Joint THEIA-STRONG2020 and JAEA/Mainz REIMEI Web-Seminar Mar. 24, 2021

Structures and production cross sections of *p*-shell Λ -hypernuclei calculated with extended shell model

Atsushi UMEYA (Nippon Inst. of Tech.)

collaborated with

Toshio MOTOBA (Osaka E-C Univ., YITP)

Kazunori ITONAGA (Gifu Univ.)

Introduction

Basic motivations

- (1) *p*-shell nuclei and hypernuclei provide a variety of interesting phenomena (shell-, cluster-, and coexistent characters), depending on E_x and mass.
- (2) High-precision experiments in hypernuclear spectroscopy are in progress.
- (3) Detailed look in Jlab (e, e'K⁺) spectroscopic data requires an extended description with multi-configuration parity-mixing mediated by hyperon.

Mar. 24, 2021

Recent $(e, e'K^+)$ reaction experiments done at the Jefferson Lab



Recent experimental result

T. Gogami et al., PRC93, 034314 (2016)

Shell-model prediction

- T. Motoba et al., PTPS117, 123 (1994)
- Core nucleus calculated with conventional *p*-shell model
- Λ in *s*-orbit

This experiment has confirmed the major peaks (#1, #2, #3, #4) predicted in DWIA by emplying the Λ particle in *s*-orbit coupled with the nuclear core states confined within the *p*-shell configuration.

However, it is interesting to observe extra strengths at $E_{\Lambda} = 0$ MeV excitation (a).

The extension of the model space is necessary and interesting challenge in view of the present hypernuclear spectroscopy.

Splitting of *p*-state in the deformed nuclei

The bump in the cross sections of ${}^{10}_{\Lambda}$ Be will be explained by the splitting of p^{Λ} -state in the deformed core-nucleus.



Deformation parameter δ

S. G. Nilsson, Mat. Fis. Medd. Dan. Vid. Selsk. 29 (1955) No. 16

Eigenvalues Ω of *z*-component of angular momentum operator and parities are good quantum numbers in the Nilsson diagram.

$$p_{3/2} \to \Omega^{\pi} = 1/2^{-}, 3/2^{-}$$

Mar. 24, 2021

 $[p^{-1}p^{\Lambda}_{\perp}]$ and $[p^{-1}p^{\Lambda}_{\prime\prime}]$ states of ${}^{9}_{\Lambda}$ Be (1)



In ${}^{9}_{\Lambda}$ Be, it is well known that the p_{Λ} -state splits into two orbital states expressed by p_{\perp} and p_{\parallel} , which is due to the strong coupling with nuclear core deformation having the α - α structure. T. Motoba *et al.*, PTPS81, 42 (1985)

The p_{\parallel} state tends to the configuration with an SU(3) classification [f]($\lambda\mu$) = [54](50) called supersymmetric. **R. H. Dalitz, A. Gal, PRL36, 362 (1976); AP131, 314 (1981)**

Mar. 24, 2021





To describe hypernuclei with deformed nuclear core such as $\alpha \alpha$, we need to extend the model space.

This talk

We have calculated the energy levels and the production cross sections for *p*-shell hypernuclei by using the extended shell model.

We focus on the *p*-state Λ hyperon in the *p*-shell Λ hypernuclei.

- Energy levels and cross sections for ${}^9_{\Lambda}Be$, ${}^{10}_{\Lambda}Be$ (${}^{10}_{\Lambda}B$), ${}^{11}_{\Lambda}B$ and ${}^{12}_{\Lambda}B$ (${}^{12}_{\Lambda}C$)
- p_{\perp}^{Λ} and p_{\parallel}^{Λ} states in hypernuclei with A = 9, 10 and 11
- Mixing of natural- and unnatural-parity nuclear-core states

We show the numerical cross sections of (γ, K^+) reactions for ${}^9_{\Lambda}$ Li and ${}^{11}_{\Lambda}$ Be which have T = 1.

- ⁹Be $(e, e'K^+)$ experiment has been done at JLab.
- ¹¹B ($e, e'K^+$) experiment may be performed.

