High-energy photons from relativistic electrons in ultraintense laser fields

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Outline

Radiation friction in the Landau-Lifschitz approach
- equation of relativistic electron motion

Numerical integration and analytical estimates in cases with a test electron in the field(s) of a:

- single running wave
- standing wave
- counterpropagating identical waves
- laser piston

Spectral characteristics

Concluding remarks

Radiation friction of relativistic electrons

Equation of motion:
(in cgs units)

$$m_e c \frac{du^i}{ds} = -\frac{e}{c} F^{ik} u_k + g^i$$

Self force: $$g^i = \frac{2e^3}{3m_e c^3} \frac{\partial F^{ik}}{\partial x^l} u_k u^l - \frac{2e^4}{3m_e^2 c^5} F^{il} F_{kl} u^k + \frac{2e^4}{3m_e^2 c^5} \left(F_{kl} u^l\right)\left(F^{km} u_m\right) u^i$$

$$x^l = \left(ct, \vec{x}\right)$$
$$u^l = \left(\gamma, \gamma\vec{\beta}\right)$$

$$m_e c \gamma \frac{d\left(\gamma, \gamma\vec{\beta}\right)}{cdt} = -\frac{e}{c} F^{ik} u_k + g^i$$

Validity criteria: $$\lambda >> \frac{e^2}{m c^2} = r_e \sim 10^{-13}\,\mathrm{cm},\ E << \frac{m^2 c^4}{e^3} = \frac{e}{r_e^2} \sim 10^{18} \frac{\mathrm{V}}{\mathrm{cm}} \Rightarrow a \sim 10^7$$

from quantum theory: $$\lambda >> \lambda_c = \frac{\hbar}{m c} \sim 10^{-11}\,\mathrm{cm},\ E << \frac{m^2 c^3}{\hbar e} \sim 10^{15} \frac{\mathrm{V}}{\mathrm{cm}} \Rightarrow a \sim 10^5$$

Normalized equations

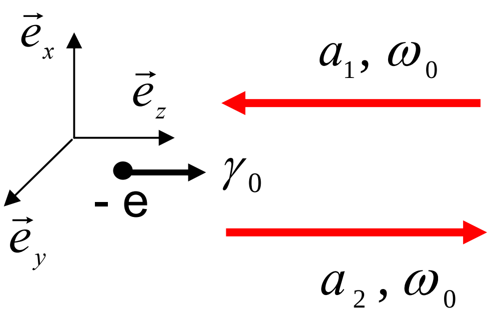
Three-vector momentum:

$$\frac{d\left(\vec{\beta}\gamma\right)}{dt} = -\vec{F}_L + \frac{2}{3}\tau_R\left\{\gamma\frac{d\vec{E}}{dt} + \vec{u}\times\frac{d\vec{H}}{dt} + \left(\vec{\beta}\vec{E}\right)\vec{E} + \vec{F}_L\times\vec{H} + \gamma^2\vec{\beta}\left[\left(\vec{\beta}\vec{E}\right)^2 + \vec{F}_L^2\right]\right\}$$

Energy balance:

$$\frac{d\gamma}{dt} = -\vec{\beta}\vec{E} + \frac{2}{3}\tau_R\left\{\gamma\vec{\beta}\frac{d\vec{E}}{dt} + \vec{F}_L\vec{E} + \gamma^2\left[\left(\vec{\beta}\vec{E}\right)^2 + \vec{F}_L^2\right]\right\}, \qquad \tau_R = k_0 r_e \sim 10^{-8}$$

E-m waves:



circular polarization

$$\vec{a}_1 = a_0\left(\vec{e}_x\sin\varphi_1 + \vec{e}_y\cos\varphi_1\right) \qquad \varphi_1 = t + z$$

$$\vec{a}_2 = a_0\left(\pm\vec{e}_x\sin\varphi_2 - \vec{e}_2\cos\varphi_2\right) \qquad \varphi_2 = t - z$$

$+$ single wave comoving with the electron; 2 waves with opposite directions

$-$ standing wave

Spectral characteristics

Totally emitted energy and power:

$$\Delta\varepsilon_{\text{rad}} = \frac{2}{3}\tau_R\gamma^2\int_{-\infty}^{\infty}\left[\left(\frac{d\vec{u}}{dt}\right)^2 - \left(\frac{d\gamma}{dt}\right)^2\right]dt\,, \quad \text{in units } m_e c^2$$

$$P_{\text{rad}} = \frac{d\varepsilon_{\text{rad}}}{dt}, \quad \text{in units } m_e c^2\omega_0$$

Total radiation power in a running wave:

$$P_{\text{rad}} = \frac{2}{3}\tau_R a_0^2 h_0^2, \quad h_0 = \sqrt{\gamma_0^2 + a_0^2} \pm \beta_0\gamma_0$$

\+ counterpropagating wave
\- comoving wave

Spectral intensity

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2\omega_0^2}{4\pi^2 c^2}\left|\int_{-\infty}^{\infty}\left[\vec{n}\times\left(\vec{n}\times\vec{\beta}\right)\right]\, e^{i\omega_0(t-\vec{n}\vec{r}/c)}dt\right|^2, \quad \text{in cgs units}$$

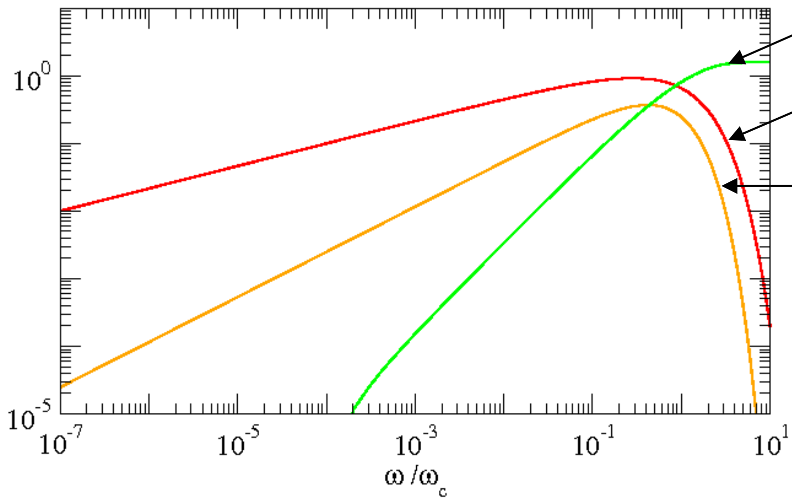
Spectrum cutoff

Ultraintense field $a_0 >> 1$: large number of closely spaced harmonics $n >> 1$

critical number $n_c \approx 3a_0^3/2\sqrt{2}$

and corresponding critical frequency $\omega_c = n_c \frac{M_0+1}{2}\omega_0, \; M_0 = \frac{h_0^2}{1+a_0^2}$

total radiated energy: $\Delta\varepsilon_{\text{rad}} \approx \sqrt{3}N_0 \frac{e^2\gamma\omega_c}{2c} \int_0^\infty d\left(\frac{\omega}{\omega_c}\right)\frac{\omega}{\omega_c}\int_{\omega/\omega_c}^\infty d\xi\, K_{5/3}(\xi)$



$$\left(\frac{\omega}{\omega_c}\right)^2 K_{2/3}^2(\omega/\omega_c) \sim \frac{d^2 I}{d\omega\, d\Omega}$$

$a_0 = 100, \; \gamma = 300$:

$n_c \approx 10^6, \; \omega_c \approx 2\times 10^7 \omega_0$

$\Delta\varepsilon_{\text{rad}} \approx 2.4\,\text{GeV} \quad (N_0 = 22)$

Analytical estimates for electron motion

Equations with a simplified friction term:

$$\frac{d\left(\vec{\beta}_\perp \gamma - \vec{a}\right)}{dt} = \frac{2}{3}\tau_R\, \vec{\beta}_\perp \gamma^2 R$$

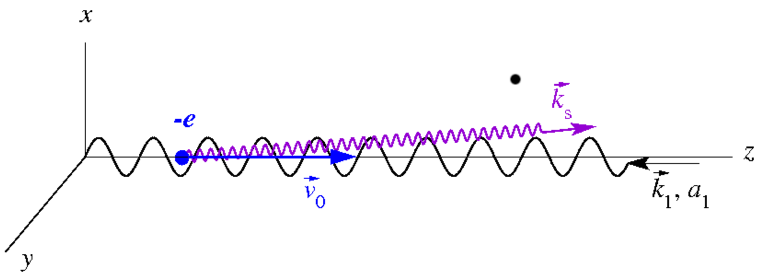
$$\frac{du_z}{dt} = -\vec{\beta}_\perp \frac{\partial \vec{a}}{\partial z} + \frac{2}{3}\tau_R\, \beta_z \gamma^2 R$$

$$\frac{d\gamma}{dt} = \vec{\beta}_\perp \frac{\partial \vec{a}}{\partial t} + \frac{2}{3}\tau_R\, \gamma^2 R$$

$$R = -\left[\left(\frac{d\vec{a}}{dt}\right)^2 + \left(\vec{\beta}_\perp \frac{\partial \vec{a}}{\partial z}\right)^2 - \left(\vec{\beta}_\perp \frac{\partial \vec{a}}{\partial t}\right)^2\right]$$

Counterpropagating wave

E.-m. field: $\vec{a} = a_0 \left(\vec{e}_x \sin\varphi + \vec{e}_y \cos\varphi \right), \quad \varphi = t + z$



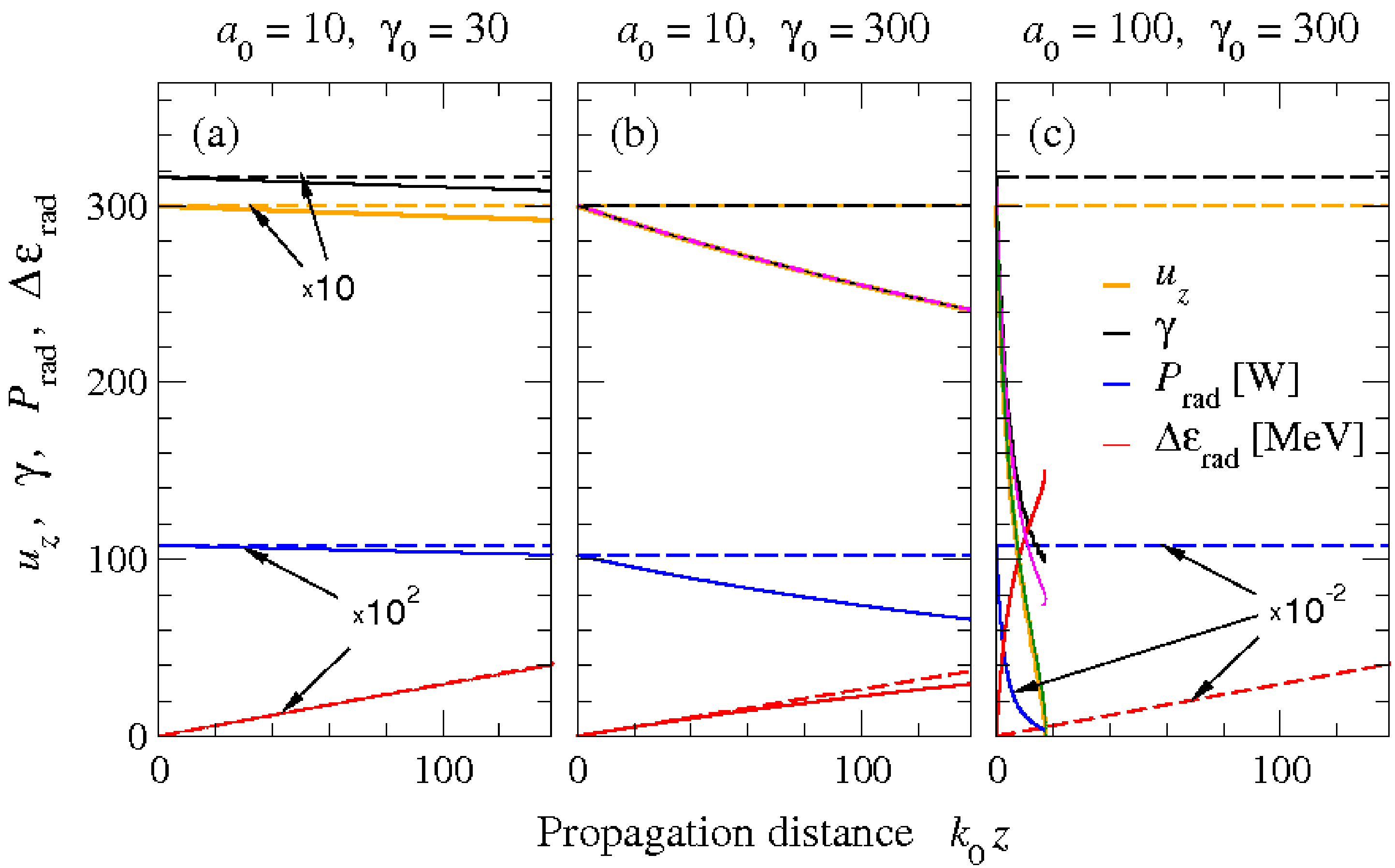
$$a^2 = a_0^2, \quad R = -\left(\frac{d\vec{a}}{dt} \right)^2 = -a_0^2 \left(1+\beta_z\right)^2$$

$$\Delta \vec{u}_\perp = \vec{u}_\perp - \vec{a} = \frac{2}{3} \tau_R \gamma a_0^3 \left(1+\beta_z\right)\left(\vec{e}_x \cos\varphi - \vec{e}_y \sin\varphi\right)$$

$$\frac{du_z}{dt} \approx -\frac{4}{3}\tau_R a_0^4 \left(\frac{2u_z^2}{a_0^2} + 1 \right) \quad \rightarrow \quad u_z = \frac{u_z(0) - \frac{a_0}{\sqrt{2}} \tan(\kappa t)}{1 + u_z(0) \frac{\sqrt{2}}{a_0} \tan(\kappa t)}, \quad \kappa = \frac{4\sqrt{2}}{3} \tau_R a_0^3$$

