$O(\alpha_s v^2)$ Corrections to $^1S_0$ Heavy Quarkonium Hadronic and Electromagnetic Decay

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NRQCD effective field theory

- Field of $O(m_Q)$ (or larger) is integrated in NRQCD effective field theory and the remained fields are organized by the power of v:

$$\mathcal{L}_{\text{NRQCD}} = \mathcal{L}_{\text{light}} + \mathcal{L}_{\text{heavy}} + \delta\mathcal{L}_{\text{4-fermion}} + \delta\mathcal{L},$$

where the four-fermions term is

$$\delta\mathcal{L}_{4-\text{fermion}} = \sum_n \frac{f_n(\mu_\Lambda)}{m_Q^{d_n-4}} \mathcal{O}_n(\mu_\Lambda),$$

- Operator

$$\mathcal{O}_n(\mu_\Lambda) = \begin{cases} \psi^\dagger \mathcal{K}'_n \chi \chi^\dagger \mathcal{K}_n \psi & \text{light hadron (LH) decay} \\ \psi^\dagger \mathcal{K}'_n \chi |0\rangle\langle 0| \chi^\dagger \mathcal{K}_n \psi & \text{electromagnetic (EM) decay} \end{cases}$$

NRQCD factorization

- Decay width of heavy quarkonium is given by the following NRQCD factorization formula:

$$\Gamma(H \to \mathrm{LH}/\mathrm{EM}) = \sum_n \frac{2\mathrm{Im} f_n(\mu_\Lambda)}{m_Q^{d_n-4}} \langle H|\mathcal{O}_n(\mu_\Lambda)|H\rangle. \qquad (1)$$

  where $f_n(\mu_\Lambda)$ can be calculated perturbatively by matching QCD to NRQCD, and $\langle H|\mathcal{O}_n(\mu_\Lambda)|H\rangle$ are non-perturbative long-distance matrix elements (LDMEs).
- Based on NRQCD effective field theory, it can be argued that NRQCD factorization holds to all order in α_s for heavy quarkonium inclusive annihilation decay.

Relation between LH decay and EM decay

- Up to order v^2, both LH decay and EM decay have two LDMEs. The corresponding operators are

$$\begin{aligned}
\mathcal{O}(^1S_0^{[1]}) &= \psi^\dagger\chi\chi^\dagger\psi, \\
\mathcal{P}(^1S_0^{[1]}) &= \frac{1}{2}\left[\psi^\dagger\chi\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\mathbf{D}})^2\psi + \mathrm{h.c.}\right], \\
\mathcal{O}_{\mathrm{EM}}(^1S_0^{[1]}) &= \psi^\dagger\chi|0\rangle\langle 0|\chi^\dagger\psi, \\
\mathcal{P}_{\mathrm{EM}}(^1S_0^{[1]}) &= \frac{1}{2}\left[\psi^\dagger\chi|0\rangle\langle 0|\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\mathbf{D}})^2\psi + \mathrm{h.c.}\right],
\end{aligned}$$

- Vacuum-saturation approximation:

$$\begin{aligned}
\langle H|\mathcal{O}_n|H\rangle &= \sum_X \langle H|\psi^\dagger\mathcal{K}'_n\chi|X\rangle\langle X|\chi^\dagger\mathcal{K}_n\psi|H\rangle \\
&\approx \langle H|\psi^\dagger\mathcal{K}'_n\chi|0\rangle\langle 0|\chi^\dagger\mathcal{K}_n\psi|H\rangle(1+O(v^4)),
\end{aligned}$$

that is, LDMEs for the two decay processes are the same.