Framework of calculations

Shell-model configurations ($^{10}_{\Lambda}$ Be case)









 $(0s)^4(0p)^4(sd)^1(0s^{\Lambda})$





Mar. 24, 2021

Extension of the model space in the shell model $\binom{10}{\Lambda}$ Be case)

Model space for ⁹Be core

- (A) conventional model space J_{core}^{-} $(0s)^4 (0p)^5$ (0p-0h)
- **(B) extended model space**

 $J_{\text{core}}^{+} (0s)^{3} (0p)^{6} \oplus (0s)^{4} (0p)^{4} (sd)^{1} (1p-1h)$

Conventional model space for ¹⁰_^Be

(I)
$$J_{\text{core}}^- \otimes 0s^{\Lambda} \Rightarrow {}_{\Lambda}^{10}\text{Be}(J^-)$$
 (II) $J_{\text{core}}^- \otimes 0p^{\Lambda} \Rightarrow {}_{\Lambda}^{10}\text{Be}(J^+)$

Extension (1) 1*p*-1*h* (1 $\hbar\omega$) core excitation is taken into account

(a)
$$J^{-}_{\text{core}} \otimes 0s^{\Lambda} \Rightarrow {}^{10}_{\Lambda} \text{Be}(J^{-})$$
 (b) $J^{-}_{\text{core}} \otimes 0p^{\Lambda} \Rightarrow {}^{10}_{\Lambda} \text{Be}(J^{+})$
(c) $J^{+}_{\text{core}} \otimes 0s^{\Lambda} \Rightarrow {}^{10}_{\Lambda} \text{Be}(J^{+})$ (d) $J^{+}_{\text{core}} \otimes 0p^{\Lambda} \Rightarrow {}^{10}_{\Lambda} \text{Be}(J^{-})$

Extension (2) Configrations mixed by ΛN **interaction**

$$\begin{array}{c}
J^{-}_{\text{core}} \otimes 0s^{\Lambda} \\
J^{-}_{\text{core}} \otimes 0p^{\Lambda} \\
\end{bmatrix} \oplus \begin{array}{c}
J^{+}_{\text{core}} \otimes 0p^{\Lambda} \\
J^{-}_{\text{core}} \otimes 0p^{\Lambda} \\
\end{bmatrix} \oplus \begin{array}{c}
J^{+}_{\text{core}} \otimes 0s^{\Lambda} \\
J^{+}_{\Lambda}Be(J^{+}) \\
\end{array}$$

Mar. 24, 2021

Configration mixing in ${}^{10}_{\Lambda}$ Be unnatural parity states



In the conventional shell model, only natural-parity nuclaer-core states (J_{core}^-) are taken into account. A particle is in the 0s orbit in ${}^{10}_{\Lambda}\text{Be}(J^-)$.

In ${}^{10}_{\Lambda}$ Be(J^+), the energy difference between $\Lambda(0s)$ and $\Lambda(0p)$ is $1\hbar\omega$, and the energy difference between 9 Be(J^-_{core}) and 9 Be(J^+_{core}) is $1\hbar\omega$.

By ΛN interaction, natural-parity nuclaer-core configurations and unnatural-parity nuclaer-core configurations can be mixed.

Mar. 24, 2021

Extended model space for target nucleus ¹⁰B



Extension of model space for target nucleus ¹⁰B up to 2*p*-2*h* (2 $\hbar\omega$) allows the ¹⁰_{Λ}Be production through various configurations.

