Radiative Improvement of the NRQCD Action with Applications to Bottomonium Hyperfine Splitting

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in collaboration with
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Introduction

Predict bottomonium hyperfine splitting (hfs) from lattice QCD:

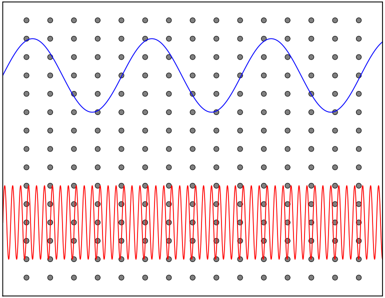
- treat heavy quarks in lattice NRQCD
- coefficients only known at tree level so far
- lattice results for hfs disagree with experiment
- becomes worse when adding $O(v^6)$ terms [Meinel 2010]

Here:

- determine matching coefficients relevant for hfs at one-loop level
- corrections to hfs lead to greatly improved agreement with experiment

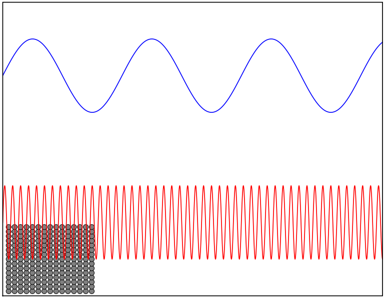
Problems for heavy quarks on the lattice

- Multi-scale problem: Need $L \gg m_\pi^{-1} \gg m_Q^{-1} \gg a$
- $a \gtrsim m_Q^{-1}$: large discretisation effects



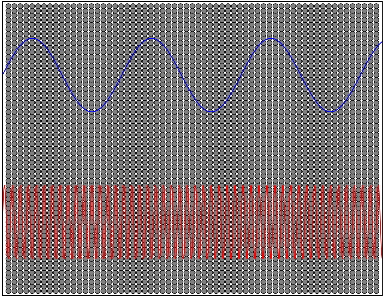
Problems for heavy quarks on the lattice

- Multi-scale problem: Need $L \gg m_\pi^{-1} \gg m_Q^{-1} \gg a$
- $L \lesssim m_\pi^{-1}$: large finite-volume effects



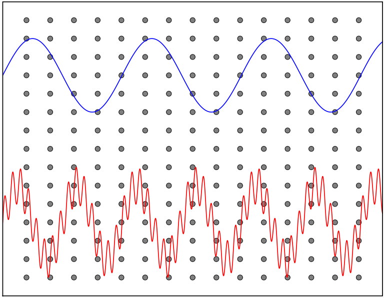
Problems for heavy quarks on the lattice

- Multi-scale problem: Need $L \gg m_\pi^{-1} \gg m_Q^{-1} \gg a$
- Large L/a leads to big computational effort



Problems for heavy quarks on the lattice

- Multi-scale problem: Need $L \gg m_\pi^{-1} \gg m_Q^{-1} \gg a$
- Another way: effective theories (HQET, NRQCD, mNRQCD)



NRQED and NRQCD

- Non-Relativistic theory: double expansion in α and v^2
- NRQED highly successful, especially for muonium/positronium hyperfine structure [Kinoshita, Nio (1996); Labelle, Zebarjad, Burgess (1997)]
- radiative corrections to NRQED have been computed in detail
- NRQCD has been successfully applied to heavy quarkonia
- until now only tree-level NRQCD is used
- Here: we compute radiative corrections to coefficients in the lattice NRQCD action using the background field method (BFM)
- BFM not needed for radiative improvement to NRQED, but vital for NRQCD

NRQCD

NRQCD action is

$$S = \sum_{\vec{x},\tau} \psi^\dagger(\vec{x},\tau)\left[\psi(\vec{x},\tau) - K(\tau)\psi(\vec{x},\tau)\right]$$

with the kernel

$$K(\tau) = \left(1 - \frac{\delta H|_\tau}{2}\right)\left(1 - \frac{H_0|_\tau}{2n}\right)^n U_4^\dagger(\tau-1)\left(1 - \frac{H_0|_{\tau-1}}{2n}\right)^2\left(1 - \frac{\delta H|_{\tau-1}}{2}\right)$$