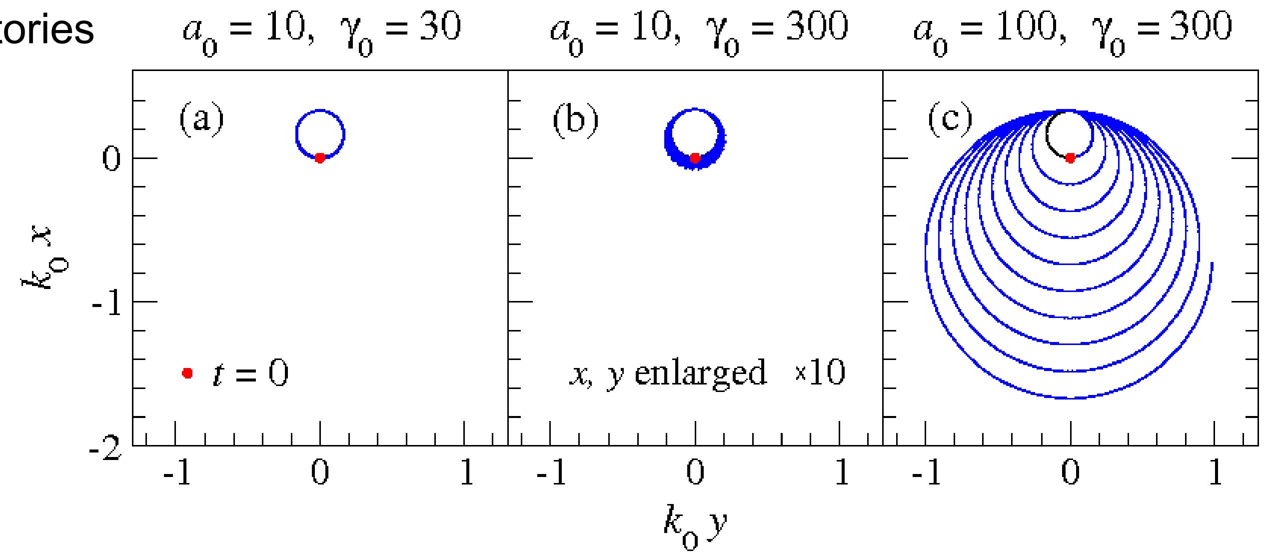
$$\frac{d\gamma}{dt} \approx -\frac{4}{3}\tau_R a_0^4 \left(\frac{2\gamma^2}{a_0^2} - 1 \right) \quad \rightarrow \quad \gamma = \frac{1 + \gamma(0) \frac{\sqrt{2}}{a_0} \coth(\kappa t)}{\frac{2}{a_0^2}\gamma(0) - \frac{\sqrt{2}}{a_0} \coth(\kappa t)}$$

Radiation energy



Electron orbits

Electron trajectories

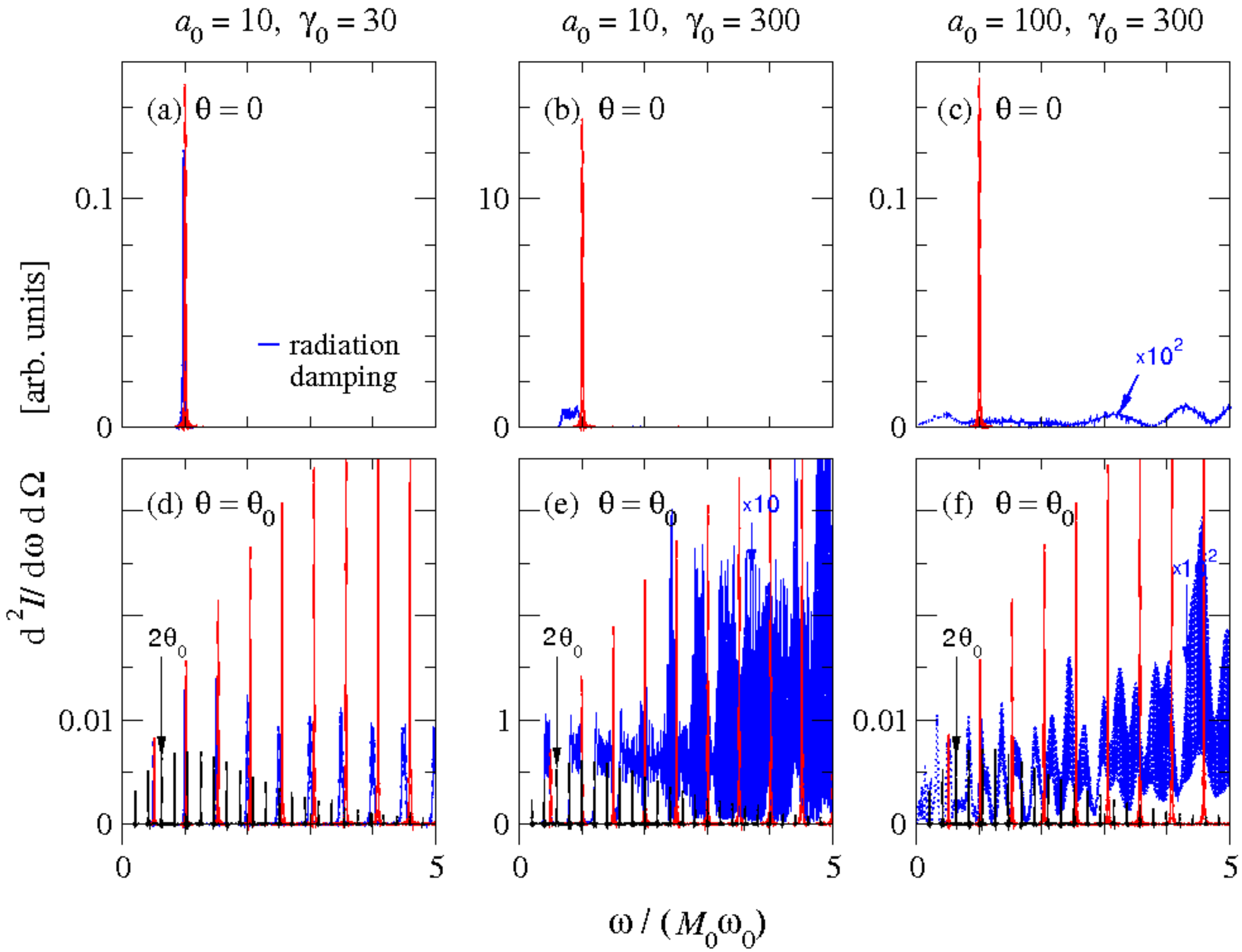


electron proper frame: $a_x = a_0 \sin\varphi',\ a_y = a_0 \cos\varphi',\quad \varphi' = \sqrt{\frac{1+\beta_z}{1-\beta_z}}\, t'$

without rad. friction: $k_0 x(t') = \frac{a_0}{\sqrt{1+a_0^2}} \sqrt{\frac{1-\beta_z}{1+\beta_z}} (1-\cos\varphi'),\ k_0 y(t') = \frac{a_0}{\sqrt{1+a_0^2}} \sqrt{\frac{1-\beta_z}{1+\beta_z}} \sin\varphi'$

$$r_\perp = \frac{a_0}{h_0},\ h_0 = \sqrt{\gamma_0^2 + a_0^2} + \gamma_0 \beta_0$$