NN interaction and nucleon single-particle energy

NN effective interactions

- $\langle p^2 | V | p^2 \rangle$ Cohen-Kurath (8–16) TBME S. Cohen, D. Kurath, NP73, 1 (1965)
- $\langle (sd)^2 | V | (sd)^2 \rangle$ modified Kuo-Brown G-matrix T. T. S. Kuo, G. E. Brown, NP85, 40 (1966)
- <p(sd)|V|p(sd)> Millener-Kurath
 D. J. Millener, D. Kurath, NPA255, 315 (1975)
- $\langle p^2 | V | (sd)^2 \rangle$ modified Kuo-Brown G-matrix (SFO) T. Suzuki, R. Fujimoto, T. Otsuka, PRC67, 044302 (2003)

Other part (contains a role of removal of spurious center-of-mass motion effects)

Anantaraman-Toki-Bertsch G-matrix N. Anantaraman, H. Toki, G. F. Bertsch, NPA398, 269 (1983)

Single-particle energies

adjusting to reproduce the experimental low-lying energy levels

ΛN interaction and Λ single-particle energy

 $\langle N\Lambda | V | N\Lambda \rangle$ Nijmegen NSC97e

Th. A. Rijken, V. G. J. Stoks, Y. Yamamoto, PRC59, 21 (1999)

 ε_s^{Λ} and ε_p^{Λ} are determined to reproduce the #1 (2⁻) and #6 (3⁺) peaks in ${}_{\Lambda}^{12}$ B production cross-section.

 $\varepsilon^{\Lambda}_{s}$ and $\varepsilon^{\Lambda}_{p}$ are applied to $^{10}_{\Lambda}$ Be.

JLab Hall C, E05-115 L. Tang *et al.*, PRC90, 034320 (2014)

Theoretical calculation T. Motoba *et al.*, PTPS185, 224 (2010)



Mar. 24, 2021

Numerical values of B_{Λ} for A = 9-12 hypernuclei

unit in MeV

	$^{9}_{\Lambda}$ Li	$^{9}_{\Lambda}$ Be	$^{10}_{\Lambda}\mathrm{Be}$	$^{11}_{\Lambda}\mathrm{Be}$	$^{11}_{\Lambda}\text{B}$	$^{12}_{\Lambda}\mathrm{B}$
J^{π}	3/2+, 5/2+	1/2+	1-,2-	1/2+	5/2+,7/2+	1-, 2-
E _{g.s.}	-21.87	-37.19	-40.72	-49.49	-51.38	-64.26
$E_{1 \text{st ex.}}$	-21.56	—	-40.55		-51.09	-64.14
$\Delta E(\text{cal})$	0.31	—	0.17		0.29	0.12
$\Delta E(\exp)$	0.57	—	0.10		0.26	0.16
Core	$^{8}\text{Li}(2^{+})$	${}^{8}\text{Be}(0^{+})$	$^{9}\text{Be}(3/2^{-})$	$^{10}\text{Be}(0^{+})$	$^{10}B(3^{+})$	$^{11}B(3/2^{-})$
$E_{\rm core}({\rm cal})$	-14.50	-30.30	-32.09	-39.47	-41.07	-52.67
$E_{\rm core}(\exp)$		-30.84	-32.50		-41.06	-52.52
$B_{\Lambda}(\text{cal})$	7.37	6.89	8.63	10.02	10.31	11.59
$B_{\Lambda}(\exp)$	8.36	6.71	8.55		10.24	11.52

E denotes the binding energy with respect to the ⁴He core.

The same two-body matrix elements of the ΛN interaction are used in the A = 9-12 hypernucei.

Our results of binding energies and doublet spacing are good agreement with the experimental data.

A = 9 hypernucleus, ${}^{9}_{\Lambda}Be$

Mar. 24, 2021

Results : Energy levels of ⁸**Be and** ⁹**Be**



Mar. 24, 2021

Results : Comparison with the cluster model (1) – Energy level –

Cluster model T. Motoba et al., PTPS81, 42 (1985)



This work



Mar. 24, 2021

Results : Comparison to the cluster model (2) – Cross section –



Mar. 24, 2021

Results : Cross sections of (K^-, π^-) and (π^+, K^+) reactions



A = 10 hypernucleus, ${}^{10}_{\Lambda}$ Be



Mar. 24, 2021

Results : Energy levels of ⁹**Be and** ¹⁰_^**Be**



Mar. 24, 2021

Results : Energy levels of ${}^{10}_{\Lambda}$ **Be (comparison with JLab experiments)**



T. Gogami et al., PRC93, 034314 (2016)

Mar. 24, 2021

Results : Cross sections of the ¹⁰B (γ , K^+) ¹⁰Be reaction (1)



Recent experimental result T. Gogami *et al.*, PRC93, 034314 (2016)

For hypernucleus ${}^{10}_{\Lambda}$ Be (1) 1*p*-1*h* (1 $\hbar\omega$) core excitation (2) Configration mixing by ΛN int. are taken into account

DWIA calculation by using Saclay-Lyon model A

Our new calculation reproduces the four major peaks (#1, #2, #3, #4).

