NRQCD Matrix Elements for S-wave Bottomonia and $\Gamma[\eta_b(nS) \to \gamma\gamma]$ with Relativistic Corrections

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Outline

- NRQCD Matrix Elements for *S*-wave bottomonia
- $\Gamma[\eta_b(nS) \to \gamma\gamma]$ with relativistic corrections
- Summary

NRQCD Matrix Elements for *S*-wave bottomonia

NRQCD Matrix Elements

- NRQCD factorization formula

$$\mathcal{A} = \sum_n c_n \langle \boldsymbol{q}^{2n} \rangle \langle O_1 \rangle^{1/2}$$

- Matrix elements at leading order in the heavy-quark velocity $\boldsymbol{v}$

$$\langle \mathcal{O}_1 \rangle_\Upsilon = \left| \langle \Upsilon | \psi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} \chi | 0 \rangle \right|^2 \qquad \langle \mathcal{O}_1 \rangle_{\eta_b} = \left| \langle \eta_b | \psi^\dagger \chi | 0 \rangle \right|^2$$

- Ratios of matrix elements of higher orders in $\boldsymbol{v}$ to the leading-order matrix elements

$$\langle \boldsymbol{q}^{2n} \rangle_\Upsilon = \frac{\langle \Upsilon | \psi^\dagger (-\frac{i}{2} \overleftrightarrow{\boldsymbol{D}})^{2n} \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} \chi | 0 \rangle}{\langle \Upsilon | \psi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} \chi | 0 \rangle}$$

$$\langle \boldsymbol{q}^{2n} \rangle_{\eta_b} = \frac{\langle \eta_b | \psi^\dagger (-\frac{i}{2} \overleftrightarrow{\boldsymbol{D}})^{2n} \chi | 0 \rangle}{\langle \eta_b | \psi^\dagger \chi | 0 \rangle}$$

$\Rightarrow$ *Relativistic Corrections*

NRQCD Matrix Elements

- $\Upsilon(nS)$ leptonic decay rates are accurately measured:

$$\Gamma[\Upsilon(1S) \to e^+e^-] = 1.340 \pm 0.018 \text{ keV}$$
$$\Gamma[\Upsilon(2S) \to e^+e^-] = 0.612 \pm 0.011 \text{ keV}$$
$$\Gamma[\Upsilon(3S) \to e^+e^-] = 0.443 \pm 0.008 \text{ keV}$$

It is tempting to obtain MEs from the leptonic decay rate

- NRQCD factorization formula for $\Upsilon(nS)$ leptonic decay:

Hard scattering amplitude

$$\mathcal{A}[\Upsilon \to e^+e^-] = \sum_n \left[\frac{1}{n!} \left(\frac{\partial}{\partial \boldsymbol{q}^2} \right)^n H(\boldsymbol{q}^2) \right] \Bigg|_{\boldsymbol{q}^2=0} \langle \boldsymbol{q}^{2n} \rangle_\Upsilon \langle \mathcal{O}_1 \rangle_\Upsilon^{1/2}$$

short distance coefficients

☞one equation, infinite variables

Resummation of Relativistic Corrections

- Generalized Gremm-Kapustin relation for *S*-wave quarkonia:

$$\langle \boldsymbol{q}^{2n} \rangle = \langle \boldsymbol{q}^2 \rangle^n$$

Bodwin, Kang, and Lee, PRD74, 014014 (2006)

  - Holds up to corrections of relative order $\boldsymbol{v^2}$
  - Static potential must be sums of constants times powers of the $Q\bar{Q}$ separation
  - Consistent with dimensional regularization

- This allows one to resum a class of relativistic corrections to all orders.

