International workshop on Heavy Quarkonium 2011
@GSI, Oct. 4-7 (2011).

Inter-quark Potentials (from Nambu-Bethe-Salpeter Amplitudes)

partly based on Ikeda, Iida, arXiv:1102.2097[hep-lat](2011).

Yoichi IKEDA
(Tokyo Institute of Technology)

in collaboration with

Hideaki Iida (RIKEN, Nishina Center)

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- Why inter-quark potentials from BS amplitudes?
- Formula of Lattice QCD potentials
- Q^{bar}-Q potentials with finite masses
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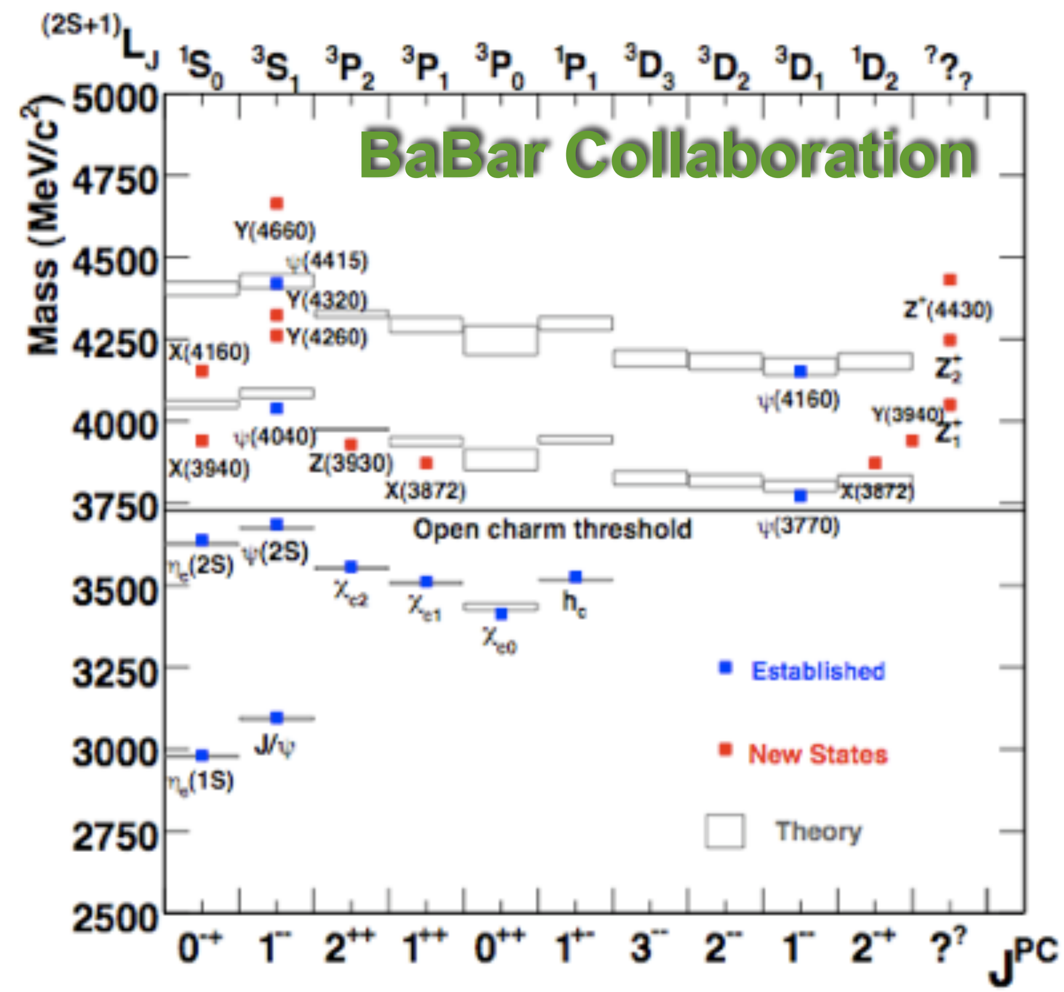
Mass spectrum of charmonium system

- Quark potential models well describe mass spectra below open charm threshold

  Godfrey, Isgur, PRD 32 (1985).
  Barnes, Godfrey, Swanson, PRD 72 (2005).