Our new calculation explains the
 ¹⁰ new bump (a) as a sum of cross sections of some J⁺ states.

Mar. 24, 2021

Results : Cross sections of the ¹⁰B (γ , K^+) ¹⁰_{Λ}Be reaction (2)

						$E_{\gamma} = 1.5$ ([,] = 1.5 GeV		EXP = T. Gogami et al, PRC93 (2016		
	⁹ Be (<i>J</i> _i)		۸ ¹⁰	$Be\left(J_{k}\right) C$	AL	$\theta = 7 \deg$	9			EXP	Fit I
Ji	E _i (exp)	<i>E</i> i (cal)	J _k	E _x	<i>−B</i> ∧	dơ/	⁄dΩ	exp	E _x	<i>−B</i> ∧	dσ/dΩ
	C2S	C2S		[MeV]	[MeV]	[nb	/sr]	peak	[MeV]	[MeV]	[nb/sr]
3/2-	0.000	0.000	1-	0.000	-8.600	9.609	21 62	#1	0.00	8 55+0 07	17 0+0 5
	1.0(rel)	1.0(rel)	<mark>2⁻</mark>	0.165	-8.435	12.008	21.02	#1	0.00	-0.33±0.07	17.0±0.5
- /0-	0.400	0.044	0-	0.710	F 000						
5/2	2.429	2.644	2	2.712	-5.888	11.654	21.05	#2	2.78±0.11	-5.76±0.09	16.5±0.5
	0.958	1.020	3-	2.860	-5.740	9.391				·	
7/2-	6.380	6.189	3-	6.183	-2.417	7.625		#3	6.26±0.16	-2.28±0.14	10.5±0.3
	0.668	0.942	4 ⁻	6.370	-2.230	13.505	21.13				
			2+(3)	7.807	-0.793	4.495	9.46		8.34±0.41	-0.20±0.40	23.2±0.7
			1+(3)	7.935	-0.665	4.968	5.40				
			3+(2)	8.712	0.112	6.150		#2			
			2+(4)	8.828	0.228	1.431	19.91	#a			
			2+(5)	9.002	0.402	9.893	(29.37)				
			3+(3)	9.059	0.459	2.434					
7/2-	11.283	10.241	<mark>3</mark> _	10.105	1.505	3.913	21 90	#4	10 83+0 10	2 28+0 07	17 2+0 5
	1.299	1.355	4-	10.455	1.855	17.985	21.30		10.00±0.10	2.2010.07	17.2±0.5
			1+(5)	10.828	2.228	4.598	20 54				
			4+(3)	11.318	2.718	11.185	29.04 (51 AA)				
			3+(5)	11.543	2.943	13.759	(31.44)				