$$\mathcal{A}[\Upsilon \to e^+e^-] = \sum_n \left[\frac{1}{n!} \left(\frac{\partial}{\partial \boldsymbol{q}^2} \right)^n H(\boldsymbol{q}^2) \right] \Bigg|_{\boldsymbol{q}^2=0} \langle \boldsymbol{q}^{2n} \rangle_\Upsilon \langle \mathcal{O}_1 \rangle_\Upsilon^{1/2}$$

$$= H(\langle \boldsymbol{q}^2 \rangle_\Upsilon) \langle \mathcal{O}_1 \rangle_\Upsilon^{1/2}$$

☞one equation, two variables

Potential Model

- Cornell potential

$$V(r) = -\frac{\kappa}{r} + \sigma r$$

fits well to lattice measurements of $Q\bar{Q}$ static potential

- By numerically solving the Schrödinger equation, with σ and the 1*S*-2*S* splitting as input parameters, $\langle \mathcal{O}_1 \rangle$ and $\langle \boldsymbol{q}^2 \rangle$ are expressed by a single variable

  ☞one equation, one variable :

  both $\langle \mathcal{O}_1 \rangle$ and $\langle \boldsymbol{q}^2 \rangle$ can be obtained from the leptonic decay rate.

Bodwin, Chung, Kang, Lee, and Yu, PRD77, 094017 (2008)

NRQCD Matrix Elements for *S*-wave bottomonia

- Compute matrix elements from the resummed decay rate formula

$$\Gamma[\Upsilon \to e^+e^-] = \frac{8\pi\alpha^2}{27m_\Upsilon^2}\left[1 - f(\langle v^2\rangle_\Upsilon) - \frac{8\alpha_s}{3\pi}\right]^2 \langle \mathcal{O}_1\rangle_\Upsilon$$

$$\langle v^2\rangle_\Upsilon \equiv \langle \boldsymbol{q}^2\rangle_\Upsilon / m_b^2 \qquad f(x) = x/[3(1+x+\sqrt{1+x})]$$

Input parameters :

- measured leptonic decay rate
- meson masses, m_b, $\alpha(m_\Upsilon)$, $\alpha_s(m_\Upsilon)$
- σ from lattice measurements of the string tension

10% error in $\langle \boldsymbol{q}^2\rangle$ for ignorance of the spin-dependent interactions

: Sources of uncertainties

NRQCD Matrix Elements for *S*-wave bottomonia

- Relativistic corrections at order-α_s have been calculated to all orders in v :
for J/ψ the corrections are about 0.3%.
- For bottomonia, v is even smaller, and therefore the corrections of order $\alpha_s v^{2n}$ can be ignored.

NRQCD Matrix Elements for *S*-wave bottomonia

- Two-loop corrections have been calculated at leading order in v. The corrections contain a strong dependence on the factorization scale $\mu_{\mathbf{f}}$.

  Czarnecki and Melnikov, PRL80, 2531 (1998)
  Beneke, Signer and Smirnov, PRL80, 2535 (1998)
- If we compute the MEs to two-loop order accuracy, the MEs will also have strong dependence on $\mu_{\mathbf{f}}$.
- If we use those MEs in quarkonium decay or production processes, the strong dependence on $\mu_{\mathbf{f}}$ will only cancel if the short-distance coefficients are calculated to relative order α_s^2.
- Generally, short-distance coefficients have not been calculated beyond relative order α_s : therefore ***we omit the two-loop corrections*** in the calculation of the MEs.
- Numerically the two-loop corrections enhance the leading order MEs by about 40%.

