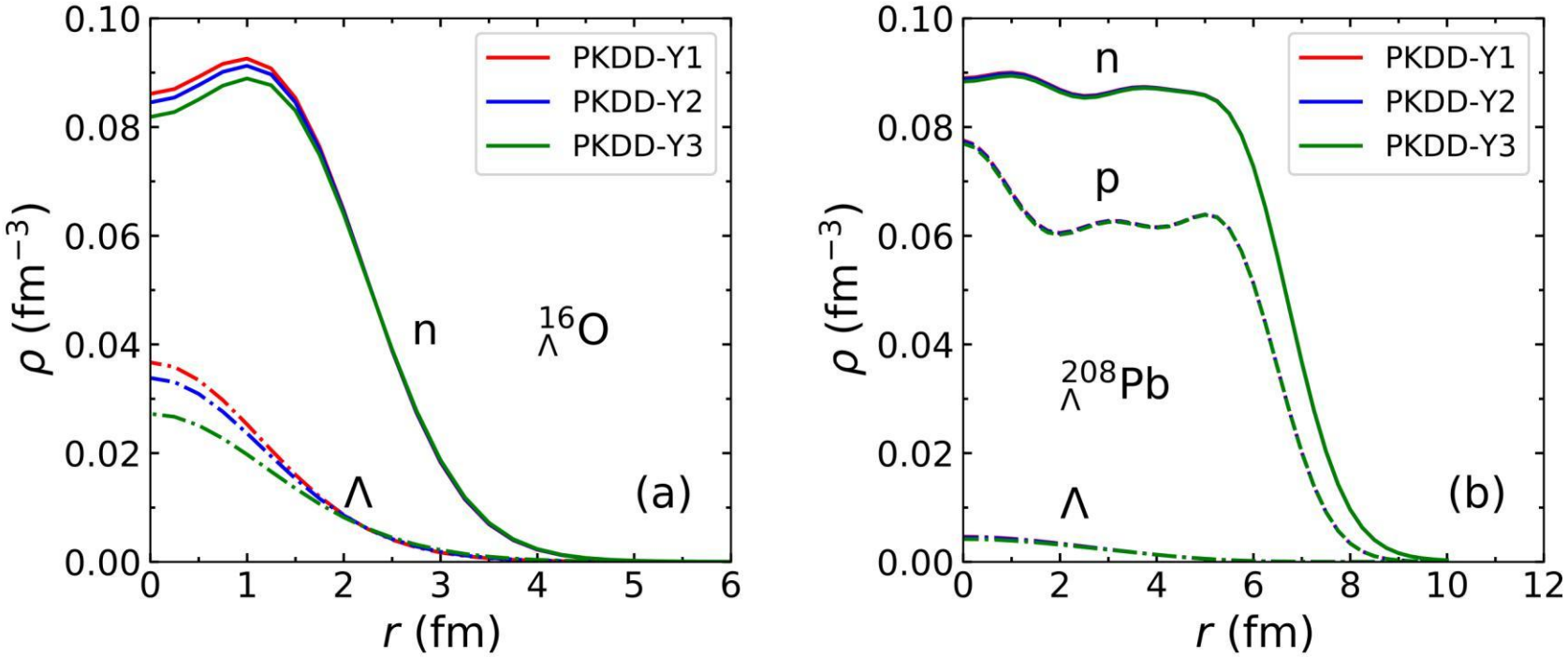
$$R_\sigma = 0.367^{+0.080}_{-0.088} \quad \text{for PKDD-Y1,}$$

$$R_\sigma = 0.464^{+0.092}_{-0.093} \quad \text{for PKDD-Y2,}$$

$$R_\sigma = 0.659^{+0.084}_{-0.085} \quad \text{for PKDD-Y3.}$$

Results: Hypernuclear density

Impact on density —> change the Λ density in light hyernuclei



Results: Hyperon star

Baryons: $p,\ n,\ \Lambda,\ \Sigma^+, \Sigma^0, \Sigma^-, \Xi^+, \Xi^+$

leptons: $e^-,\ \mu^-$

mesons: $\sigma,\ \omega,\ \rho$

Coupling constants:

$$2g_{\omega\Xi} = g_{\omega\Sigma} = 2g_{\omega N}/3,$$
$$g_{\rho\Sigma} = 2g_{\rho\Xi} = 2g_{\rho N},$$
$$U_Y^{(N)} = g_{\sigma Y}\sigma + g_{\omega Y}\omega + g_{\rho Y}\rho + \Sigma_R,$$
$$U_\Sigma^{(N)} = +30\ \text{MeV},\ U_\Xi^{(N)} = -15\ \text{MeV}.$$

Equation of state:

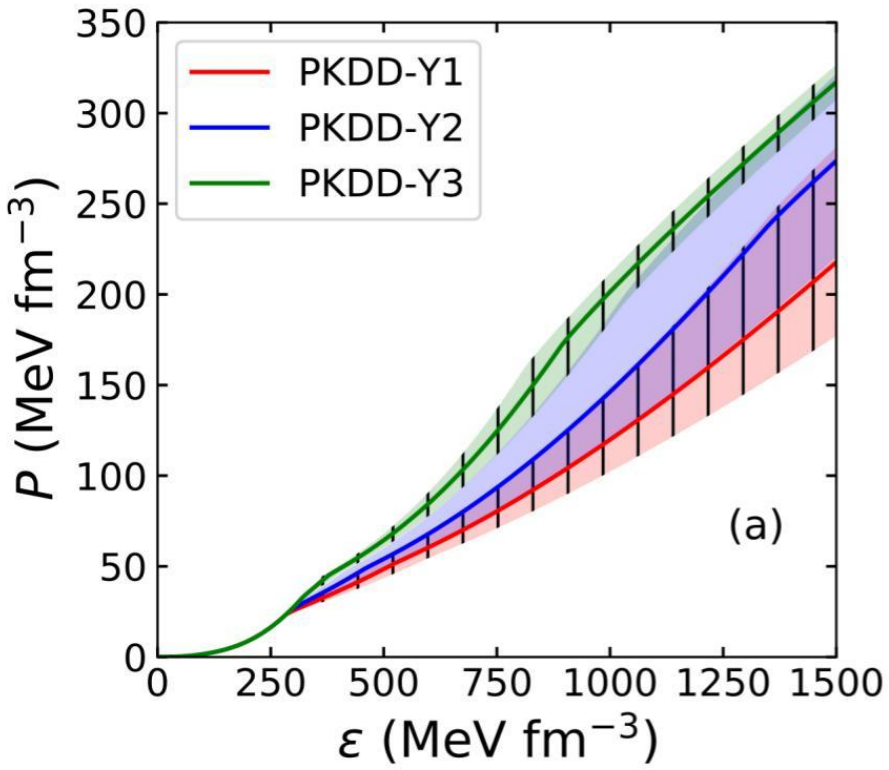
$$\varepsilon = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2\omega^2 + \frac{1}{2}m_\rho^2\rho^2 + \sum_B \varepsilon_{\text{kin}}^B,$$
$$P = -\frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2\omega^2 + \frac{1}{2}m_\rho^2\rho^2 + \rho^\upsilon\Sigma_R + \sum_B P_{\text{kin}}^B.$$

