Relativistic laser transparency and propagation in plasma: Is it governed by dispersion relation or energy balance?

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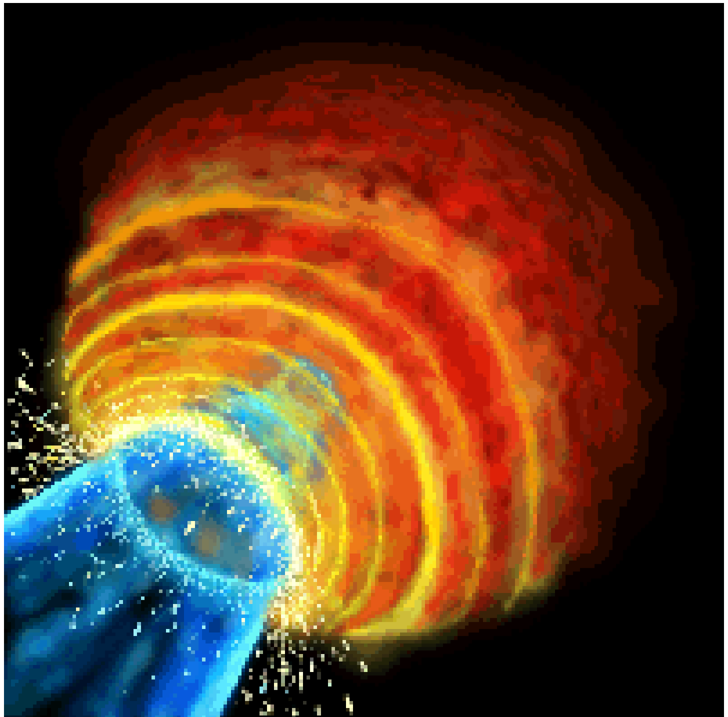
Preface



"Open Sesame",

"*Ali baba and the forty thieves*"

Who opens the door for relativistic intense laser pulse propagating into an overdense plasma?

How does it work?

Outline

- **Theoretical background**
  - **Classical eletromagnetic (EM) wave propagation**
  - **Relativistic induced transparency**
- **Numerical simulations**
  - **Relativistic critical density increase**
  - **Relativistic laser pulse propagation**
- **Applications**
  - **Ion acceleration and Fast ignition**
  - **Relativistic plasma shutter**
  - **Shortening of laser pulses**
- **Conclusion**

Classical EM wave propagation

- **Wave Equation**

$$\nabla^2 \mathbf{E} + \frac{\omega^2}{c^2} \varepsilon \mathbf{E} = 0 \text{ (in a uniform plasma)}$$

- **Dispersion relation**

$$\omega^2 = \omega_p^2 + c^2 k^2,$$

plasma frequency ω_p is the minimum frequency for EM wave propagation in a plasma.

the electrons will shield the EM field when $\omega < \omega_p = \sqrt{4\pi e^2 n / m_e}$

- **Critical density**

the condition $\omega_p = \omega$ defines the so-called critical density n_c,

$$n_c = m_e \omega^2 / 4\pi e^2 = 1.1 \times 10^{21} / (\lambda[\mu m])^2 \text{ cm}^{-3}$$

- **Group velocity (or propagation velocity)**

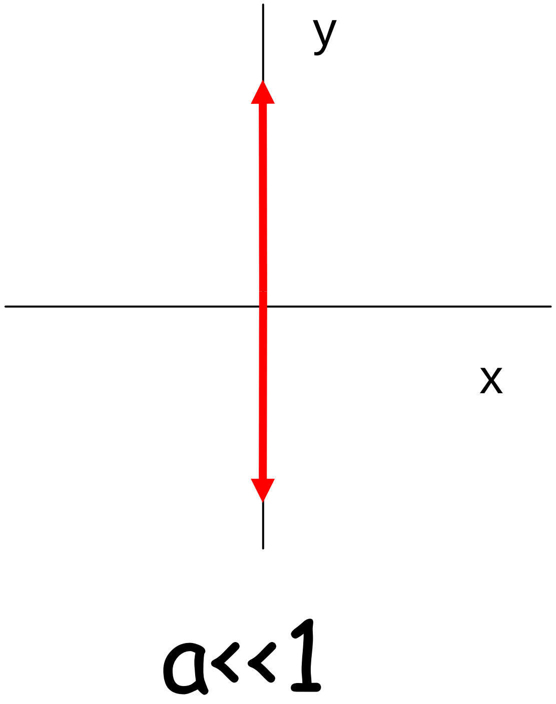
$$\frac{v_g}{c} \equiv \frac{1}{c}\frac{\partial \omega}{\partial k} = \sqrt{\varepsilon} = \left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2} = \left(1 - \frac{n}{n_c}\right)^{1/2}$$

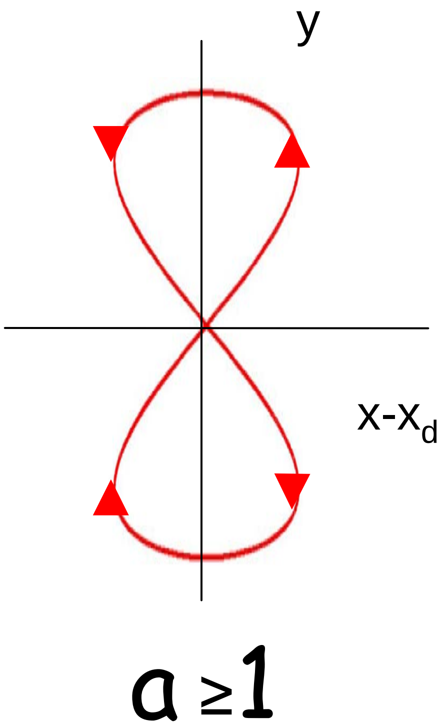
Relativistic induced transparency

- **Dimensionless laser amplitude a:**

$$I\lambda^2 = \frac{\pi}{2} c A^2 = \left[1.37\times10^{18} \frac{\mathrm{W}}{\mathrm{cm}^2} \mu\mathrm{m}^2 \right] a^2$$

- **Single particle's 8-like motion for a ≥ 1**





T. C. Pesch and H. –J. Kull, Phys. Plasmas 14, 083103 (2007).

Relativistic induced transparency

- **If |v| ~ c,**

$$m_e = \gamma m_{e0} = (1 - v^2/c^2)^{-1/2} m_{e0}$$

- **Relativistic critical density**

$$n_{cr} = m_e \omega^2 / 4\pi e^2 = \langle \gamma \rangle n_c$$

the Lorentz factor averaged from the single particle's 8-like motion

$$\langle \gamma \rangle \approx [1 + a_t^2/2]^{1/2},\ a_t \text{ the local total field amplitude.}$$

