High-quality short electron bunches in laser wakefield accelerators

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ExtreMe Matter Institute EMMI

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EMMI

Outline

- **Introduction**
  - *expected potential and achieved results of the laser – plasma electron acceleration*
- **Quality of accelerated electron bunches**
  - *energy spread*
  - *loading effect and bunch charge*
- **Electron bunch injection**
  - *trapping and compression*
  - *nonlinear laser pulse dynamics*
- **Symmetry violation of the laser pulse focusing and propagation**
  - *point stability and alignment*
- **Conclusions**

Expected potential

of laser – plasma acceleration of electrons

Electric field of plasma wave (*with phase velocity ~ c,* $\lambda_p = 2\pi c/\omega_p$):

$$E_P\,[\mathrm{V/m}] \approx 10^2\,\alpha\,\left(n_e\,[\mathrm{cm}^3]\right)^{1/2} \propto \gamma_g^{-1} = \omega_p/\omega_0$$

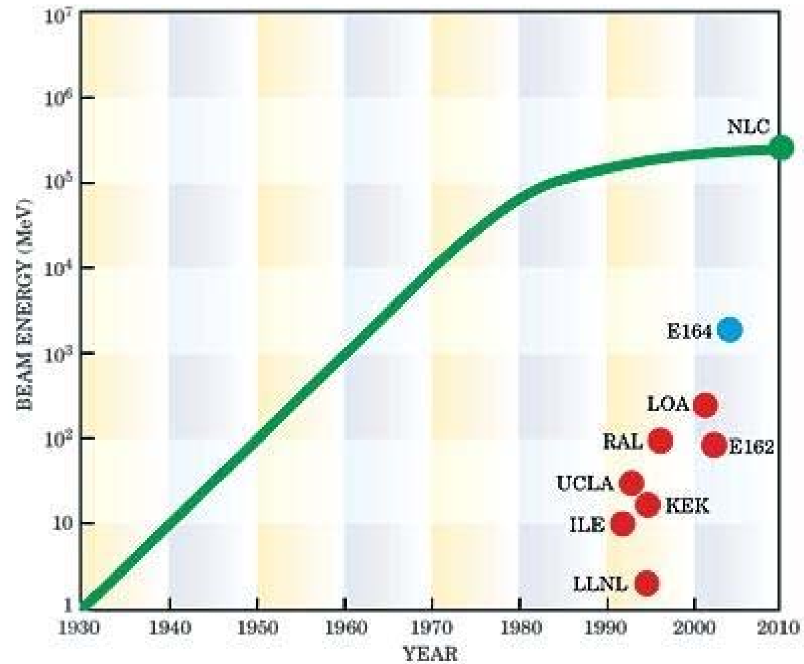
$\alpha = \delta n/n_0$ – *plasma wave amplitude; at* $\alpha = 0.3 \div 1.0$, $n_e = 10^{17} \div 10^{18}$ cm^{-3}:

$$E_P = 10 \div 100\ \mathrm{GV/m}$$

maximum of accelerating gradient in traditional accelerators (RF linac):

$E_{RF} \sim 10 - 100$ MV/m

Exponential growth of “the Livingston curve” began tapering off around 1980



Physical Restrictions on the Energy of Accelerated Electrons in Laser-Plasma Accelerators

Maximum length of acceleration:

$$l_a \le L_{deph} = \frac{\lambda_p}{2(c - v_{ph})} c = \gamma_g^2 \lambda_p, \quad \gamma_g = \frac{\omega_0}{\omega_p} = \sqrt{\frac{n_c}{n_e}}$$

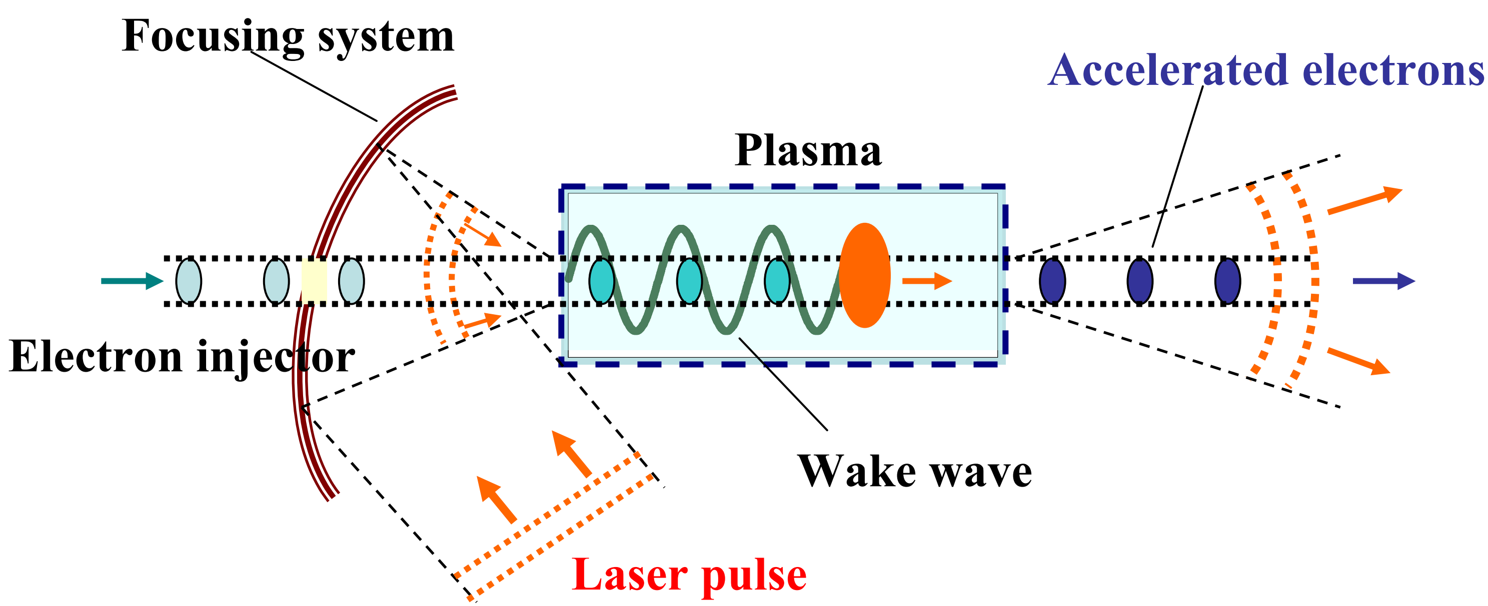
Energy gain: $$\Delta W_e = eE_a l_a \cong \frac{e\Delta\varphi}{\lambda_p / 2} l_a = 2mc^2 \gamma_g^2 \alpha$$

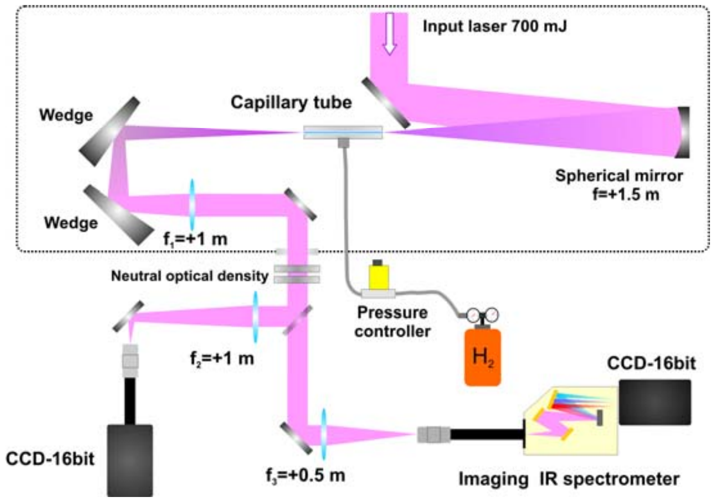
Laser Pulse Diffraction:

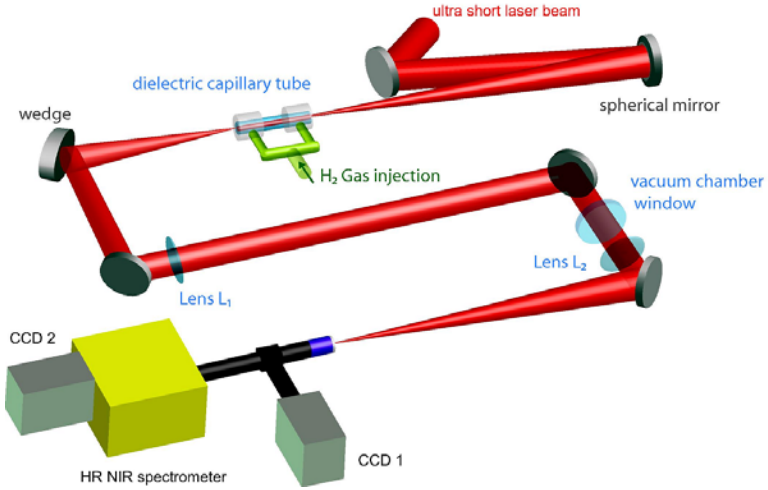
$$L_{diff} = \pi Z_R = \frac{2\pi^2 r_L^2}{\lambda}, \quad a_0^2 \propto P_L / r_L^2 \qquad \alpha \propto a_0^2$$

for LWFA $$\Delta W_{diff}[\mathrm{MeV}] = 960 \frac{\kappa \lambda[\mu\mathrm{m}]}{\tau[\mathrm{fs}]} P[\mathrm{TW}]$$

Scheme of one cascade of the laser wake-field accelerator





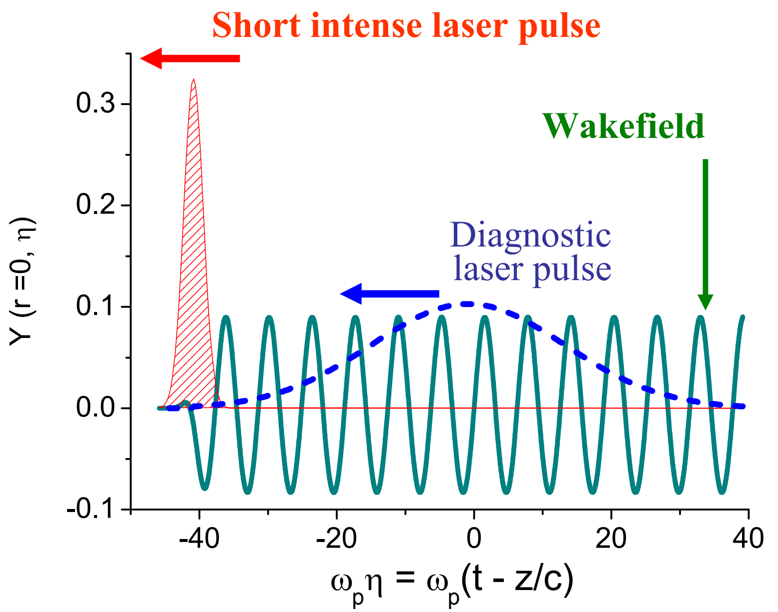


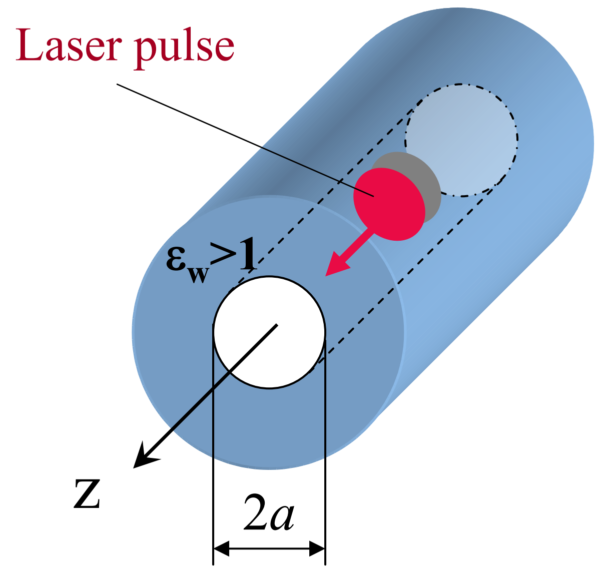
B. Cros, et al. Schematic view of the experiment **at the Lund Laser Center**

Wakefield generation by guided laser pulses
➢ *spectroscopic diagnostics of the wakefield*

$$\widetilde{E}(r) = \sum_{n=1}^{N_m} C_n J_0(k_{\perp n} r), \quad k_{\perp n} = \frac{u_n}{a} - i\frac{u_s}{k_{w\perp} a^2}$$

$$C_n = \frac{2}{[a J_1(u_n)]^2} \int_0^a E(r) J_0(u_n r / a)\, r dr, \quad J_0(u_n) = 0$$





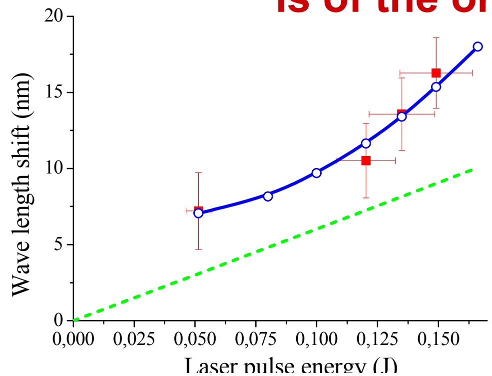
Laser energy leakage:

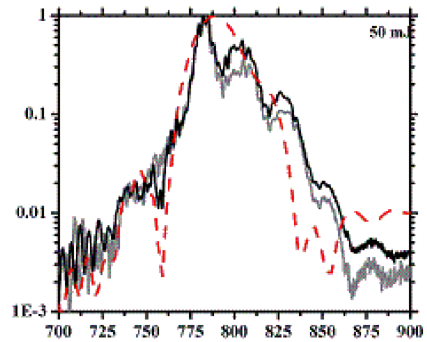
$$I_L(z) = I_0 \exp(- z / L_D)$$

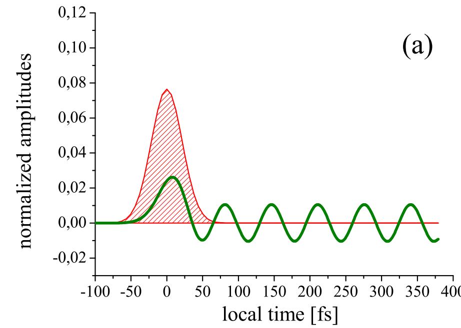
$$L_{D,n}^{-1} = \frac{u_n^2}{k_0^2 a^3} \frac{1+\varepsilon_w}{\sqrt{\varepsilon_w - 1}}$$

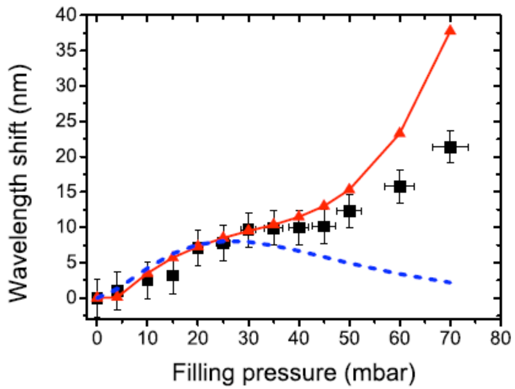
Spectral diagnostics of the laser wake fields in capillary tubes