- Exotic states (X, Y, Z) can be expected as non-standard c^{bar}-c mesons

- All exotic states reveal as resonances above open charm threshold



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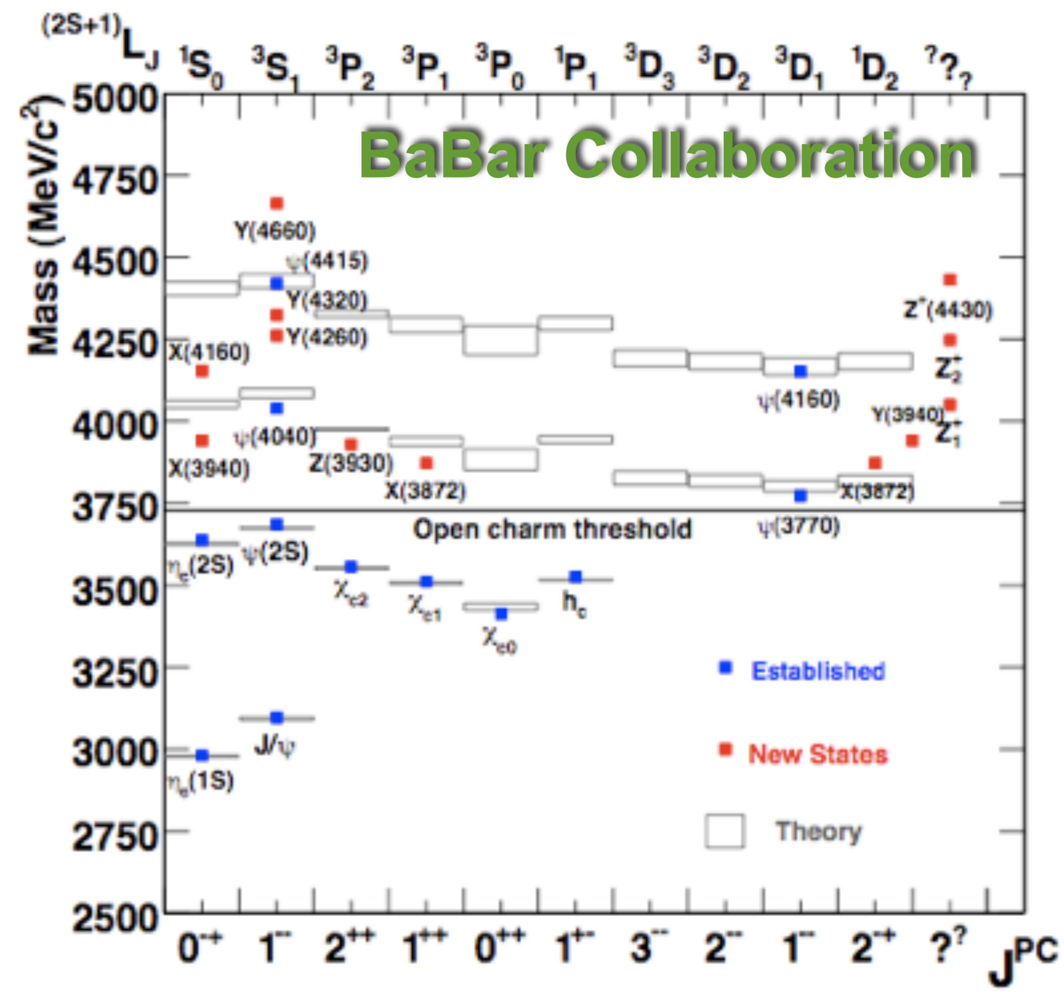

