# Pseudoscalar Quarkonium Exclusive Decays to Vector Meson Pair 

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## I. $\eta_{b}$ Exclusive Decays to $\mathrm{J} / \Psi$ Pair

The lowest energy state in $Y$ family, the $\eta_{b}$, is very elusive. The existence of the $\eta_{b}$ is a solid prediction of the quark model

About thirty year after it spin triplet partner being found, recently it was observed for the first time by Babar through $\mathrm{Y}(3 \mathrm{~s})-->\eta_{b} \mathrm{Y}$

[Aubert, et al., Babar Collaboration, 2008]



In recent years, the search for $\eta_{b}$, has been conducted at CLEO, LEP, and CDF, B-factories, using both inclusive and exclusive methods

It is worth noting that both Babar and CLEO-c measurements are indirect ones. For further study on $\eta_{b}$ physics, direct measurements on its decay products are necessary

## Direct Search for $\eta_{b}$ at CDF [Tseng, CDF, 2002]

$\eta_{b} \rightarrow \mathrm{~J} / \psi \mathrm{J} / \psi$
A small cluster of 7 events can be seen, where 1.8 events are expected from background
If this cluster is due to decay, then the product of its production cross-section and decay branching fractions are near the lower limit of expectation from Braaten et al.

According Braaten, Fleming and Leibovich (hep-ph/0008091) , though helicity suppressed,

$$
\operatorname{Br}\left[\eta_{b} \rightarrow J / \psi+J / \psi\right]=7 \times 10^{-3} \sim 7 \times 10^{-5}
$$

Which seems to be overestimated, since

$$
\operatorname{Br}\left[\eta_{b} \rightarrow C+C+\bar{C}+\bar{C}\right] \sim 10^{-5}
$$

[Maltoni and Polosa,hep-ph/0405082]

A recent analysis shows:
[Jia, hep-ph/0611130]

$$
\begin{aligned}
& \operatorname{Br}\left[\eta_{b} \rightarrow \phi+\phi\right] \approx(0.9-1.4) \times 10^{-9} \\
& \operatorname{Br}\left[\eta_{b} \rightarrow J / \psi+J / \psi\right]=2.4_{-1.9}^{+4.2} \times 10^{-8}
\end{aligned}
$$

If so, such a rare decay mode perhaps will not be observed in the foreseeable future in experiment

More recently, the $\eta_{b} \rightarrow \mathrm{~J} / \psi \mathrm{J} / \psi$ process was calculated at the next-to-leading order accuracy and find the NLO correction many enhance the branching fraction to the same level of relativistic correction
[Bin Gong, Yu Jia, and J.X.Wang, PLB, 2009]

However, there exists a disagreement in this result
[Braguta \& Kartvelishvili, arXiv:0907.2772]

The radiative correction to $\eta_{b} \rightarrow \mathrm{~J} / \psi \mathrm{J} / \psi$ process is recalculated by a third party. Result shows there is no agreement with either initial calculation or with the
criticism

Nevertheless, numerical results are all in a similar order
[Sun, Hao, QCF, PLB, 2011]


FIG. 1: Typical Feynman diagrams of the exclusive process $\eta_{b}\left(P_{\eta_{b}}\right) \rightarrow J / \psi\left(P_{J / \psi_{1}}\right)+J / \psi\left(P_{J / \psi_{2}}\right)$ at the one-loop level.

Because of parity and Lorentz invariance, the decay amplitude possesses the following unique tensor structure

$$
\mathcal{M}\left(\lambda_{1}, \lambda_{2}\right)=\mathcal{A} \varepsilon_{\mu \nu \rho \sigma} \varepsilon_{J / \psi_{1}}^{* \mu}\left(\lambda_{1}\right) \varepsilon_{J / \psi_{2}}^{* \nu}\left(\lambda_{2}\right) P_{J / \psi_{1}}^{\rho} P_{J / \psi_{1}}^{\sigma}
$$