where

$$H_0 = \frac{\Delta^{(2)}}{2M_0}, \quad \delta H = -c_1\frac{(\Delta^{(2)})^2}{8M_0^3} + c_2\frac{ig}{8M_0^2}\left(\vec{\Delta}^\pm\cdot\vec{E} - \vec{E}\cdot\vec{\Delta}^\pm\right)$$

$$-c_3\frac{g}{8M_0^2}\vec{\sigma}\cdot\left(\vec{\Delta}^\pm\times\vec{E} - \vec{E}\times\vec{\Delta}^\pm\right) - c_4\frac{g}{2M_0}\vec{\sigma}\cdot\vec{B} + c_5 a^2\frac{\Delta^{(4)}}{24M_0} + c_6 a\frac{(\Delta^{(2)})^2}{16nM_0^2}$$

and $n \geq 3/(M_0 a)$ is a stability parameter. At tree level $c_i = 1$.
Beyond tree-level, there are additional four-fermi terms including

$$\mathcal{L}_{4f} = d_1\frac{\alpha^2}{M_0^2}(\psi^\dagger\chi^*)(\chi^T\psi) + d_2\frac{\alpha^2}{M_0^2}(\psi^\dagger\vec{\sigma}\chi^*)\cdot(\chi^T\vec{\sigma}\psi)$$

Matching

Determine radiative corrections to c_i and determine d_i by demanding that RQCD and NRQCD give the same effective potential after non-relativistic reduction, i.e. the following diagram commutes:

$$\begin{tikzcd}
\text{QCD} \arrow[r, "\text{NR reduction}"] \arrow[d, "\text{1PI}"'] & \begin{array}{c}\text{tree-level}\\ \text{NRQCD}\end{array} \arrow[d, "\text{1PI}"] \\
\Gamma \arrow[r, "\text{NR reduction}"'] & \Gamma^{\text{NR}}
\end{tikzcd}$$

$\rightsquigarrow$ Need effective potential for gauge theory.

Alternative would be to fix c_i non-perturbatively by matching to experiment.

Background Field Method

Effective potential is defined by

$$\Gamma[\Phi] = \int_{1\mathrm{PI}} D\phi \, \mathrm{e}^{S[\Phi+\phi]}$$

and only makes sense in perturbation theory restricted to 1PI diagrams. In gauge theories,

- decompose $A_\mu = B_\mu + g q_\mu$.
- BRST invariance guarantees $D \leq 4$ operators in Γ are gauge covariant $\rightsquigarrow$ renormalizability, but
- $D > 4$ operators are not necessarily gauge covariant

NRQCD is an effective theory with $D > 4$ operators. Gauge covariance

- can be imposed at tree level,
- must be retained at the loop level to avoid serious complications,
- is preserved by using background field gauge [DeWitt 1981; Barvinsky, Vilkovitsky 1983; ...]

Background Field Gauge

Background field gauge (BFG) is defined by the gauge fixing function

$$f(A) = D_\mu^B q^\mu = (\partial_\mu + iB_\mu)q^\mu$$

and hence qqB and $qqBB$ vertices are gauge-parameter dependent.
Lattice gauge theories in BFG are renormalizable [Lüscher, Weisz, 1995].
BFG leads to

- QED-like Ward identities, and
- finite counterterms.

and hence we

- can compute all diagrams numerically, and
- do not need to calculate gauge field renormalization.

Practical for checking gauge invariance of results.
c_4 is gauge-parameter independent for on-shell quarks (hfs is physical)

Kinetic and $\mathcal{O}(a^2)$ terms

The operators multiplied by the coefficients c_1, c_5, c_6 have a purely fermionic part $\rightsquigarrow$ gauge covariance ensures they can be computed from quark self-energy alone, no need for BFG

Contributions to different operators behave like $(p^2)^2$ and $\sum_i p_i^4$ $\rightsquigarrow$ can be isolated by looking at different fourth-order partial derivatives of self-energy w.r.t. momentum

Implemented using TaylUR 3 automatic differentiation package [GvH, 2009]

