A. Pukhov, D. an der Brügge
Institute for Theoretical Physics I
University of Dusseldorf, Germany

Coherent Short Wavelength Radiation: Relativistic High Harmonics from Overdense Plasmas

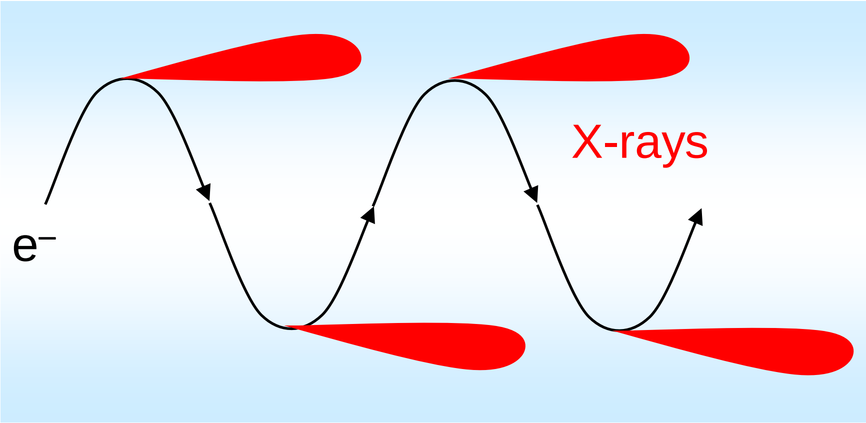
EMMI-Workshop, GSI Darmstadt, 2011

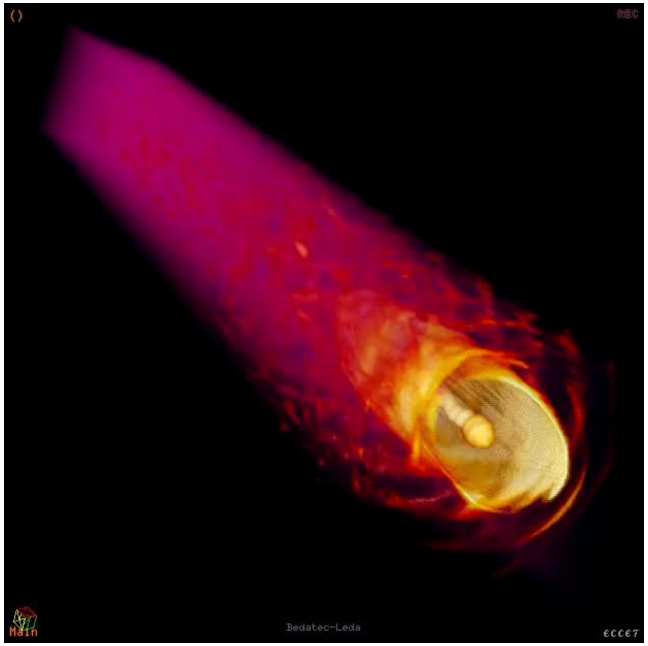
Outline

- **X-ray emission by relativistic electrons**
- **Relativistic harmonics and attosecond pulses from overdense plasmas, spectrum $n^{-8/3}$**
- **Coherent Synchrotron Emission (CSE) spectrum $n^{-6/5}$**
- **Spectral modulations and the femtosecond plasma surface dynamics**

Synchrotron radiation from the Laser-Plasma Bubble

see Kiselev, Pukhov, Kostyukov, Phys. Rev. Lett. **93**, 135004 (2004)

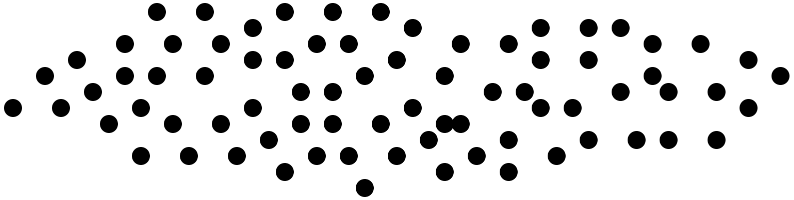




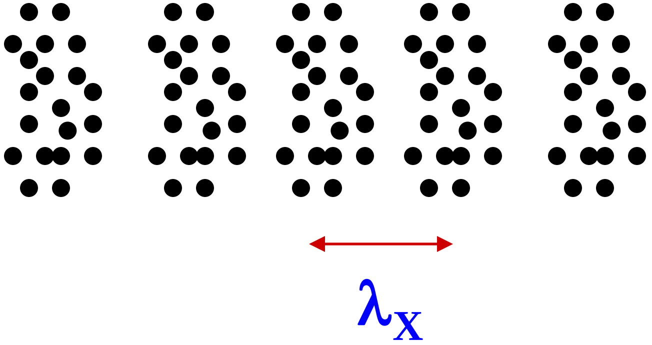
Electron moving on a circular orbit in the ultrarelativistic limit emits synchrotron radiation (high harmonics) when **$\Delta\theta>1/\gamma$.**

Incoherent vs coherent emission

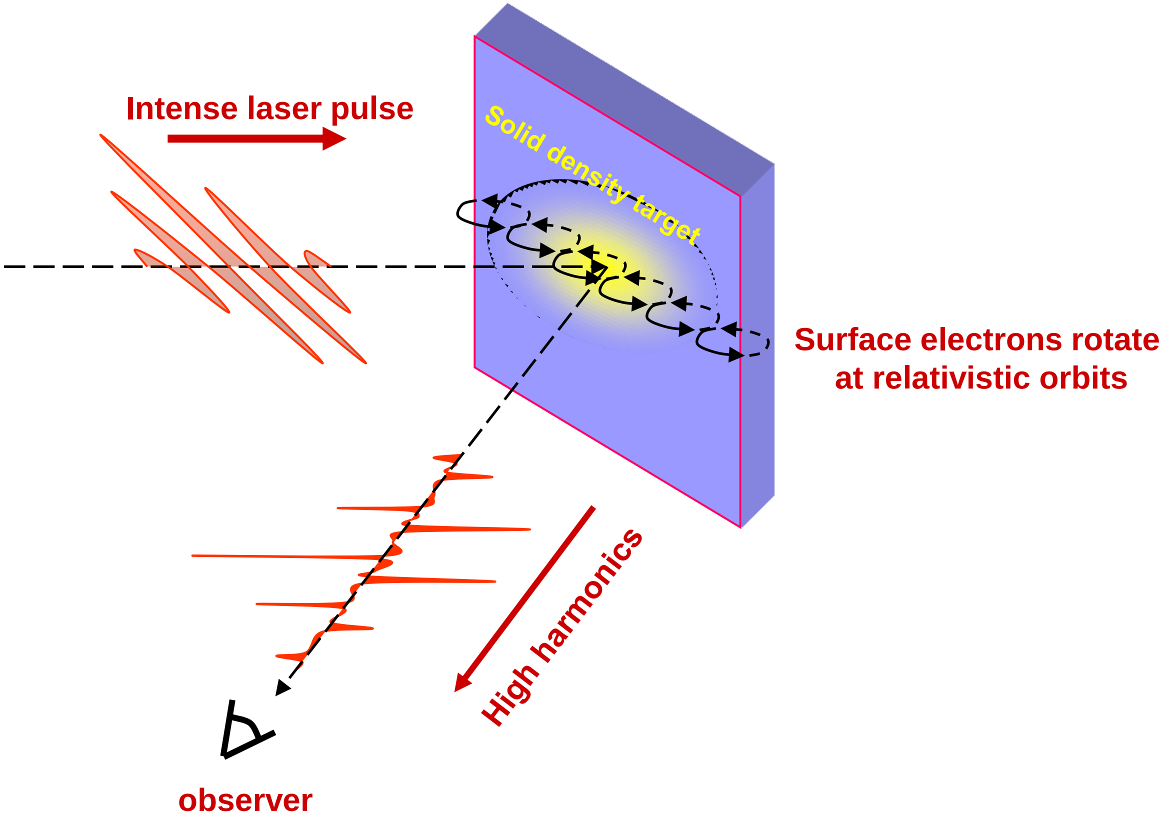
Unbunched electrons: incoherent emission: N_{ph}~N_e.