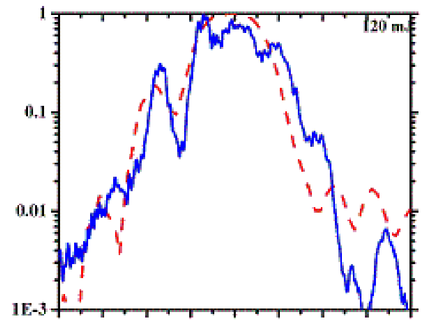
The average product of gradient and length achieved in this experiment is of the order of 0.4 GV at a pressure of 50 mbar

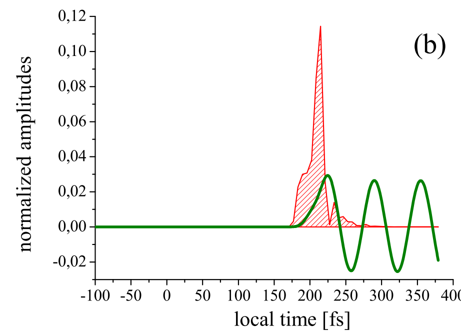


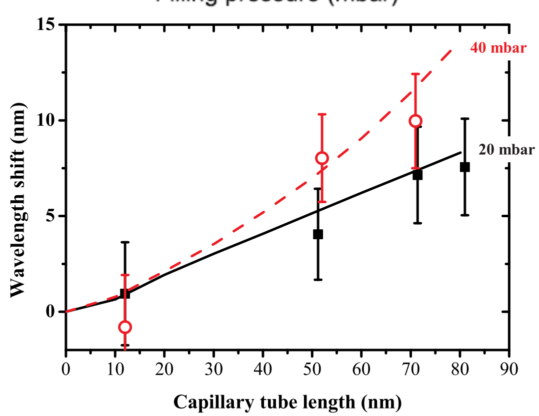


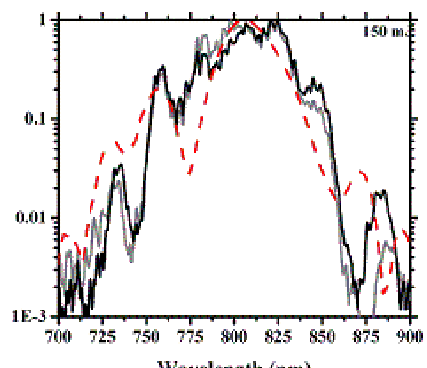


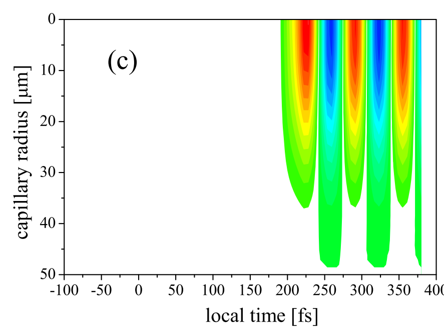












N.E. Andreev, et al, NJP, v. 12, 045024 (2010)

Energy spread in LWFA of short e-bunches

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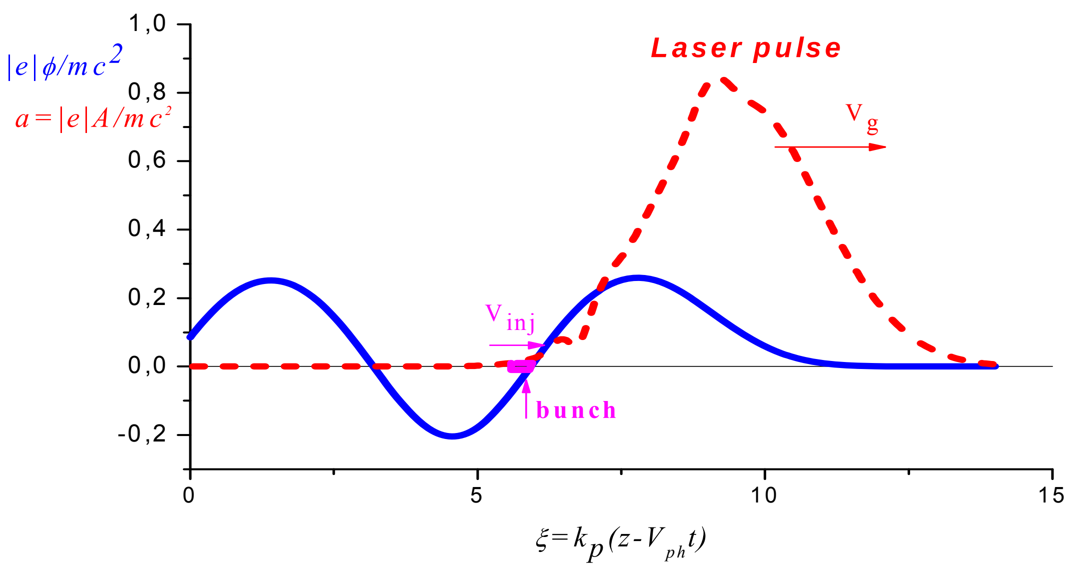
Electron bunch injection into LWFA at the maximum of accelerating field

Parameters of the laser pulse and electron bunch

$$a_0 = \frac{|e|E_L}{mc\omega} = 1.0 \qquad \gamma_{ph} = \omega/\omega_p = 50 \qquad E_{inj} = 80\,mc^2 \qquad L_b = 0.1 k_p^{-1}$$



$$E_{max} \approx 2mc^2\gamma_{ph}^2\varphi_{max}$$

$$|\Delta E| \approx 2mc^2\gamma_{ph}^2 k_p L_{b0}\left\{\frac{d\phi(\xi_{inj})}{d\xi}\right\}$$

without loading effect

$$\Delta E / E_{max} \sim k_p L_b \sim 10\%$$

for $L_b \approx$ 1mkm (3fs !)

The wake field of a cylindrical electron bunch moving with the velocity V(t)

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$$\delta\varphi_b \equiv \frac{e}{mc^2}\delta\Phi_b = \frac{n_b}{n_0}\left[1 - \mathrm{I}_0(\rho)\mathrm{K}_1(\rho_b)\rho_b\right](1-\cos\varsigma)$$

$$\varsigma = k_p\left[z - \int_0^t V(t')dt'\right]$$

$$where\ \rho < \rho_b = k_p R_b, \quad -k_p L_b \le \varsigma \le 0, \quad k_p = \omega_p / c$$

For a wide electron bunch $R_b >> k_p^{-1}$ ***and*** $r_\perp < R_b,\ k_p(R_b - r_\perp) > 1$
the wake field of electron bunch can be approximated by 1-D distribution :

$$\delta\varphi_b \equiv \frac{e\delta\Phi_b}{mc^2} = \frac{1}{2}\frac{n_b}{n_0}\varsigma^2$$

where ζ=0 corresponds to the leading front of the bunch,
and $\varsigma = -k_p L_b < 1$ corresponds to the trailing edge

Loading effect doesn't influence substantially the maximum energy of accelerated electrons under condition

$$\frac{n_b}{n_0} k_p L_b << \varphi_{\max}$$

An electron motion in the laser and e-bunch wake fields

$$\frac{dq}{d\tau} = \frac{\partial}{\partial \bar{z}}\left(\varphi + \delta\varphi_b\right)$$

$$\left[E/mc^2 - \beta_{ph} q - \varphi\right]_{\xi_{inj}}^{\xi} = \frac{n_b}{n_0}(\xi - \xi_{inj})\varsigma$$

$$where\; q = P/mc, \quad \tau = \omega_p t, \quad \bar{z} = k_p z$$

The energy spread at the end of acceleration

$$\frac{\Delta E}{mc^2} = 2\gamma_{ph}^2 k_p L_b \left\{ \left(\frac{d\varphi}{d\xi_{inj}} - \frac{d\varphi}{d\xi} \right) + \frac{k_p L_b}{2} \left(\frac{d^2\varphi}{d\xi^2} - \frac{d^2\varphi}{d\xi_{inj}^2} \right) - \frac{n_b}{n_0}(\xi - \xi_{inj}) \right\}$$