NRQCD Matrix Elements for *S*-wave bottomonia

$$\langle \mathcal{O}_1 \rangle_{\Upsilon(1S)} = 3.069^{+0.207}_{-0.190}\ \mathrm{GeV}^3$$
$$\langle \mathcal{O}_1 \rangle_{\Upsilon(2S)} = 1.623^{+0.112}_{-0.103}\ \mathrm{GeV}^3$$
$$\langle \mathcal{O}_1 \rangle_{\Upsilon(3S)} = 1.279^{+0.090}_{-0.083}\ \mathrm{GeV}^3$$

- Relativistic corrections are small (-0.4%, 3%, 6% for 1*S*, 2*S*, 3*S*)
- The errors are much smaller than available lattice QCD calculations

$$\langle \mathcal{O}_1 \rangle_{1S} = 4.10(1)(9)(41)\ \mathrm{GeV}^3 \quad \text{(unquenched)}$$

Bodwin, Sinclair and Kim, PRD65, 054504 (2002)

NRQCD Matrix Elements for *S*-wave bottomonia

$$\langle \boldsymbol{q}^2 \rangle_{\Upsilon(1S)} = -0.193^{+0.072}_{-0.073} \text{ GeV}^2 \qquad \langle v^2 \rangle_{\Upsilon(1S)} = -0.009^{+0.003}_{-0.003}$$

$$\langle \boldsymbol{q}^2 \rangle_{\Upsilon(2S)} = 1.898^{+0.210}_{-0.210} \text{ GeV}^2 \qquad \langle v^2 \rangle_{\Upsilon(2S)} = 0.090^{+0.011}_{-0.011}$$

$$\langle \boldsymbol{q}^2 \rangle_{\Upsilon(3S)} = 3.283^{+0.353}_{-0.352} \text{ GeV}^2 \qquad \langle v^2 \rangle_{\Upsilon(3S)} = 0.155^{+0.018}_{-0.018}$$

- $\langle v^2 \rangle$ roughly agrees with the typical estimate $v^2 \sim 0.1$ except that $\langle v^2 \rangle_{\Upsilon(1S)} = -0.009^{+0.003}_{-0.003}$ is tiny
- The errors are significantly smaller than the lattice QCD calculations

$$-5 \text{ GeV}^2 < \langle \boldsymbol{q}^2 \rangle_{1S} < 2 \text{ GeV}^2$$ (unquenched)

Bodwin, Sinclair and Kim, PRD65, 054504 (2002)

- These numbers have been used in the analysis of relativistic corrections to the axial vector and vector currents in the $\bar{b}c$ meson system at order α_s

Lee, Sang and Kim, JHEP 1101,113 (2011)

$$\Gamma[\eta_b(nS) \to \gamma\gamma]$$

with relativistic corrections

$\Gamma[\eta_b(nS) \to \gamma\gamma]$

- $\eta_b(1S)$ has been discovered, little was known except for its mass
$$m_{\eta_b(1S)} = 9390.9 \pm 2.8 \text{ MeV}$$

BABAR, PRL101, 071801 (2008)
BABAR, PRL103, 161801 (2009)
CLEO, PRD81, 031104 (2010)

- BELLE collaboration reported more accurate measurement of the mass, and also measured the total width :
$$m_{\eta_b(nS)} = 9401.0 \pm 1.9^{+1.4}_{-2.4} \text{ MeV}$$
$$\Gamma[\eta_b(1S)] = 12.4^{+5.5+11.5}_{-4.6-3.4} \text{ MeV}$$

BELLE Preliminary

See Roman Mizuk's talk on Wednesday

- We present a prediction of the two-photon decay rate $\Gamma[\eta_b(nS) \to \gamma\gamma]$ as an application of our matrix elements and resummation.

Strategy of Calculation

- Heavy-quark spin symmetry :

$$\langle \mathcal{O}_1 \rangle_{\eta_b} = \langle \mathcal{O}_1 \rangle_{\Upsilon}[1 + O(v^2)]$$

$$\langle \boldsymbol{q}^2 \rangle_{\eta_b} = \langle \boldsymbol{q}^2 \rangle_{\Upsilon}[1 + O(v^2)]$$

  Use $\langle \mathcal{O}_1 \rangle_{\Upsilon}$ and $\langle \boldsymbol{q}^2 \rangle_{\Upsilon}$ for $\langle \mathcal{O}_1 \rangle_{\eta_b}$ and $\langle \boldsymbol{q}^2 \rangle_{\eta_b}$,
  and introduce additional errors of $v^2 \sim 0.1$

- Order-α_s^2 corrections account for $-2.64\alpha_s^2$ in the ratio $\Gamma[\eta_b \to \gamma\gamma]/\Gamma[\Upsilon \to e^+e^-]$:
  take this as the error for omitting the order-α_s^2 corrections.