Electrons bunched at λ_X: coherent emission: N_{ph}~N_e^2.

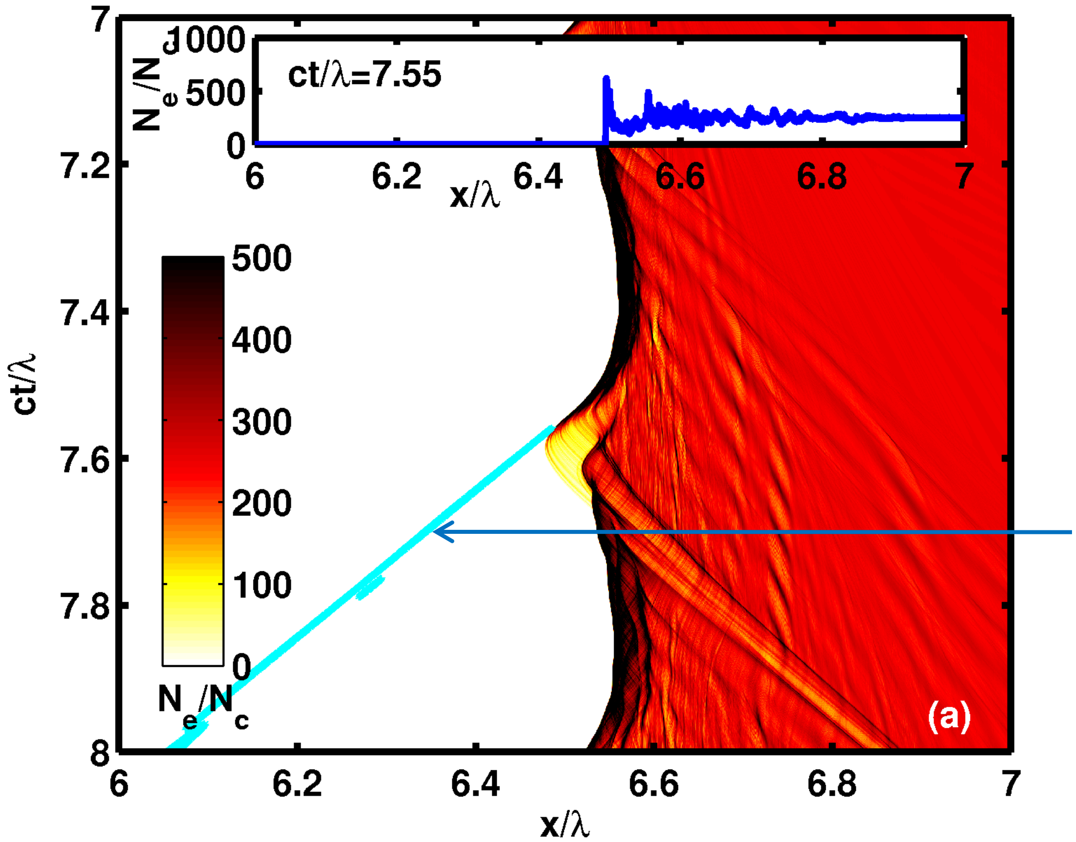


Relativistic High Harmonics from solid state target surfaces



pukhov@tp1.uni-duesseldorf.de

Relativistic Harmonics



Parameters:

$N_e = 250\ N_c$

step profile

a_0=60

Harmonic emission

pukhov@tp1.uni-duesseldorf.de

Analytic description of HHG

Wave equation

$$-\frac{1}{c^2}\partial_t^2\mathbf{A}(t,x)+\partial_x^2\mathbf{A}(t,x)=-\frac{4\pi}{c}\mathbf{j}_\perp(t,x)$$

Solution

$$\mathbf{A}(t,x)=4\pi\int_{-\infty}^{+\infty}\mathbf{j}_\perp(t',x')G(t,x,t',x')dt'dx'$$

Green function satisfying boundary conditions

$$G(t,x,t',x')=\frac{1}{2}\left[\theta\left(t-t'-\frac{|x-x'|}{c}\right)-\theta\left(t-t'-\frac{x-x'}{c}\right)\right]$$

pukhov@tp1.uni-duesseldorf.de

Analytic description of HHG

Electric field

$$\mathbf{E}_\perp(t,x)=\frac{2\pi}{c}\int_x^{+\infty}\left[\mathbf{j}_\perp\left(\underset{\uparrow}{t+\frac{x-x'}{c}},x'\right)-\mathbf{j}_\perp\left(\underset{\uparrow}{t-\frac{x-x'}{c}},x'\right)\right]dx'$$

Reflected wave

Incoming wave

So that at the left of plasma $\mathbf{E}_\perp(t,x)=\mathbf{E}_i(t-\frac{x}{c})+\mathbf{E}_r(t+\frac{x}{c})$

pukhov@tp1.uni-duesseldorf.de

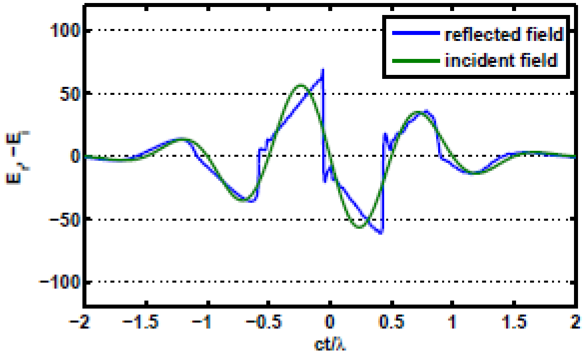
The ROM boundary condition

Baeva, Gordienko, Pukhov, *Phys. Rev. E***74**, 046404 (2006)