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Optimization of bunch acceleration

The energy spread of the bunch has a minimum at the condition:

$$\frac{d\varphi}{d\xi_{inj}} - \frac{d\varphi}{d\xi} + \frac{k_p L_b}{2}\left(\frac{d^2\varphi}{d\xi^2} - \frac{d^2\varphi}{d\xi_{inj}^2}\right) - \frac{n_b}{n_0}(\xi - \xi_{inj}) = 0$$

The optimal bunch density for a minimum energy spread :

$$n_b = \frac{n_0}{\xi_{\max} - \xi_{inj}}\left\{\frac{d\varphi}{d\xi_{inj}} + \frac{k_p L_b}{2}\frac{d^2\varphi}{d\xi_{\max}^2}\right\}$$

The minimal energy spread for optimal bunch density:

$$\frac{|\Delta E_{\min}|}{mc^2} = \gamma_{ph}^2 \frac{(k_p L_b)^2}{4}\left|\frac{d^2\varphi}{d\xi_{\max}^2}\right|$$

N.E. Andreev and S.V. Kuznetsov, IEEE Trans. on Plasma Sci., vol. 36, No.4. pp. 1765-1772, 2008

Computer simulation and comparison with analytic predictions

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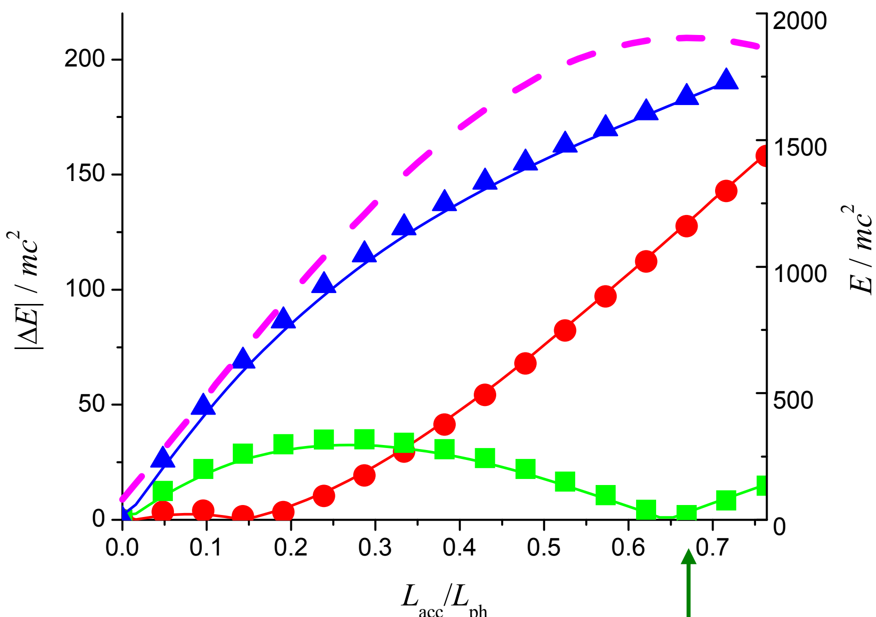
Parameters of laser pulse and electron bunch

$$a_0 = \frac{|e|E_L}{mc\omega} = 1.0 \qquad \gamma_{ph} = \omega/\omega_p = 50 \qquad E_{inj} = 80\, mc^2 \qquad L_b = 0.1 k_p^{-1}$$

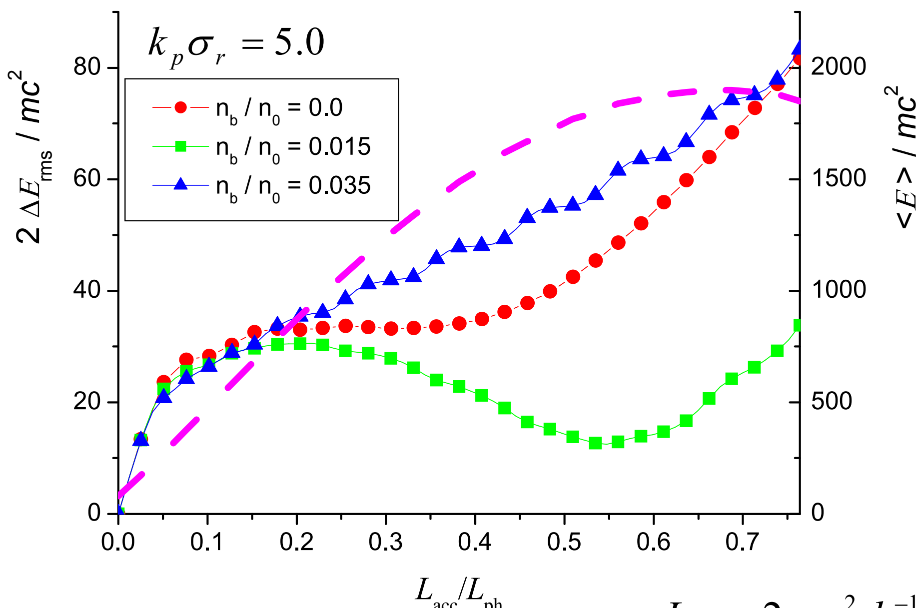


$|\Delta E_{min}| / mc^2 = 1.29$

in agreement with analytical prediction

Solid lines are analytical prediction; markers are results of numerical modeling for different bunch densities:

$n_b / n_0 =$ **0 – circles; 0.3 – triangles; 0.121 – squares.**



$$L_{ph} = 2\pi\, \gamma_{ph}^2 k_p^{-1}$$

$$|a(r_\perp, z, t)| = a(\xi) \exp\left[-\rho^2 / \left(k_p r_0\right)^2\right]$$

$$\tau_L = 1.1 \omega_p^{-1} \qquad r_0 = 14.14\, k_p^{-1}$$

Electron Bunch Injection in Front of the Laser Pulse

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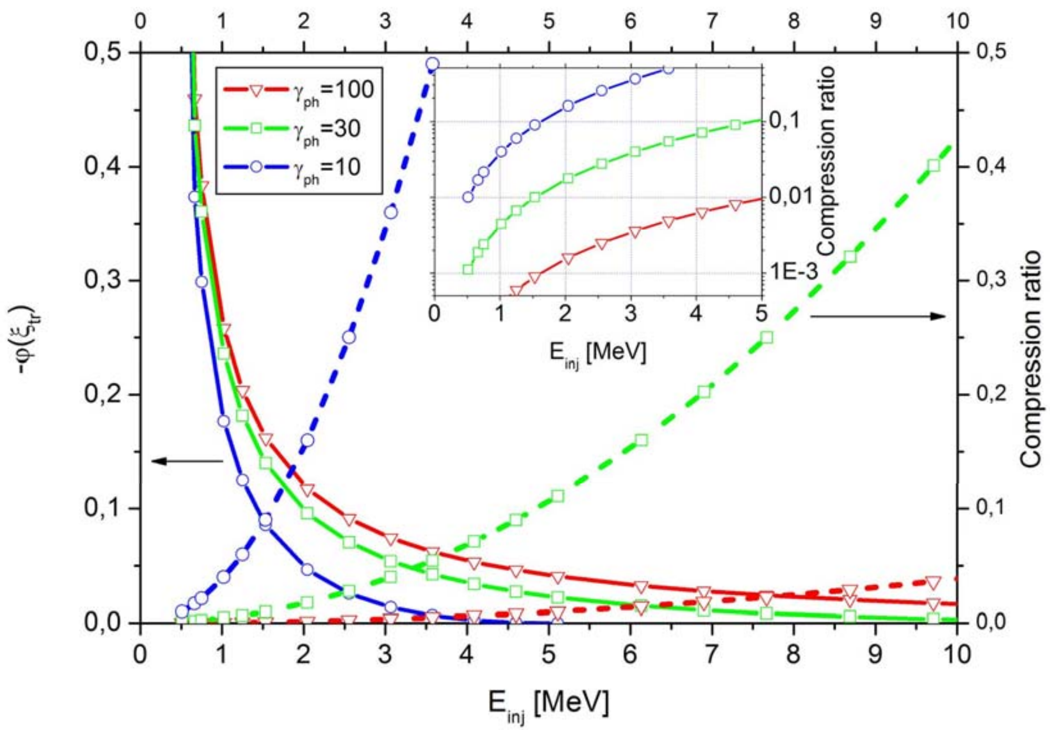
trapping and compression

