Factorization and Quarkonium Production

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Based on work done with Kang, Nayak, Sterman, and …

International workshop on heavy quarkonium (QWG2011)
GSI, Darmstadt, Germany, October 4-7, 2011

Outline of my talk

- Production mechanisms
- Surprises + anomalies
- What can we learn from the surprises and anomalies?
- Perturbative QCD factorization approach
- Connect pQCD factorization to NRQCD factorization
- Summary

A long history for the production

- Discovery of J/ψ – November revolution – 1974
- Color singlet model: 1975 –

  Einhorn, Ellis (1975),
  Chang (1980),
  Berger and Jone (1981), …

  Only the pair with right quantum numbers

  Effectively No free parameter!

- Color evaporation model: 1977 –

  Fritsch (1977), Halzen (1977), …

  All pairs with mass less than open flavor heavy meson threshold

  One parameter per quarkonium state

- NRQCD model: 1986 –

  Caswell, Lapage (1986)
  Bodwin, Braaten, Lepage (1995)
  QWG review: 2004, 2010

  All pairs with various probabilities – NRQCD matrix elements

  Infinite parameters – organized in powers of v and α_s

- pQCD factorization approach: 2005 –

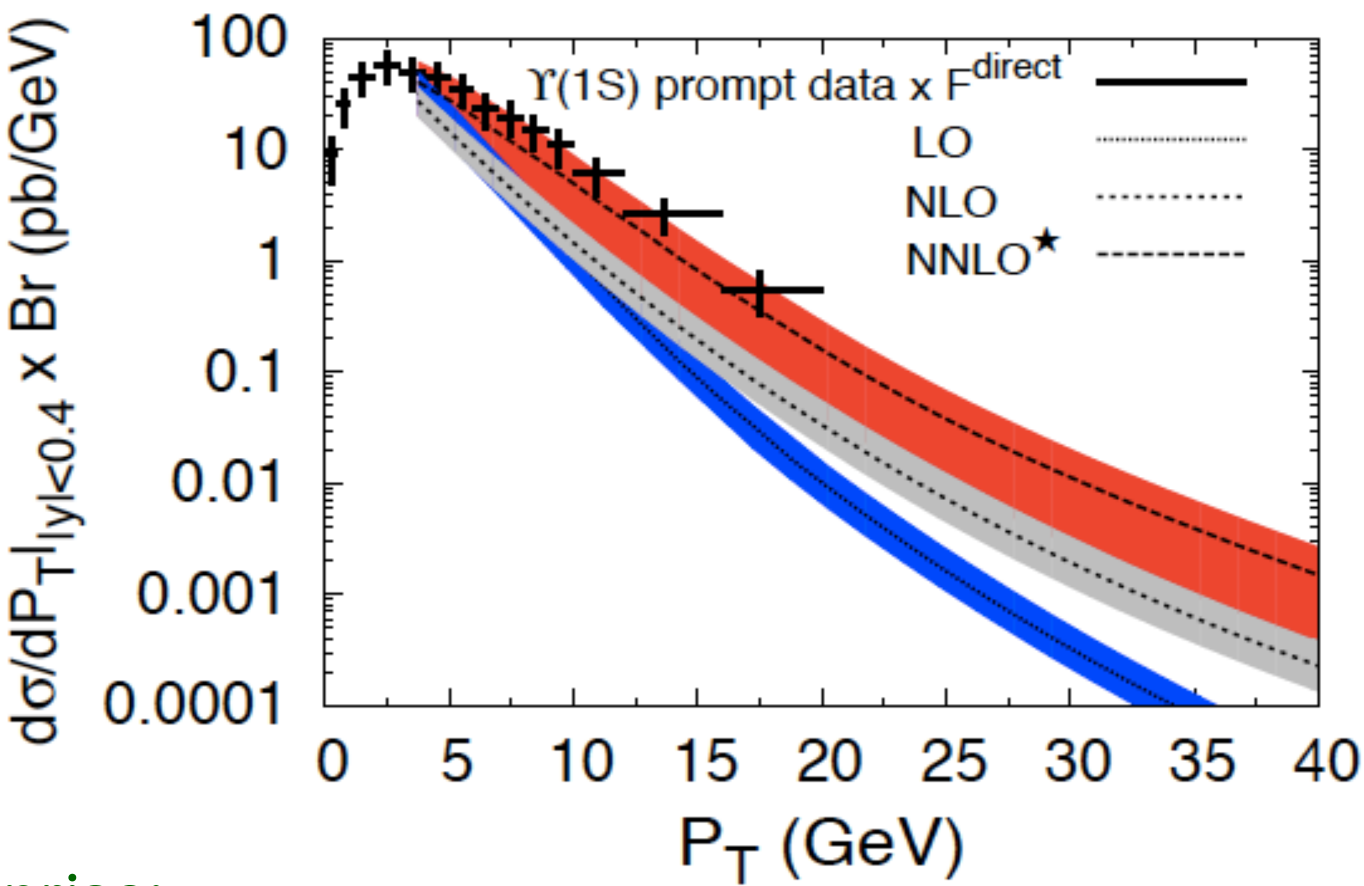
  Nayak, Qiu, Sterman (2005), …
  Kang, Qiu, Sterman (2010), …

  $P_T \gg M_H$: M_H/P_T power expansion + α_s – expansion

  Universal fragmentation functions – evolution/resummation

Color singlet model – huge HO contribution

Campbell, Maltoni, Tramontano (2007), Artoisenet, Lansburg, Maltoni (2007)
Artoisenet, Campbell, Lansburg, Maltoni, Tramontano (2008)

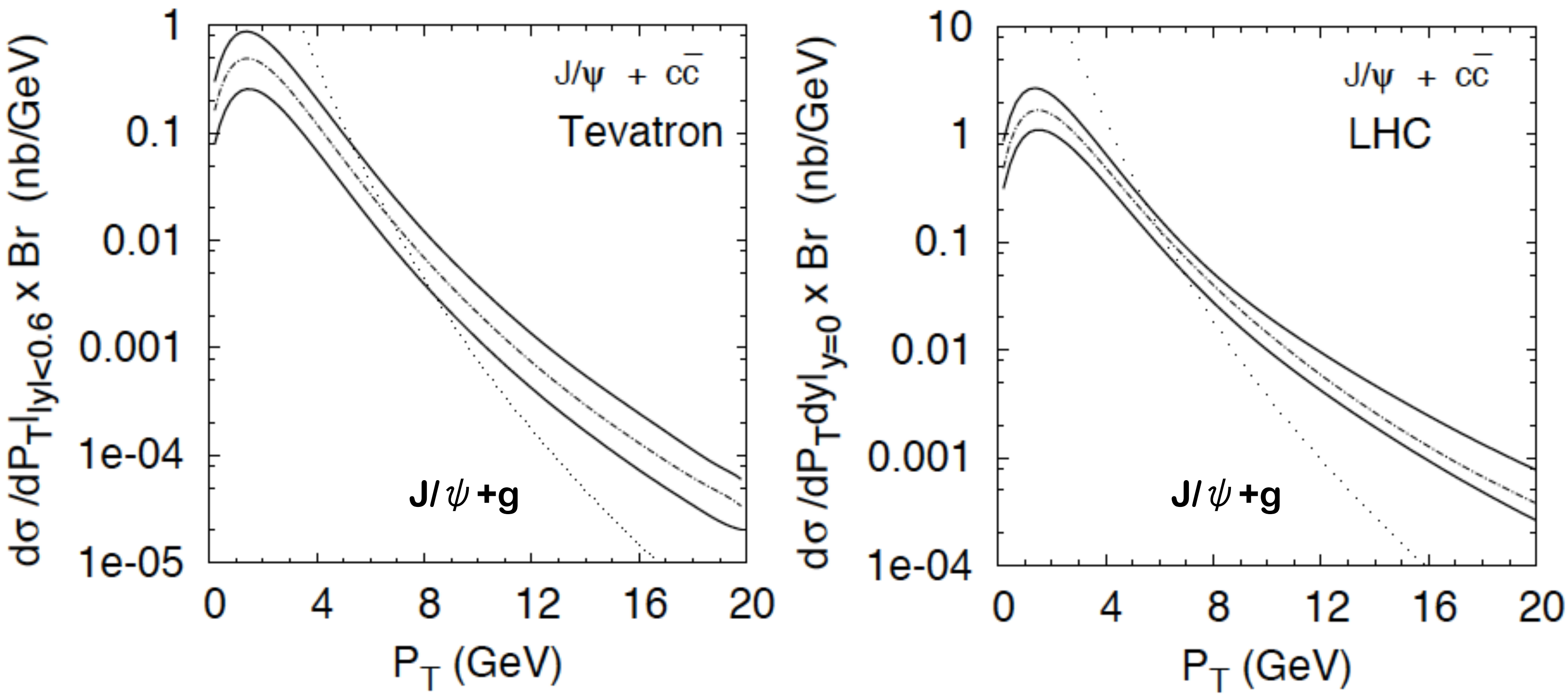


- ❑ **Surprise:**

Order of magnitude enhancement from high orders?