  Czarnecki and Melnikov, PRL80, 2531 (1998)
  Beneke, Signer and Smirnov, PRL80, 2535 (1998)
  Czarnecki and Melnikov, PLB519, 212 (2001)

- Order-$\alpha_s v^2$ correction is tiny, and is omitted.

  See Yan-Qing Ma's talk at 12:35 today

  Ma, Wang and Chao, arXiv:1012.1030 [hep-ph]
  Jia, Yang, Sang and Xu, arXiv:1104.1418 [hep-ph]

- Assume hyperfine splitting $m_{\Upsilon(nS)} - m_{\eta_b(nS)} = 50 \text{ MeV}$ for n=2 and 3.

$\boldsymbol{\Gamma[\eta_b(nS) \to \gamma\gamma]}$

- Resumed two-photon decay rate formula

$$\Gamma[\eta_b \to \gamma\gamma] = \frac{2\pi\alpha^2}{81 m_b^2}\left[1 - g(\langle v^2\rangle_{\eta_b}) - \frac{(20-\pi^2)\alpha_s}{6\pi}\right]^2 \langle \mathcal{O}_1\rangle_{\eta_b}$$

$$g(x) = 1 - \{\log[1 + 2\sqrt{x(1+x)} + 2x]\}/[2\sqrt{x(1+x)}]$$

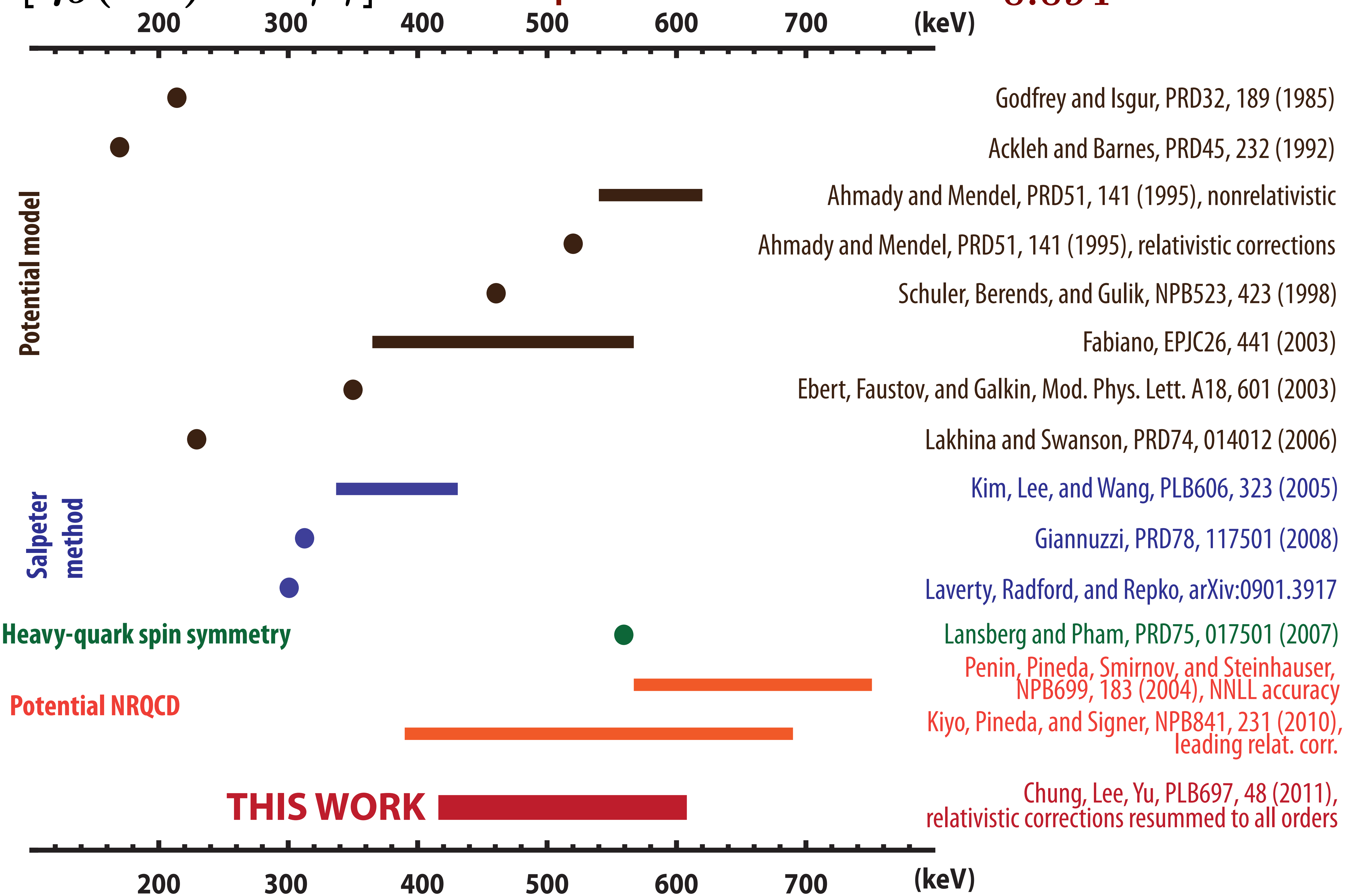
- Our predictions of two-photon widths:

$$\boldsymbol{\Gamma[\eta_b(1S) \to \gamma\gamma] = 0.512^{+0.096}_{-0.094}\ \mathrm{keV}}$$
$$\boldsymbol{\Gamma[\eta_b(2S) \to \gamma\gamma] = 0.235^{+0.043}_{-0.043}\ \mathrm{keV}}$$
$$\boldsymbol{\Gamma[\eta_b(3S) \to \gamma\gamma] = 0.170^{+0.031}_{-0.031}\ \mathrm{keV}}$$

- Roughly, $\mathrm{Br}[\eta_b(1S) \to \gamma\gamma] \sim \mathbf{6.9 \times 10^{-5}}$.

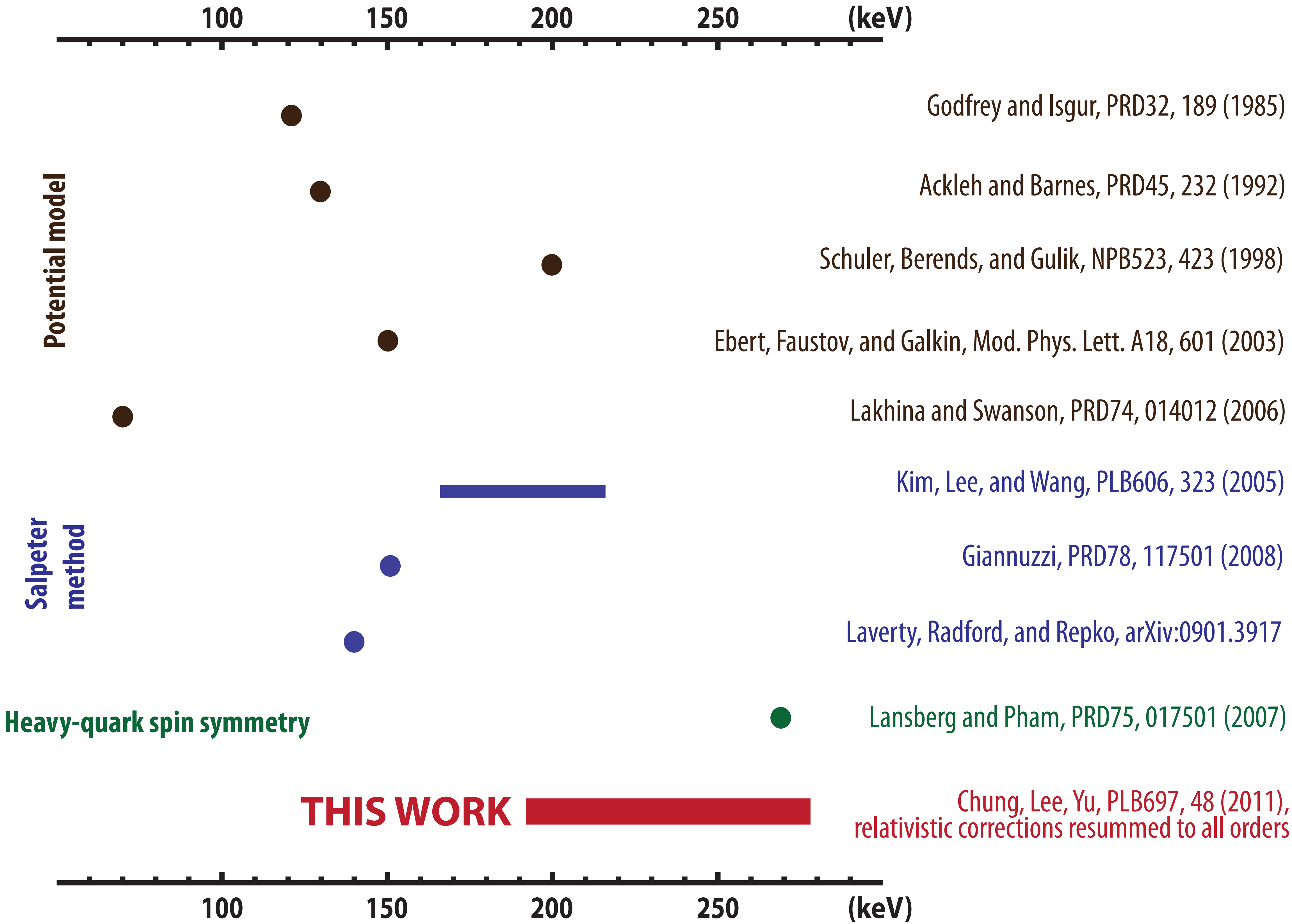
Comparison with Previous Predictions

$\Gamma[\eta_b(1S) \to \gamma\gamma]$ Our prediction = $0.512^{+0.096}_{-0.094}$ keV