- Skin layer evolution time long compared to skin length (e.g. step-like profile), then follows ARP boundary condition

$$E_i\left(x_{ARP}(t)-ct\right)+E_r\left(x_{ARP}(t)+ct\right)=0$$

- Reflected field phase modulation of incident field
  - same maxima and minima
  - same sequence of monotonic intervals



Normal incidence, $N_e = 250\, N_c$, sharp edged profile, $a_0 = 60$

pukhov@tp1.uni-duesseldorf.de

Analytical derivation of the BGP spectrum

Fourier transform of the reflected wave

$$E_r(\omega) = -\int E_i\left(t - \frac{x_{ARP}}{c}\right) e^{i\omega(t + x/c)} \left(1 + \frac{\dot{x}_{ARP}}{c}\right) dt$$

We assume for the incident wave $E_i(t) = a(t)\left[e^{i\omega_0 t} + e^{-i\omega_0 t}\right]/2$

The reflected wave is thus $E_r(\omega) = E_+ + E_-$

$$E_\pm(\omega) = -\int a\left(t - \frac{x_{ARP}}{c}\right) e^{i\left[\omega\left(t + \frac{x_{ARP}}{c}\right) \pm \omega_0\left(t - \frac{x_{ARP}}{c}\right)\right]} \left(1 + \frac{\dot{x}_{ARP}}{c}\right) dt$$

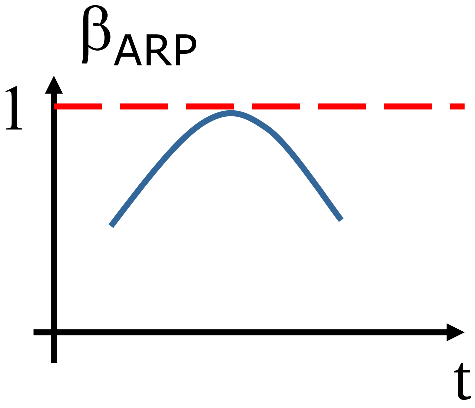
pukhov@tp1.uni-duesseldorf.de

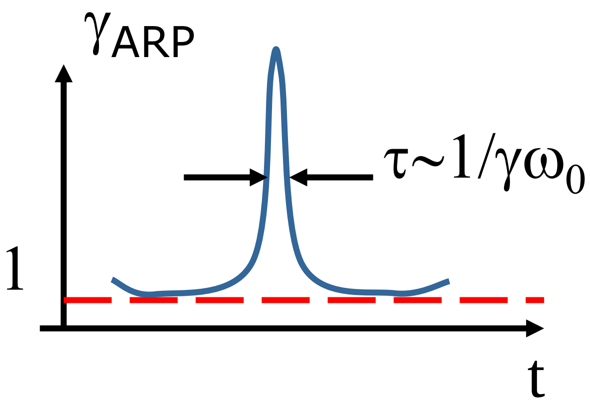
Stationary points and the γ-spikes

The stationary phase points correspond to the instants when the apparent reflection point (ARP) moves towards the observer with maximum velocity.

The corresponding ARP gamma factor

$$\gamma_{\text{ARP}} = \left(1 - \dot{x}_{\text{ARP}}^2\right)^{-1/2}$$



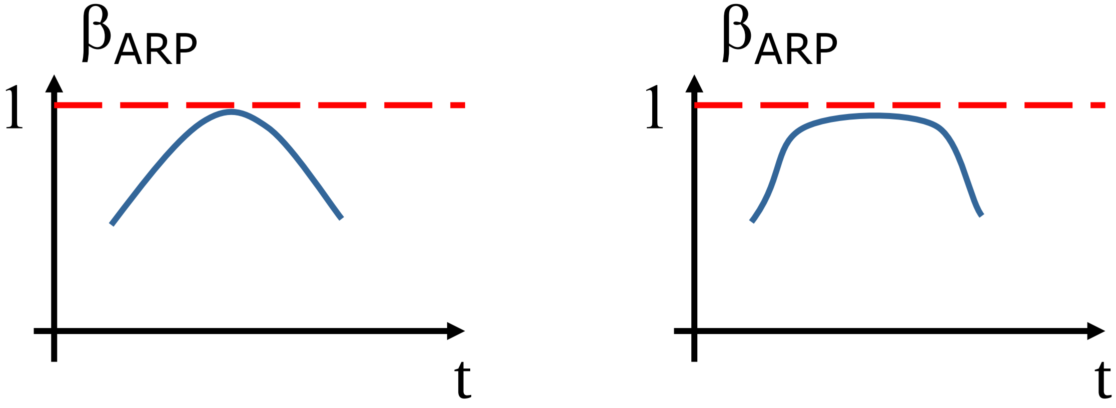


Order of the γ-spikes

The spectrum depends on the exact behavior of the ARP in the neighborhood of the γ-spike.

The 2*n*-th order maximum of the surface velocity

$$\frac{d^k}{dt^k} x_{\mathrm{ARP}} = 0, \quad \text{for all } 1 < k < 2n - 1$$



pukhov@tp1.uni-duesseldorf.de

General form of the spectrum

The spectrum can be calculated for an arbitrary order *n* of the surface velocity maximum

$$I_{\mathrm{n}}(\omega) \propto \omega^{-\frac{4n+4}{2n+1}} \left[\mathrm{Gai}_n\left(\frac{\omega\gamma^{-2} - 4\omega_0}{2(\alpha\omega)^{1/(2n+1)}} \right) - \mathrm{Gai}_n\left(\frac{\omega\gamma^{-2} + 4\omega_0}{2(\alpha\omega)^{1/(2n+1)}} \right) \right]^2$$