Color singlet model – huge associate production

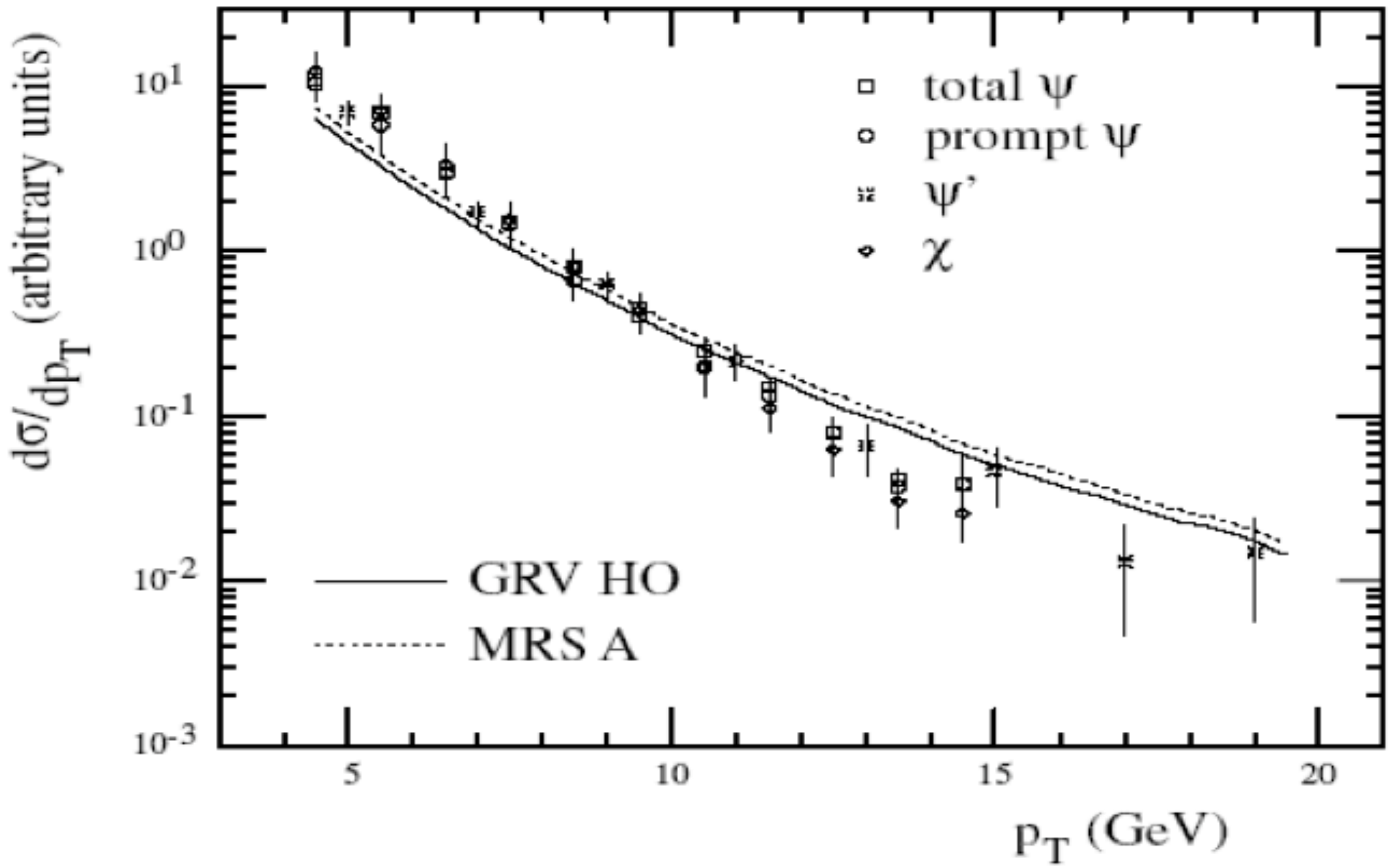
Artoisenet, Lansburg, Maltoni (2007)



- ❑ **More surprises and question:**
  - ✧ **More than order of magnitude larger than leading order – shape?**
  - ✧ **Much larger than leading power single charm fragmentation**

Color evaporation model

- Good for total cross section, ok for p_T distribution:



Amundson et al, PLB 1997

- Question:

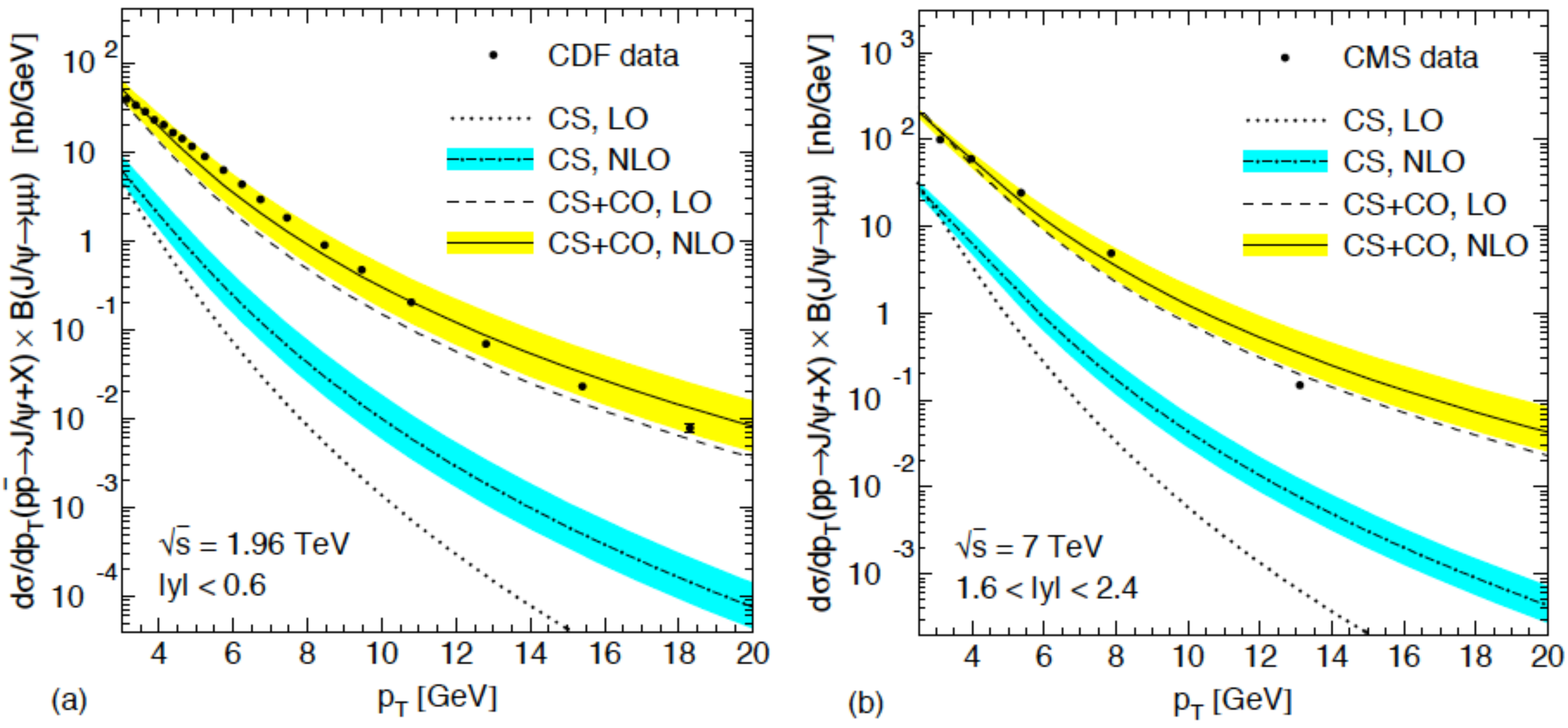
Better p_T distribution – the shape – polarization?

NRQCD – most successful so far

- NLO color octet contributions – becoming available:

  Most hard calculations were done in China and Germany!

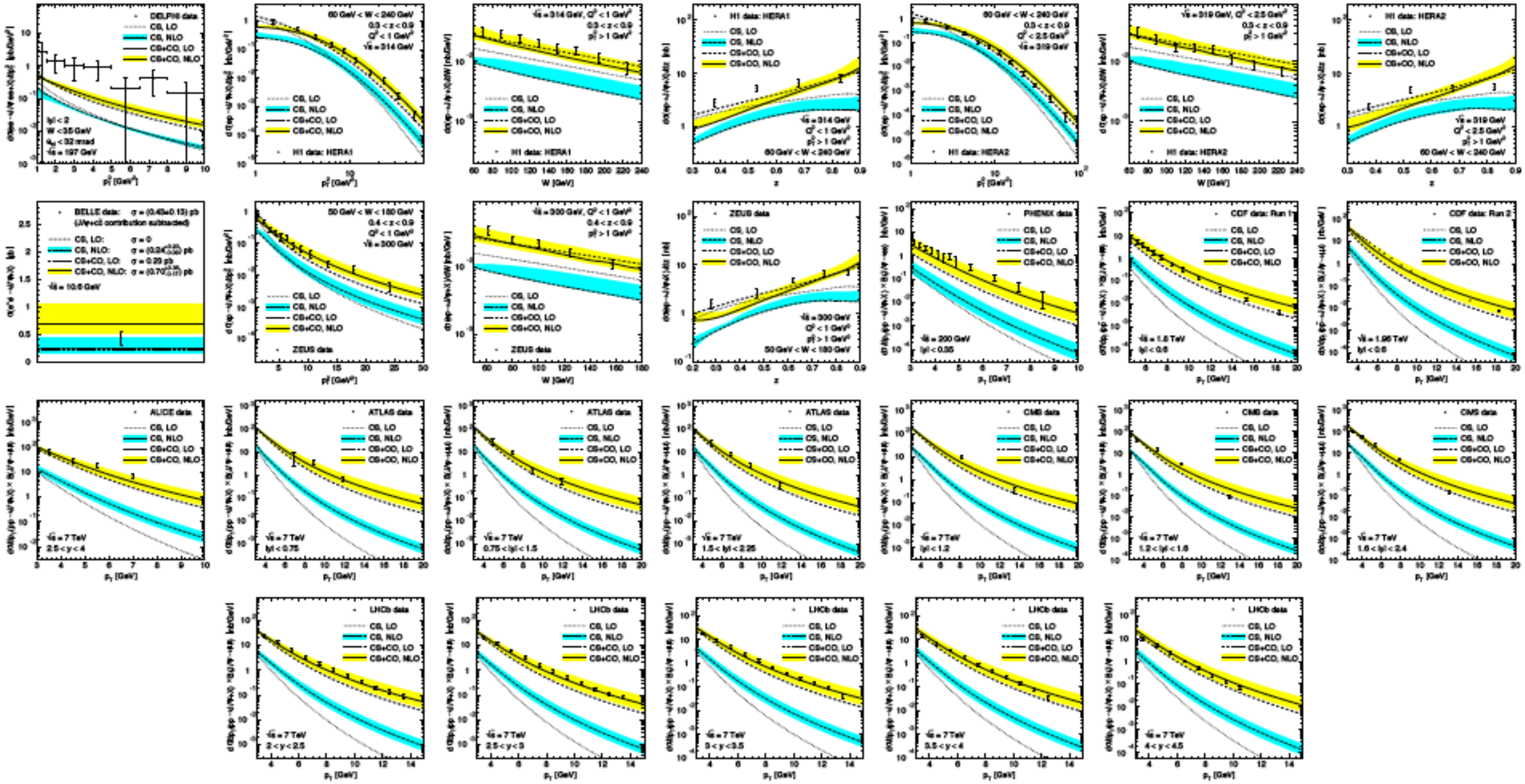
- Phenomenology:



- Fine details – shape?

PRL 106, 022003 (2011)

NRQCD – global analysis



194 data points from 10 experiments, fix singlet $\langle O[^3S_1^{[1]}]\rangle = 1.32\ \mathrm{GeV}^3$

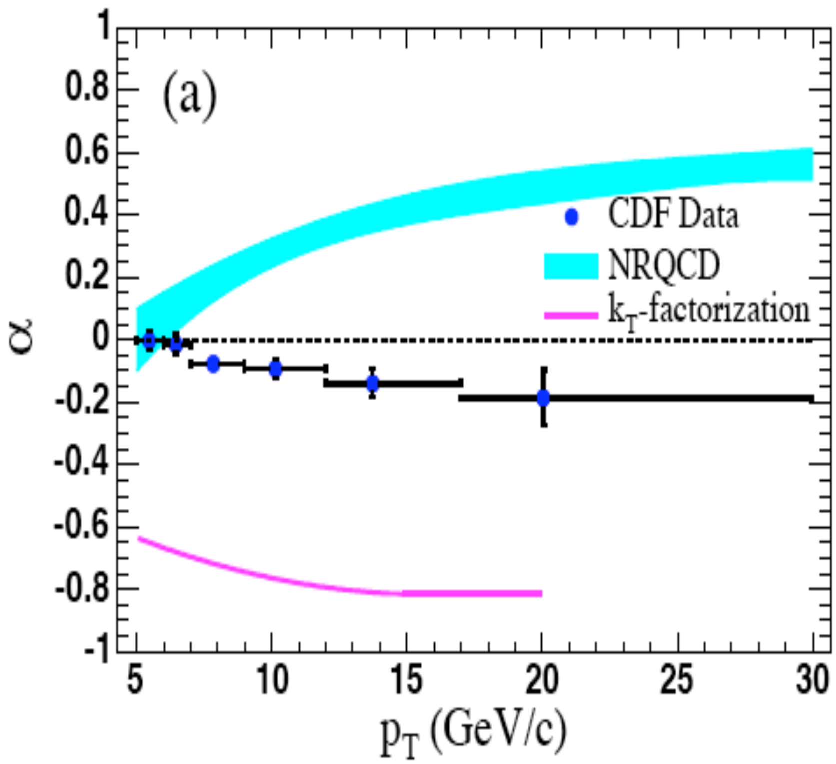
$\langle \boldsymbol{O}[^1S_0^{[8]}]\rangle = (4.97 \pm 0.44)\cdot 10^{-2}\ \mathrm{GeV}^3$ $\langle \boldsymbol{O}[^3S_1^{[8]}]\rangle = (2.24 \pm 0.59)\cdot 10^{-3}\ \mathrm{GeV}^3$

$\langle \boldsymbol{O}[^3P_0^{[8]}]\rangle = (-1.61 \pm 0.20)\cdot 10^{-2}\ \mathrm{GeV}^5$