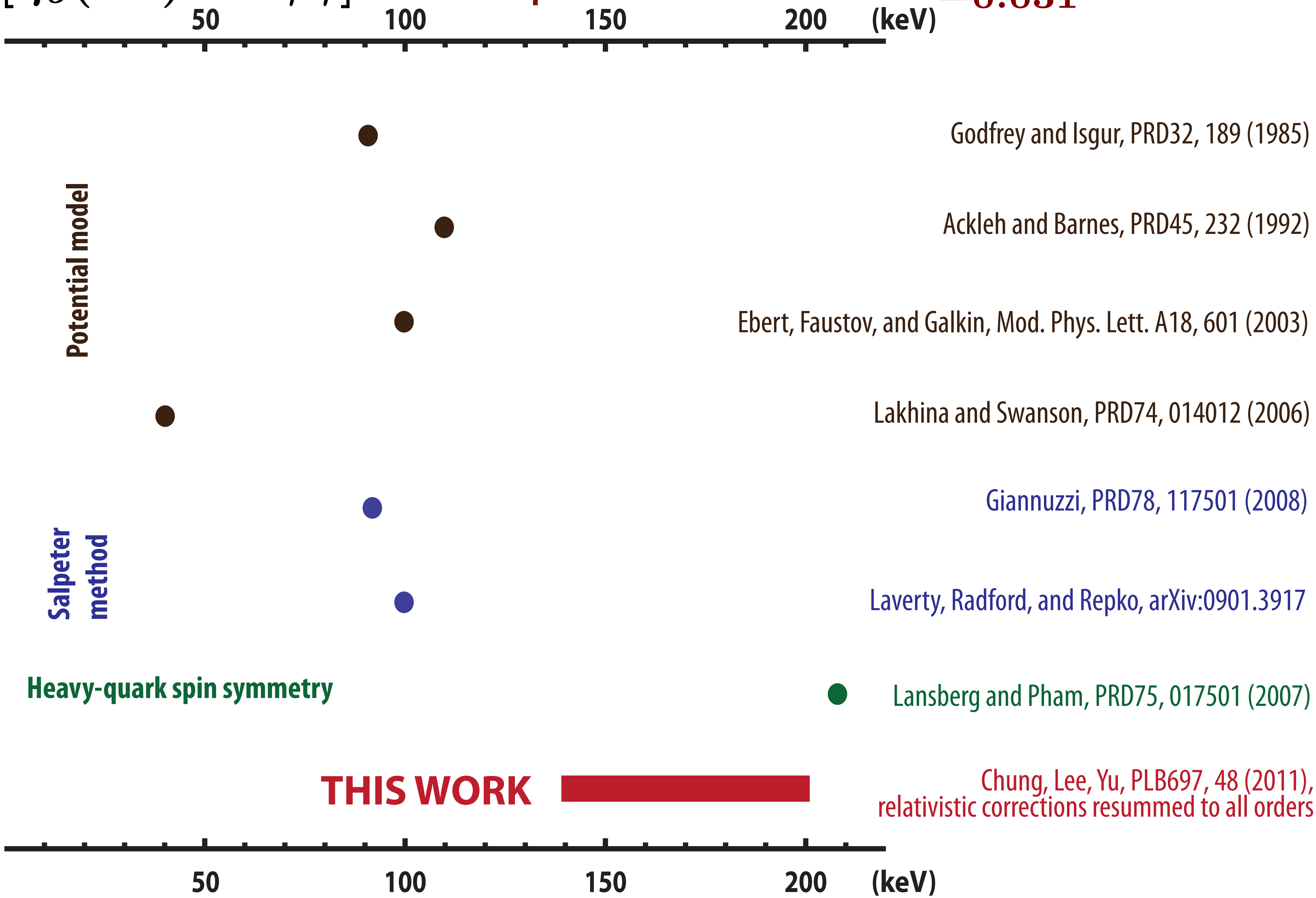
Comparison with Previous Predictions

$\Gamma[\eta_b(2S) \to \gamma\gamma]$ Our prediction = $0.235^{+0.043}_{-0.043}$ keV



Comparison with Previous Predictions

$\Gamma[\eta_b(3S) \to \gamma\gamma]$ Our prediction = $0.170^{+0.031}_{-0.031}$ keV



Summary

- We computed NRQCD matrix elements $\langle \mathcal{O}_1 \rangle$ and the ratio $\langle \boldsymbol{q}^2 \rangle$ for *S*-wave bottomonia with relativistic corrections resummed to all orders in $\boldsymbol{v}$.
- Our results have much smaller uncertainties compared to previous results from lattice calculations.
- By assuming approximate heavy-quark spin symmetry we make predictions for the the two-photon widths of $\eta_b(nS)$.
- Our prediction for $\Gamma[\eta_b(1S) \to \gamma\gamma]$ is consistent with recent potential NRQCD predictions.
- We expect $\Gamma[\eta_b(1S) \to \gamma\gamma]$ to be measurable at the superKEKB or superB factory, and the CERN LHC.