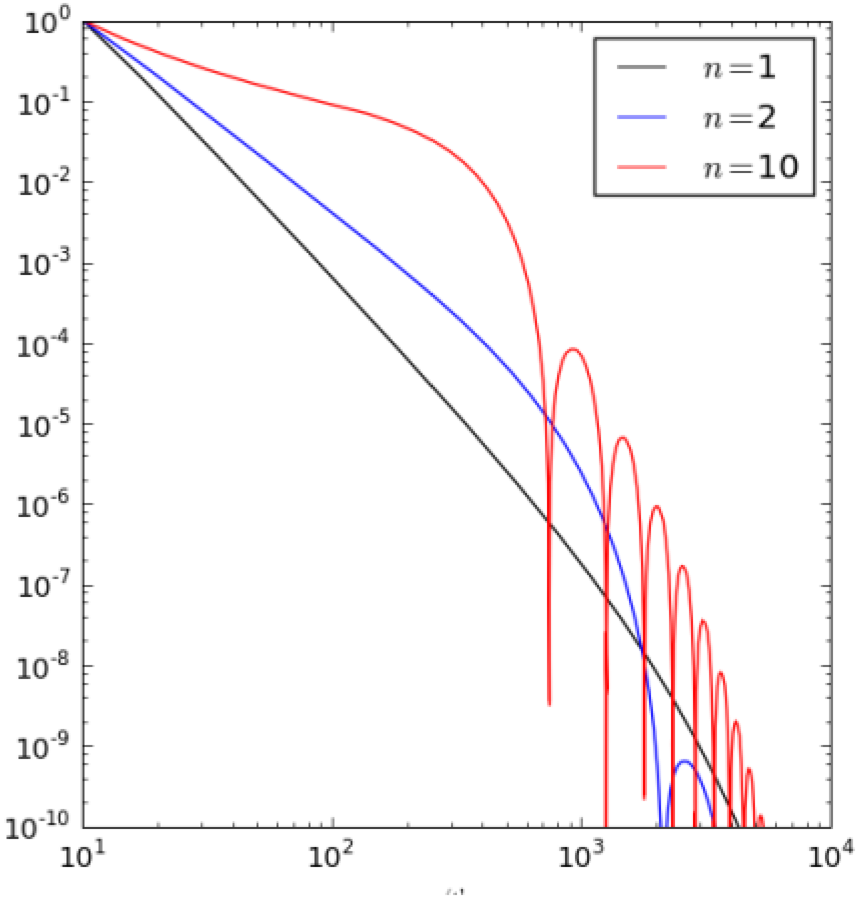
Where $\mathrm{Gai}_{\mathrm{n}}$ is the generalized Airy function:

$$\mathrm{Gai}_{\mathrm{n}}(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp\left[i\left(xt + t^{2n+1}/(2n+1)\right)\right] dt$$

pukhov@tp1.uni-duesseldorf.de

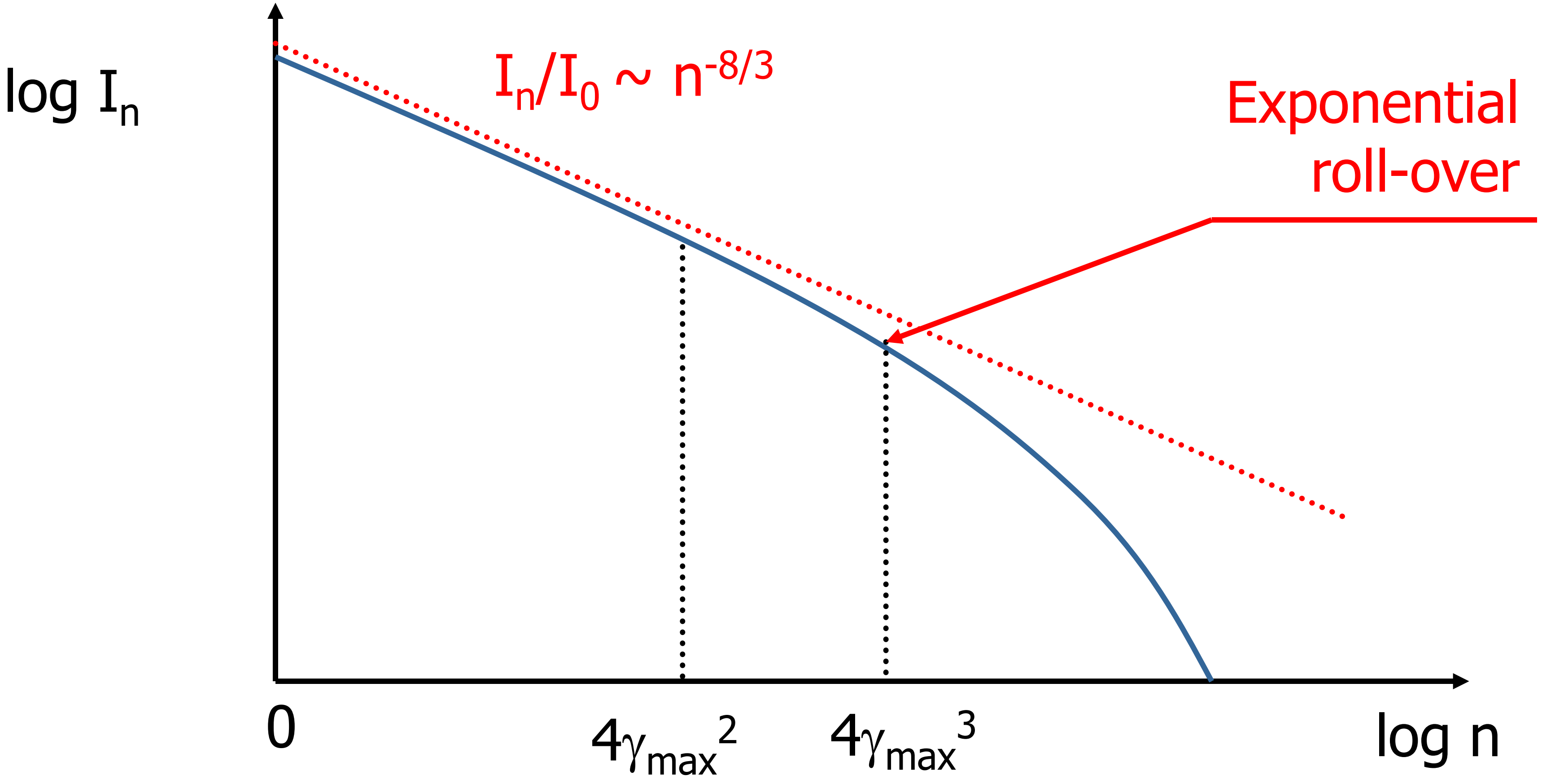
General form of the spectrum

The spectrum shape
for orders $n=1,2,10$
of the surface
velocity maximum.
$\alpha=1$, $\gamma=8$ have been used



The BGP Spectrum

Baeva, Gordienko, Pukhov, *Phys. Rev. E* **74**, 046404 (2006)