All states are characterized by pole, residue and unitarity cut of T-matrix

$$T^{-1}(\sqrt{s}) = \sum_i \frac{R_i}{\sqrt{s}-W_i} + a(s_0) + \frac{s-s_0}{2\pi}\int_{s_+}^{\infty} ds' \frac{\rho(s')}{(s'-s)(s'-s_0)}$$

(General form of amplitudes from N/D method)

Interaction parts cannot be determined within the scattering theory

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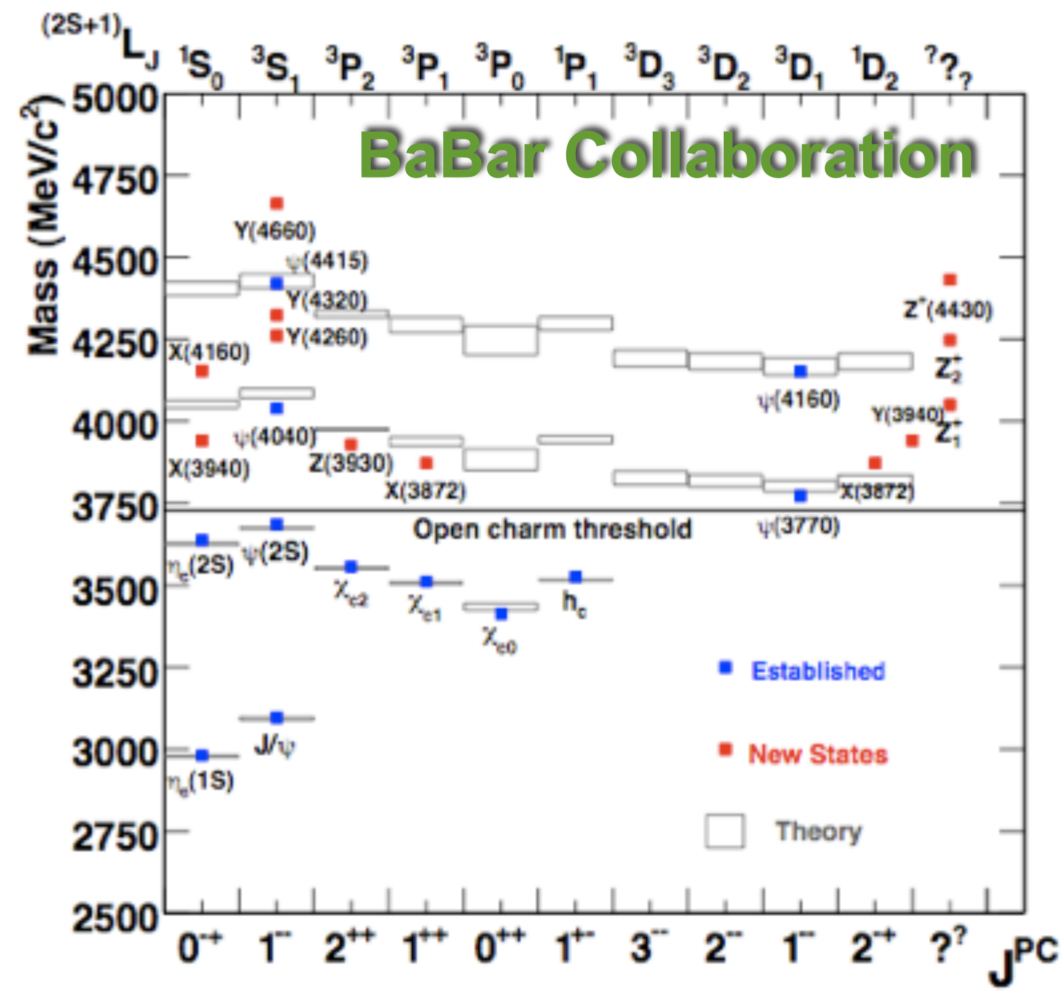