Low-energy spectra



Doppler upshift factor:

$$M_0 = \frac{h_0^2}{1 + a_0^2} = \frac{1+\beta_0}{1-\beta_0}$$

Angle of max. radiation:

$$\theta_0 = 2 \big/ \sqrt{M_0}$$

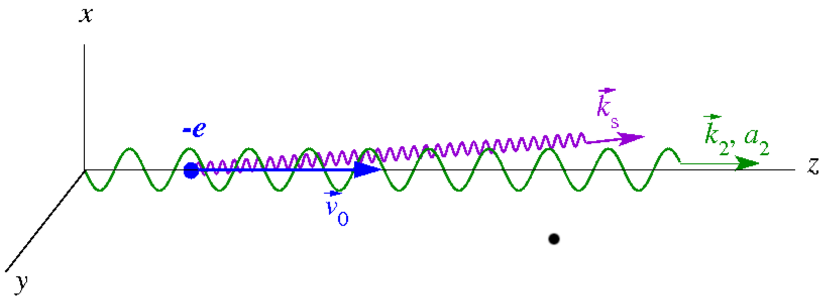
Harmonics:

$$\omega_n = \frac{n M_0 \omega_0}{1 - \beta_1 (1 - \cos\theta)}$$

$$\beta_1 = \frac{M_0 - 1}{2 M_0}$$

Copropagating wave

E.-m. field: $\vec{a} = a_0\left(\vec{e}_x \sin\varphi - \vec{e}_y \cos\varphi\right), \quad \varphi = t - z$



$$a^2 = a_0^2, \quad R = -\left(\frac{d\vec{a}}{dt}\right)^2 = -a_0^2\left(1-\beta_z\right)^2$$

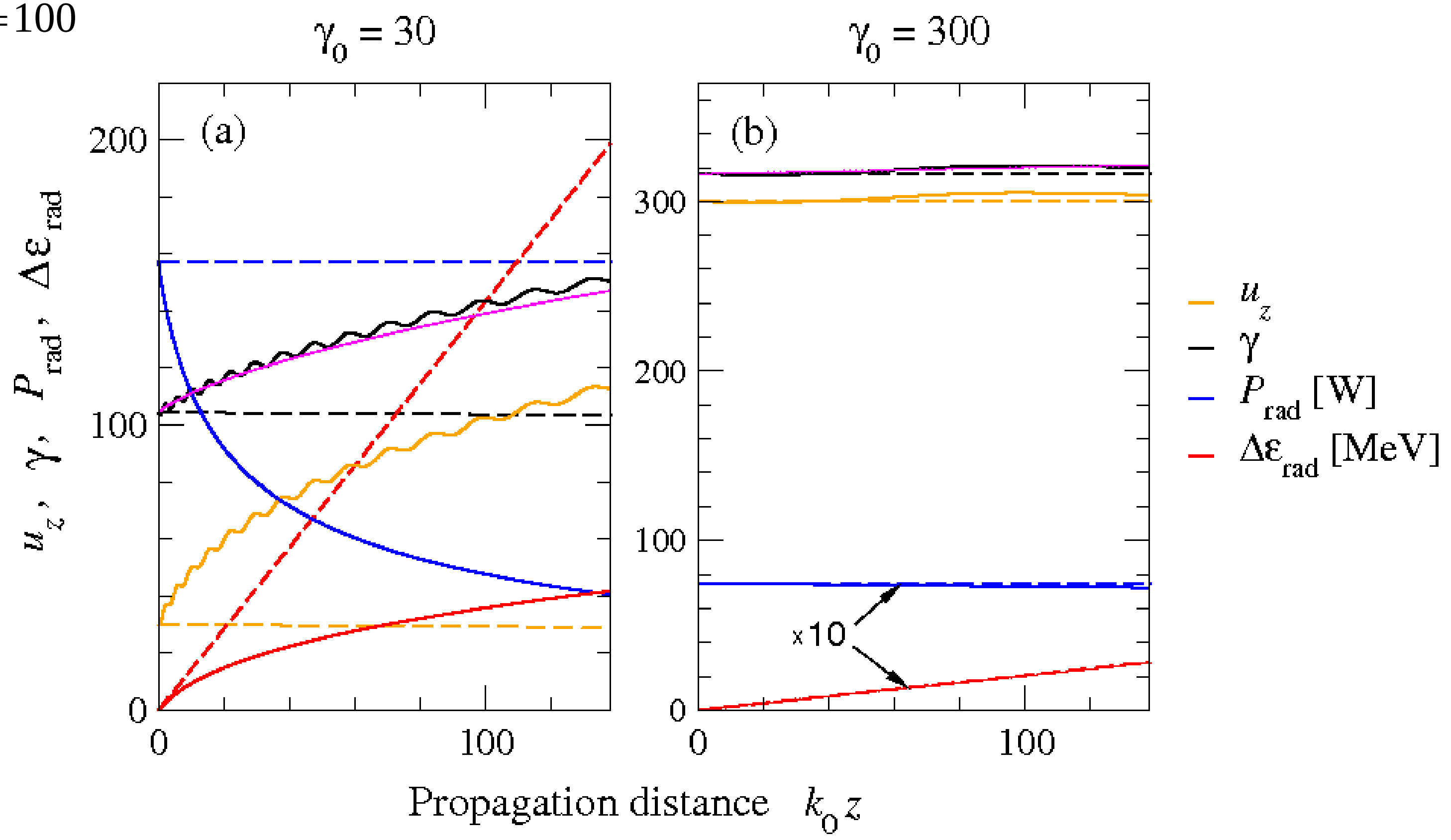
$$\Delta\vec{u}_\perp = \vec{u}_\perp - \vec{a} = \frac{2}{3}\tau_R\,\gamma\,a_0^3\left(1-\beta_z\right)\left(\vec{e}_x\cos\varphi + \vec{e}_y\sin\varphi\right)$$

$$\frac{du_z}{dt} = \frac{2}{3}\tau_R\,a_0^2\left(1-\beta_z\right)^2\gamma^2$$

$$\frac{d\gamma}{dt} = \frac{2}{3}\tau_R\,a_0^2\,\beta_z\left(1-\beta_z\right)^2\gamma^2 \quad \rightarrow \quad \gamma \approx \left[\gamma(0)^3 + \frac{1}{2}\tau_R\,a_0^6\,t\right]^{1/3}$$

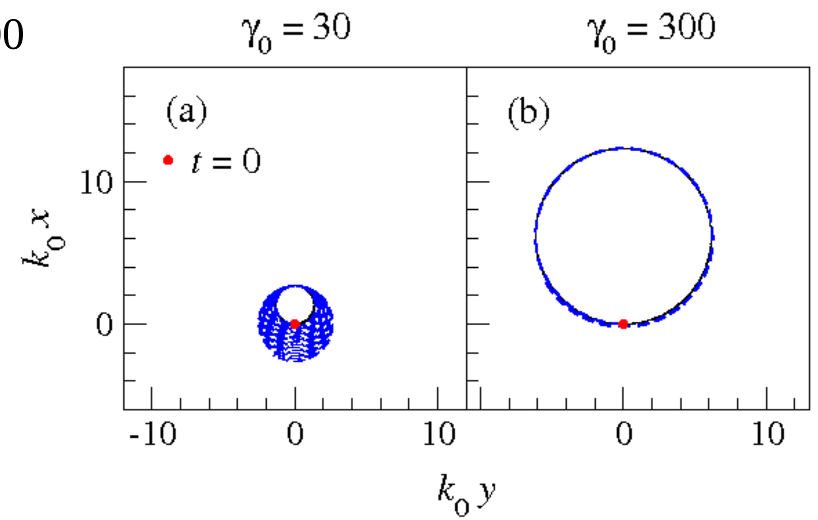
Radiation energy

$a_0 = 100$



Electron orbits

Electron trajectories $a_0 = 100$



electron proper frame: $a_x = a_0 \sin\varphi', \; a_y = a_0 \cos\varphi', \quad \varphi' = \sqrt{\frac{1-\beta_z}{1+\beta_z}}\, t'$

without rad. friction: $k_0 x(t') = \frac{a_0}{\sqrt{1+a_0^2}} \sqrt{\frac{1+\beta_z}{1-\beta_z}} (1-\cos\varphi'), \; k_0 y(t') = \frac{a_0}{\sqrt{1+a_0^2}} \sqrt{\frac{1+\beta_z}{1-\beta_z}} \sin\varphi'$