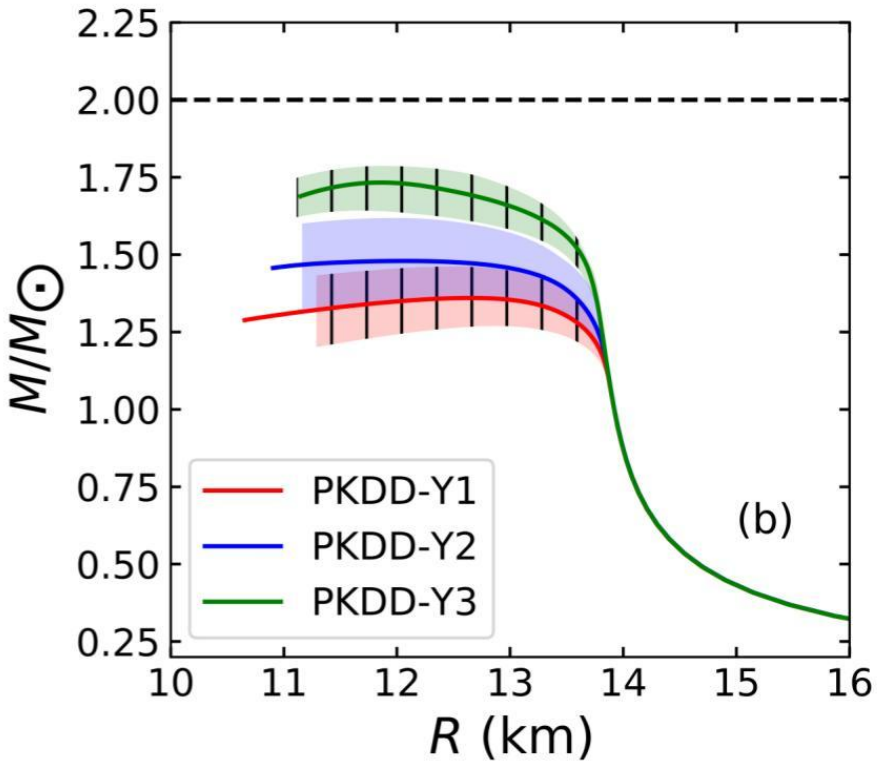
TOV equation:

$$\frac{dP}{dr} = -\frac{[P(r)+\varepsilon(r)][M(r)+4\pi r^3 P(r)]}{r(r-2M(r))},$$
$$\frac{dM}{dr} = 4\pi^2\varepsilon(r).$$

Results: Hyperon star

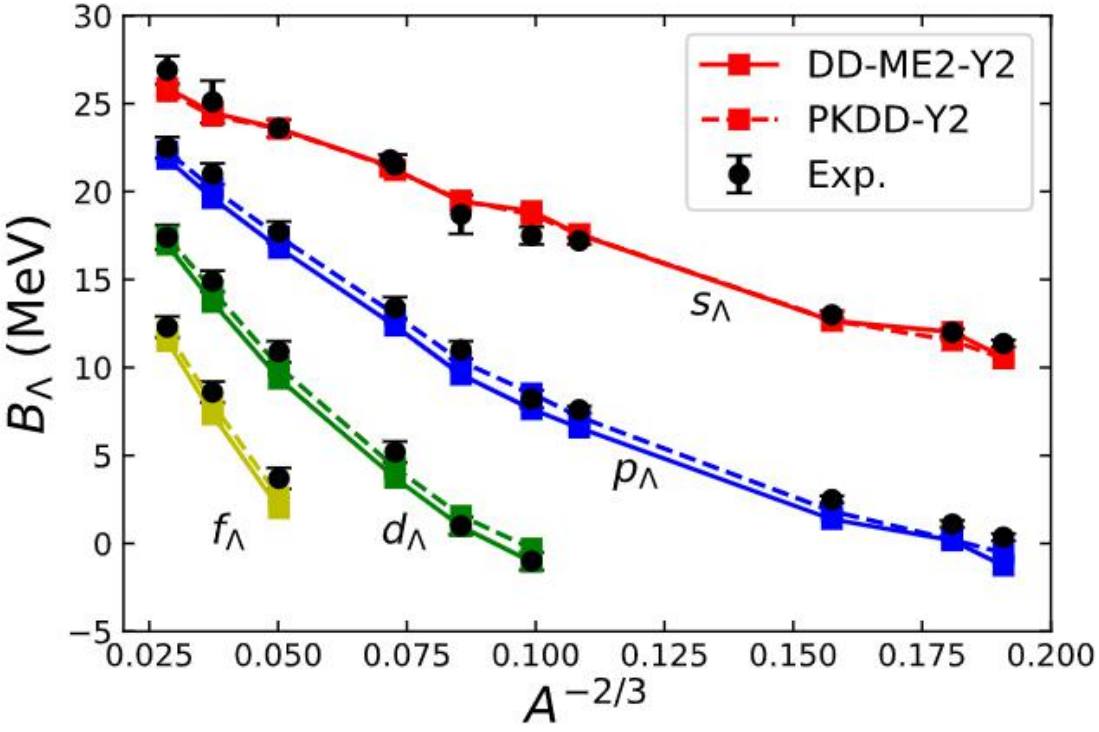
Impact on hyperon star —>The lighter Rσ, the softer EoS and smaller maximum mass of neutron star.