bunch injected in front of the laser pulse can be trapped and compressed in the wake field, if the condition

$$-\varphi(\xi_{tr}) = E_{inj}/mc^2 - \left[\left(1-\gamma_{ph}^{-2}\right)\left(E_{inj}^2/m^2c^4 - 1\right)\right]^{1/2} - 1/\gamma_{ph}$$

is fulfilled in the focusing phase of the wakefield



$$\frac{L_{b,rms}}{L_{b0}} \cong \frac{c - V_{ph}}{V_{ph} - u_{inj}} \approx \frac{E_{inj}^2}{\gamma_{ph}^2 m^2 c^4}$$

N.E. Andreev, S.V. Kuznetsov, et al,
Plasma Phys. Control. Fusion, 52 (2010)

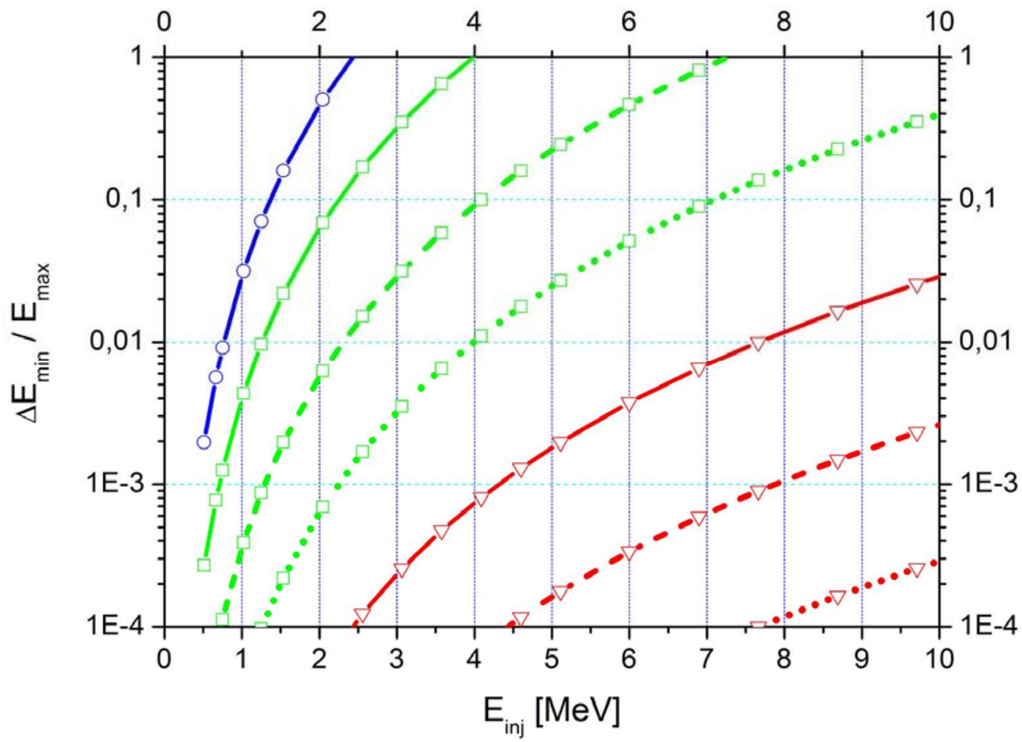
Electron Bunch Injection in Front of the Laser Pulse

energy spread at the end of acceleration

$$E_{\max} \cong 2\gamma_{ph}^2 mc^2 \varphi_{\max}$$

$$\frac{\Delta E_{\min}}{E_{\max}} \cong \frac{1}{2}(k_p L_{b,rms})^2 \cong 2\pi^2 \gamma_{ph}^{-6} \left(\frac{E_{inj}}{mc^2}\right)^4 (L_{b0}/\lambda_0)^2$$

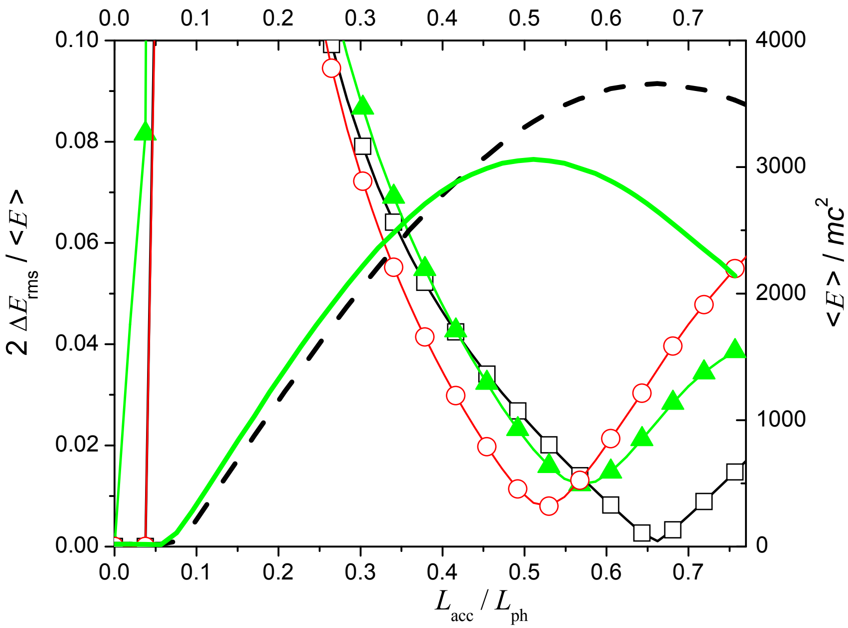
$$\frac{\Delta E_{\min}}{mc^2} = \gamma_{ph}^2 (k_p L_{b,rms})^2 \left|\frac{d^2\varphi}{d\xi_{\max}^2}\right|$$