- **Group velocity (relativistic)**

$$\frac{v_g}{c} = \left(1 - \frac{n}{n_{cr}}\right)^{1/2} = \left(1 - \frac{n}{\langle \gamma \rangle n_c}\right)^{1/2}$$

P. Mulser and D. Bauer, *"High Power Laser-Matter Interaction"*, Springer, 2010.

A new diagnostics for determining the critical density

- **Laser and plasma parameters**
- **Cycle-averaged propagation appears very regular, laser is mainly reflected at the relativistic critical surface the steady state relativistic wave equation is satisfied well**

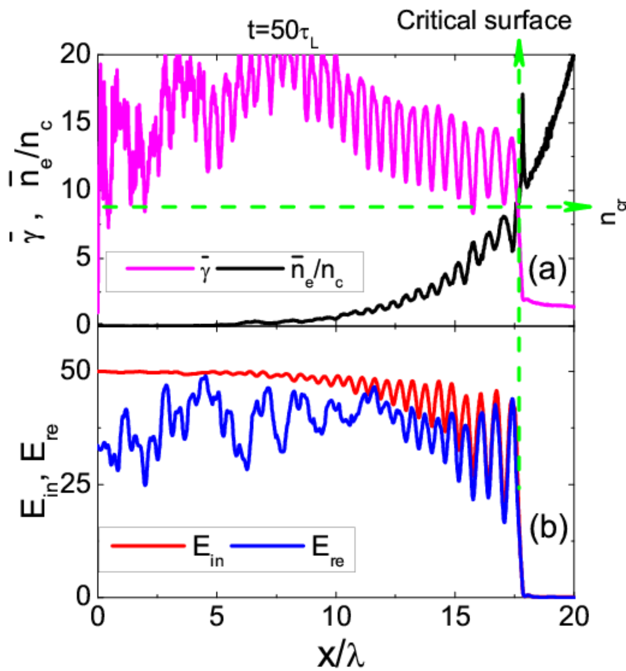
Infinite laser pulse

$$n_e = \begin{cases} 0, & x < 5\lambda \\ n_{\max}, & x > 20\lambda \\ n_{\max}\exp[(x-20)/L], & \text{otherwise}; \end{cases}$$

$n_{\max} = 2(1+a^2)^{1/2}$, L is the scale length.

relativistic wave equation:

$$\nabla^2\mathbf{E} + \frac{\omega^2}{c^2}\left(1 - \frac{n_e}{n_{cr}}\right)\mathbf{E} = 0$$



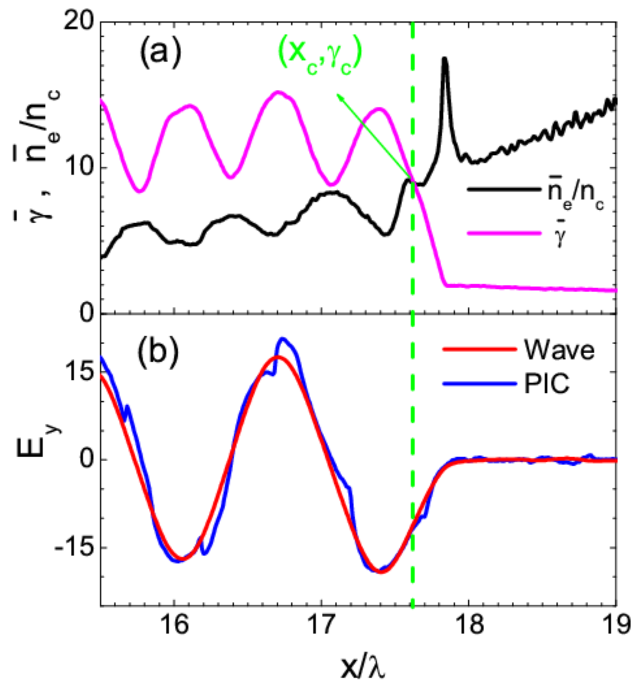
$a = 10$, incident angle $\theta = 0$

Incident wave field energy density

$$E_{in} = (E_y + B_z)^2/4 + (E_z - B_y)^2/4$$

Reflected wave field energy density

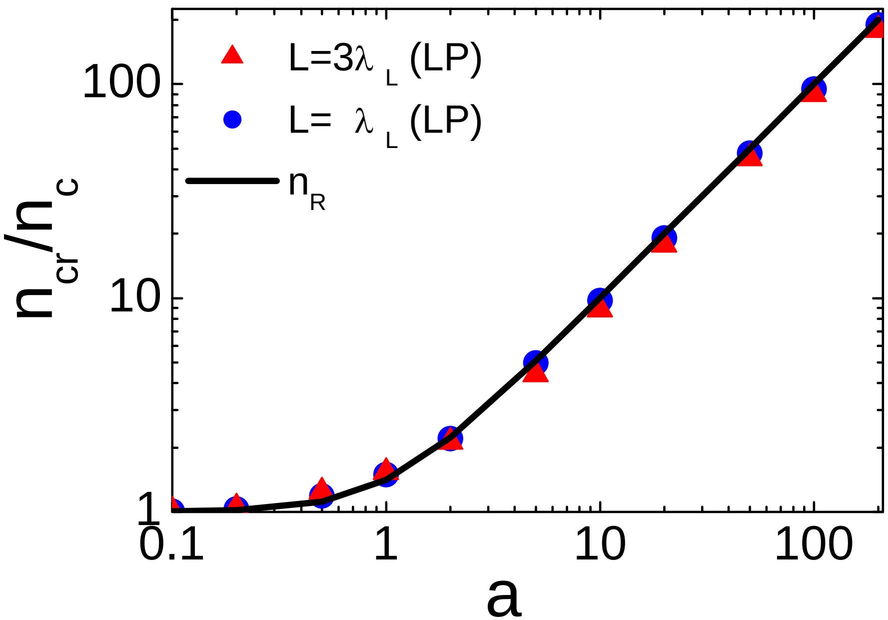
$$E_{re} = (E_y - B_z)^2/4 + (E_z + B_y)^2/4$$