Butenschoen and Kniehl, arXiv: 1105.0820

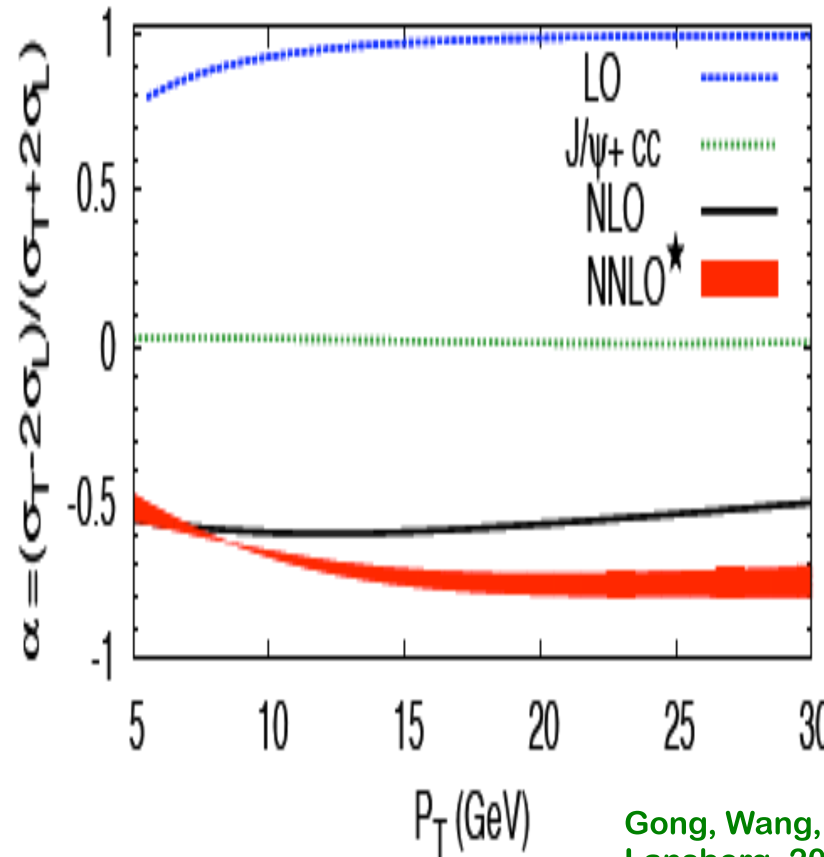
Anomalies from J/ψ polarization

NRQCD



Cho & Wise, Beneke & Rothstein, 1995, …

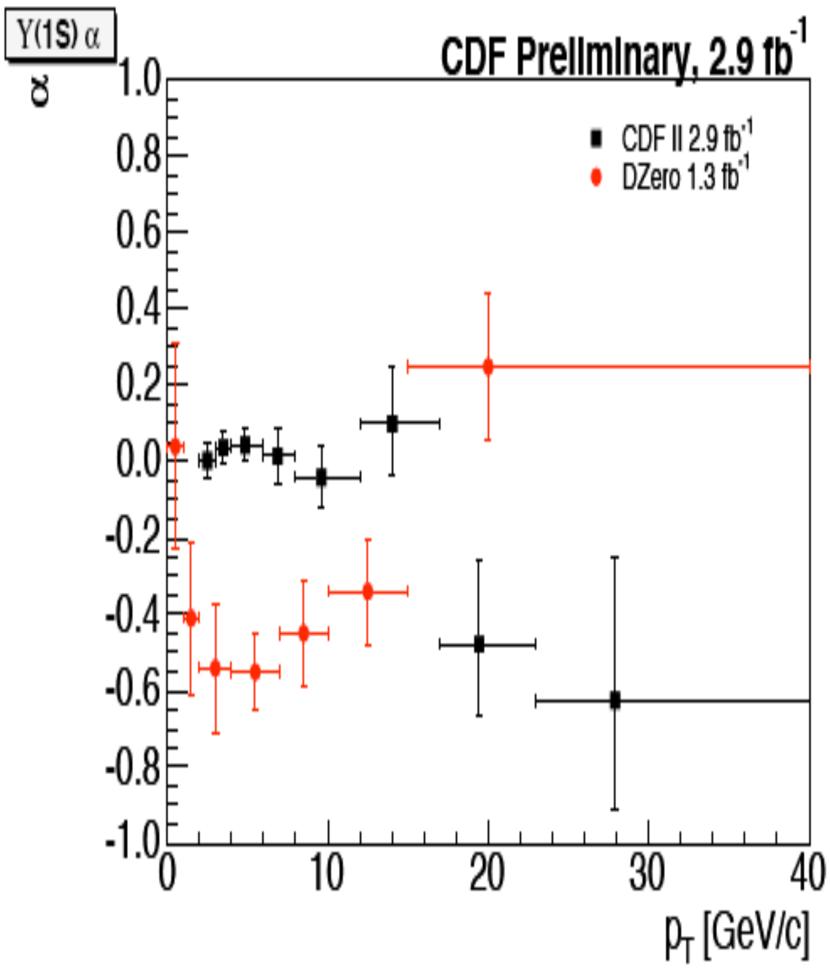
CSM

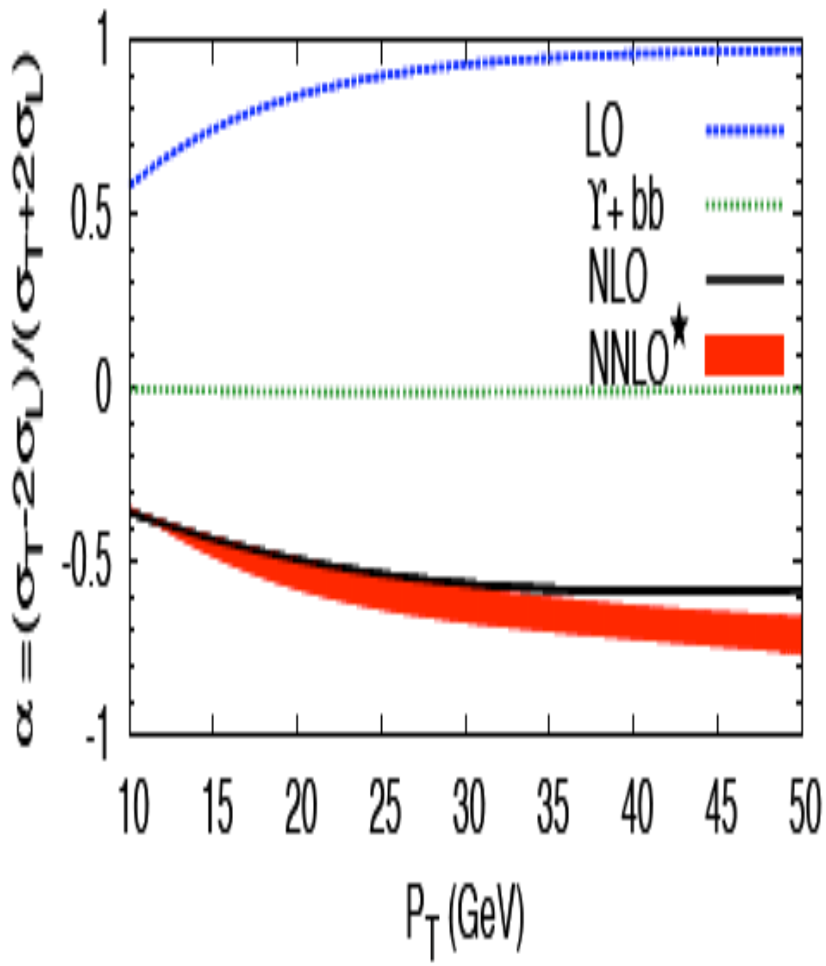


Gong, Wang, 2008
Lansberg, 2009

- ✧ **NRQCD: Dominated by color octet – NLO is not a huge effect**
- ✧ **CSM: Huge NLO – change of polarization?**

Confusions from Upsilon polarization





- ✧ **Resolution between CDF and D0?**
- ✧ **Change of polarization from LO to NLO?**

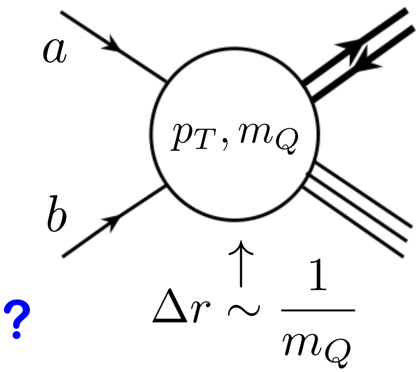
Gong, Wang, 2008

Artoisenet, et al. 2008

Lansberg, 2009

What can we learn from these surprises?

- What these calculations have in common?
  - ✧ Perturbative production of at least one heavy quark pair
  - ✧ Feynman diagram expansion in powers of α_s
- What is the key difference between these calculations?
  - ✧ The color and spin states of the heavy quark pair



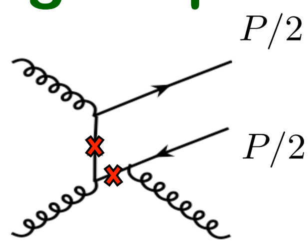
- What is missing in these calculations?
  - ✧ Where was the high p_T heavy quark pair produced?
- The active heavy quark pair (transforms into quarkonium) can be produced at $1/p_T$, $1/m_Q$, or somewhere between
  - ✧ The p_T-dependence of the production rate is sensitive to where the pair was produced!

Why high orders in CSM are so large?

Kang, Qiu and Sterman, 2011

❑ LO in α_s but higher power in $1/p_T$:

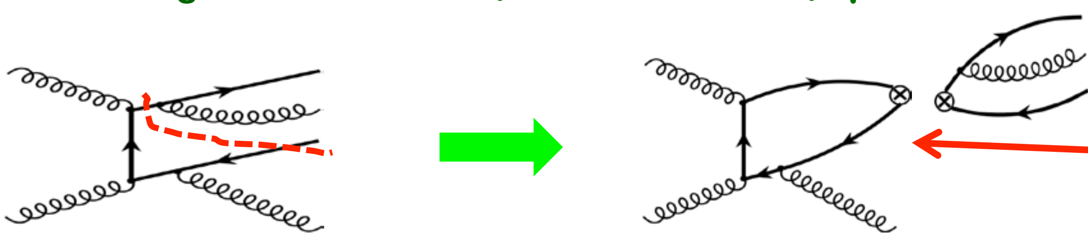
LO in α_s:



$$\hat{\sigma}^{\mathrm{LO}} \propto \frac{\alpha_s^3(p_T)}{p_T^8}$$

NNLP in $1/p_T$!

❑ NLO in α_s but lower power in $1/p_T$:

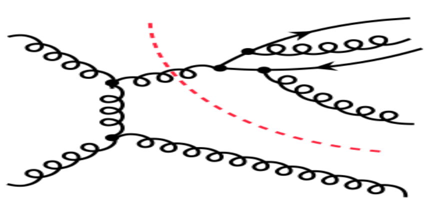


Leading power projection to "octet pair"

$$\hat{\sigma}^{\mathrm{NLO}} \to \frac{\alpha_s^3(p_T)}{p_T^6} \otimes \alpha_s(\mu) \log(\mu^2/\mu_0^2)$$

$$\mu_0 \gtrsim 2m_Q$$

❑ NNLO in α_s but leading power in $1/p_T$:



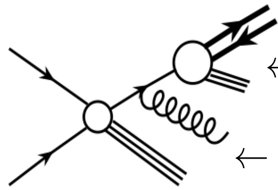
$$\hat{\sigma}^{\mathrm{NNLP}} \to \frac{\alpha_s^2(p_T)}{p_T^4} \otimes \alpha_s^3(\mu) \log^m(\mu^2/\mu_0^2)$$

Leading order in α_s-expansion =\= leading power in $1/p_T$-expansion!

PQCD power counting

Kang, Qiu and Sterman, 2011

- IF $p_T >> m_Q$, the pair produced
  - ✧ at $1/m_Q$:



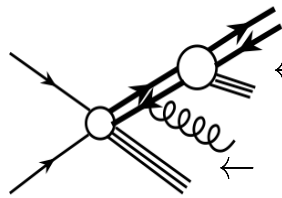
$\leftarrow \frac{1}{m_Q}$

$\leftarrow \Delta r \sim \frac{1}{p_T}$

$$\frac{1}{p_T^4}\sum_n \left[\log(\frac{p_T^2}{\mu_0^2})\right]^n$$

Only final-state fragmentation

  - ✧ at $1/P_T$:



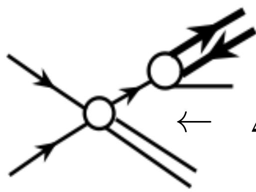
$\leftarrow \frac{1}{m_Q}$

$\leftarrow \Delta r \sim \frac{1}{p_T}$

$$\frac{1}{p_T^6}\sum_n \left[\log(\frac{p_T^2}{\mu_0^2})\right]^n$$

Short-distance Production

  - ✧ between: [$1/m_Q$, $1/P_T$]

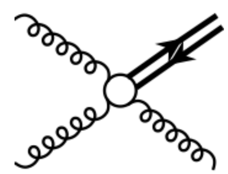


$\leftarrow \Delta r \sim \frac{1}{p_T}$

$$\frac{1}{p_T^4}$$

Modified evolution + pair production

- Role of color:
  - ✧ Color can be perturbatively resolved between m_Q and P_T
  - ✧ Factorize into a singlet or octet pair
  - ✧ Color affects p_T-dependence



$$\begin{cases} \frac{1}{p_T^8} & \text{LO singlet} \\ \frac{1}{p_T^6} & \text{LO octet} \end{cases}$$

Perturbative factorization approach

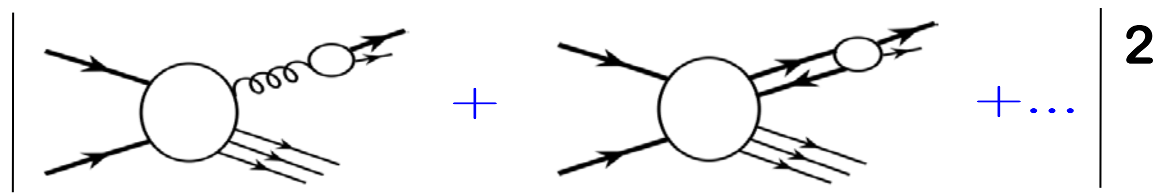
Nayak, Qiu, and Sterman, 2005
Kang, Qiu and Sterman, 2010-11