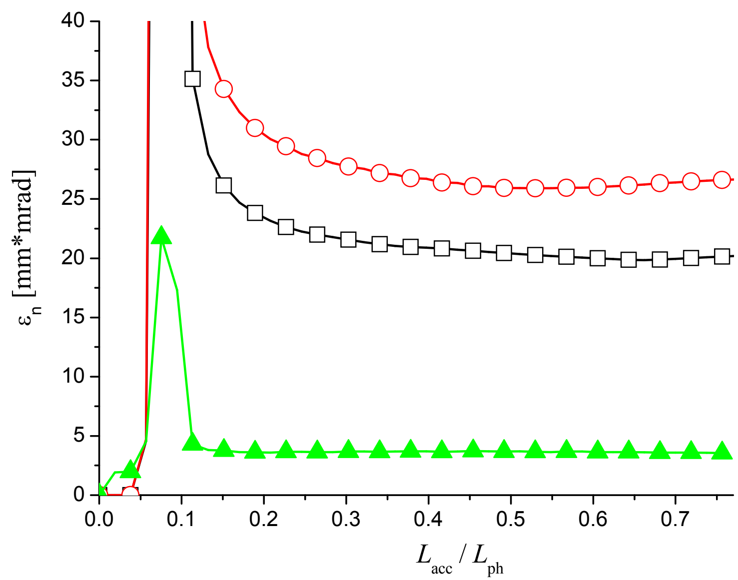


$\gamma_{ph} = 100$, 30, and 10 marked by triangles, squares and circles respectively, and for three initial bunch lengths $L_{b0} = 100$, 30, and 10 μm (solid, dashed and dotted lines respectively) for the laser wave length $\lambda_0 = 1$ μm

Computer simulation by the code LAPLAC

Results of full scale modeling including laser pulse dynamics, gas ionization and bunch loading





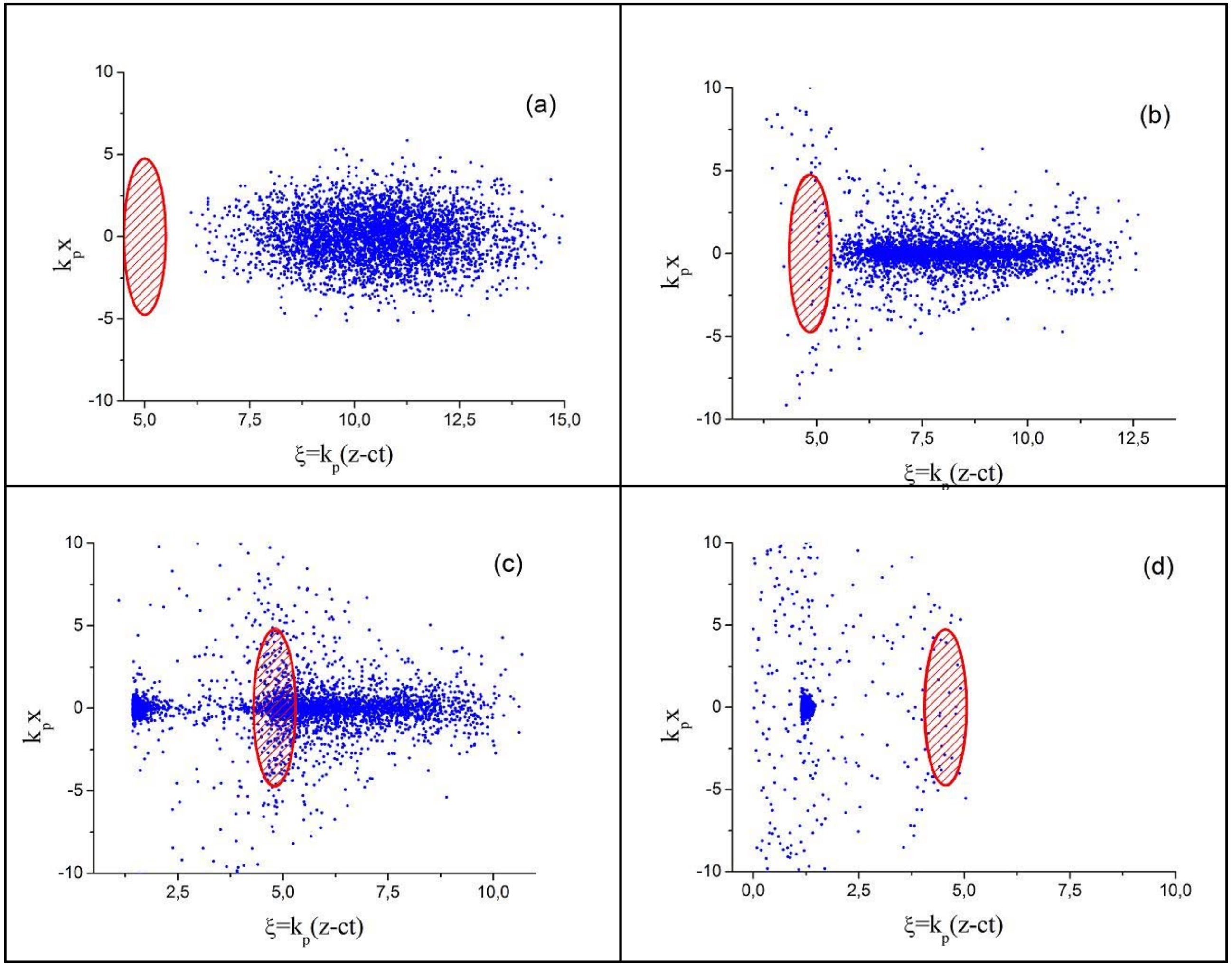
Parameters of the laser pulse and electron bunch

$$a_0 = \frac{|e|E_L}{mc\omega} = 1 \quad \gamma_{ph} = \omega/\omega_p = 100 \quad E_{inj} = 10\ \mathrm{MeV} \quad L_b = 1.26\,k_p^{-1} \quad R_b = 0.15\,k_p^{-1}$$

N.E. Andreev and S.V. Kuznetsov, IEEE Trans. on Plasma Sci., vol. 36, No.4. pp. 1765-1772, 2008

Computer simulation by the code LAPLAC

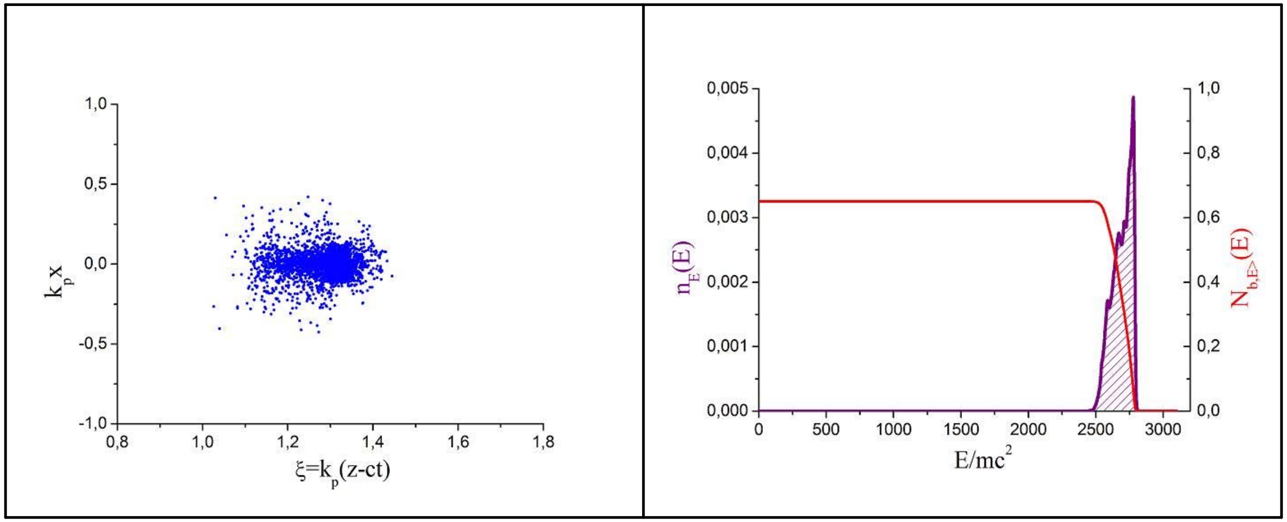
trapping and compression



Computer simulation by the code LAPLAC

accelerated electron bunch

the bunch has acquired an energy of 1.4 GeV with a narrow energy spectrum and low emittance 4.8 mm mrad