Mar. 24, 2021

Results : Configrations of J^+ **states corresponding to the new bump**

$J_n^{\pi}(-B_{\Lambda}[\text{MeV}])$	$[J_{\rm core}^{\pi}]j^{\Lambda}$	$[J_{\rm core}^{\pi}]j^{\Lambda}$	$[J_{\rm core}^{\pi}]j^{\Lambda}$
XS [nb/sr]			
$2^+_3(-0.739)$		$[3/2_1^-](p_{3/2}p_{1/2})^{\Lambda}$	$[5/2_1^-](p_{3/2}p_{1/2})^{\Lambda}$
4.49		82.5%	15.8%
$1_3^+(-0.665)$		$[3/2_1^-](p_{3/2}p_{1/2})^{\Lambda}$	$[5/2_1^-]p_{3/2}^{\Lambda}$
4.97		79.5%	17.9%
$2^+_4(0.228)$	$[5/2^+_2]s^{\Lambda}_{1/2}$	$[3/2_1^-](p_{3/2}p_{1/2})^{\Lambda}$	$[5/2_1^-](p_{3/2}p_{1/2})^{\Lambda}$
1.43	87.5%	9.4%	2.4%
$2^+_5(0.402)$	$[5/2^+_2]s^{\Lambda}_{1/2}$	$[3/2_1^-](p_{3/2}p_{1/2})^{\Lambda}$	$[5/2_1^-](p_{3/2}p_{1/2})^{\Lambda}$
9.89	11.3%	70.9%	10.8%
$3_2^+(0.112)$	$[5/2^+_2]s^{\Lambda}_{1/2}$	$[3/2_1^-]p_{3/2}^{\Lambda}$	$[5/2_1^-](p_{3/2}p_{1/2})^{\Lambda}$
6.15	31.6%	55.4%	9.7%
$3_3^+(0.459)$	$[5/2^+_2]s^{\Lambda}_{1/2}$	$[3/2_1^-]p_{3/2}^{\Lambda}$	$[5/2_1^-](p_{3/2}p_{1/2})^{\Lambda}$
2.43	67.5%	27.1%	2.7%

Mar. 24, 2021

Results : Cross sections of the ¹⁰B (K^-, π^-) ¹⁰_AB reaction (1)



In the (K^-, π^-) reaction, the large peak at $E_{\Lambda} = 4.4$ MeV is a *p*-substitutional state via the $p_{3/2}^N \rightarrow p_{3/2}^{\Lambda}$, which is strongly excited by recoilless reaction.

The small peak at $E_{\Lambda} = 0 \text{ MeV}$ corresponds to the new bump and is explained as a mixture of s^{Λ} and p^{Λ} states.

The large peak at $E_{\Lambda} = 4.4 \text{ MeV}$ in ${}^{10}_{\Lambda}\text{Be}$ corresponds to the $[p^{-1}p_{\perp}^{\Lambda}]$ state in ${}^{9}_{\Lambda}\text{Be}$ (⁹Be analog state).

The small peak at $E_{\Lambda} = 0 \text{ MeV}$ in ${}^{10}_{\Lambda}\text{Be}$ corresponds to the $[p^{-1}p^{\Lambda}_{//}]$ state in ${}^{9}_{\Lambda}\text{Be}$.

Mar. 24, 2021

Results : Cross sections of the ¹⁰B (K^-, π^-) ¹⁰_AB reaction (2)



CONCLUDE:

 $\alpha \alpha$ -like core deformation causes splitting of p^{Λ} -states, then lowenergy $p^{\Lambda}_{//}$ can mix with s^{Λ} -states.

 $[{}^{9}\text{Be}(J^{-}) \times \Lambda(p_{//})] + [{}^{9}\text{Be}(J^{+}) \times \Lambda(s)]$

These parity-mixed wave functions at $E_{\Lambda} = 0 \text{ MeV}$ can explain the extra peak #a.

A = 12 hypernucleus, ${}^{12}_{\Lambda}B$

Mar. 24, 2021

Results : Energy levels of ^{11}B and $^{12}_\Lambda B$



Mar. 24, 2021

Results : Cross sections of the ¹²C (γ , K^+) ¹²_AB reaction



JLab Hall C, E05-115 L. Tang *et al.*, PRC90, 034320 (2014)

Theoretical calculation T. Motoba *et al.*, PTPS185, 224 (2010)

New results show no peak for #4.

 p^{Λ} -state does not split because the core is spherical.

A new question if the extra peak #4 in Hall C experiment can survive in future experiment with better statistics

A = 11 hypernucleus, ${}^{11}_{\Lambda}B$

Mar. 24, 2021

Results : Energy levels of ${}^{10}B$ and ${}^{11}_{4}B$ (1)



Mar. 24, 2021

Results : Energy levels of ${}^{10}B$ and ${}^{11}_{\Lambda}B$ (2)



3rd and 4th column D. J. Millener, NPA804, 84 (2008).