Alternatively fix non-perturbatively from dispersion relation of quarkonia

The $\sigma \cdot B$ term

Continuum QCD effective action

$$\Gamma[\Psi, \overline{\Psi}, A] = Z_2^{-1} \overline{\Psi} \not{D} \Psi + \delta Z_\sigma \overline{\Psi} \frac{\sigma^{\mu\nu} F_{\mu\nu}}{2m} \Psi + \ldots$$

after renormalization of the first term gives

$$\Gamma[\Psi_R, \overline{\Psi}_R, A] = \overline{\Psi}_R \not{D} \Psi_R + b_\sigma \overline{\Psi}_R \frac{\sigma^{\mu\nu} F_{\mu\nu}}{2m_R} \Psi_R + \ldots$$

with

$$b_\sigma = \delta Z_\sigma Z_2 Z_m = \sum_{n=1} b_\sigma^{(n)} \alpha^n ,$$

where leading correction is $O(\alpha_s)$ and comes from δZ_σ alone.
Non-relativistic reduction gives

$$(1 + b_\sigma)\, \psi_R^\dagger \frac{\vec{\sigma} \cdot \vec{B}}{2m_R} \psi_R \; .$$

for $\sigma \cdot B$ term.
Straightforward continuum calculation gives

$$b_\sigma = \left(\frac{3}{2\pi} \log \frac{\mu}{m} + \frac{13}{6\pi} \right) \alpha$$

Effective action for NRQCD contains spin-dependent term

$$\Gamma_\sigma[\psi, \psi^\dagger, A] = c_4 Z_\sigma^{\mathrm{NR}} \psi^\dagger \frac{\vec{\sigma}\cdot\vec{B}}{2M}\psi$$

after renormalization becomes

$$\Gamma_\sigma[\psi_R, \psi_R^\dagger, A] = c_4 Z_\sigma^{\mathrm{NR}} Z_2^{\mathrm{NR}} Z_m^{\mathrm{NR}} \psi_R^\dagger \frac{\vec{\sigma}\cdot\vec{B}}{2M_R}\psi_R \; .$$

Require that anomalous chromomagnetic moment equal to QCD gives matching condition

$$c_4 Z_\sigma^{\mathrm{NR}} Z_2^{\mathrm{NR}} Z_m^{\mathrm{NR}} = 1 + b_\sigma \; .$$

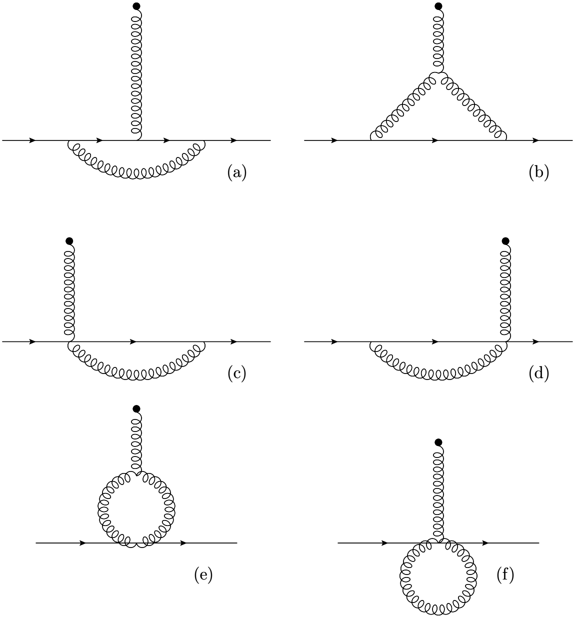
At tree level and one-loop order we find

$$\begin{aligned} c_4^{(0)} &= 1 \, , \\ c_4^{(1)} &= b_\sigma^{(1)} - \delta Z_\sigma^{\mathrm{NR},(1)} - \delta Z_2^{\mathrm{NR},(1)} - \delta Z_m^{\mathrm{NR},(1)} \; . \end{aligned}$$

NRQCD contribution contains IR logarithm $\frac{3\alpha}{2\pi}\log(\mu a)$, combines with QCD IR log to give $-\frac{3\alpha}{2\pi}\log(Ma)$.
Also need to take into account mean-field improvement $U \mapsto U/u_0$: effect on $\delta Z_\sigma^{\mathrm{NR},(1)}$ and $\delta Z_m^{\mathrm{NR},(1)}$. Final result for one-loop correction to c_4 is

$$\begin{aligned} c_4^{(1)} &= \tfrac{13}{6\pi} - \delta\tilde{Z}_\sigma^{\mathrm{NR},(1)} - \delta\tilde{Z}_2^{\mathrm{NR},(1)} - \delta\tilde{Z}_m^{\mathrm{NR},(1)} \, , \\ &- \delta Z_m^{\mathrm{tad},(1)} - \delta Z_\sigma^{\mathrm{tad},(1)} - \tfrac{3}{2\pi}\log Ma \end{aligned}$$

where $\delta\tilde{Z}_X$ denotes a finite diagrammatic contribution.