Critical density VS laser intensity

- **In a normally incident and linearly polarized laser pulse, the total field amplitude a_t at critical surface and the incident laser amplitude a approximately satisfy** $\Theta = a_t^2 / 2a^2 \approx 1$

$$n_{cr} = [1 + a_t^2 / 2]^{1/2} n_c \approx n_R \equiv [1 + a^2]^{1/2} n_c$$

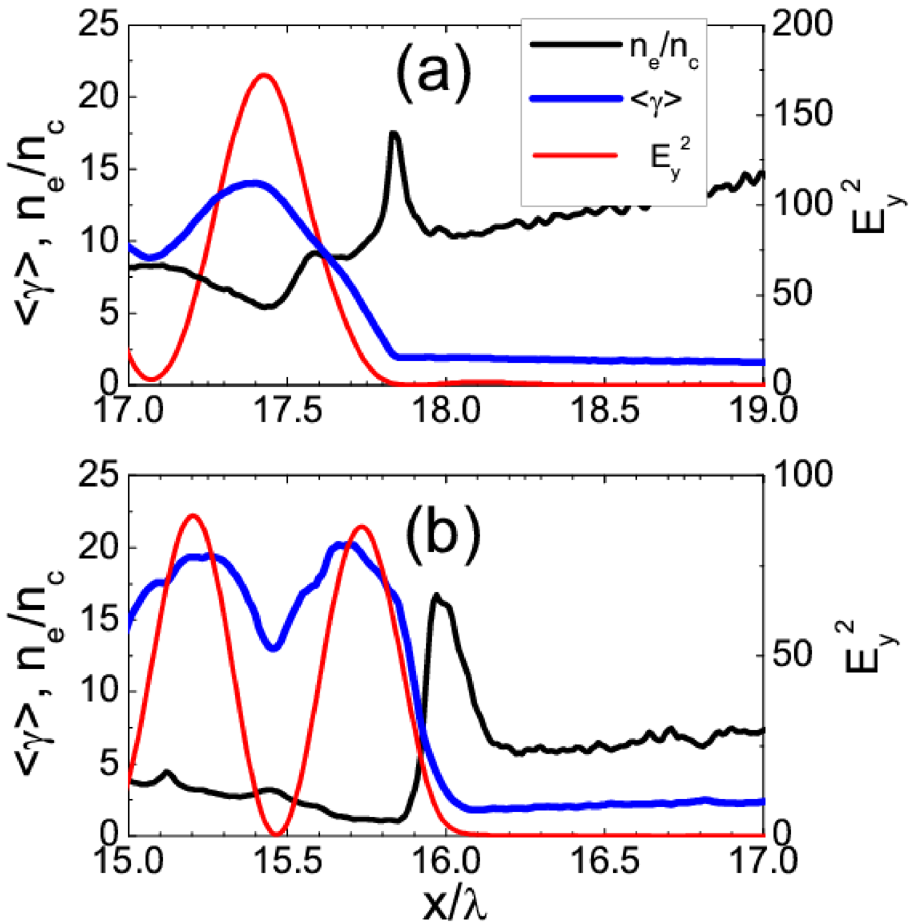


if density scale length $L \geq \lambda$

n_{cr} almost of no dependence on L

Effect of laser polarization

- For circular polarization, a sharp density peak restricts the critical density increase and prevents the laser propagation



for $a=10,\ \theta=0$

(a) Linear polarization

$$\gamma_c = 8.96 = [1+\Theta a^2]^{1/2},$$

$$\Theta \approx \frac{a_t^2}{2a^2} = 0.793$$

(b) Circular polarization

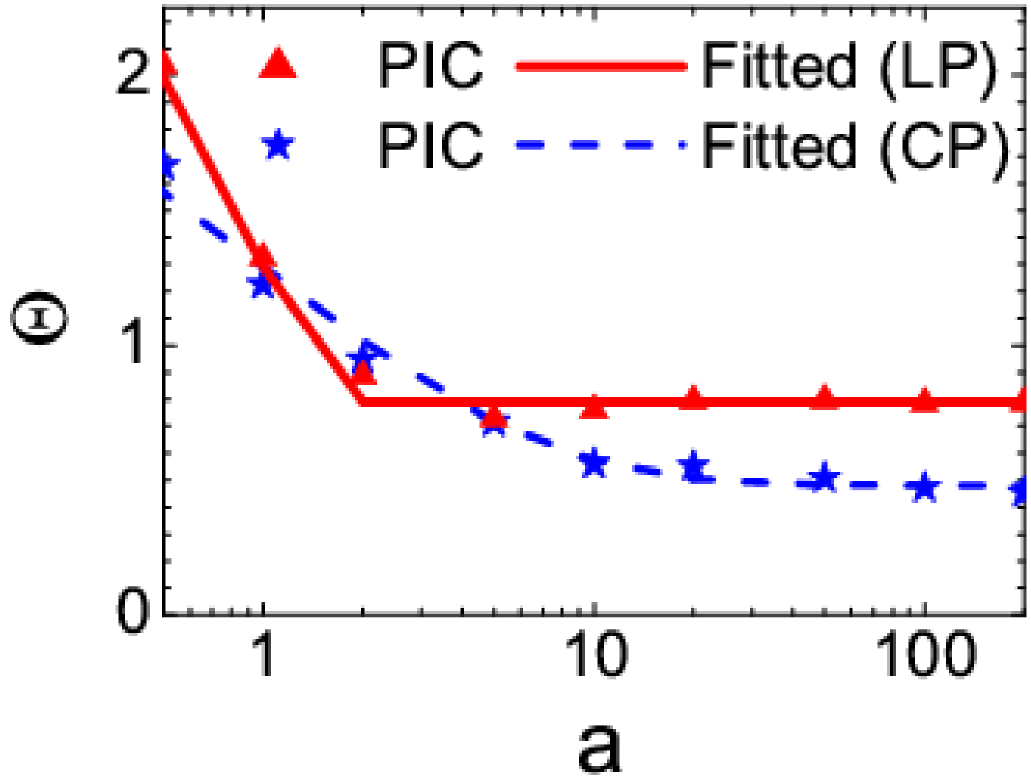
$$\gamma_c = 7.29 = [1+\Theta a^2]^{1/2},$$

$$\Theta \approx \frac{a_t^2}{2a^2} = 0.521$$

Effect of laser polarization

- For normal incident, the relativistic critical density increase can be well fitted by $\gamma_c = [1+\Theta a^2]^{1/2}$,

$$\text{with } \Theta \approx \begin{cases} 0.79+1.36\exp(-a^3) & \text{(linear polarization)} \\ 0.48+2.15\exp(-a^{1/2}) & \text{(circular polarization)} \end{cases}$$