The total trapped and accelerated number of particles in the bunch is about 65% of the injected electrons

$E_{inj} = 10$ MeV $\quad L_{b0} = 2\sigma_z = 50\,\mu\text{m}$ $\quad r_0 = 80\,\mu\text{m}$ $\quad I_L = 1.2\times10^{18}\,\text{W/cm}^2$ $\quad P_L / P_{cr} = 0.72$

Q_b=5 pC $\quad \sigma_\perp = r_{rms}/\sqrt{2} = 25\,\mu\text{m}$ $\quad \tau_{\text{FWHM}} = 33$ fs $\quad$ laser energy 4.3 J $\quad n_0(0) = 10^{17}\,\text{cm}^{-3}$

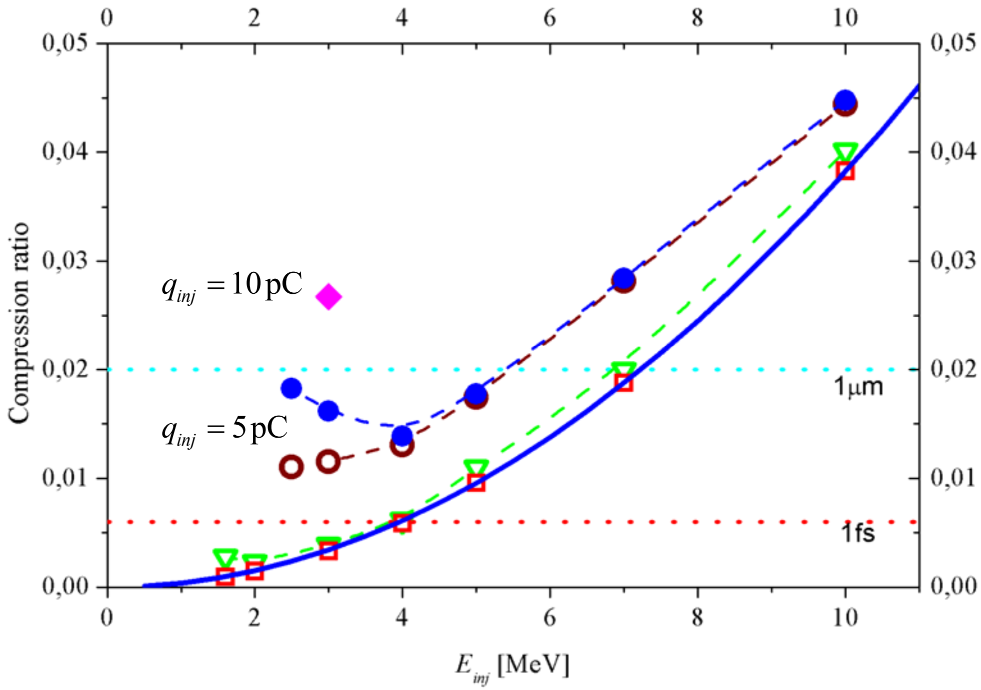
$\varepsilon_{N,r} = 4 r_{rms}\sigma_{P_r/mc} = 1\,\text{mm mrad}$

$L_b \approx R_b \approx 2.5\ \mu\text{m}$

N.E. Andreev, S.V. Kuznetsov, B. Cros, V.E. Fortov, G. Maynard and P. Mora, Plasma Phys. Control. Fusion 52 (2010)

Restrictions on the e-bunch compression

Initial emmitance and loading effect

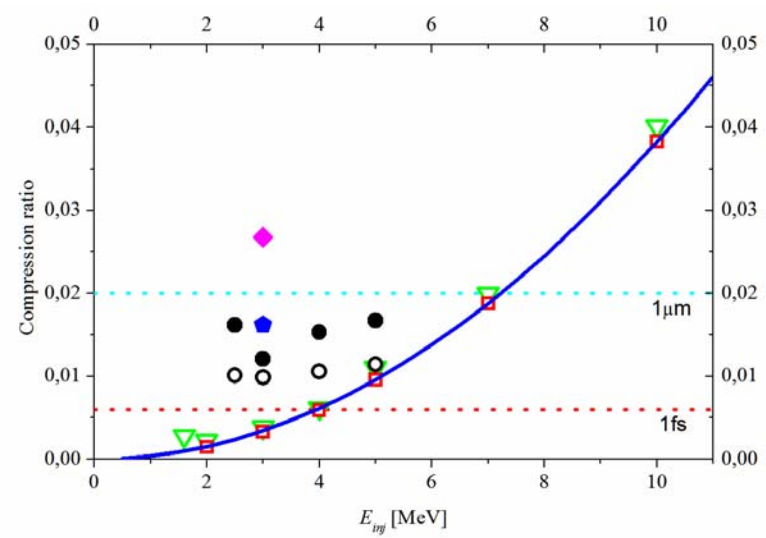


$$\gamma_{ph} = 100$$

L_{b0}=47μm
R_{b0}=33μm

$$\varepsilon_{n0} = 1\,\mathrm{mm}\times\mathrm{mrad}$$

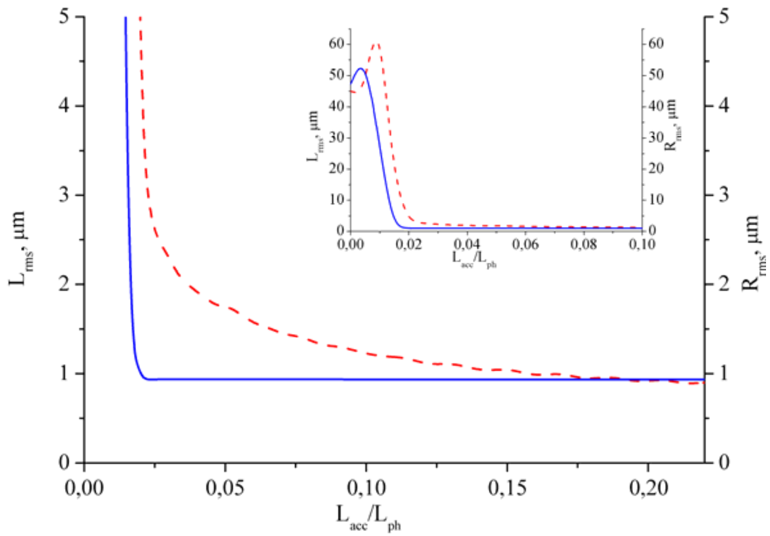
open symbols represent 3-D modelling results for the prescribed laser pulse

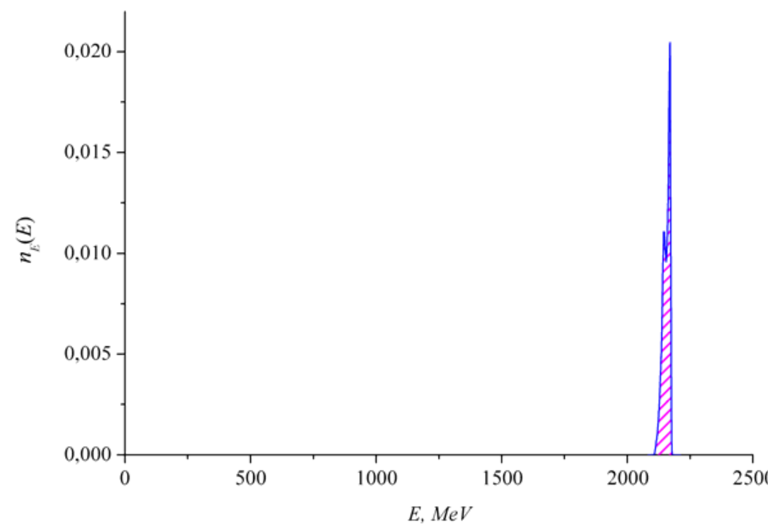


Computer simulation by the code LAPLAC

accelerated electron bunch

the bunch has acquired an energy of **2.2 GeV** with a narrow energy spectrum and low emittance 5.4 mm×mrad