(a)
(b)
(c)
(d)
(e)
(f)

Four-fermion interactions

Beyond tree-level, four-fermion terms

$$\mathcal{L}_{4f} = d_1 \frac{\alpha^2}{M_0^2}(\psi^\dagger \chi^*)(\chi^T \psi) + d_2 \frac{\alpha^2}{M_0^2}(\psi^\dagger \vec{\sigma} \chi^*) \cdot (\chi^T \vec{\sigma} \psi)$$

also contribute to hfs:

- d_1 term contributes to η_b and includes $Q\overline{Q}$ annihilation contribution from QCD,
- d_2 term contributes to Υ.

Renormalisation constants are

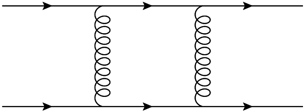
$$\begin{aligned} Z_{f1} &= \alpha^2 \left(A_{f1} - \log \tfrac{\mu}{m} - \tfrac{16\pi}{27} \tfrac{m}{\mu} \right) , \\ Z_{f2} &= -\frac{1}{3} Z_{f1} , \end{aligned}$$

and for QCD a simple calculation gives

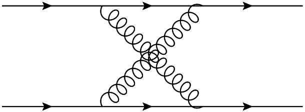
$$A_{f1}^R = \frac{8}{27} ,$$

giving matching parameters

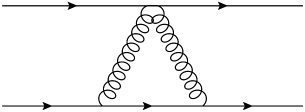
$$\begin{aligned} d_1 &= -3 d_2 - \frac{2}{9}(2 - 2\log 2) , \\ d_2 &= \frac{8}{81} - \frac{1}{3} A_{f1}^{NR} + \frac{1}{3} \log Ma . \end{aligned}$$



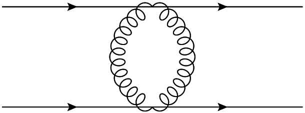
(a)



(b)



(c)



(d)

Implementation notes

On the lattice decompose the link as the ordered product

$$U_\mu(x) = \mathrm{e}^{g_0 q_\mu(x+\frac{1}{2}\hat{\mu})} \mathrm{e}^{B_\mu(x+\frac{1}{2}\hat{\mu})}$$

leading to a dependence of the Feynman rules on the number of background and quantum fields (qqq, Bqq, BBq, etc.), and different terms for different orderings (Bqq, qBq, qqB, etc.)

- BFG fixing term now affects all vertices with exactly two quantum gluons
- Implemented in HiPPY and HPsrc for automated lattice perturbation theory [Hart, Horgan, vH et al., arXiv:1011.2696]
- Use suitable IR subtraction functions to analytically subtract IR logs from numerical results

Checks:

- correct divergences,
- gauge dependences match for individual terms,
- sum is non-trivially gauge independent,

give confidence in the correctness of the results.

Bottomonium Hyperfine Splitting

Include these radiative corrections in simulations [R. Dowdall's talk]
Both operators' contribution to hfs is dominated by contact term: may estimate multiplicative change in hfs by