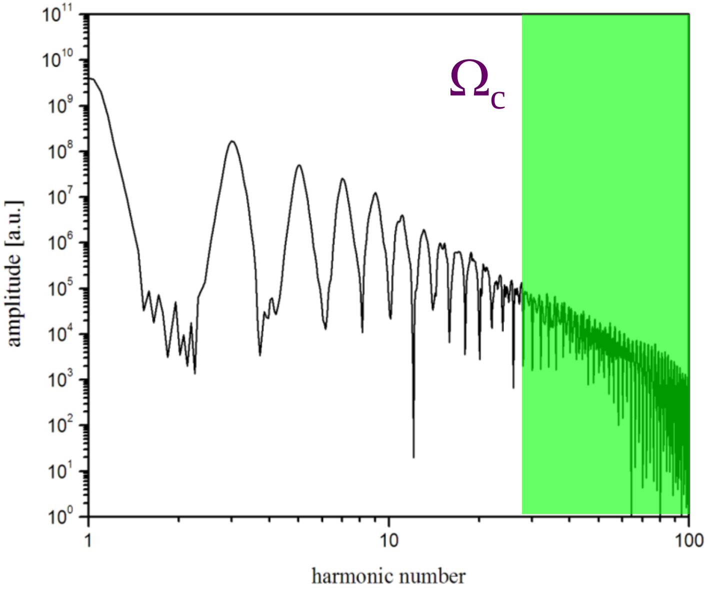


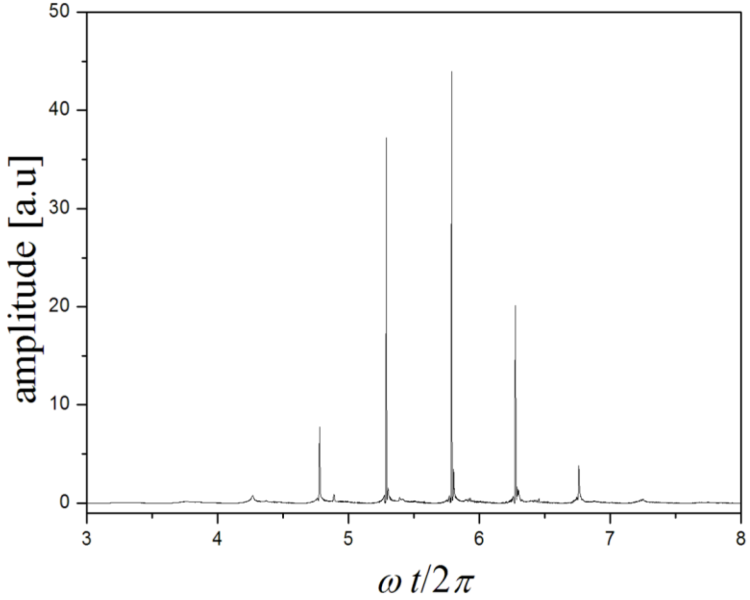
pukhov@tp1.uni-duesseldorf.de

(Sub-)Attosecond Pulses

Baeva, Gordienko, Pukhov, *Phys. Rev. E* **74**, 046404 (2006)

- After proper filtering of RHHG one obtains *a train of (sub-)attosecond pulses*

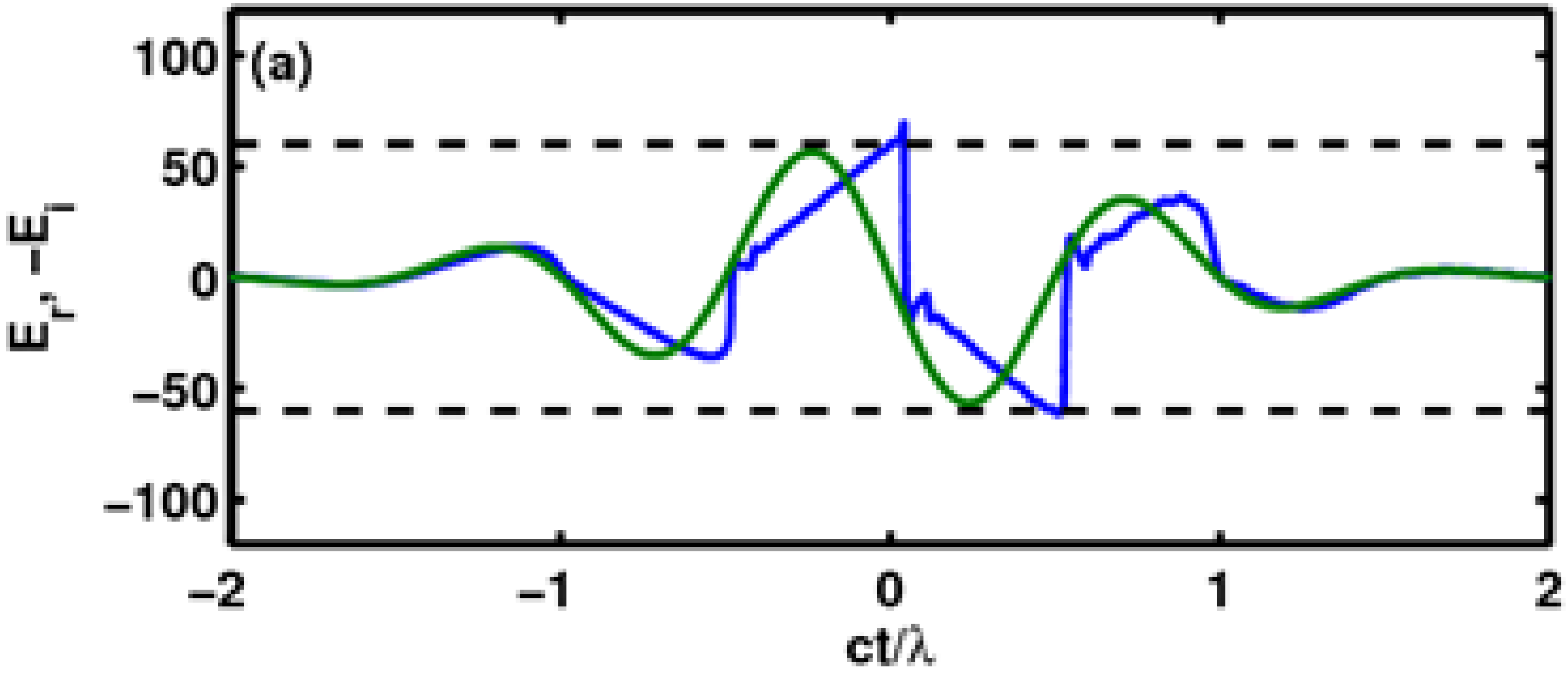




pukhov@tp1.uni-duesseldorf.de

How universal is the -8/3 spectrum?