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(General form of amplitudes from N/D method)

Interaction parts cannot be determined within the scattering theory

Reliable input based on QCD becomes powerful tool to analyze spectra

Q^{bar}-Q interquark potential

- Q^{bar}-Q potentials can be expected having the following form:

$$V_{\bar{Q}Q}(r) = \boxed{\sigma r - \frac{4}{3}\frac{\alpha_s}{r}} + \boxed{V_{\mathrm{spin}}(r)\vec{S}_{\bar{Q}}\cdot\vec{S}_{Q} + V_{T}(r)\hat{S}_{12} + V_{LS}(r)\vec{L}\cdot\vec{S}} + \cdots$$

Spin-independent Spin-dependent

☑ Effective field theory approach (pNRQCD) for charmonium spectra :
Wilson loop + relativistic correction ($1/m_Q$, v (velocity), $1/m_Q v$ expansion)

Bali, Phys. Rept. 343 (2001).
Brambilla, Pineda, Soto, Vairo, NPB 566 (2000); Rev. Mod. Phys. 77 (2005).
Koma et al., PRL 97 (2006).
Koma et al., NPB 769 (2007).

☑ Our approach through Nambu-Bethe-Salpeter (NBS) amplitude :
We define effective inter-quark potential with finite quark mass
which becomes faithful to QCD T-matrix

Lin et al., NPB 619 (2001).
Aoki, Hatsuda, Ishii, PTP 123 (2010).

Reliable input based on QCD for quark potential models can be extracted

How to define Q^{bar}-Q interquark potential

- **We start with NBS equation for invariant amplitudes at meson rest frame :**

$$\boxed{\mathcal{M}(p,p';P) = K(p,p') + \int d^4k K(p,k) G(k;P) \mathcal{M}(k,p';P)}$$

P : meson 4-momentum P=(M, 0) at center-of-mass frame
p, p', k : relative 4-momentum of Q^{bar}-Q system
K(p,p') : irreducible kernel
G(k;P) : product of free quark propagator w/ assumption of **constant quark mass m_Q**

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- Non-relativistic reduction through Levy-Klein-Macke (LKM) method

Reviewed in Klein, Lee, PRD 10 (1974).

Concept of LKM method :
Replacement of free-propagator G(k;P) to non-relativistic one leads to rearrangement of interaction kernel of original NBS equation

$$\boxed{\mathcal{M} = I + I\overleftarrow{\mathcal{P}}G_{\mathrm{N.R.}}\overrightarrow{\mathcal{P}}\mathcal{M}}$$

I(p,p') is "new" kernel and satisfying $I = K + K(G - \overleftarrow{\mathcal{P}}G_{\mathrm{N.R.}}\overrightarrow{\mathcal{P}})I$

$$\boxed{\overrightarrow{\mathcal{P}}f(p;P) \equiv \frac{1}{2\pi i}\int_{\mathrm{UHP}} dp^0\left[(p^0 - P^0/2 + E(\vec{p}) - i\epsilon)^{-1} + (p^0 \to -p^0)^{-1}\right]f(p;P)}$$

Note :
We do not require instantaneous NBS kernel K(p,p') in LKM method

How to define Q^{bar}-Q interquark potential

- 3-dimensional LKM equation for NBS invariant amplitude:

Reviewed in Klein, Lee, PRD 10 (1974).

$$\boxed{\overrightarrow{\mathcal{P}}\mathcal{M}(p,p';P) = \overrightarrow{\mathcal{P}}I(p,p';P) + \overrightarrow{\mathcal{P}}\int d^4k I(p,k)\overleftarrow{\mathcal{P}}G_{\mathrm{N.R.}}(\vec{k};P)\overrightarrow{\mathcal{P}}\mathcal{M}(k,p';P)}$$