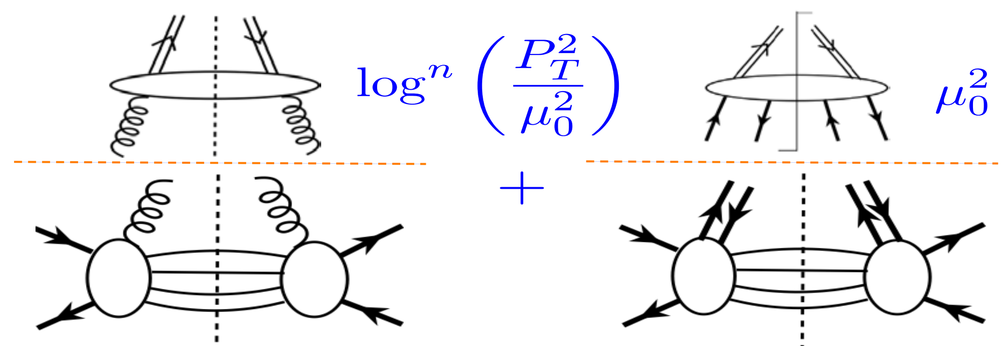
❑ Basic ideas:

- ✧ Expand cross section in powers of μ_0^2/p_T^2 with $\mu_0 \gtrsim 2m_Q$
- ✧ Resum logarithmic contribution into "fragmentation functions"
- ✧ Apply NRQCD to input fragmentation functions at $\mu_0 \sim 2m_Q$

❑ Factorization – all orders in α_s:

$E\dfrac{d\sigma_{J/\psi}}{d^3P}$:





$\log^n\left(\dfrac{P_T^2}{\mu_0^2}\right)$ $\quad$ $\mu_0^2\log^n\left(\dfrac{P_T^2}{\mu_0^2}\right)$

$\mu_0 \sim 2m_Q$

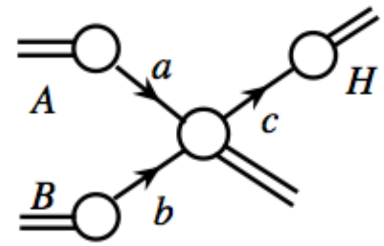
$\mathcal{O}\left(\dfrac{1}{P_T^4}\right)$ $\quad$ $\mathcal{O}\left(\dfrac{1}{P_T^6}\right)$

Power series in α_s without large logarithms

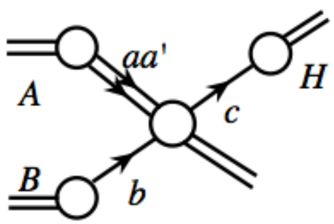
Why such power correction important?

- **Leading power in hadronic collisions:**

$$d\sigma_{AB\to H} = \sum_{a,b,c} \phi_{a/A} \otimes \phi_{b/B} \otimes d\hat{\sigma}_{ab\to cX} \otimes D_{c\to H}$$



- **1st power corrections in hadronic collisions:**



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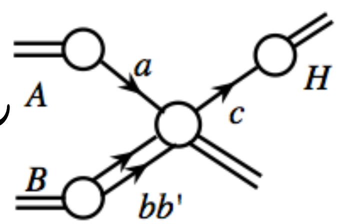

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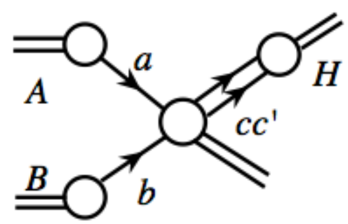
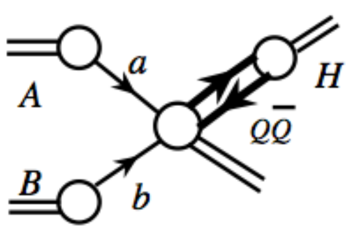


$\sim \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_T^2}\right) \otimes D_{c\to H}$

- **Dominated 1st power corrections:**



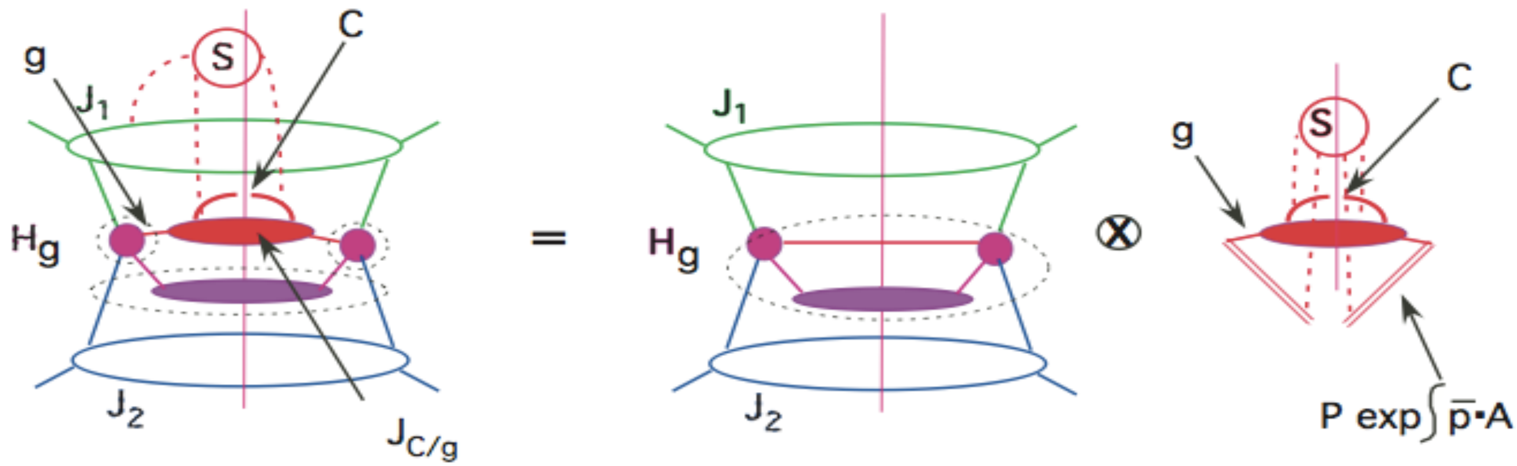
$\sim \mathcal{O}\left(\frac{(2m_Q)^2}{P_T^2}\right) \otimes D^{(2)}_{[Q\bar{Q}]\to H}$

Key: competition between $P_T^2 \gg (2m_Q)^2$ **and** $D^{(2)}_{[Q\bar{Q}]\to H} \gg D_{c\to H}$

PQCD Factorization

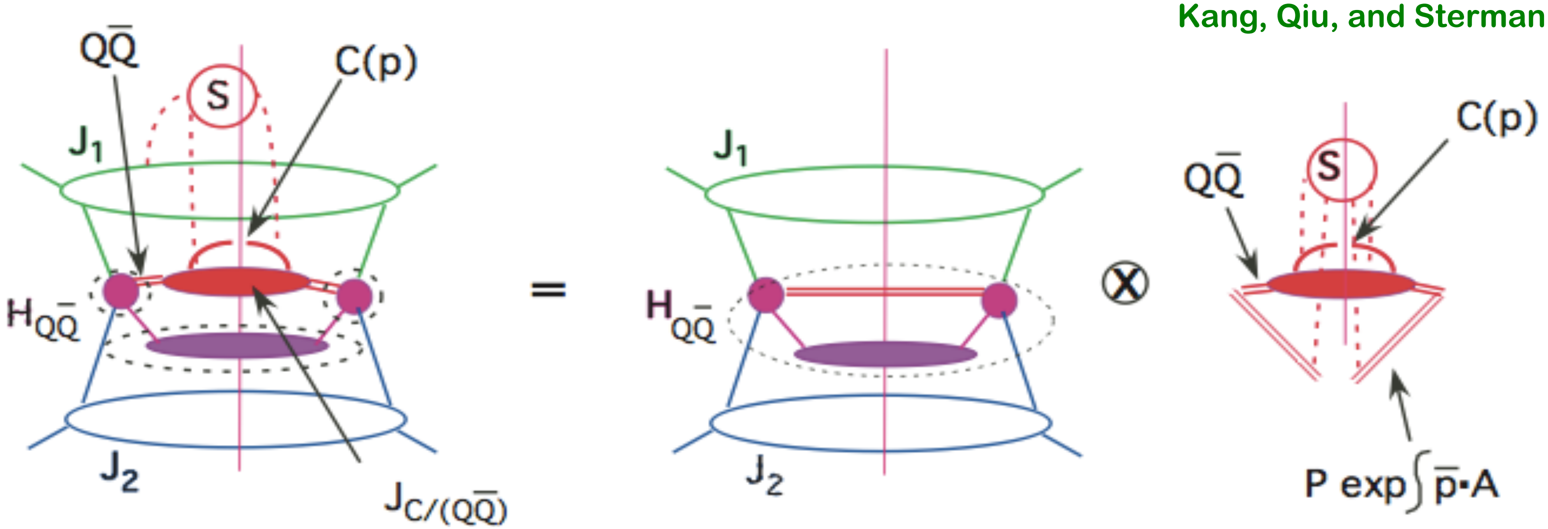
Nayak, Qiu, and Sterman, 2005

❑ Leading power – single hadron production



❑ Next-to-leading power – $Q\bar{Q}$ channel:

Qiu, Sterman, 1991
Kang, Qiu, and Sterman, 2010



Formalism and production of the pairs

❑ Factorization formalism:

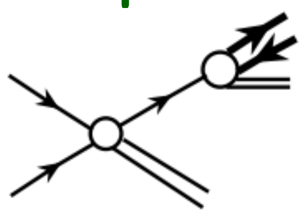
Kang, Qiu and Sterman, 2010

$$d\sigma_{A+B\to H+X}(p_T) = \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q)$$
$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q)$$
$$+ \mathcal{O}(m_Q^4/p_T^4)$$

$$\hat{p}_Q = \frac{1+\zeta}{2z}\hat{p}\,, \quad \hat{p}_{\bar{Q}} = \frac{1-\zeta}{2z}\hat{p}$$

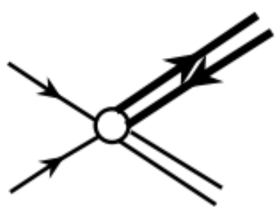
❑ Production of the pairs:

✧ at $1/m_Q$:



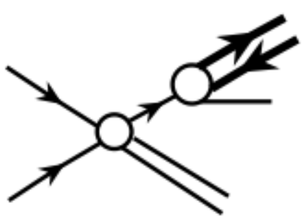
$$D_{i\to H}(z, m_Q, \mu_0)$$

✧ at $1/P_T$:



$$d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(\hat{p}_{[Q\bar{Q}(\kappa)]}, m_Q = 0, \mu)$$

✧ between: $[\,1/m_Q\,,\,1/P_T]$



$$\frac{d}{d\ln(\mu)} D_{i\to H}(z, m_Q, \mu) = \ldots$$
$$+\frac{1}{\mu^2}\Gamma(z,\zeta,\zeta') \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q)$$

Predictive power

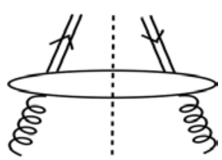
- **Calculation of short-distance hard parts in pQCD:**

  Power series in α_s, without large logarithms

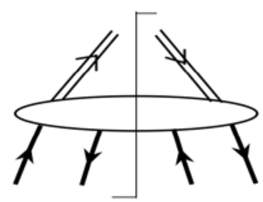
- **Calculation of evolution kernels in pQCD:**

  Power series in α_s, scheme in choosing factorization scale μ
  Could affect the term with mixing powers

- **Universality of input fragmentation functions at μ_0:**



$D_{H/f}(z, m_Q, \mu_0)$



$\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z, \zeta, \zeta', m_Q, \mu_0)$

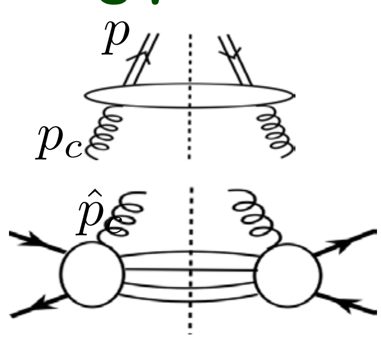
- **Physics of $\mu_0 \sim 2m_Q$ – a parameter:**

  Evolution stops when $\log\left[\frac{\mu_0^2}{(4m_Q^2)}\right] \sim \left[\frac{4m_Q^2}{\mu_0^2}\right]$

  Different quarkonium states require different input distributions!

Cut vertices and projection operators

❑ Leading power:

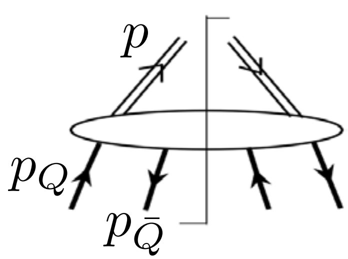


$$\widetilde{\mathcal{P}}_{\mu\nu}(p) = \frac{1}{2}\left[-g_{\mu\nu} + \frac{p_\mu n_\nu + n_\mu p_\nu}{p\cdot n} - \frac{p^2}{(p\cdot n)^2} n_\mu n_\nu\right]$$

$$\mathcal{P}_{\mu\nu}(p) = -g_{\mu\nu} + \bar{n}_\mu n_\nu + n_\mu \bar{n}_\nu \equiv d_{\mu\nu}$$

Hard parts available = that of pion production

❑ Next-to-leading power – QQ-channel with m_Q = 0:



$$\widetilde{\mathcal{P}}^L_v(p) = \frac{1}{4p\cdot n}\gamma\cdot n$$

$$\widetilde{\mathcal{P}}^L_a(p) = \frac{1}{4p\cdot n}\gamma\cdot n\gamma^5$$

$$\widetilde{\mathcal{P}}^L_t(p) = \frac{1}{4p\cdot n}\gamma\cdot n\gamma^\alpha_\perp$$

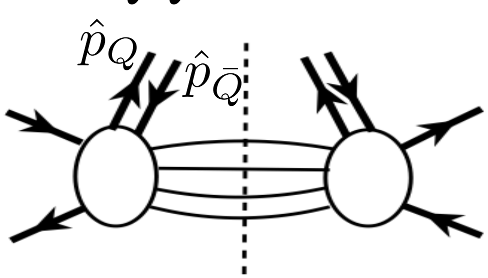
PQCD – relativistic:

Upper components

NRQCD – nonrelativistic:

Lower components

For a $Q\bar{Q}$ pair:



$$\mathcal{P}^L_v(\hat{p}_Q,\hat{p}_{\bar{Q}}) = \gamma\cdot\hat{p} = \gamma\cdot(\hat{p}_Q + \hat{p}_{\bar{Q}})$$

$$\mathcal{P}^L_a(\hat{p}_Q,\hat{p}_{\bar{Q}}) = \gamma_5\gamma\cdot\hat{p} = \gamma_5\gamma\cdot(\hat{p}_Q + \hat{p}_{\bar{Q}})$$

$$\mathcal{P}^L_t(\hat{p}_Q,\hat{p}_{\bar{Q}}) = \gamma\cdot\hat{p}\gamma^\alpha_\perp = \gamma\cdot(\hat{p}_Q + \hat{p}_{\bar{Q}})\gamma^\alpha_\perp$$

Hard part is insensitive to the difference in quarkonium states!

Short-distance hard parts

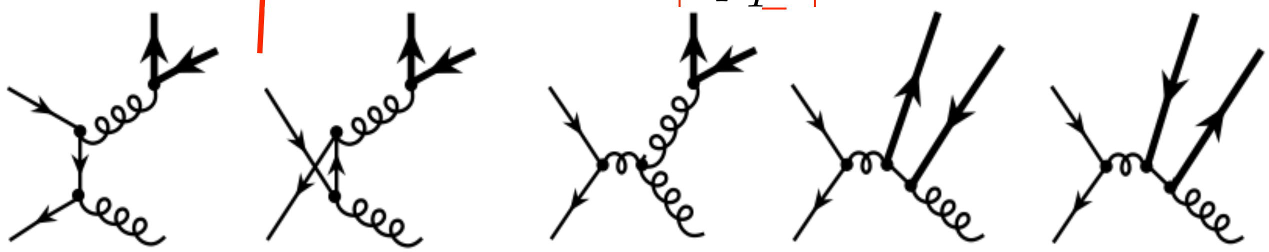
❑ Even tree-level needs subtraction:

$$\sigma^{(3)}_{q\bar{q}\to[Q\bar{Q}(c)]g} = \hat{\sigma}^{(3)}_{q\bar{q}\to[Q\bar{Q}(\kappa)]g} \otimes D^{(0)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(c)]} + \hat{\sigma}^{(2)}_{q\bar{q}\to gg} \otimes D^{(1)}_{g\to[Q\bar{Q}(c)]}$$

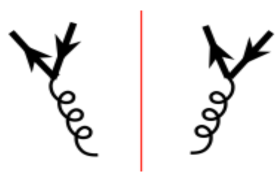
$$\Rightarrow \quad \hat{\sigma}^{(3)}_{q\bar{q}\to[Q\bar{Q}]g} = \sigma^{(3)}_{q\bar{q}\to[Q\bar{Q}]g} - \sigma^{(2)}_{q\bar{q}\to g} \otimes D^{(1)}_{g\to[Q\bar{Q}]}$$

$$\boxed{\frac{\alpha_s^3(\mu)}{p_T^6}} \qquad \boxed{\frac{\alpha_s^2(\mu)}{p_T^4}} \qquad \boxed{\frac{\alpha_s(2m_Q)}{(2m_Q)^2}}$$

$\sigma^{(3)}_{q\bar{q}\to[Q\bar{Q}(c)]g}$:



$D^{(1)}_{g\to[Q\bar{Q}]}$:



$$\widetilde{\mathcal{P}}_{\mu\nu}(p) = \frac{1}{2}\left[-g_{\mu\nu} + \frac{p_\mu n_\nu + n_\mu p_\nu}{p\cdot n} - \frac{p^2}{(p\cdot n)^2} n_\mu n_\nu\right]$$

$$H^{(3)}_{q\bar{q}\to[Q\bar{Q}(a8)]g} = \frac{8\pi\alpha_s}{\hat{s}} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2} \frac{1}{(1-\zeta^2)(1-\zeta'^2)} \frac{N^2-1}{N}\left[1 + \zeta\zeta' - \frac{4}{N^2}\right]$$

Normalized to 2 → 2 amplitude square

Evolution of fragmentation functions

Kang, Qiu and Sterman, 2011

- **Independence of the factorization scale:**

$$\frac{d}{d\ln(\mu)}\sigma_{A+B\to HX}(P_T)=0$$

✧ **at Leading power in 1/P$_T$:**

DGALP evolution

$$\frac{d}{d\ln\mu^2}D_{H/f}(z,m_Q,\mu)=\sum_j\frac{\alpha_s}{2\pi}\gamma_{f\to j}(z)\otimes D_{H/j}(z,m_Q,\mu)$$

✧ **next-to-leading power in 1/P$_T$:**

$$\frac{d}{d\ln\mu^2}D_{H/f}(z,m_Q,\mu)=\sum_j\frac{\alpha_s}{2\pi}\gamma_{f\to j}(z)\otimes D_{H/j}(z,m_Q,\mu)$$
$$+\frac{1}{\mu^2}\sum_{[Q\bar{Q}(\kappa)]}\frac{\alpha_s^2}{(2\pi)^2}\Gamma_{f\to[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta')\otimes\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q,\mu)$$

$$\frac{d}{d\ln\mu^2}\mathcal{D}_{H/[Q\bar{Q}(c)]}(z,\zeta,\zeta',m_Q,\mu)=\sum_{[Q\bar{Q}(\kappa)]}\frac{\alpha_s}{2\pi}K_{[Q\bar{Q}(c)]\to[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta')$$
$$\otimes\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q,\mu)$$

- **Evolution kernels are perturbative:**

✧ **Set mass:** $m_Q\to 0$ **with a caution**

NRQCD for input distributions

- **Input distributions are universal, non-perturbative:**

  Should, in principle, be extracted from experimental data

- **Use low energy QCD effective theory to calculate them:**

  $\mu_0 \sim 2m_Q$ – reduce unknown functions to a few unknown numbers!