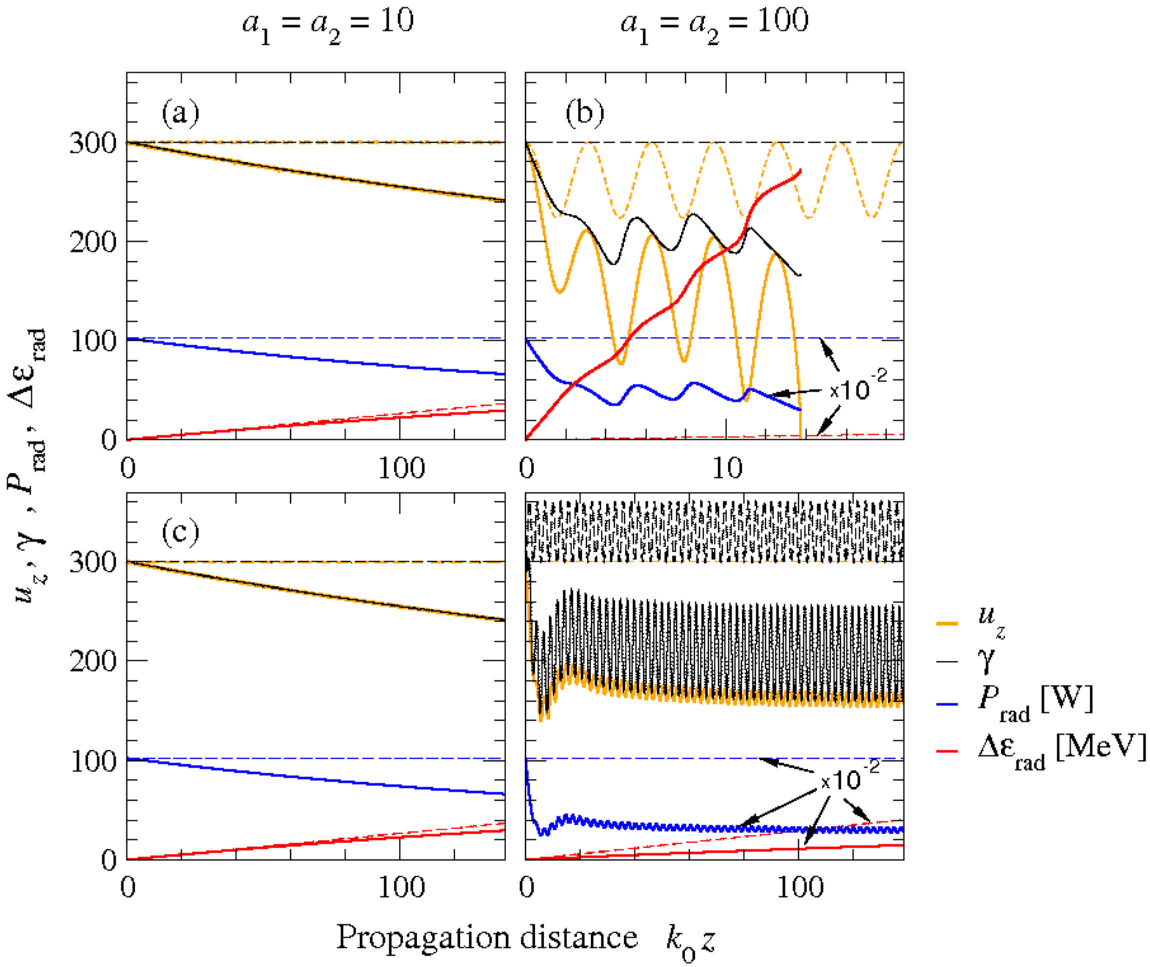
$$r_\perp = \frac{a_0}{h_0}, \; h_0 = \sqrt{\gamma_0^2 + a_0^2} - \gamma_0 \beta_0 \qquad \sqrt{\frac{1-\beta_0}{1+\beta_0}} = \sqrt{M_0} = \frac{h_0}{\sqrt{1+a_0^2}}$$

Superposition of two waves with opposite directions

$\gamma_0 = 300$

standing wave

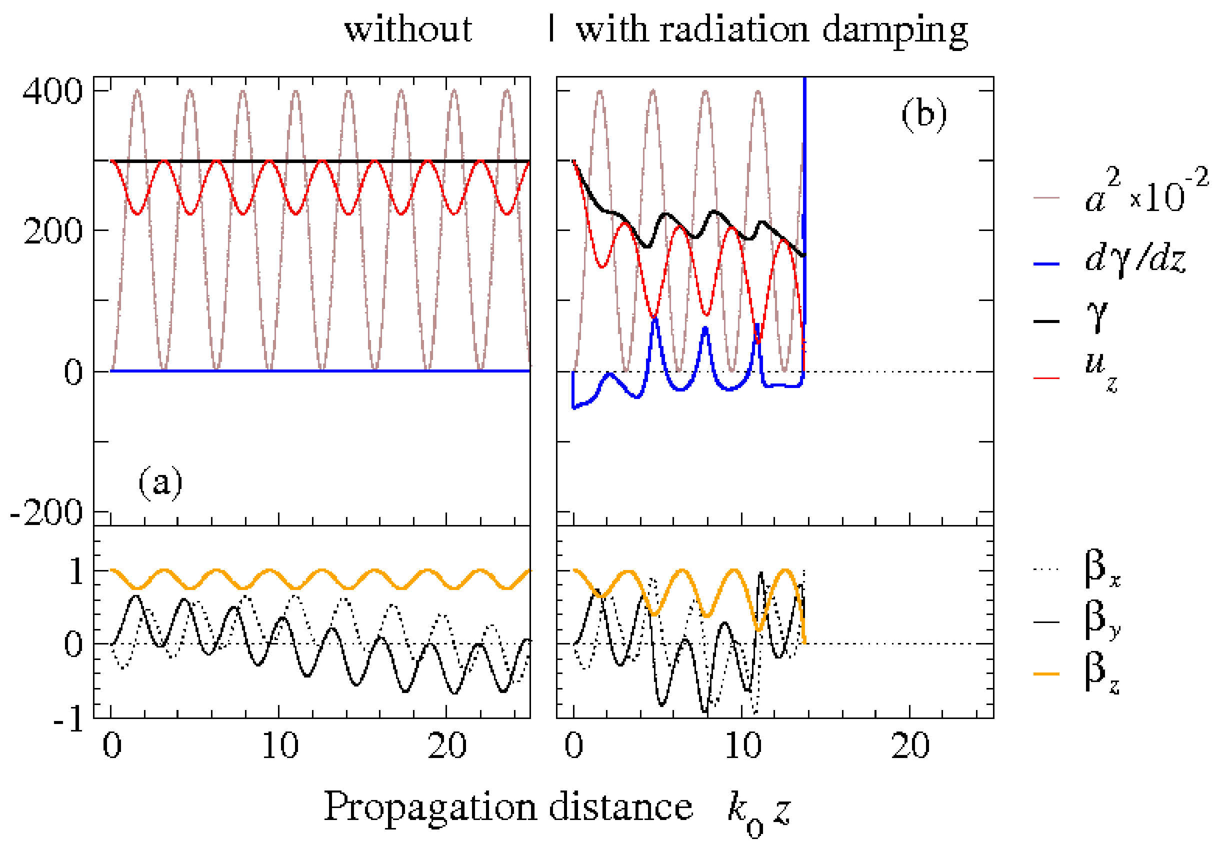
identical counterpropagating waves



Standing wave

$a_1 = a_2 = 100$

$\gamma_0 = 300$



Electron orbits - estimates

Electron trajectories $\gamma_0 = 300$

electron proper frame:

$$a_x = a_0 \left(\sin\varphi_1' \mp \sin\varphi_2' \right), \;\; \varphi_1' = \sqrt{\frac{1+\beta_z}{1-\beta_z}}\, t'$$