L.H.S. of LKM equation is found as equal-time NBS wave function

-> Schrödinger-type equation for NBS wave function is easily derived :

$$\boxed{(E-2E(\vec{p}))\tilde{\phi}_E(\vec{p}) = \int d^3p' U(\vec{p},\vec{p}')\tilde{\phi}_E(\vec{p}')} \qquad E = P^0 = M_{\mathrm{meson}}$$

with non-local, energy-independent potential U(p,p') satisfying $U(\vec{p},\vec{p}') = \overrightarrow{\mathcal{P}}I(p,p')\overleftarrow{\mathcal{P}}$

$$\Rightarrow \quad \boxed{(E-H_0)\phi_E(\vec{r}) = \int d^3r' U(\vec{r},\vec{r}')\phi_E(\vec{r}')}$$

Schrödinger-type equation for NBS wave function in r-space

How to define Q^{bar}-Q interquark potential

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Schrödinger-type equation for NBS wave function in r-space

Summary of LKM method :

- Relativistic 3-dimensional equation are extracted from equal-time NBS wave function -> suitable for LQCD simulation
- Obtained 3-dimensional equation is Schrödinger-type equation (LKM equation)
- "Effective potential" of LKM equation is related to irreducible kernel K(p.p') of NBS equation -> potential model based on QCD can be constructed

Q^{bar}-Q interquark potential on lattice

Aoki, Hatsuda, Ishii, PTP 123 (2010).
Ikeda, Iida, arXiv:1102.2097[hep-lat](2011).

1. Measure equal-time Nambu-Bethe-Salpeter wave function

$$\boxed{\phi_E(\mathbf{r}) = \sum_{\mathbf{x}} \langle 0|\bar{q}(\mathbf{x})\Gamma q(\mathbf{x}+\mathbf{r})|E; J^{PC}\rangle}$$

Spacial correlation of 4-point function

$$\begin{aligned} G^{(2)}(\mathbf{r}, t - t_{\mathrm{src}}) &= \sum_{\mathbf{x},\mathbf{X},\mathbf{Y}} \langle 0|\bar{q}(\mathbf{x},t)\Gamma q(\mathbf{x}+\mathbf{r},t)\Big(\bar{q}(\mathbf{X},t_{\mathrm{src}})\Gamma q(\mathbf{Y},t_{\mathrm{src}})\Big)^{\dagger}|0\rangle \\ &= \sum_{\mathbf{x}}\sum_{E_n} A_{E_n}\langle 0|\bar{q}(\mathbf{x})\Gamma q(\mathbf{x}+\mathbf{r})|E_n\rangle e^{-E_n(t-t_{\mathrm{src}})} \\ &\rightarrow A_{E_0}\phi_{E_0}(\mathbf{r})e^{-E_0(t-t_{\mathrm{src}})} \qquad (E_0 = M, \ \ t \gg t_{\mathrm{src}}) \end{aligned}$$

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$$\boxed{(E - H_0)\phi_E(\mathbf{r}) = \int d^3r' U(\mathbf{r},\mathbf{r}')\phi_E(\mathbf{r}')}$$

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3. Velocity expansion of non-local potential

$$U(\mathbf{r},\mathbf{r}') = \Big(\underbrace{V_{\rm C}(r) + V_{\rm spin}(r)\vec{S}_{\bar{\rm Q}}\cdot\vec{S}_{\rm Q} + V_{\rm T}(r)\hat{S}_{12}}_{\text{Leading order}} + \underbrace{V_{\rm LS}(r)\vec{L}\cdot\vec{S}}_{\text{NLO}} + \cdots\Big)\delta(\mathbf{r}-\mathbf{r}')$$

Q^{bar}-Q interquark potential on lattice

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We examine s-wave "effective LO potentials" in pseudo-scalar and vector channels

LQCD setup

Y.I., Iida, arXiv:1102.2097[hep-lat](2011).