## After a lengthy calculation, we obtain:

[Sun, Hao, QCF, 2010]

$$
\mathcal{A}=\frac{512 \sqrt{2} \pi \alpha_{s}^{3} m_{c} \psi_{\eta_{s}}(0) \psi_{J / \psi}^{2}(0)}{9 \sqrt{3} m_{b}^{9 / 2}\left(m_{b}^{2}-4 m_{c}^{2}\right)} F\left(m_{c}^{2}, m_{b}^{2}\right)
$$

with the real and imaginary parts reading as

$$
\begin{aligned}
\operatorname{Re}\left(F\left(m_{c}^{2}, m_{b}^{2}\right)\right)_{a s y}= & \frac{19}{32} \log ^{2}(a)-\frac{1}{8} \log (2) \log (a)+\frac{5}{4} \log (a)+\frac{5}{16} \log ^{2}(2) \\
& +\frac{1}{2} \log (2)+\frac{29 \pi^{2}}{96}-\frac{3 \sqrt{3}}{8} \pi+\frac{3}{4}
\end{aligned}
$$

$$
\operatorname{Im}\left(F\left(m_{c}^{2}, m_{b}^{2}\right)\right)_{a s y}=\frac{19 \pi}{16} \log (a)+\frac{7 \pi}{16} \log (2)+\pi
$$

in small $m_{c}$ limit

The double and single logarithmic terms agree with Gong et al. result, but not the finite terms

## With the following in puts

$$
\psi_{J / \psi}(0)=0.263 \mathrm{GeV}^{3 / 2}, m_{c}=1.5 \mathrm{GeV}, m_{b}=4.7 \mathrm{GeV}, \alpha_{s}=0.18 \sim 0.26
$$

we have

$$
B r\left[\eta_{b} \rightarrow J / \psi J / \psi\right]=5.93 \times 10^{-8} \sim 2.58 \times 10^{-7}
$$

## Higher twist contributions

In the light-cone formalism, the leading twist term has no contribution to the $\eta_{b} \rightarrow \mathrm{~J} / \psi \mathrm{J} / \psi$ process

We expand the LCDA projector in momentum space given by Beneke and Feldmann(2001) to twist-4, which yields more terms than what employed by Braguta et al.

## With the asymptotic form for twist-2

 distribution amplitudes, i.e.$$
\phi_{\perp}(u)=\phi_{\|}(u)=\phi_{A S}(u)=6 u(1-u)
$$

The analytical decay amplitude turns out to be pretty simple, it reads

$$
\begin{aligned}
\mathcal{M}_{\perp \perp}= & T_{0} \varepsilon_{\mu \nu \rho \sigma} \varepsilon_{1 \perp}^{* \mu} \varepsilon_{2 \perp}^{* \nu} n_{-}^{\rho} n_{+}^{\sigma} \frac{9}{256 E_{1}^{2} E_{2}^{2}} \times\left[\left(\pi^{2}-4\right) m_{V_{1}} m_{V_{2}}\left(f_{V_{1}} \tilde{f}_{V_{2}}+f_{V_{1}} \tilde{f}_{V_{2}}\right)\right. \\
& \left.+2 \pi^{2}\left(m_{V_{2}}^{2} f_{V_{1}}^{T} \tilde{f}_{V_{2}}^{T}+m_{V_{1}}^{2} f_{V_{2}}^{T} \tilde{f}_{V_{1}}^{T}\right)\right] .
\end{aligned}
$$

## Here,

$$
\tilde{f}_{V}=f_{V}-f_{V}^{T} \frac{m_{1}+m_{2}}{m_{V}}, \quad \hat{f}_{V}^{T}=f_{V}^{T}-f_{V} \frac{m_{1}+m_{2}}{m_{V}}
$$

and decay constants $f_{J / \psi}$ and $f_{J / \psi}^{T}$ can be obtained through experiment and NRQCD

$$
f_{J / \psi}=416 \mathrm{MeV} \quad \boldsymbol{\quad} \quad f_{J / \psi}^{T}=379 \mathrm{MeV}
$$

OThen the numerical result reads

$$
B r\left[\eta_{b} \rightarrow J / \psi J / \psi\right]=(1.1 \sim 2.3) \times 10^{-6}
$$

## II. $\eta_{c}$ Exclusive Decays to Vector Meson Pair

Theoretically, the description of $\eta_{c} \rightarrow \mathrm{~V} \mathrm{~V}$ process is a long-standing unsolved problem

Perturbatively, this process is helicity suppressed

- HSR prediction: $\lambda_{1}+\lambda_{2}=0$;
- Parity conservation $\Rightarrow \mathcal{M} \sim \mathcal{A} \epsilon_{\mu \nu \rho \sigma} \varepsilon_{1}^{\mu} \varepsilon_{2}^{\nu} p_{1}^{\rho} p_{2}^{\sigma}$

$$
\varepsilon_{L}=a p_{1}+b p_{2} \Rightarrow \text { No longitudinal polarization! }
$$