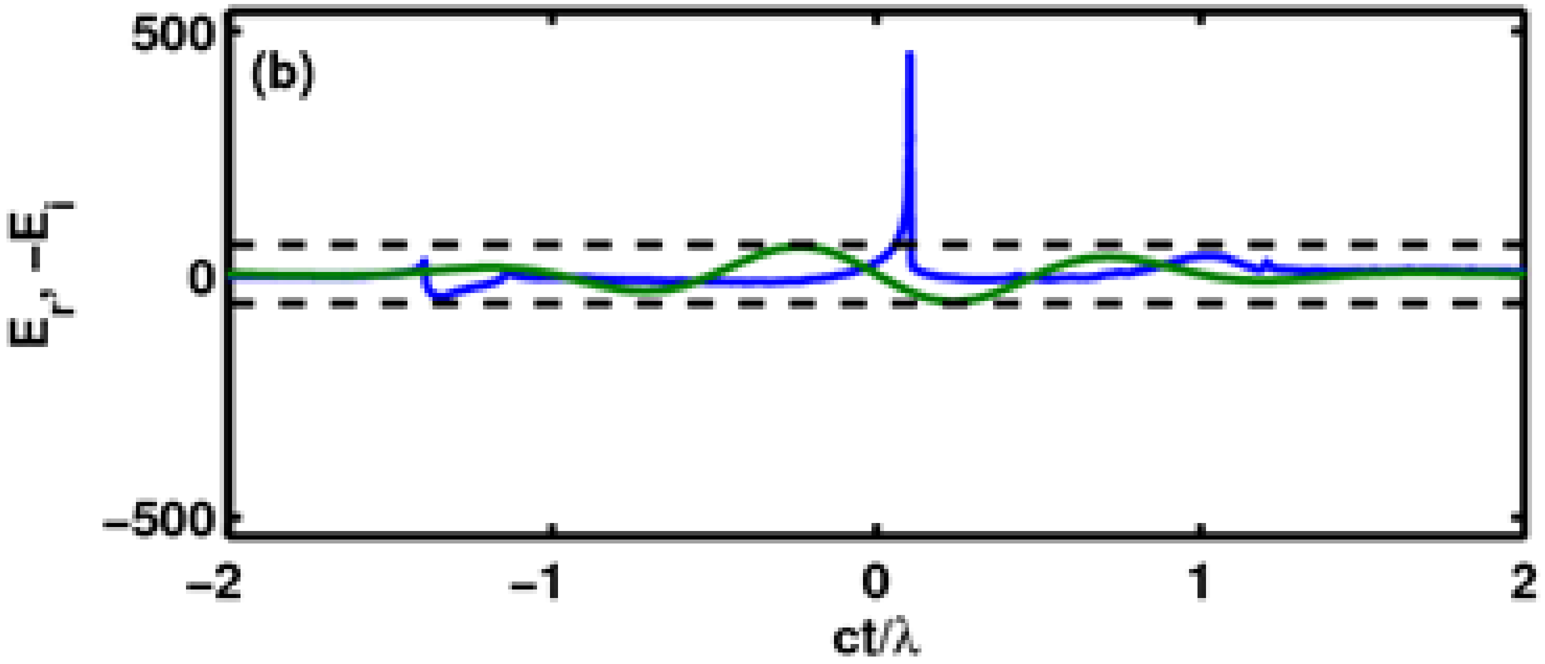
BGP case: pure phase modulation of the laser field



pukhov@tp1.uni-duesseldorf.de

Giant single attosecond pulse

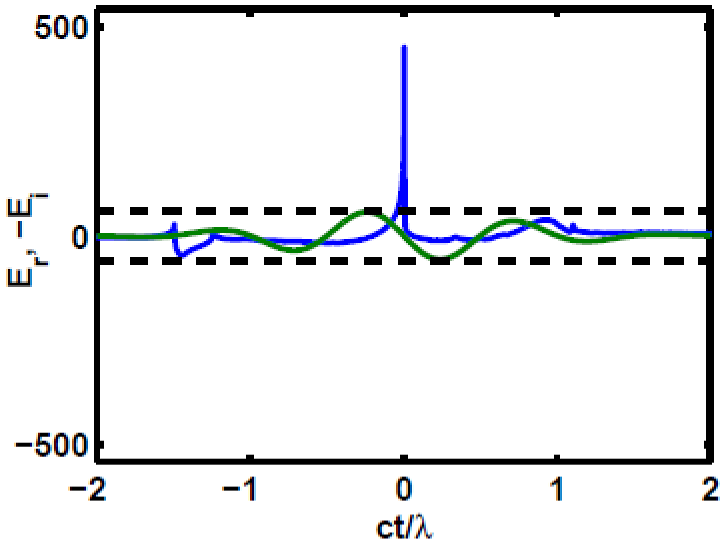
63° oblique incidence, selected CEP and density gradient.

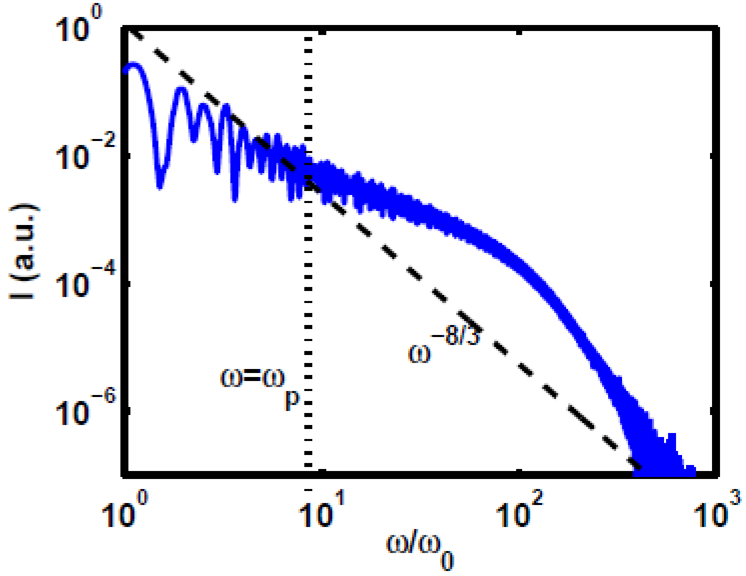


pukhov@tp1.uni-duesseldorf.de

Violation of the boundary condition. Much flatter spectrum, super intense isolated pulse

Simulation parameters: plasma density ramp $\propto \exp(x/(0.33\,\lambda))$ up to a maximum density of $N_e = 95\,N_c$ (lab frame), oblique incidence at 63° angle (p-polarised). Laser field amplitude is $a_0 = 60$.