Present situation of study about ${}^1S_0$ state decay

- ${}^1S_0$ state heavy quarkonium EM decay

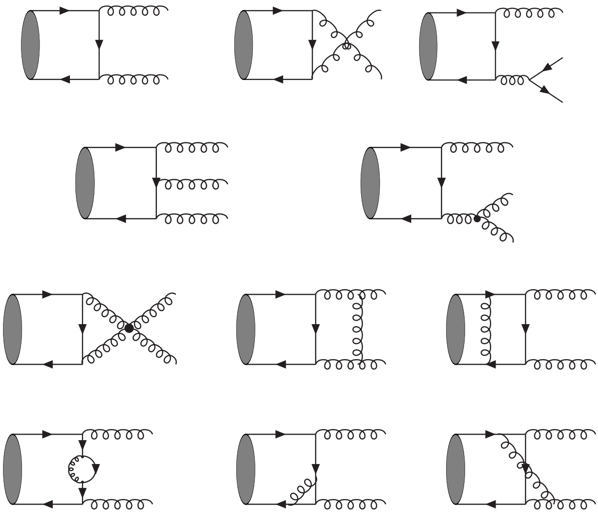
	1	v^2	v^4
1	$\surd$	$\surd$	$\surd$
α_s	$\surd$	?	...
α_s^2	$\surd$	...	...
number of LDMEs	1	2	7

- ${}^1S_0$ state heavy quarkonium LH decay

	1	v^2	v^4
1	$\surd$	$\surd$	$\surd$
α_s	$\surd$	?	...
α_s^2	...	...	...
number of LDMEs	1	2	4

- NLO calculation is important to control the theoretic uncertainty, for both $O(v^0)$ and $O(v^2)$ terms.
- $\alpha_s(m_Q) \sim v^2$, thus $O(\alpha_s v^2)$ is important for the next step.

Feynman diagrams (LH decay)



Expansion in v

- In the rest frame of $Q\overline{Q}$

$$p_Q = \frac{1}{2}P + q, p_{\overline{Q}} = \frac{1}{2}P - q,$$
$$P = (2E_{\mathbf{q}}, \mathbf{0}), q = (0, \mathbf{q}), E_{\mathbf{q}} = \sqrt{m_Q^2 + \mathbf{q}^2}.$$

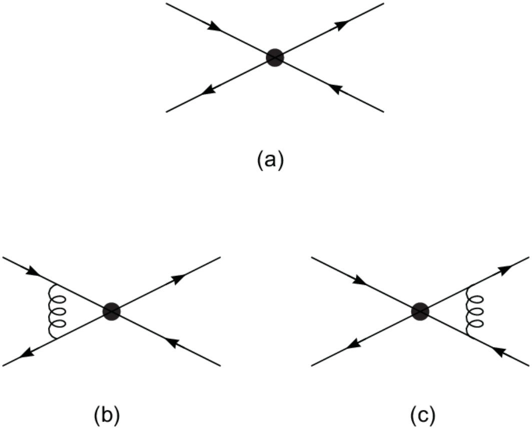
- Redefinition for all momentum: $p_i \ \rightarrow \ p_i' E_{\mathbf{q}}/m$,

$$\prod_i \left(\frac{d^{(D-1)}k_i}{2(k_i)_0}\right) \delta^D(P - \sum_i k_i) \ = \ \prod_i \left(\frac{d^{(D-1)}k_i'}{2(k_i')_0}\right) \delta^D \left(P' - \sum_i k_i'\right) f(\mathbf{q}^2).$$

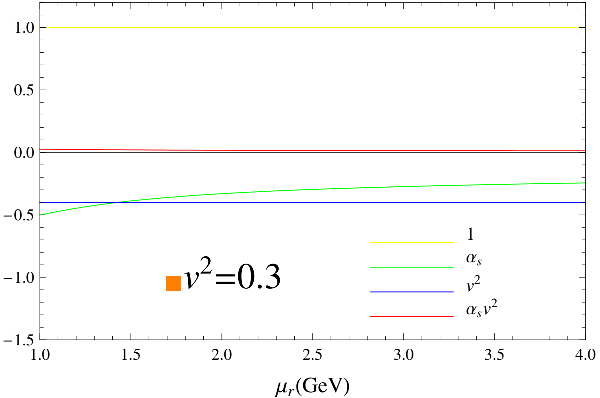
- Because $P'^2 = 4m_Q^2$ and $\partial p_i' \cdot p_j'/\partial \mathbf{q} = 0$, we expand $\mathbf{q}$ before doing phase space integration and loop integration.
- After the expansion, there is no additional term of order v^2 shows up in subsequent integration.

perturbative NRQCD calculation

- To order $\alpha_s v^2$ in NRQCD:

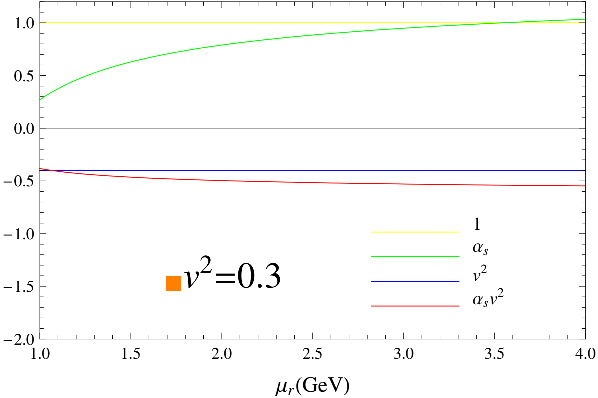


Result: EM decay



- $\alpha_s v^2$ correction is small.

Result: LH decay



- $\alpha_s v^2$ is large, larger than order v^2 result.

LDMEs

- Fit data: maybe no enough data to constrain these LDMEs.
- Lattice calculation: large uncertainty for order v^2 LDME.
- Using potential model.

LDMEs in potential model

- Leading order LDME can be related to wave function at the origin:

$$\frac{1}{2N_c}\langle\mathcal{O}(^1S_0^{[1]})\rangle_H = \frac{1}{4\pi}|\mathcal{R}_H(0)|^2, \tag{3}$$

- Assuming a spin independent potential and regularize the ultravoilet divergence using dimensional regularization, one finds the following relation for LDME at higher oreder in v:

$$\langle \boldsymbol{q}^{2r}\rangle_H = (m\,\epsilon_{nS})^r[1+\mathcal{O}(v^2)], \tag{4}$$

  where ϵ_{nS} is bounding energy.
- We will choose Cornell potential

$$V(r) = -\frac{\kappa}{r} + \sigma r, \tag{5}$$

  where σ can be calculated by Lattice QCD. To determine κ, we need at least one experiment data.

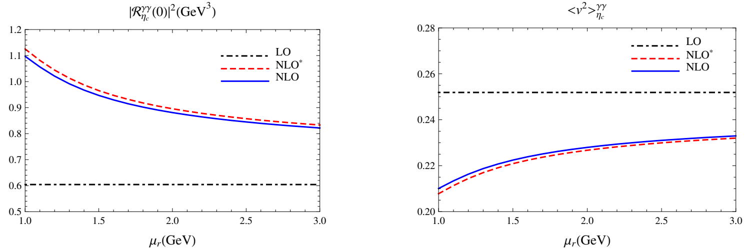
LDMEs for η_c (1)

- Using $\Gamma^{\gamma\gamma}(\eta_c) = 7.2 \pm 0.7 \pm 2.0\ \mathrm{KeV}$ as input

$$|\mathcal{R}^{\gamma\gamma}_{\eta_c}(0)|^2 = 0.881^{+0.382}_{-0.313}\ \mathrm{GeV}^3, \quad (6a)$$

$$\langle v^2\rangle^{\gamma\gamma}_{\eta_c} = 0.228^{+0.126}_{-0.100}, \quad (6b)$$

Renormalization scale dependence



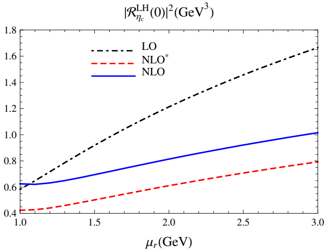
LDMEs for η_c (2)

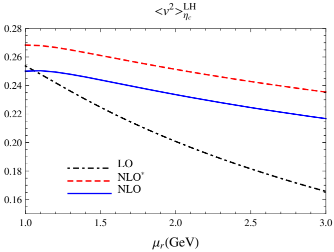
- Assume the total decay width $\Gamma^{\rm total}(\eta_c) = 28.6 \pm 2.2\ \mathrm{MeV}$ approximately equals to LH decay width, we have

$$|\mathcal{R}^{\rm LH}_{\eta_c}(0)|^2 = 0.814^{+0.332}_{-0.256}\ \mathrm{GeV}^3, \quad (7a)$$

$$\langle \boldsymbol{v}^2\rangle^{\rm LH}_{\eta_c} = 0.234^{+0.121}_{-0.099}, \quad (7b)$$