- Quench QCD simulation

- Plaquette gauge action & Standard Wilson quark action
- β=6.0 (a=0.104 fm, a^{-1}=1.9GeV)
- Box size : 32^3 x 48 -> L=3.3 (fm)
- Four different hopping parameters (κ=0.1320, 0.1420, 0.1480, 0.1520)

  -> M_{PS}=2.53, 1.77, 1.27, 0.94 (GeV), M_V=2.55, 1.81,1.35, 1.04 (GeV)
- N_{conf}=100

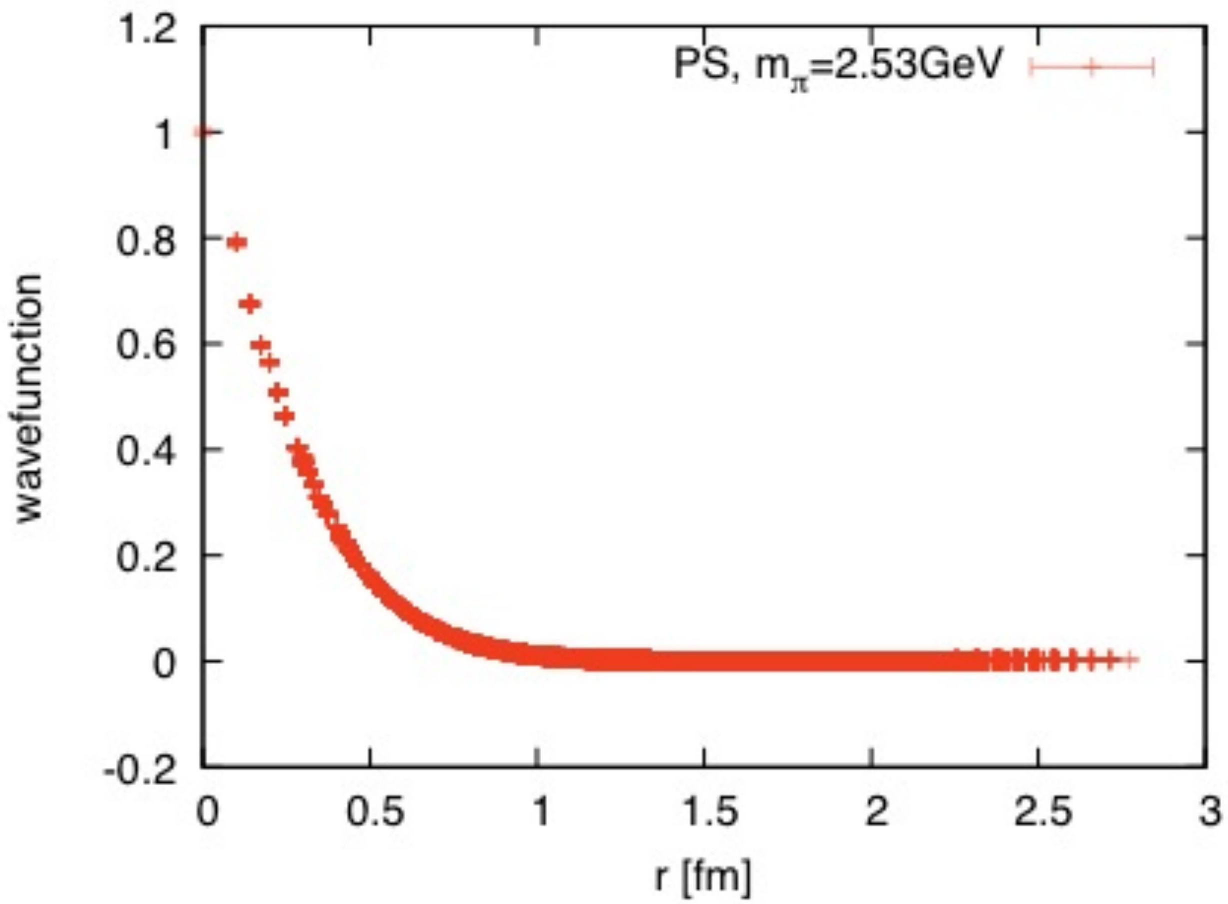