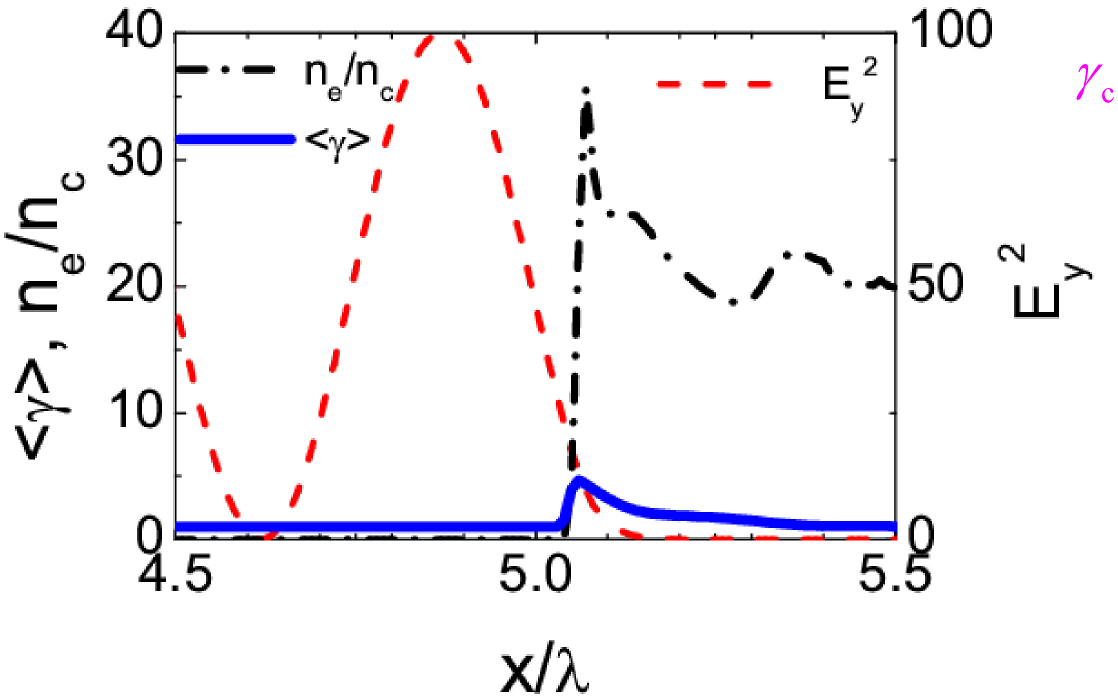
Effect of plasma density profile

- For normal incident, if density scale length $L>\lambda$,

$$\gamma_c = n_{cr}/n_c \text{ is almost independent of density profile}$$

- For a very steep and relativistically overdense plasma.

$$\gamma_c = n_{cr}/n_c \text{ is strongly suppressed}$$



γ_c is only about 3.9 for $a = 10$,

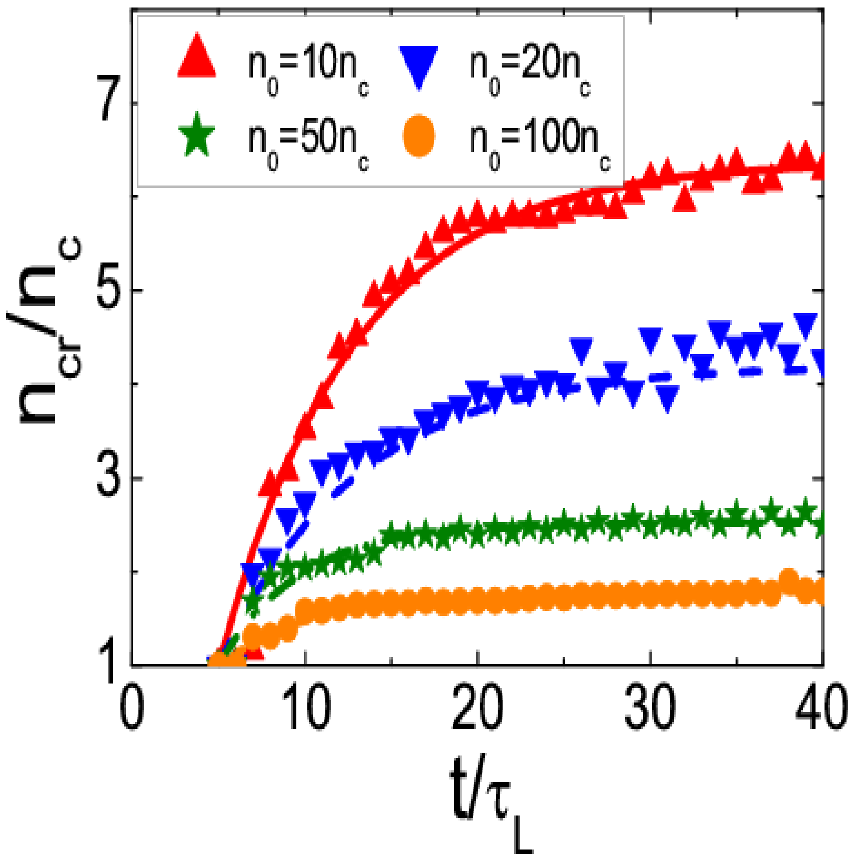
with step-like profile

$$n_e = \begin{cases} 0, & x < 5\lambda \\ 20n_c, & x > 5\lambda \end{cases}$$

electric field at the surface

and skin depth $\propto 1/n_e^{1/2}$

Response time of critical density increase



Kinetic energy density,

$$E_{\rm kin} = (\gamma_{\rm c} - 1) n_0 {\rm m_e c}^2,$$

For relativistic transparency,

$E_{\rm kin}$ can be larger than $E_{\rm em}$

Skin depth,

$$l_d = \lambda / (n_0 / n_{cr} - 1)^{1/2}$$

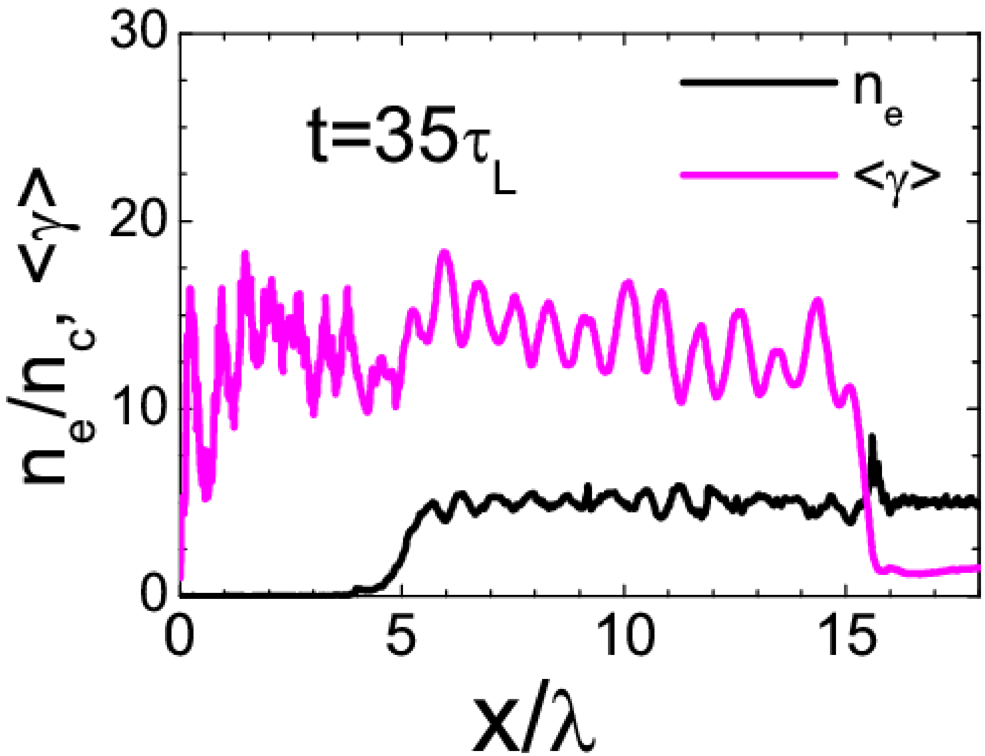
From energy balance, response time *t*

$$t / \tau_L = l_{\rm d} E_{\rm kin} / (1 - R) I,$$

for n_0=10n_c

$$t \approx 15 \tau_L$$

Relativistic laser beam propagation (LP)