Renormalization dependence





LDMEs for η_c (3)

- Errors in Eq.(6) are dominated by data (experimental), and errrors in Eq.(7) are dominated by renormalization scale dependence (theoretical).
- It is possible to get better results by combining the two results. By minimizing the χ^2, we get our final results:

$$
\begin{aligned}
|\mathcal{R}_{\eta_c}(0)|^2 &= 0.834^{+0.281}_{-0.197}\ \mathrm{GeV}^3, \\
\langle \boldsymbol{v}^2 \rangle_{\eta_c} &= 0.232^{+0.121}_{-0.098},
\end{aligned}
$$

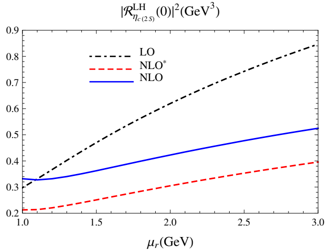
LDMEs for $\eta_c(2S)$

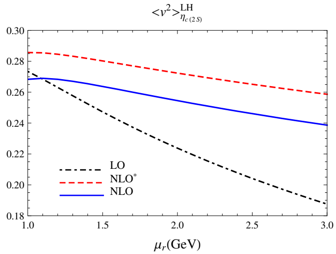
- Using total decay width 14 ± 7 Mev as input:

$$|\mathcal{R}^{\mathrm{LH}}_{\eta_c(2S)}(0)|^2 = 0.423^{+0.245}_{-0.230}\ \mathrm{GeV}^3, \tag{9a}$$

$$\langle \boldsymbol{v}^2 \rangle^{\mathrm{LH}}_{\eta_c(2S)} = 0.255^{+0.130}_{-0.109}. \tag{9b}$$

Renormalization scale dependence





Predictions

- Total decay width for η_c

	LO	*NLO**	*NLO*	Experimental data
Width(MeV)	$12.9^{+17.3}_{-6.0}$	$42.9^{+42.1}_{-18.4}$	$31.4^{+29.3}_{-14.4}$	$28.6^{+2.2}_{-2.2}$

- $\Gamma(\eta_c \to \gamma\gamma)$

	LO	*NLO**	*NLO*	Experimental data
Width(KeV)	$15.28^{+6.81}_{-8.45}$	$4.75^{+2.19}_{-2.39}$	$6.61^{+2.77}_{-2.83}$	$7.2 \pm 0.7 \pm 2.0$

- $\Gamma(\eta_c(2S) \to \gamma\gamma)$

	LO	*NLO**	*NLO*	Experimental data
Width(KeV)	$7.61^{+4.84}_{-5.55}$	$2.31^{+1.58}_{-1.48}$	$3.34^{+1.47}_{-1.58}$	$Br < 5 \times 10^{-4}$

Summary

- We calculated the order $\alpha_s v^2$ corrections for $^1S_0$ state EM decay and LH decay. It was found that $\alpha_s v^2$ correction has a small infuence for $^1S_0$ state EM decay, but has an significant influence for LH decay.
- By combining potential model, we extract LDMEs at LO and NLO in v.
- After considering the order $\alpha_s v^2$ correction, our predictions for EM and LH decay width of η_c, and EM decay width of $\eta_c(2s)$ are improved comparing with data.
- It is also possible to do the order $\alpha_s v^2$ corrections for other heavy quarkonium state production and decay.

Compare with arXiv:1104.1418

Jia, Yang, Sang and Xu do an independent work for order $\alpha_s v^2$ corrections for $^1S_0$ state EM decay. Short distance coefficients for this channel agree with each other in our two groups. Methods are not the same, including:

- They performed the matching calculation at amplitude level; We performed it in decay rate level. Matching at amplide level works for EM decay, but it does not work for LH decay.
- They expanded the **q** after the loop integration; We expanded it before loop integration. In their calculation all poles including Coulomb pole present in short distance coefficients; but in our calculation, we ignore potential region in the loop integration because result in this region will be absorbed into LDMEs exactly, thus Coulomb pole does not show up in our calculation.

Thank you!