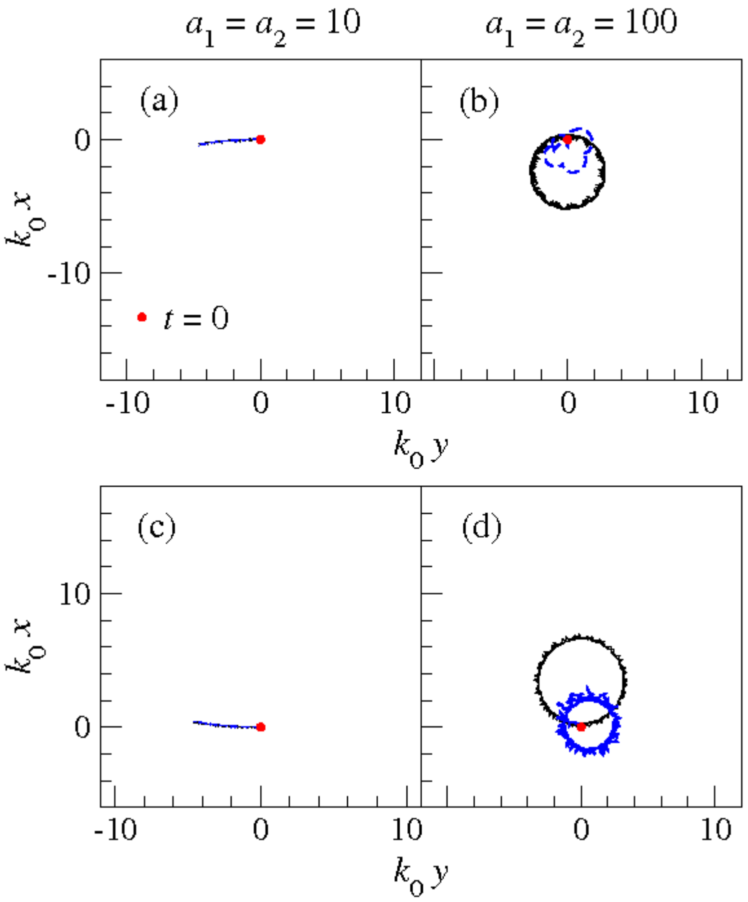
$$a_y = a_0 \left(\cos\varphi_1' - \cos\varphi_2' \right), \;\; \varphi_2' = \sqrt{\frac{1-\beta_z}{1+\beta_z}}\, t'$$

$$k_0 x(t') = \pm \frac{a_0}{\gamma_s'} \sqrt{\frac{1+\beta_z}{1-\beta_z}} \left(\cos\varphi' - 1 \right)$$

$$k_0 y(t') = - \frac{a_0}{\gamma_s'} \sqrt{\frac{1+\beta_z}{1-\beta_z}} \sin\varphi'$$

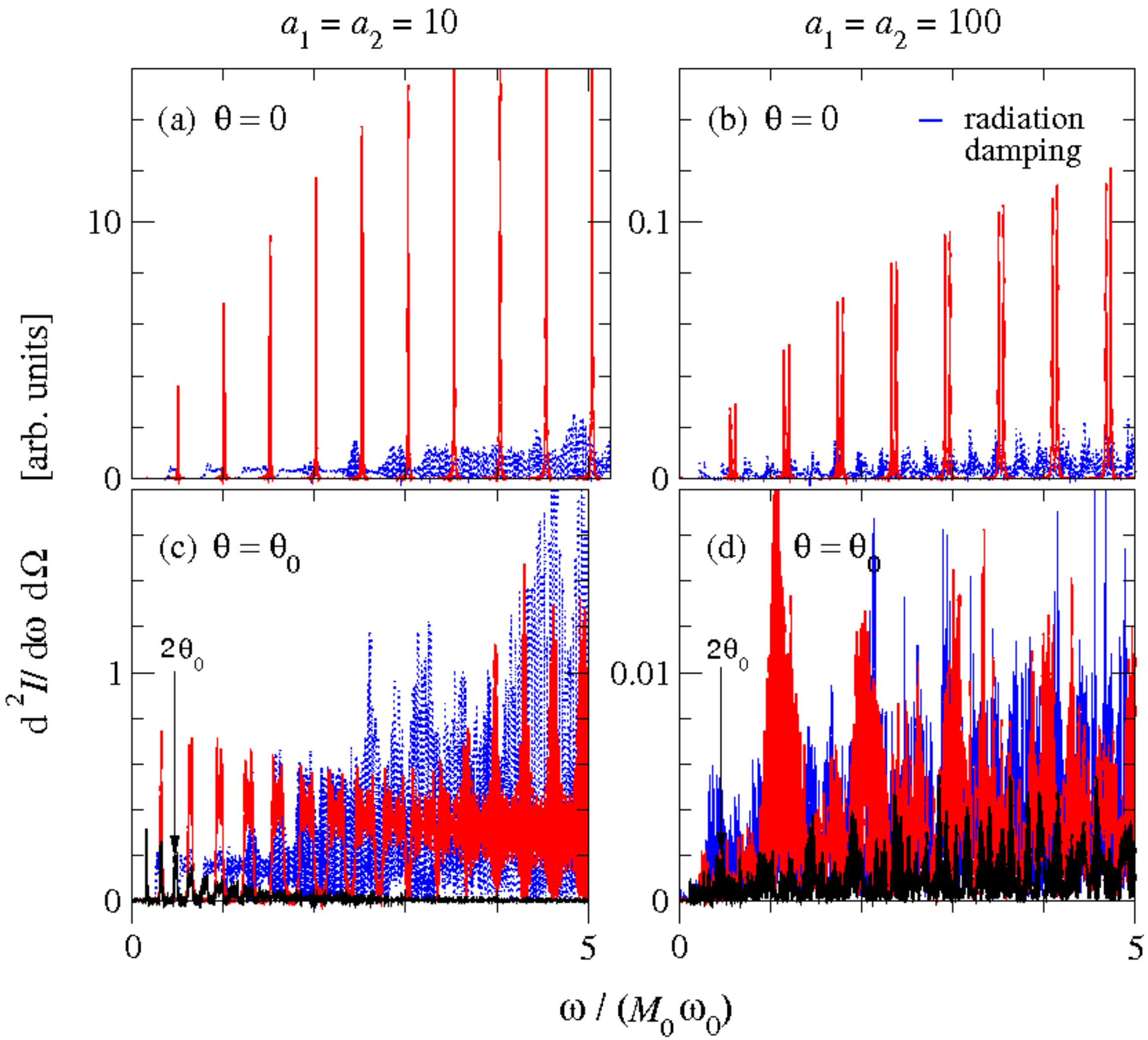
$$\gamma_s' = \sqrt{1 + 2a_0^2}$$

$$r_\perp = \frac{a_0}{\gamma_s'} \sqrt{\frac{1+\beta_z}{1-\beta_z}} = \frac{a_0}{h_0}, \quad h_0 = \sqrt{\gamma_0^2 + 2a_0^2} - \gamma_0 \beta_0$$