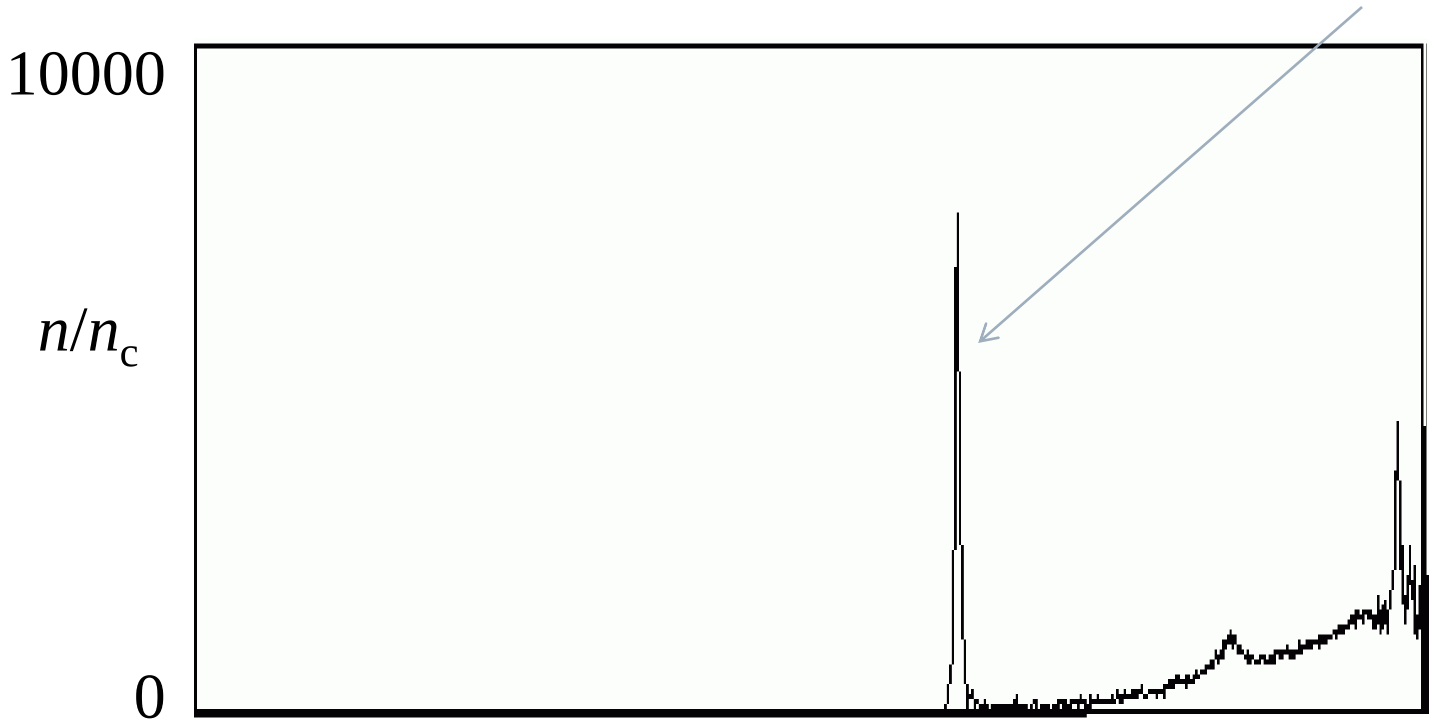


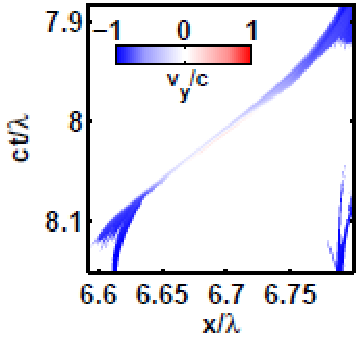
- Violation of ARP boundary condition $\Longrightarrow$ no ROM
- No spectral cut-off at plasma frequency $\Longrightarrow$ no CWE

pukhov@tp1.uni-duesseldorf.de

DadB, Pukhov, Phys. Plasmas 17, 033110 (2010)

Nanobunching in electron density





Simulation parameters: $a_0 = 60$, plasma density ramp $\propto \exp(x/(0.33\,\lambda))$, maximum density $N_e = 95\,N_c$, p-polarised incidence at 63°

- 2nd order zero in v_y
- Bunch width: $\delta_{FWHM} \approx 0.0015\,\lambda$, roughly Gaussian profile

Excellent Agreement between Theory and Simulation

- Theoretical Spectrum:

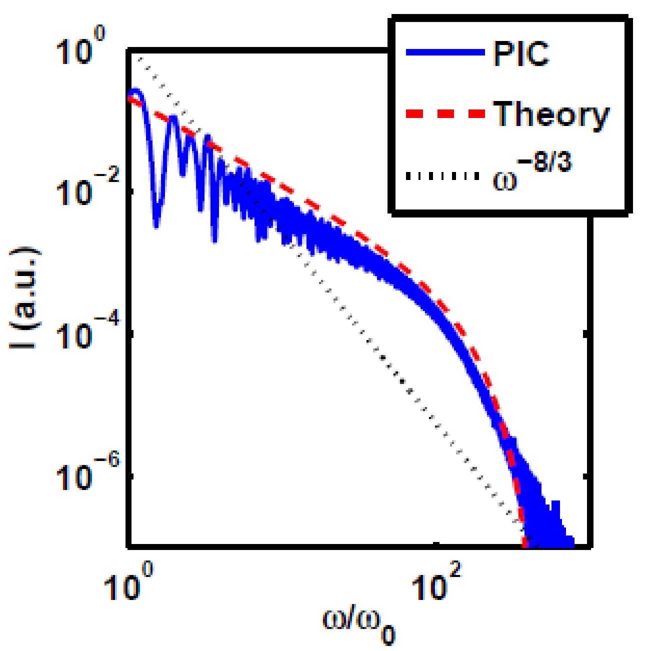
$$I(\omega) \propto \omega^{-6/5} \times \left[S''\left(\left(\frac{\omega}{\omega_{rs}}\right)^{4/5}\right)\right]^2 \times \exp\left(-\left(\frac{\omega}{\omega_{rf}}\right)^2\right)$$