Our result of the energy of the 2nd doublet $(1/2^+, 3/2^+)$ is almost the same as Millener's result and is 300 keV lower than the experimental result.

For this doublet, effect of the *LS* term of the ΛN int. is suggested. D. J. Millener, NPA804, 84 (2008).

Mar. 24, 2021



Mar. 24, 2021



Mar. 24, 2021



Mar. 24, 2021



Mar. 24, 2021

Results : Energy of $p_{\prime\prime}$ -state



The p^{Λ} -state splits into p_{\perp} - and $p_{//}$ -states due to the strong coupling with nuclear core deformation.

In ${}^{9}_{\Lambda}$ Be, the enregy of $p_{//}^{\Lambda}$ -state comes down to $E_x \approx 7 \text{ MeV}$ from the Λ single-particle energy difference $\varepsilon_p^{\Lambda} - \varepsilon_s^{\Lambda} \approx 11 \text{ MeV}$.

The bump at $E_x \approx 8 \text{ MeV}$ in the cross sections of ${}^{10}_{\Lambda}\text{Be}$ corresponds to the $p^{\Lambda}_{//}$ -state.

In the cross sections of ${}^{11}_{\Lambda}$ B, the small 5/2⁻ peak at $E_x \approx 9$ MeV corresponds to the $p^{\Lambda}_{\prime\prime}$ -state.

The energy splitting between p_{\perp} - and $p_{//}$ states in ${}^{11}_{\Lambda}B$ is smaller than that in ${}^{9}_{\Lambda}Be$, which is due to the small deformation of the nuclear core in ${}^{11}_{\Lambda}B$.

Mar. 24, 2021





In the spherical nuclear core, p^{Λ} -state does not split into $p^{\Lambda}_{//}$ and p^{Λ}_{\perp} .

The new type wave function should appear in ${}^{9,10}_{\Lambda}$ Be and ${}^{10,11}_{\Lambda}$ B due to the core deformation, but "not" in spherical systems without enough deformation.

A = 9 and T = 1 hypernucleus, ${}_{\Lambda}^{9}$ Li

Mar. 24, 2021

Results : Energy levels of 8 Be (8 Li) and ${}^{9}_{\Lambda}$ Be (${}^{9}_{\Lambda}$ Li)





Mar. 24, 2021

Results : Cross sections of the ⁹Be (γ, K^+) ⁹Li reaction



44

A = 11 and T = 1 hypernucleus, ${}^{11}_{\Lambda}Be$

Mar. 24, 2021

Results : Energy levels of ^{10}B (^{10}Be) and $^{11}_{\Lambda}B$ ($^{11}_{\Lambda}Be$)



46

Mar. 24, 2021

Results : Cross sections of the ¹¹B (γ, K^+) ¹¹Be reaction (1)



Mar. 24, 2021



Summary

Summary

We have calculated the energy levels and the production cross sections for *p*-shell hypernuclei by using the extended shell model.

- Strong coupling between *p*-state Λ and core deformation is realized in ${}^{9,10,11}_{\Lambda}\text{Be}$ and ${}^{10,11}_{\Lambda}\text{B}$.
- In these nuclei, p^{Λ} -state splits into $p^{\Lambda}_{/\!/}$ and p^{Λ}_{\perp} .
- In ${}^{10}_{\Lambda}$ Be, the lower $p_{//}^{\Lambda}$ comes down in energy and $[{}^{9}\text{Be}(J^{-}) \times \Lambda(p_{//})]$ couples easily with $[{}^{9}\text{Be}(J^{+}) \times \Lambda(s)]$.
- Such new type wave function should appear in ${}^{9,10,11}_{\Lambda}Be$ and ${}^{10,11}_{\Lambda}B$ due to the core deformation.

The finding of peak #a in ¹⁰B $(e, e'K^+)$ ¹⁰_ABe is a novel evidence for genuine hypernuclear wave function with parity-mixing realized in "deformed" hypernuclei.

Detailed analysis for ${}^{9}_{\Lambda}$ Li and ${}^{11}_{\Lambda}$ Be with T=1 are now in progress.