linear polarization $\theta = 0$,

$a = 10$, at t=35 τ_L, and

$$n_e = \begin{cases} 0, & x < 5\lambda \\ 5n_c, & x > 5\lambda \end{cases}$$

Theoretically

$$v_{prop} \approx v_g = (1 - n/n_{cr})^{1/2} c = 0.66\text{c},$$

but from PIC

$$v_{prop} \approx \frac{(15.5-5)\lambda}{(35-5)\tau_L} = 0.35\text{c},$$

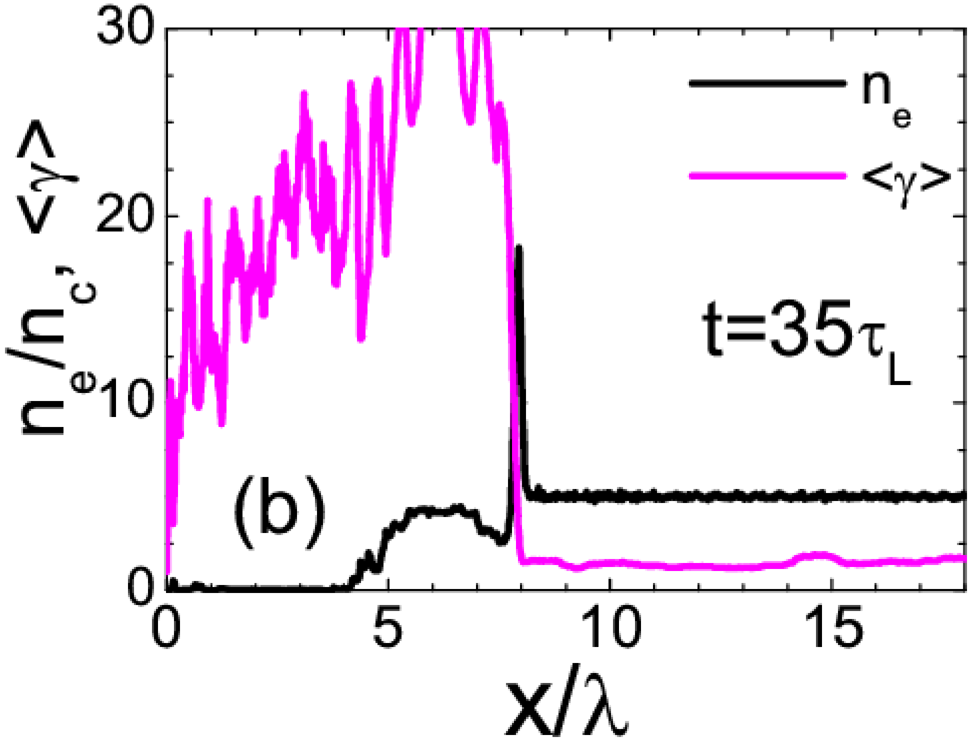
- **Previous community attributed the inhibition of the propagation velocity to the oscillation of the ponderomotive force and hence the oscillation of electron density at the laser front.[1]**

[1] H. Sakagami, K. Mima, Phys. Rev. E 54, 1870 (1996).

Relativistic laser beam propagation (CP)

- Ponderomotive force for circular polarized laser

$$f_p = -\frac{m}{4}\frac{\partial}{\partial x} v_{os}^2(x)\hat{x}, \qquad v_{os}(x) = eE/m\omega, \text{ without oscillation}$$



Theoretically

$$v_{prop} \approx v_g = \left(1 - n/n_{cr}\right)^{1/2} c = 0.7c,$$

but from PIC

$$v_{prop} \approx \frac{(8.0-5)\lambda}{(35-5)\tau_L} = 0.1c.$$

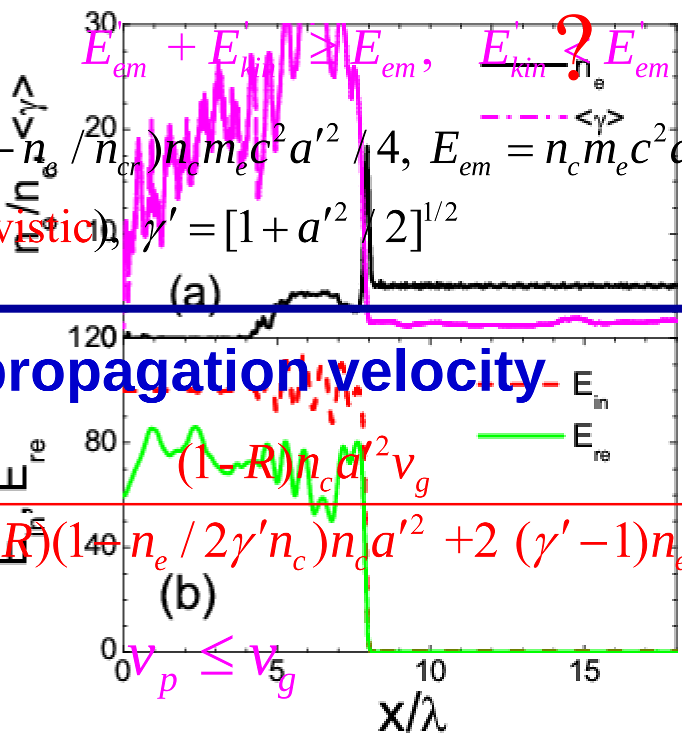
CP pulse propagates even more slowly than LP pulse.