The total trapped and accelerated number of particles in the bunch is about 25% of the injected electrons

$E_{inj} = 3$ MeV $\quad L_{b0} = 2\sigma_z = 47\,\mu$m $\quad r_0 = 37\,\mu$m $\quad I_L = 2.7\times10^{18}$ W/cm^2 $\quad P_L/P_{cr} = 0.35$

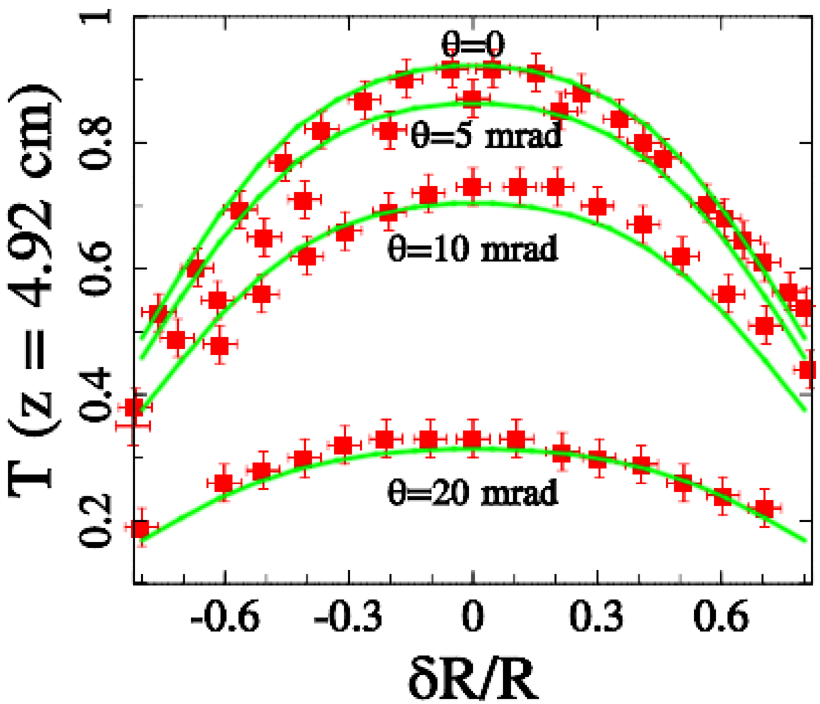
Q_b=10 pC $\quad R_{b0}$=45µm $\quad \tau_{FWHM} = 31$ fs $\quad$ laser energy 2.25 J $\quad n_0(0) = 1.1\times10^{17}$ cm^{-3}

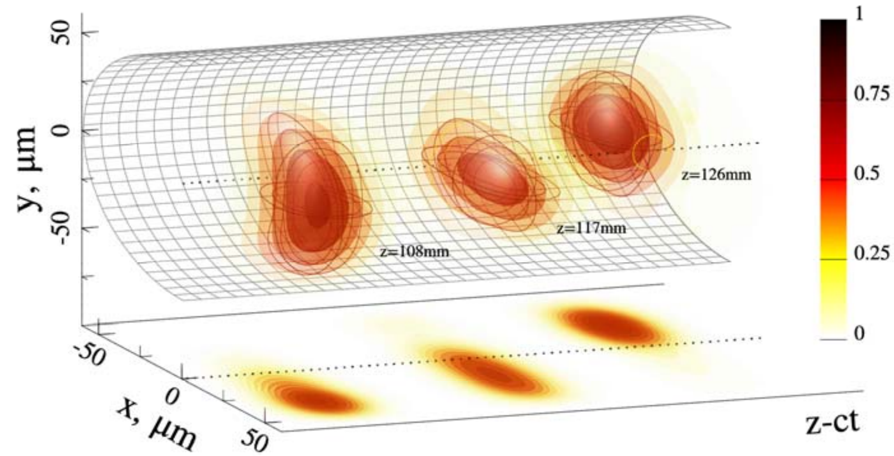
$\varepsilon_{N,r} = 4 r_{rms}\sigma_{P_r/mc} = 1$ mm mrad

$L_b \approx R_b \approx 0.9$ µm

N.E. Andreev, V.E. Baranov, B. Cros, V.E. Fortov, S.V. Kuznetsov, G. Maynard, P. Mora, NIM A (2011)

Laser pulse transmission in capillary at broken symmetry





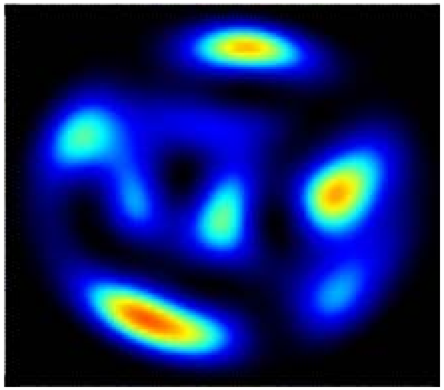
Silicon capillary, $R_{cap} = 51\mu m$, $r_0 = 32\mu m$, $\lambda_L = 0.63\mu m$, circular polarization

linearly-polarized laser pulse with r_0 = 40μm, FWHM duration 135 fs, R_{cap} = 60μm
The angle between laser and capillary axis $\theta = 6\,\text{mrad}$

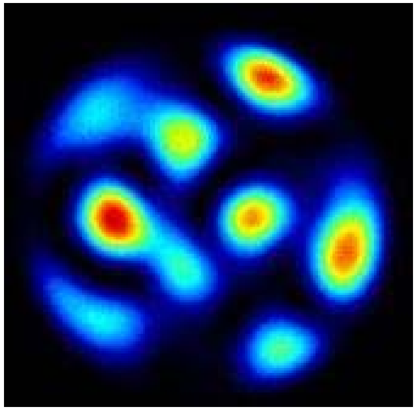
M.Veysman, N. E. Andreev, K. Cassou, Y. Ayoul, G. Maynard, and B. Cros, J. Opt. Soc. Am. B, V. 27, No. 7 (2010)

Experimental fluence distributions confirm the modelling results

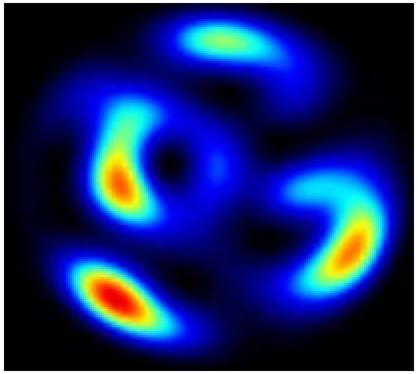
Theory, z=49.5 mm



experiment, z=49.5mm



Theory, z=48.5 mm



M. Veysman, N.E. Andreev, K. Cassou, Y. Ayoul, G. Maynard, and B. Cros, JOSA B (2010)

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Conclusions

- The effective optical diagnostics of the wakefield generated in a long capillary waveguide is elaborated and tested experimentally
- The control of the wakefield phase velocity is necessary for an effective electron bunch compression
- The transverse focusing of the bunch (lens effect), while it propagates in plasma before the laser pulse overtakes the bunch, is important for the decrease of the final bunch emittance
- The effective longitudinal bunch compression in this scheme of injection (to μm and sub-μm sizes) leads to a small relative energy spread (of order 1%) at the end of the acceleration stage
- Loading effect can be controlled and used to optimize electron bunch parameters for low energy spread

  (but it limits the bunch charge!)
- Broken symmetry of the laser pulse entrance to the waveguide will prevent regular acceleration