- **NRQCD – single parton distributions:**

  Nayak, Qiu and Sterman, 2005

$$D_{H/f}(z, m_Q, \mu_0) \to \sum_{[Q\bar{Q}(c)]} \hat{d}_{f\to[Q\bar{Q}(c)]}(z, m_Q, \mu_0)\langle \mathcal{O}^H_{[Q\bar{Q}(c)]}(0)\rangle_{\rm NRQCD}$$

– Dominated by transverse polarization

- **NRQCD – heavy quark pair distributions:**

  Kang, Qiu and Sterman, 2011

$$\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z, \zeta, \zeta', m_Q, \mu_0) \to \sum_{[Q\bar{Q}(c)]} \hat{d}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(c)]}(z, \zeta, \zeta', m_Q, \mu_0)\langle \mathcal{O}^H_{[Q\bar{Q}(c)]}(0)\rangle_{\rm NRQCD}$$

– Dominated by longitudinal polarization

- **No proof of such factorization yet!**

  Nayak, Qiu and Sterman, 2005

  Single parton case was verified to two-loops (with gauge links)!

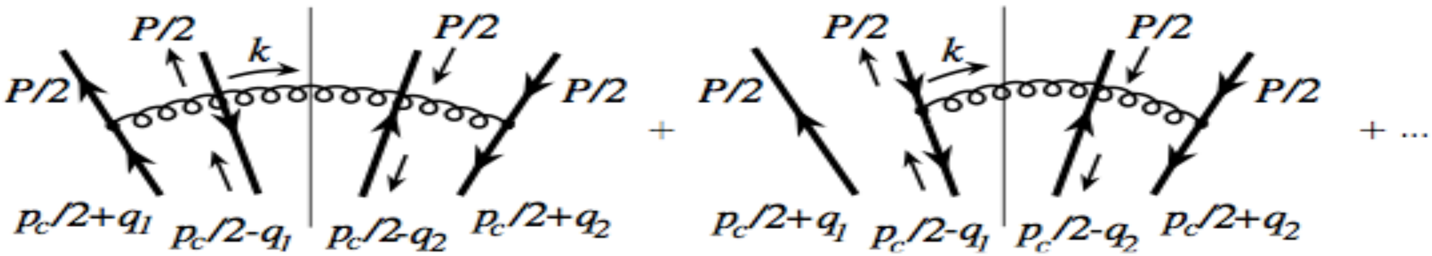
Polarization of heavy quarkonium

Kang, Qiu and Sterman, 2011

- Fragmentation functions determine the polarization

  Short-distance dynamics at r ~ 1/p_T is insensitive to the details taken place at the scale of hadron wave function ~ 1 fm

- Heavy quark pair fragmentation functions at LO:



NRQCD to a singlet pair:

$$\mathcal{D}_{[Q\bar{Q}(\kappa)]\to J/\psi} = 2\,\mathcal{D}^T_{[Q\bar{Q}(\kappa)]\to J/\psi} + \mathcal{D}^L_{[Q\bar{Q}(\kappa)]\to J/\psi}$$

$$\mathcal{D}^L_{[Q\bar{Q}(a8)]\to J/\psi}(z,\zeta,\zeta',m_Q,\mu) = \frac{1}{2N^2}\frac{\langle O^{J/\psi}_{1(^3S_1)}\rangle}{3m_c}\Delta(\zeta,\zeta')\;\frac{\alpha_s}{2\pi}z(1-z)\left[\ln\left(r(z)+1\right)-\left(1-\frac{1}{1+r(z)}\right)\right]$$

$$\mathcal{D}^T_{[Q\bar{Q}(a8)]\to J/\psi}(z,\zeta,\zeta',m_Q,\mu) = \frac{1}{2N^2}\frac{\langle O^{J/\psi}_{1(^3S_1)}\rangle}{3m_c}\Delta(\zeta,\zeta')\,\frac{\alpha_s}{2\pi}z(1-z)\left[1-\frac{1}{1+r(z)}\right]$$

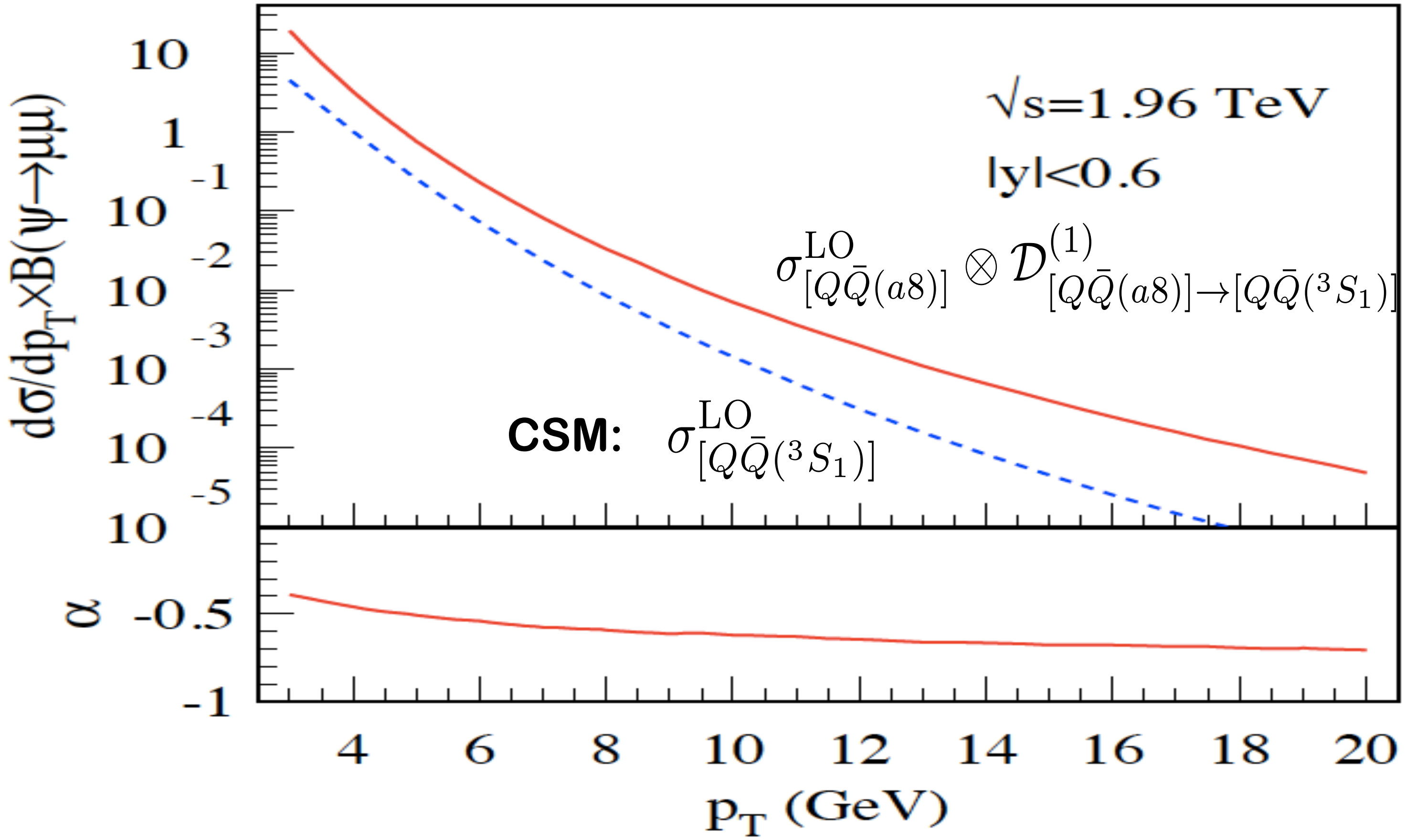
where

$$\Delta(\zeta,\zeta') = \frac{1}{4}\sum_{a,b}\delta(\zeta-a(1-z))\delta(\zeta'-b(1-z)), \qquad r(z) \equiv \frac{z^2\mu^2}{4m_c^2(1-z)^2}$$

Production rate and polarization

Kang, Qiu and Sterman, 2011

- LO hard parts + LO fragmentation contributions:

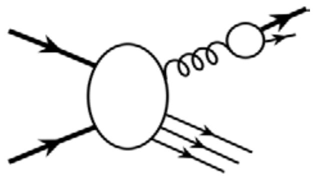


LO heavy quark pair fragmentation contribution reproduces the bulk of NLO color singlet contribution, and the polarization!