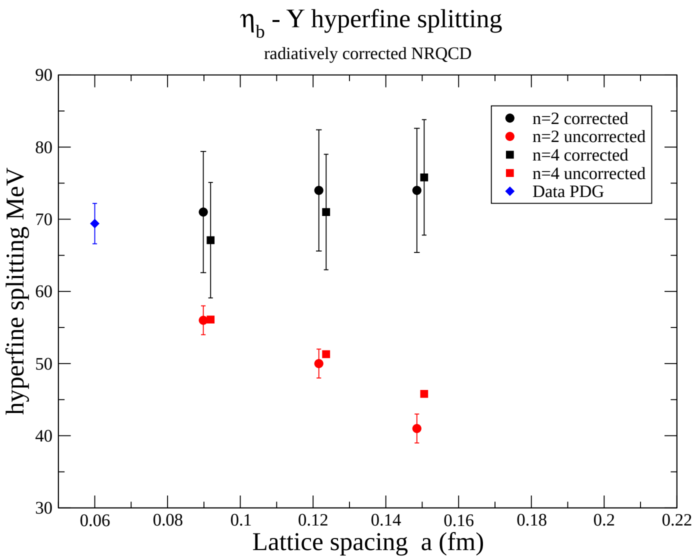
$$1 \; + \; \alpha_V(q^*) \left(2\, c_4^{(1)} - \frac{27}{16\pi} \, (d_1 - d_2) \right) \; ,$$

Take $q^* = \pi/a$, Landau mean link $u_0^{(2)} = 0.750$

Corrections to bottomonium hfs results of [Gray et al., 2005] from radiative improvement of the action:

Ma	$\alpha_V(q^*)$	Correction % 4-fermion	Correction % $\sigma \cdot B$	hfs (MeV) [Gray et al.]	hfs (MeV) corrected
1.95	0.216	-1.2(1)	+31.4(3)	56(2)	73(3)(5)(6)
2.8	0.249	+11.9(2)	+39.8(3)	50(2)	76(3)(6)(5)
4.0	0.293	+35.7(4)	+49.3(3)	41(2)	76(3)(7)(4)

Errors are statistical, $O(\alpha^2)$, and relativistic



n=2 data [Gray et al., 2005], n=4 data [Davies, Dowdall et al., 2011]

Results and Prospects

Radiative corrections have impact:

- Lattice spacing dependence greatly reduced,
- Lattice results now agree with PDG value 69.4(2.8),
- discrepancy with new BELLE value remains to be understood.

Remaining uncertainties can be further reduced:

- $O(v^6)$ terms will be included with recomputed c_4 to greatly reduce systematic error,
- [Meinel, 2010] finds significant effect on hfs (Iwa/DW) – check in our corrected theory.

Coefficients of spin-orbit coupling and Darwin term are currently being determined.

Summary

- BFG needed to match NRQCD to QCD:
  - guarantees only gauge-covariant operators with $D > 4$ are generated by renormalization,
  - renders QCD results finite,
  - reduces effort through QED-like Ward Identities.
- Successfully computed one-loop radiative corrections to c_4, d_1, d_2:
  - IR log does reduces hfs, but overall c_4 is positive
  - Greatly reduced discretization effects (four-fermion operators important)
  - New estimate for bottomonium hfs agrees well with experiment
- Radiative correction to c_1, c_5, c_6 are known [Müller, Monahan et al., 2010], c_2 and c_3 are in hand [Hammant et al., w.i.p.]

The end

Thank you for your attention

Backup slides

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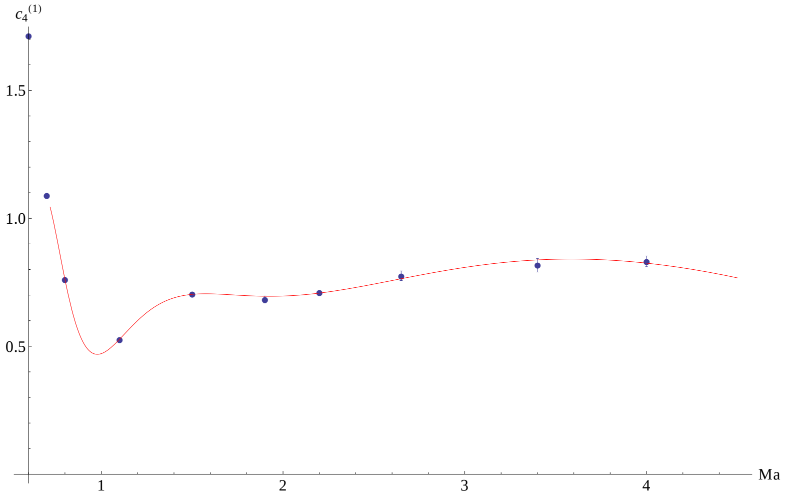
$c_4^{(1)}$
1.5
1.0
0.5
1
2
3
4
Ma