- **Inhibition of propagation velocity is not attributed to the oscillation of ponderomotive force.**

Non-relativistic	Relativistic transparency
dielectric function $\varepsilon = 1 - n_e / n_c$ is constant in plasma	$\varepsilon = 1 - n_e / n_{cr}$ behide laser front $\varepsilon = 1 - n_e / n_c$ before laser front response time for $n_c \rightarrow n_{cr}$
$R = 0$ at laser front	$R \geq 0$ at laser front
$E'_{em} + E'_{kin} = E_{em}$, $E'_{kin} < E'_{em}$	$E'_{em} + E'_{kin} \geq E_{em}$, E'_{kin} ? E'_{em}

$$E'_{kin} = (\gamma' - 1) n_e m_e c^2,\ E'_{em} = (2 - n_e / n_{cr}) n_c m_e c^2 a'^2 / 4,\ E_{em} = n_c m_e c^2 a'^2 / 2,$$

$E'_{kin} = n_e m_e c^2 a'^2 / 4$ (non-relativistic), $\gamma' = [1 + a'^2 / 2]^{1/2}$

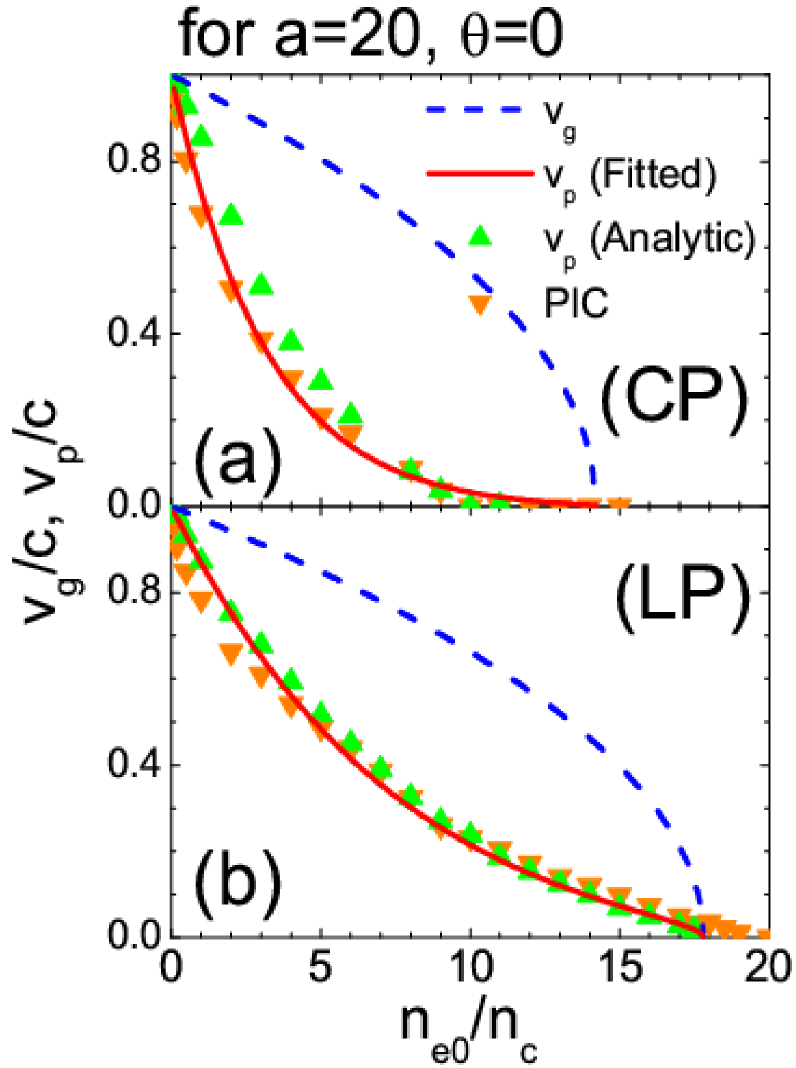


From energy balance, propagation velocity

$$v_p = \frac{(1-R)I}{E'_{em} + E'_{kin}} = \frac{(1-R) n_c a'^2 v_g}{(1+R)(1 - n_e / 2\gamma' n_c) n_c a'^2 + 2(\gamma' - 1) n_e}$$

$v_p = v_g$ $\qquad$ $v_p \leq v_g$

Relativistic propagation velocity



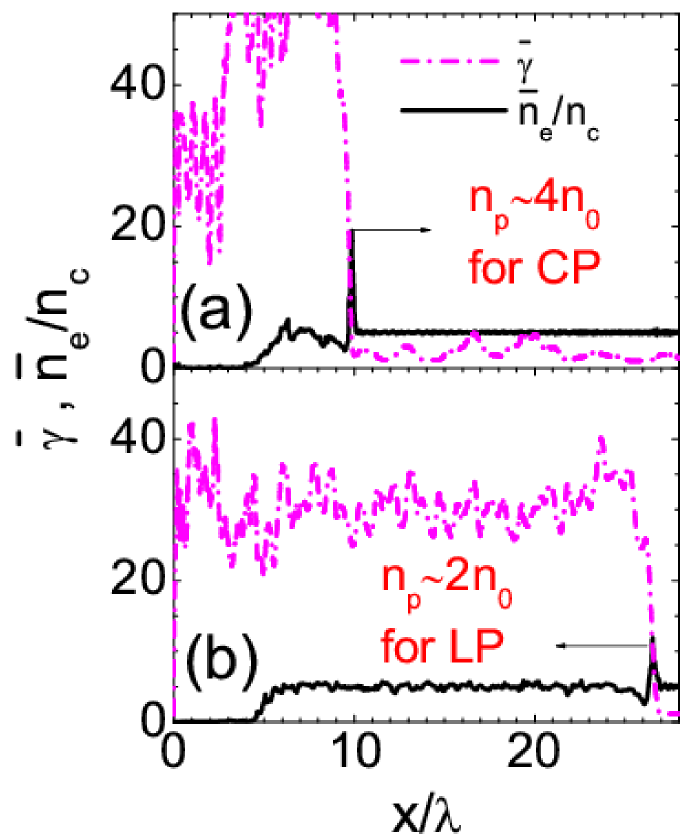
$$v_p = \frac{(1-R)n_c a'^2 v_g}{(1+R)(1-n_e/2\gamma' n_c)n_c a'^2 + 2(\gamma'-1)n_e}$$