using

$\omega_{rs} = 800$

$\omega_{rf} = 225$

compatible with PIC data



Two relativistic regimes of HHG

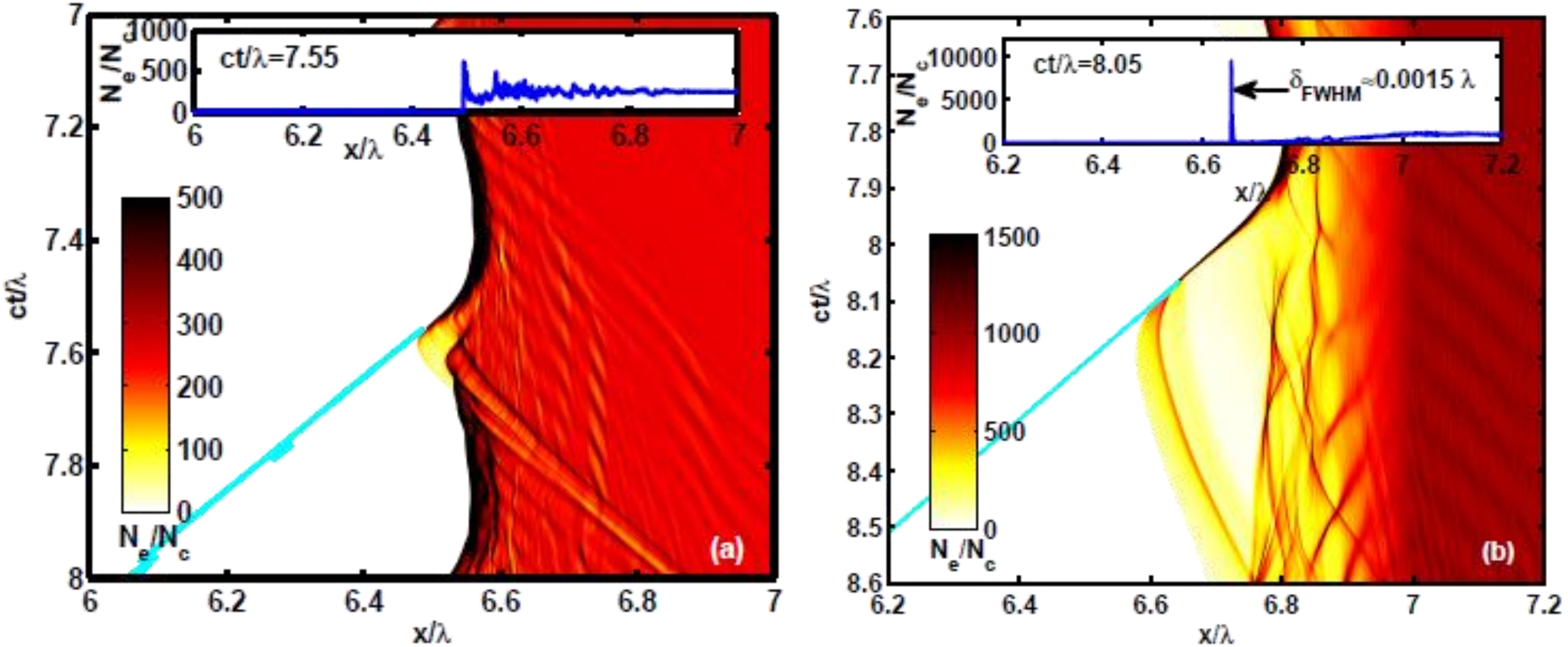
1. BGP case:
   plasma boundary stays “conjunct”
   the skin layer emits as a whole
   → the universal **$n^{-8/3}$** spectrum

2. Nanobunching of plasma electrons,
   Coherent Synchrotron Emission (**CSE**)
   → much flatter spectra, **$n^{-4/3}$ or $n^{-6/5}$**

DadB, Pukhov, Phys. Plasmas 17, 033110 (2010)

pukhov@tp1.uni-duesseldorf.de

BGP vs CSE case

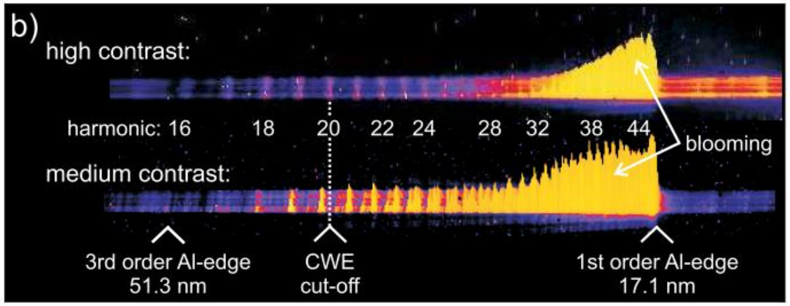


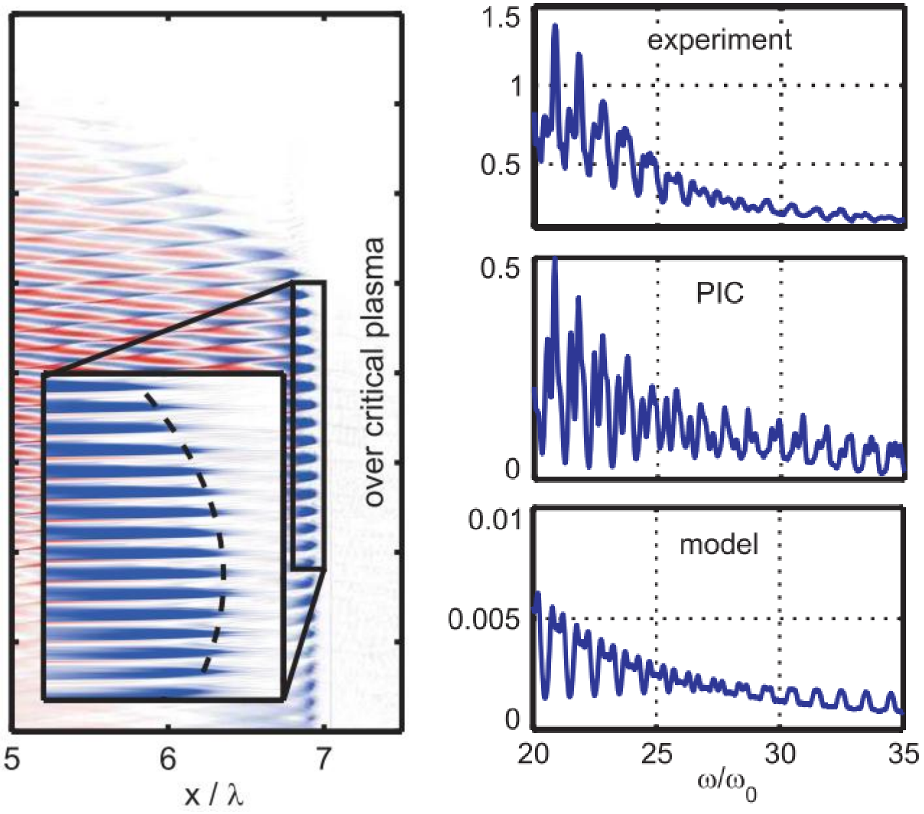
Pictures: electron density and contour lines (cyan) of the emitted harmonics radiation ($\omega > 4.5\omega_0$).
Simulation parameters: $a_0 = 60$ in both cases, (a) normal incidence, $N_e = 250\,N_c$, sharp edged profile; (b) plasma density ramp $\propto \exp(x/(0.33\,\lambda))$, maximum density $N_e = 95\,N_c$, p-polarised incidence at 63°

pukhov@tp1.uni-duesseldorf.de

HHG experiment on D’Arcturus

M. Behmke et al., *Phys. Rev. Lett.*, accepted (2011)





The harmonic spectrum contains information on the femtosecond dynamics of relativistic plasma

pukhov@tp1.uni-duesseldorf.de

Summary

- High harmonics from solid surfaces may become the very bright coherent source of XUV and X-rays

- The spectrum is a power law with an exponential roll over. The exponent is
  p=-8/3 for BGP spectrum
  p=-6/5 for CSE spectrum

- Giant intensity attosecond pulse is generated due to CSE.

- Spectral modulations encode the femtosecond plasma surface dynamics