## Experiment measurement on $\eta_{c} \rightarrow \mathrm{~V} V$ processes

- BES Collaboration, Phys.Rev.D72:072005,2005.

| Final state | Branching ratio(Br) | $\mathrm{Br} / \kappa_{12}^{3}$ |
| :---: | :---: | :---: |
| $\rho^{0} \rho^{0}$ | $4.15 \times 10^{-3}$ | $6.6 \times 10^{-3}$ |
| $K^{* 0} \bar{K}^{* 0}$ | $5.2 \times 10^{-3}$ | $5.0 \times 10^{-3}$ |
| $\phi \phi$ | $2.5 \times 10^{-3}$ | $6.0 \times 10^{-3}$ |
| $\omega \omega$ | $<4 \times 10^{-3}$ | $<6.4 \times 10^{-3}$ |
| $\omega \phi$ | $<7.1 \times 10^{-3}$ | $<6.8 \times 10^{-3}$ |

$\mathrm{K}_{12}$ is a kinematical factor

## There were lots of theoretical investigations on this issue in the literature, e.g.

- $\eta_{c}$-glueball mixing Anselmino et al., PRD42(1990)3218, PRD50(1994)595.
- three-particle wave functions for light vectors Chernyak et al., NPB348(1991)327.
- Bethe-Salpeter wave functions Jia and Chao, HEP\&NP.23(1995)765.
- $\eta_{c}-\eta^{\prime}-\eta$ mixing Feldmann and Kroll, PRD62(2000)074006.
- ${ }^{3} P_{0}$ quark-creation mechanism B.S.Zou et al., PRD71(2005)114002.
- intermediate meson exchange model Q.Zhao, PLB636(2006)197.


## In light-cone distribution formalism we calculate this process, up to the next-to-leading order in twist expansion

In addition to the asymptotic form, the LCDA form in terms of Gegenbauer polynomials is also taken into account in our calculation

$$
\phi_{\|, \perp}\left(u, \mu^{2}\right)=6 u(1-u)\left(1+\sum_{n=1}^{\infty} a_{n}^{\|, \perp}\left(\mu^{2}\right) C_{n}^{3 / 2}(2 u-1)\right)
$$

## We find

| Final state | $\operatorname{Br}[\mathrm{ex}]$ | $\operatorname{Br}[\mathrm{AS}]$ | $\operatorname{Br}[\mathrm{GP}]$ |
| :---: | :---: | :---: | :---: |
| $\rho \rho$ | $(2.0 \pm 0.7) \times 10^{-2}$ | $2.0 \times 10^{-4}$ | $2.8 \times 10^{-4}$ |
| $K^{*} \bar{K}^{*}$ | $(9.2 \pm 3.4) \times 10^{-3}$ | $7.2 \times 10^{-4}$ | $9.0 \times 10^{-4}$ |
| $\omega \omega$ | $<3.1 \times 10^{-3}$ | $9.1 \times 10^{-5}$ | $1.3 \times 10^{-4}$ |
| $\phi \phi$ | $(2.7 \pm 0.9) \times 10^{-3}$ | $6.6 \times 10^{-4}$ | $8.1 \times 10^{-4}$ |

- The results are in disagreement with the experiment measurements
[Sun, Hao, QCF, 2010]


## III. Brief Surmmany

To further study the nature of recently observed state $\eta_{b}$ direct measurement of its decay products is necessary

With in the pQCD and factorization scheme, we have calculated the helicity suppressed process
J/4 to next-to-leading order

Our result confirms the existence of double logarithm, however, others terms differs from what in the literature, though numerical difference is not big

In the light cone formalism, expanding the LCDAs of final vector mesons to twist-4, we find that the higher twist terms contribute more to the $\eta_{b}$ decay width than what from the NLO corrections
According to our twist-4 calculation, the branching fraction of $\Rightarrow J / \psi J / \psi$ process can be as large as $10^{-6}$, which enables the direct search of $\eta_{h}$ in Tevatron Run II or LHC

# In the light cone formalism we have calculated the process $\eta_{c} \rightarrow V \mathrm{~V}$ in next-to-next-leading order in twist expansion 

Result shows that the higher twist DAs indeed violate the helicity conservation rules, however it still deviate a lot from the experimental measurement

This means that the perturbative description of is not enough, and some non-perturbative mechanism may play important roles

## Thank you for

## your attention