n_p are the different heights of density ridge formed before laser front

for a

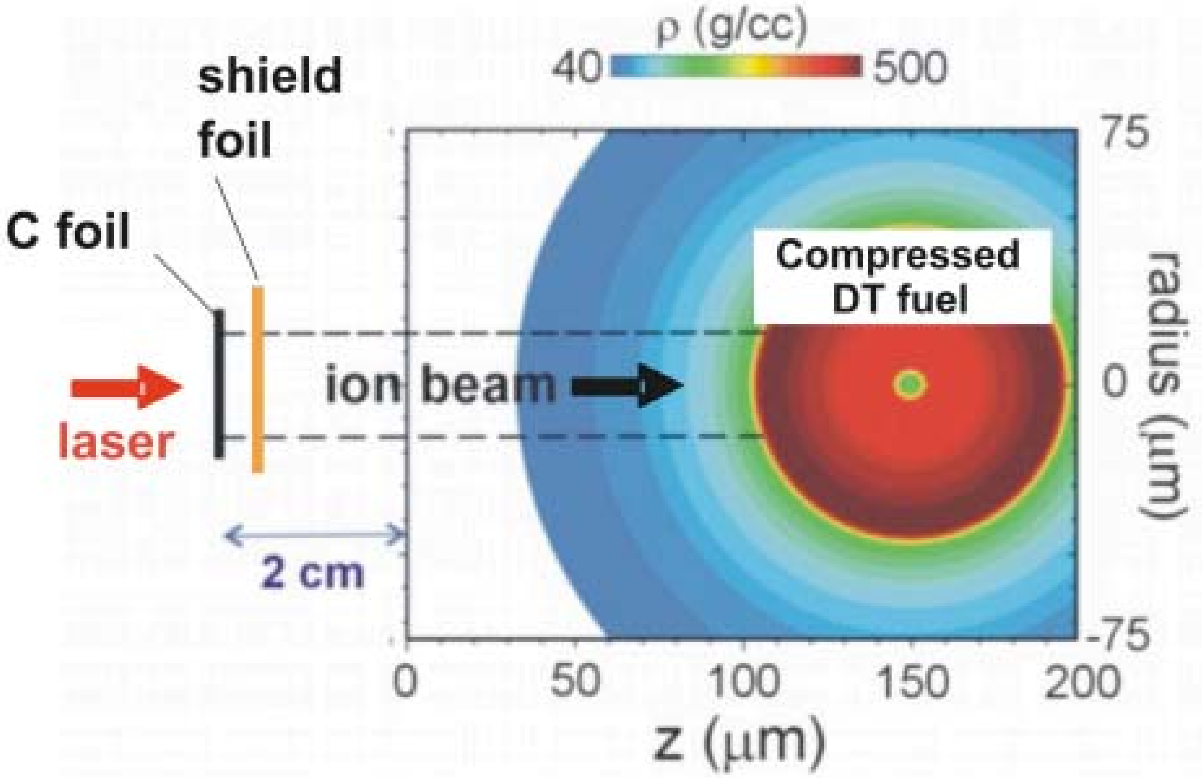
propa ... l by

$v_p = \epsilon$



Application (a): Ion acceleration and Fast ignition

- **The Break Out Afterburner is an ion acceleration technique that may achieve the fast ignition**

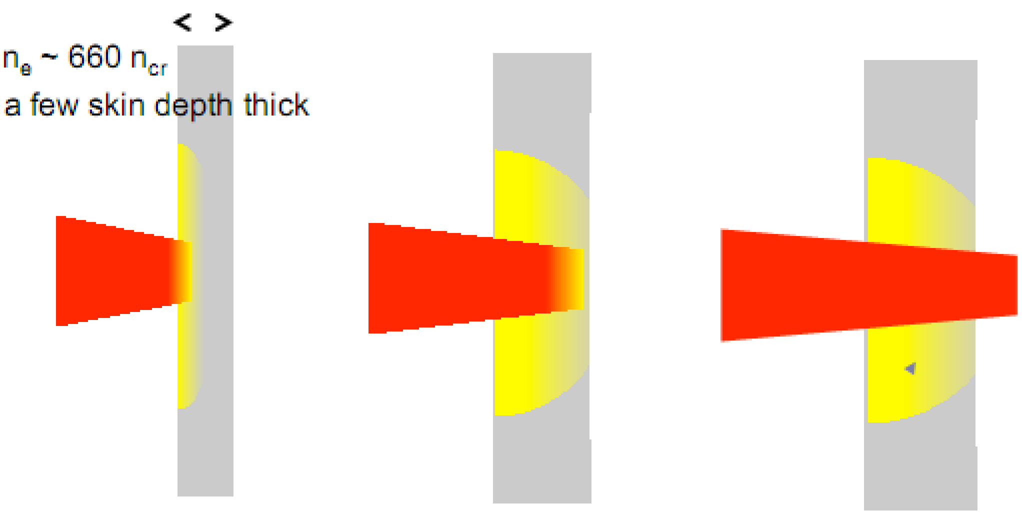


L. Yin et al, Laser Part. Beams 24, 291 (2006), Phys. Plasmas 14, 056706 (2007);

J. J. Honrubia et al, Nucl. Fusion 46, L25 (2006), J Phys. Conf. Ser. 244 (2010).

Application (a): Ion acceleration and Fast ignition

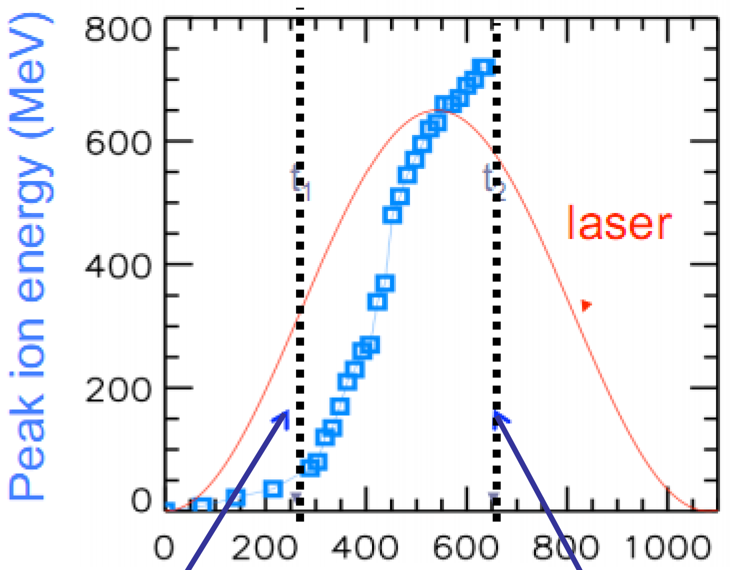
- **The Break Out Afterburner (BOA) is a robust ion acceleration mechanism that occurs (> 10^{20} W/cm^2, LP) when a nm-scale target turns relativistically transparent**