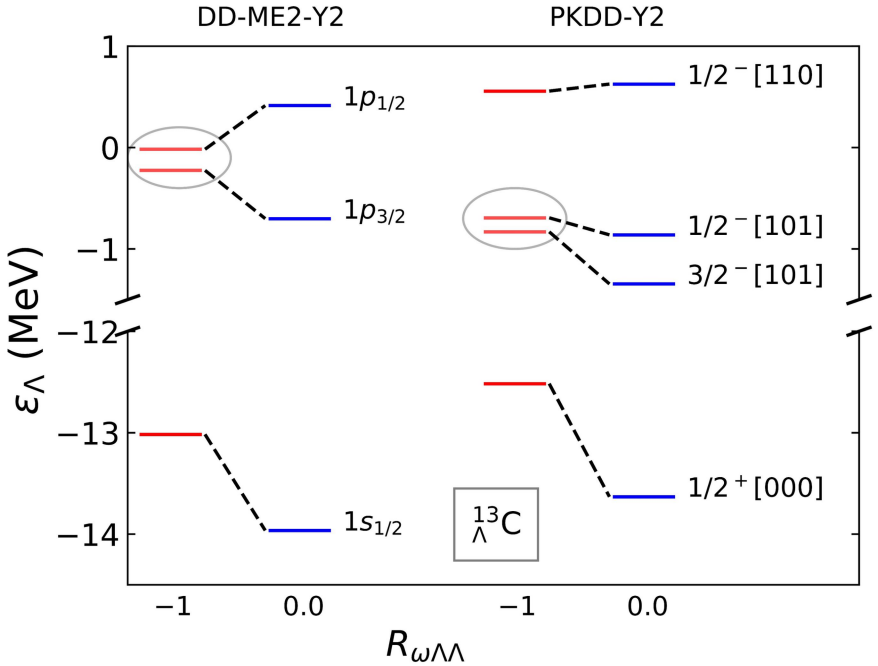




Results: Excited states and single particle levels

- Excited energies in p_Λ, d_Λ and f_Λ states are close to experimental values
- Energy splitting for spin partners are close to the experimental value with tensor coupling





$152 \pm 54(\text{stat}) \pm 36(\text{syst})$ keV.

S. Ajimura, H. Hayakawa, T. Kishimoto, et al. Phys. Rev. Lett. 86 (2001) 4255.

Summary

- The effective ΛN interactions for hypernuclei are investigated by a deformed DDRMF model and new parameter sets are proposed.
- Shape change effect is obvious in the $A \leq 40$ mass region
- Data from light hypernuclei make the ΛN coupling constants smaller
- R_σ and R_ω are strongly correlated and uncertainties for R_σ are given
- Density for Λ in light mass region and EoS of hypernuclear matter are sensitive to the parameters
- Single particle excited energies are close to the experimental values
- Spin-orbit splitting in *p* state can be well described by including tensor coupling

Thanks for discussions with
Johann Haidenbauer, Hoai Le, Andreas Nogga, Xiang-Xiang Sun and Kun Wang.

谢谢！

Thanks for your attention!

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