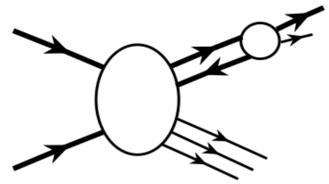
Polarization and high spin states

- **Competition between LP and NLP:**

$$E\frac{d\sigma_{J/\psi}}{d^3P}:$$



$+$



$+\ldots$

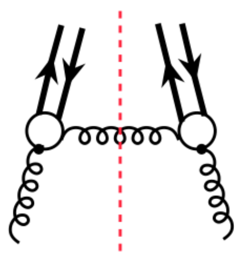
Transverse polarization

Longitudinal polarization

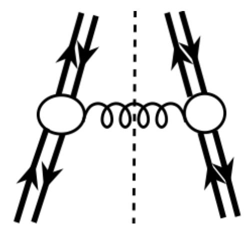
$$\frac{\langle \mathcal{O}_{\text{octet}}\rangle}{p_T^4} \text{ vs. } \frac{\langle \mathcal{O}_{\text{singlet}}\rangle}{p_T^6}$$

- **Contribution of high spin states:**

$$\frac{d^n}{dq^{\alpha_1}\ldots dq^{\alpha_n}}$$



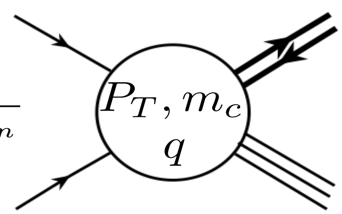
$$\frac{d^n}{dq^{\alpha_1}\ldots dq^{\alpha_n}}$$



$$P_c = P/2 + q$$
$$P_{\bar{c}} = P/2 - q$$

instead of,

$$\frac{d^n}{dq^{\alpha_1}\ldots dq^{\alpha_n}}$$



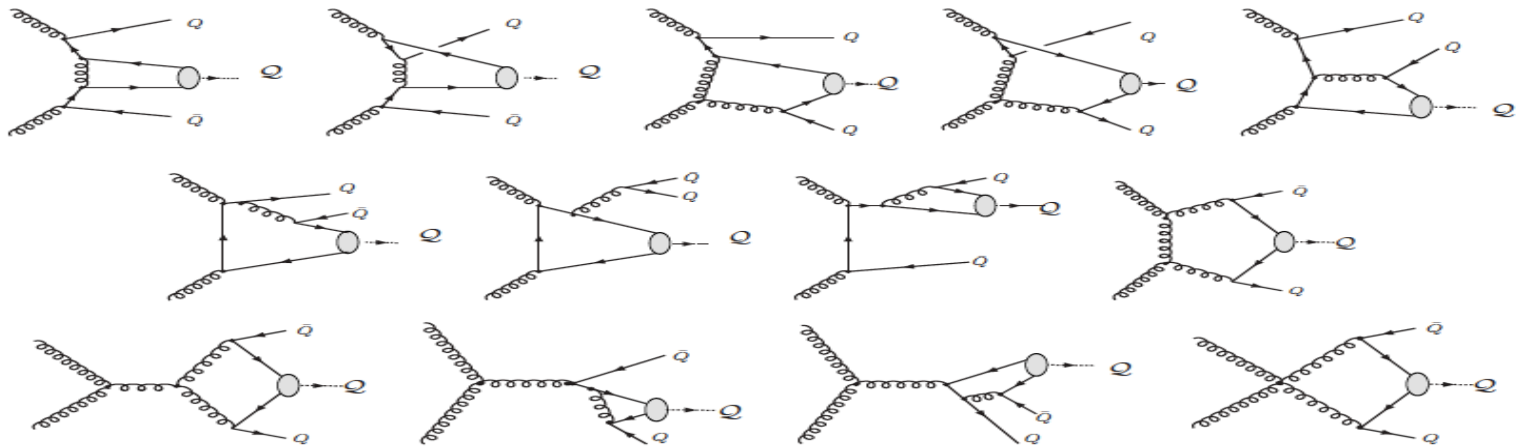
$$P_c = P/2 + q$$
$$P_{\bar{c}} = P/2 - q$$

Universal and process independent, if NRQCD factorization is valid

Associate production in CSM

- Complete set of diagrams:

Artoisenet, Lansburg, Maltoni (2007)



- Claim:

Fragmentation contribution to inclusive quarkonium production sizably underestimates the exact calculation at high p_T!

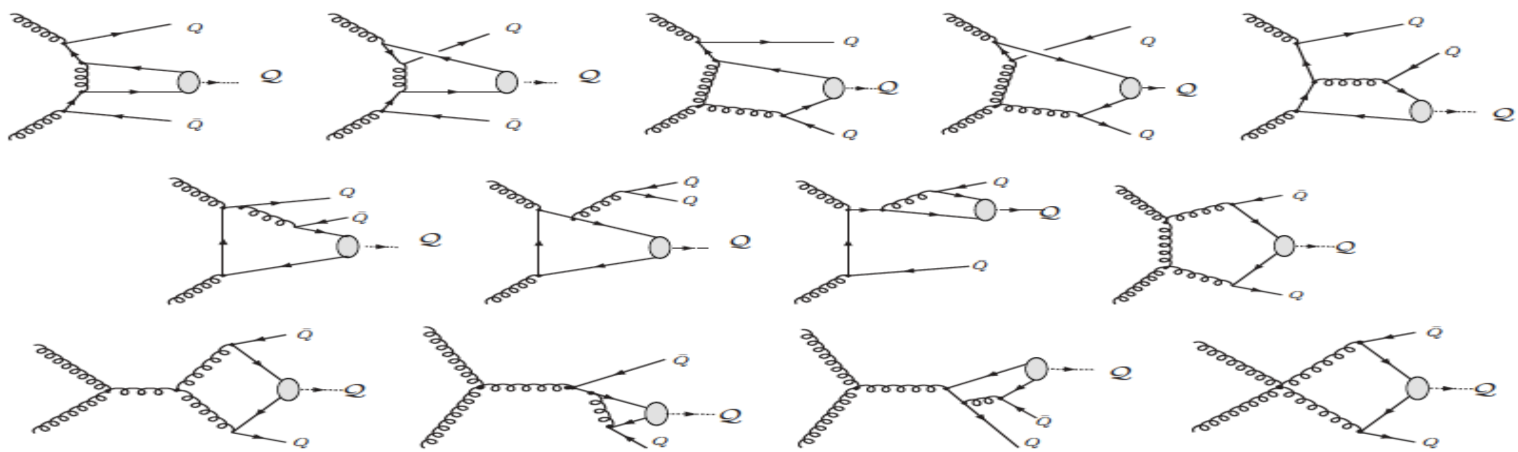
- Is there any problem for the fragmentation approach?

Answer: NO!

Associate production in CSM

- Complete set of diagrams:

Artoisenet, Lansburg, Maltoni (2007)



- Claim:

Fragmentation contribution to inclusive quarkonium production sizably underestimates the exact calculation at high p_T!

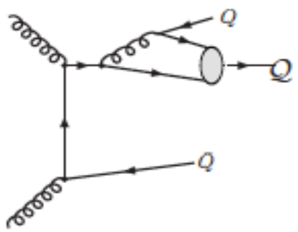
- Is there any problem for the fragmentation approach?

Answer: NO!

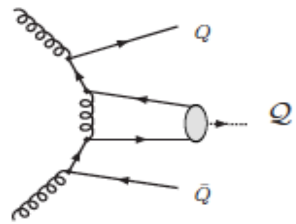
The existing CSM calculation is not consistent with pQCD power counting, and is not perturbatively stable at high p_T ($>>m_Q$)!

Definition of the associate production?

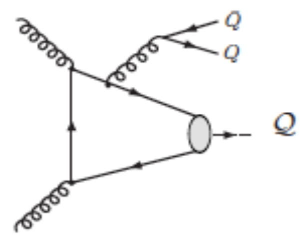
- **Unfair comparison:**
  - ✧ **CSM: extra charm can be in any part of final-state phase-space**
  - ✧ **Frag: extra charm can only be in a narrow cone around the J/ψ**
- **CSM calculation is not perturbatively stable when $p_T >> m_Q$:**



Q-fragmentation



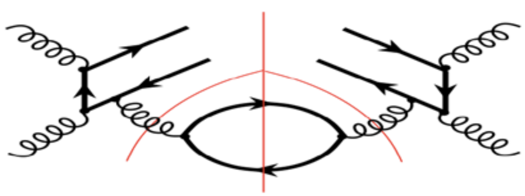
Logs in PDF



Need interference diagrams

- **Inclusive J/ψ(p):**

Need the interference



to remove the pole
when $m_Q \to 0$

- **Key: What is the physical observable one wants to calculate?**
  - ✧ **Inclusive J/ψ(p), J/ψ(p)+D(p_D), J/ψ(p)+$\overline{D}$($p_{\overline{D}}$)+D(p_D), …**

Summary

- When $p_T >> m_Q$ at collider energies, all existing models for calculating the production rate of heavy quarkonia are not perturbatively stable
  - ✧ LO in α_s-expansion may not be the LP term in $1/p_T$-expansion
  - ✧ Heavy flavor scattering channels are important when $p_T >> m_Q$

    (Resummation of initial-state logarithms)

- When $p_T >> m_Q$, $1/p_T$-power expansion before α_s-expansion

  Fragmentation approach takes care of both $1/p_T$-expansion and resummation of the large logarithms

- RHIC/LHC are offering an excellent opportunity to test the heavy quarkonium production mechanism, and QCD dynamics of heavy quarks

Thank you!