Initially, heating is confined to the target front

Target expands Skin depth widens Volumetric heating

Target becomes relativistically transparent BOA begins



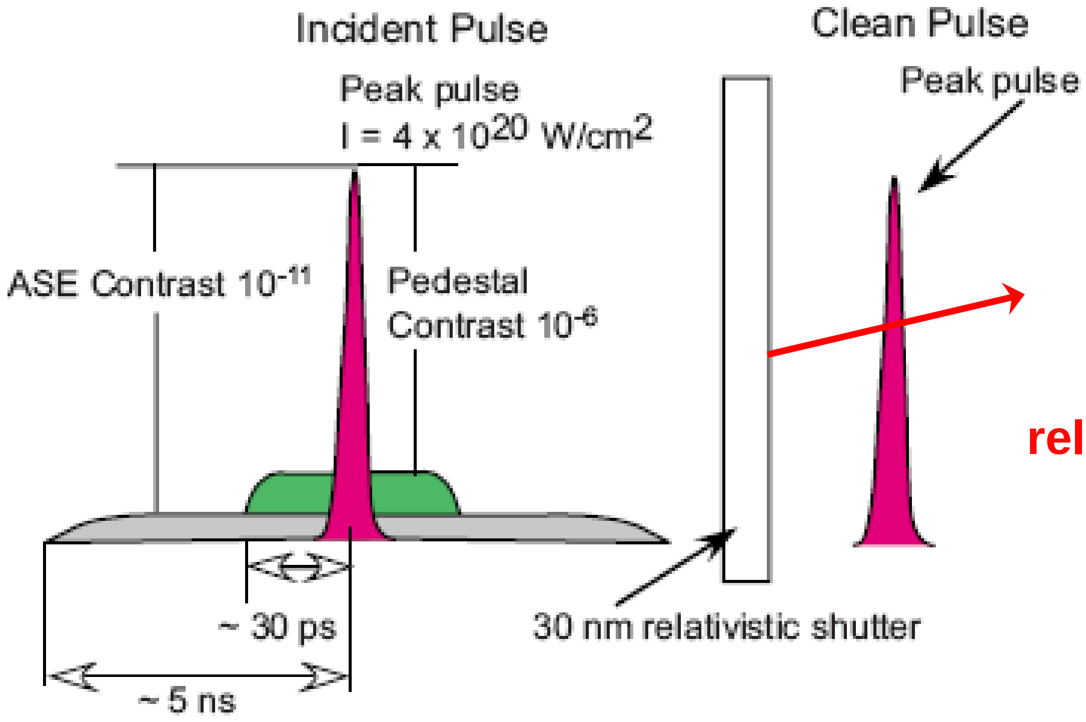
$n_e<n_{cr}$ relativistic transparency

$n_e<n_c$ classical transparency

L. Yin et al, Laser Part. Beams 24, 291 (2006), Phys. Plasmas 14, 056706 (2007).

Application (b): Relativistic plasma shutter

- A relativistic plasma shutter can remove the pre-pulse and produce a clean ultrahigh intensity pulse

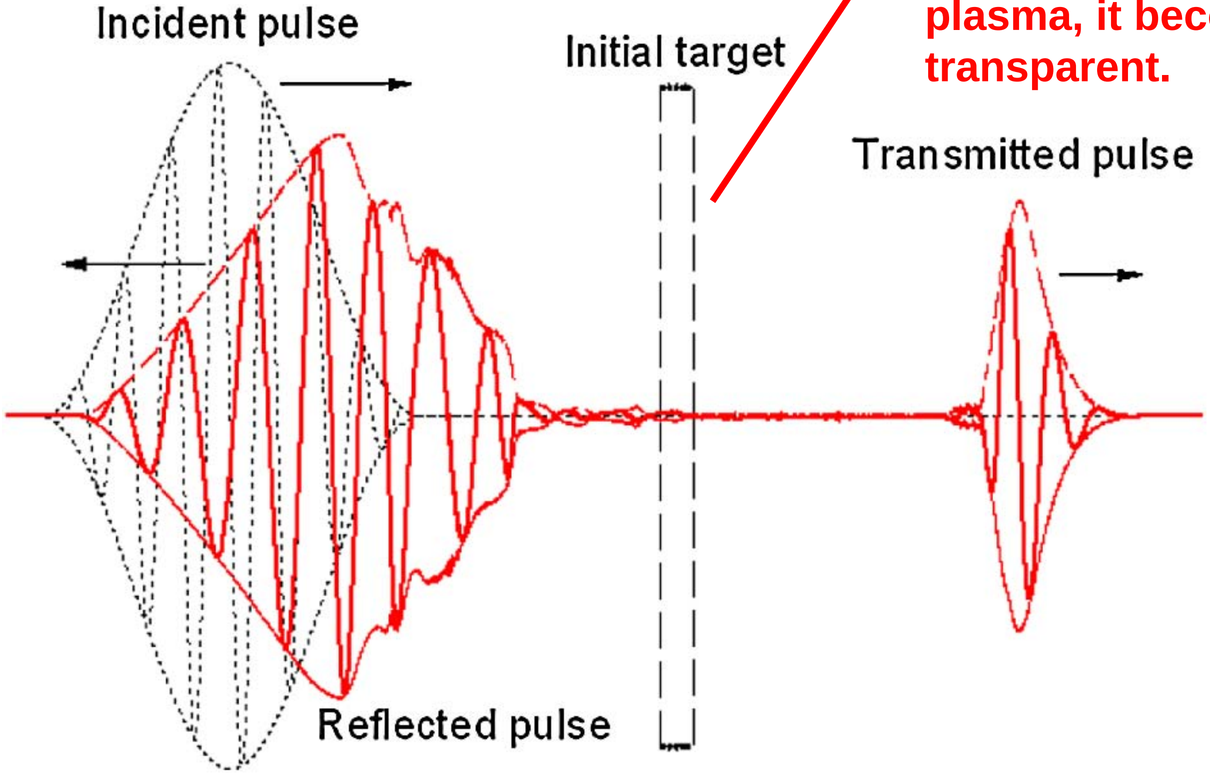


This shutter is classically overdense but relativistically underdense.

S. A. Reed *et al.*, Appl. Phys. Lett. 94, 201117 (2009).

Application (c): Shortening of laser pulses

- A quasi-single-cycle relativistic pulse can be produced by ultrahigh laser-foil interaction



The thin foil initially is relativistically overdense.

When it evolves into a thick but rare plasma, it becomes relativistically transparent.

L. L. Ji *et al.*, Phys. Rev. Lett. 103, 215005 (2009).

Conclusion

- **Relativistic induced transpancy makes the propagation of a relativistic laser pulse into an overdense plasma possible**
  - We clarify the underlying physics of the relativistic critical density increase, and propose a method for determining the relativistic critical surface and the relativistic critical density increase.
  - We have shown that the critical density increase strongly depends on the plasma density profile and laser polarization, and have discovered and explained a rather long response time for the relativistic critical density increase.
- **Relativistic laser pulse propagation is governed by energy balance**
  - The propagation velocity is much less than the group velocity from dispersion relation when the total energy density in plasma exceeds the wave energy density in vacuum.
- **The relativistic induced transparency finds wide applications in fast ignition scheme, ion acceleration, relativistic plasma shutter, and shortening of laser pulses.**

Thanks for your attention!