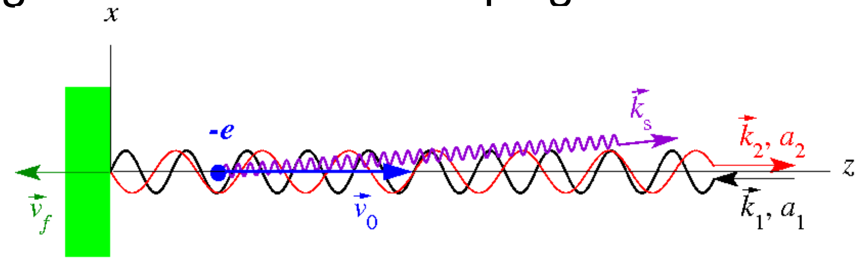
Low-energy spectra - two identical counterpropagating waves

$\gamma_0 = 300$



Laser piston

Significant radiation damping of relativistic electrons at a ~ 100



$$\beta_f = \frac{v_f}{c} = \frac{B}{1+B},$$

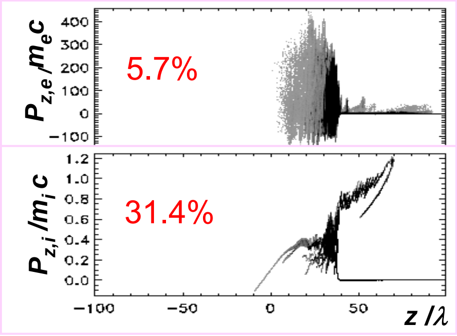
$$B = \left(I / \rho c^3\right)^{1/2}$$

$$I = a^2 n_c m_e c^3$$

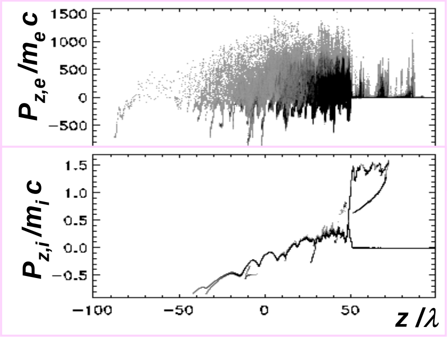
$$a = 100$$

$$n_{i0} = 10 n_c$$

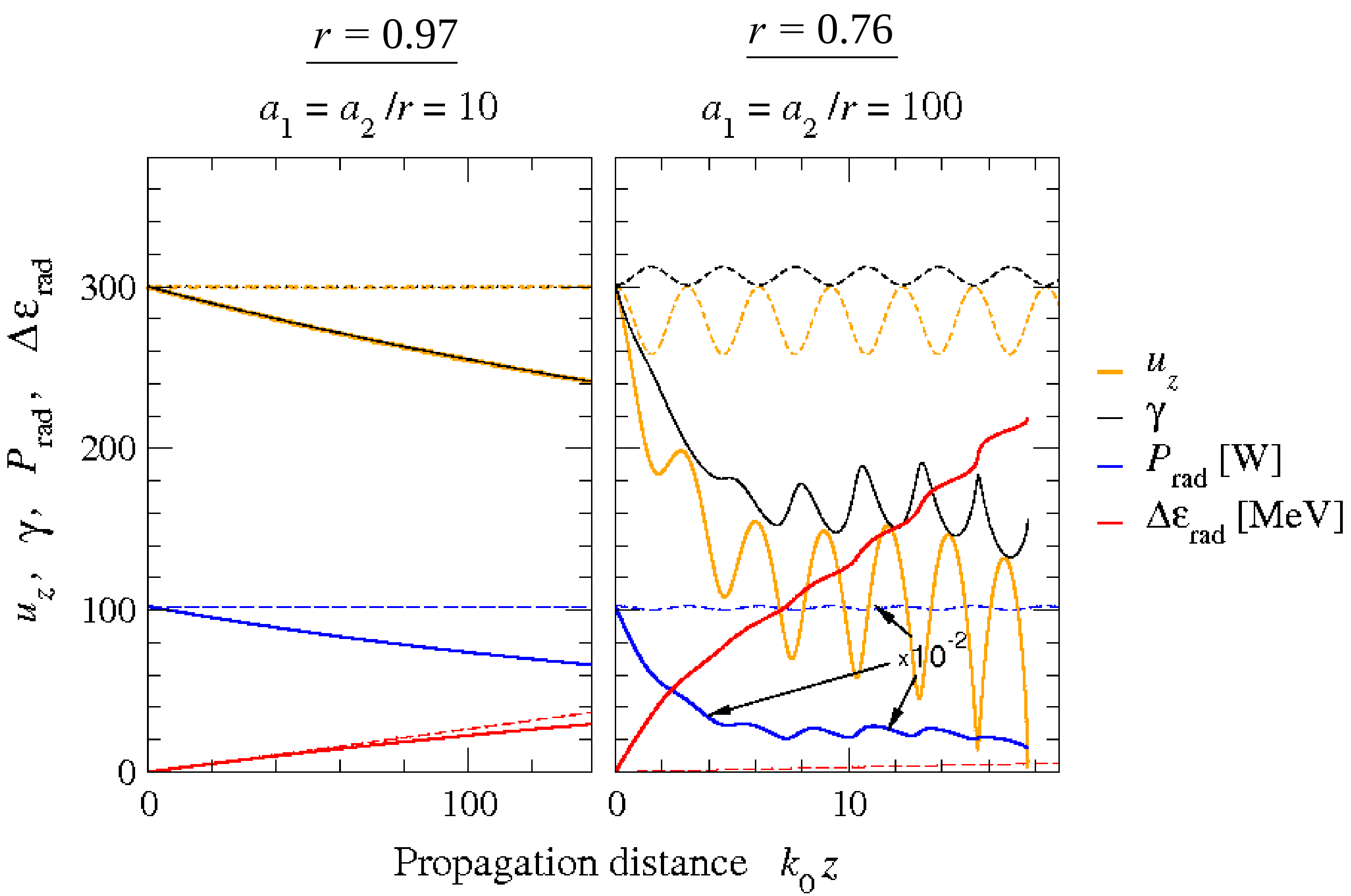
radiation losses : 43%



without radiation losses



T. Schlegel, N. Naumova et al., Phys. Plasmas ***16****, 083103 (2009)*

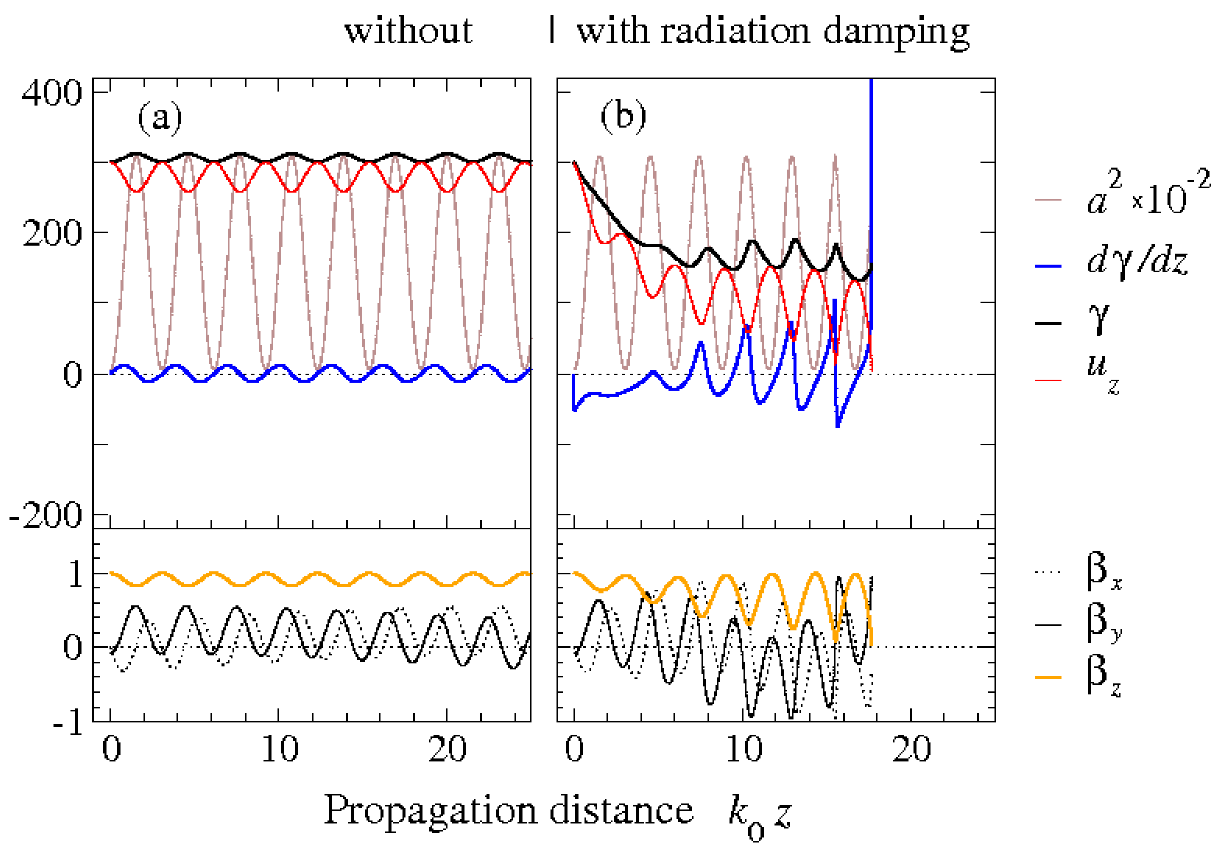


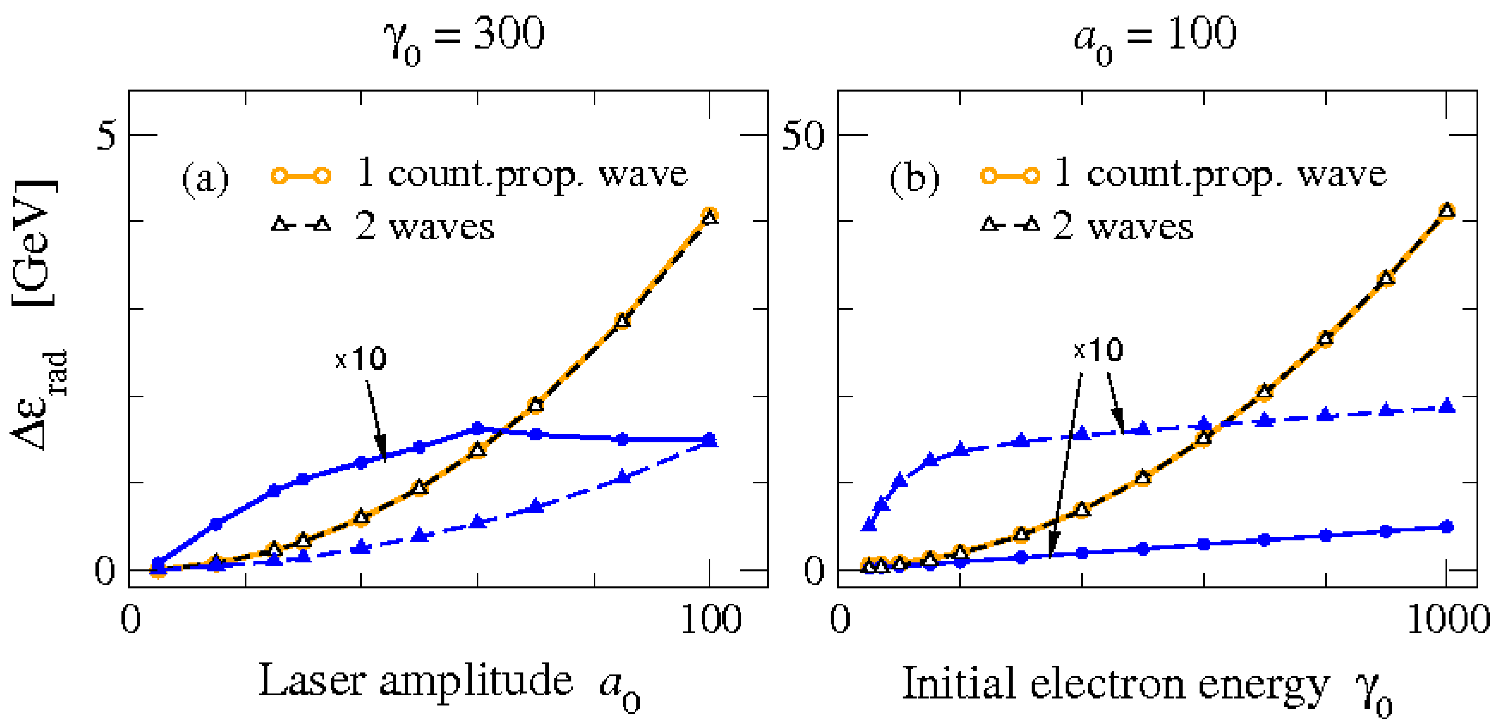
r = 0.97
a1 = a2 /r = 10
r = 0.76
a1 = a2 /r = 100
uz, γ, Prad, Δεrad
300
200
100
0
0
100
0
10
x10-2
Propagation distance k0z
uz
γ
Prad [W]
Δεrad [MeV]

Laser piston

$a_1 = 100$

$\gamma_0 = 300$





$\gamma_0 = 300$
$a_0 = 100$
$\Delta\varepsilon_{rad}$ [GeV]
5
0
(a)
1 count.prop. wave
2 waves
x10
0
100
Laser amplitude a_0
50
0
(b)
1 count.prop. wave
2 waves
x10
0
1000
Initial electron energy γ_0

Conclusion

The relativistic equation of motion with the radiation friction force in the Landau-Lifschitz description was solved numerically and treated analytically in a simplified form.

Stopping of relativistic electrons in a counterpropagating wave at amplitudes a ~ 100 after a couple of laser wave lengths → strong reduction of the total radiation energy.

Similarly, radiation friction at such field strengths stops the electron motion in a standing wave and in the vacuum field behind a quasistationary laser piston.

At lower initial energies of the electron and strong radiation damping, an acceleration regime becomes possible.

Electron stopping due to the radiation friction will be suppressed in the fields of two identical (same handedness) counterpropagating waves – a stable electron propagation at a lower energy level and reduced radiation emission was observed.

Low-energy spectra

$a_0 = 100,\ \gamma_0 = 30$

radiation damping