- Wall source
- Coulomb gauge fixing

Q^{bar}-Q wave function

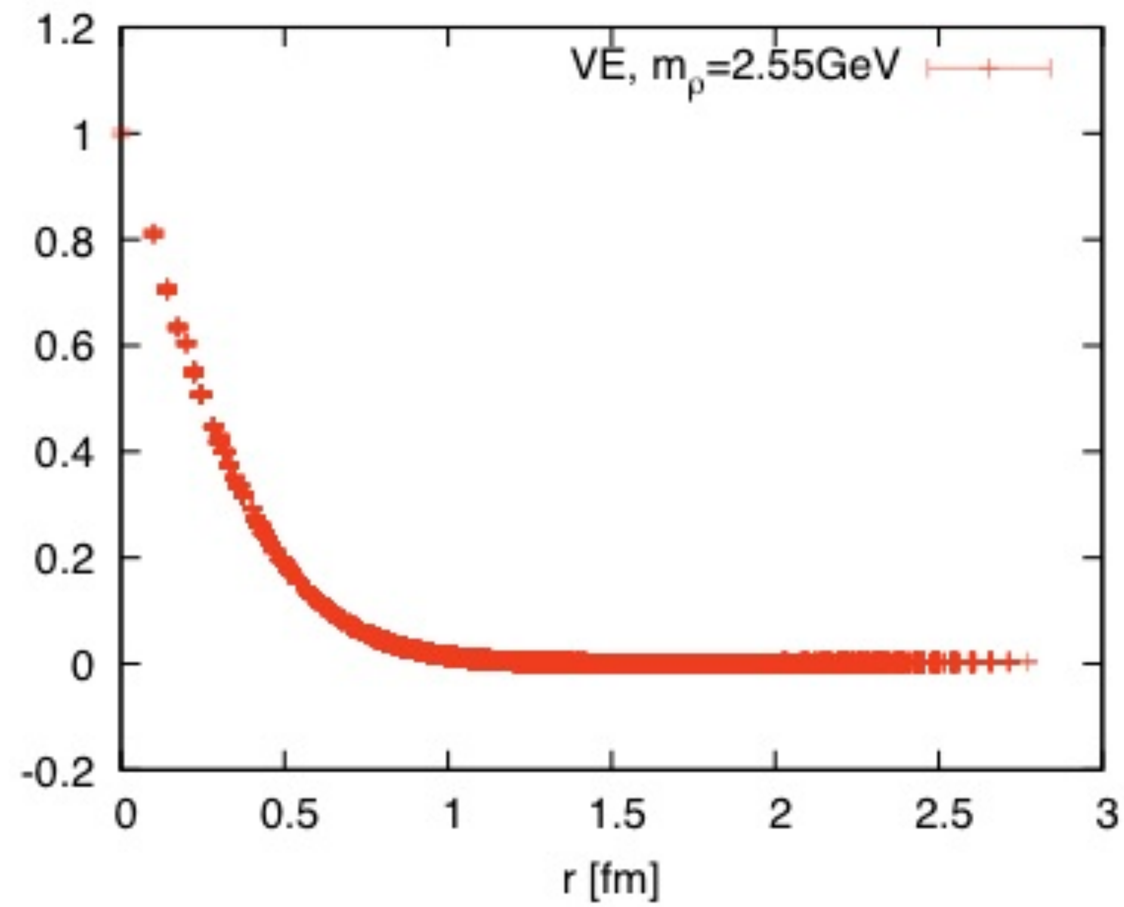
NBS wave function for heaviest quark mass in our simulation (κ=0.1320)

$$\phi_E(\mathbf{r}) = \sum_{\mathbf{x}} \langle 0|\bar{q}(\mathbf{x})\Gamma q(\mathbf{x}+\mathbf{r})|E; J^{PC}\rangle$$

Pseudoscalar channel



Vector channel



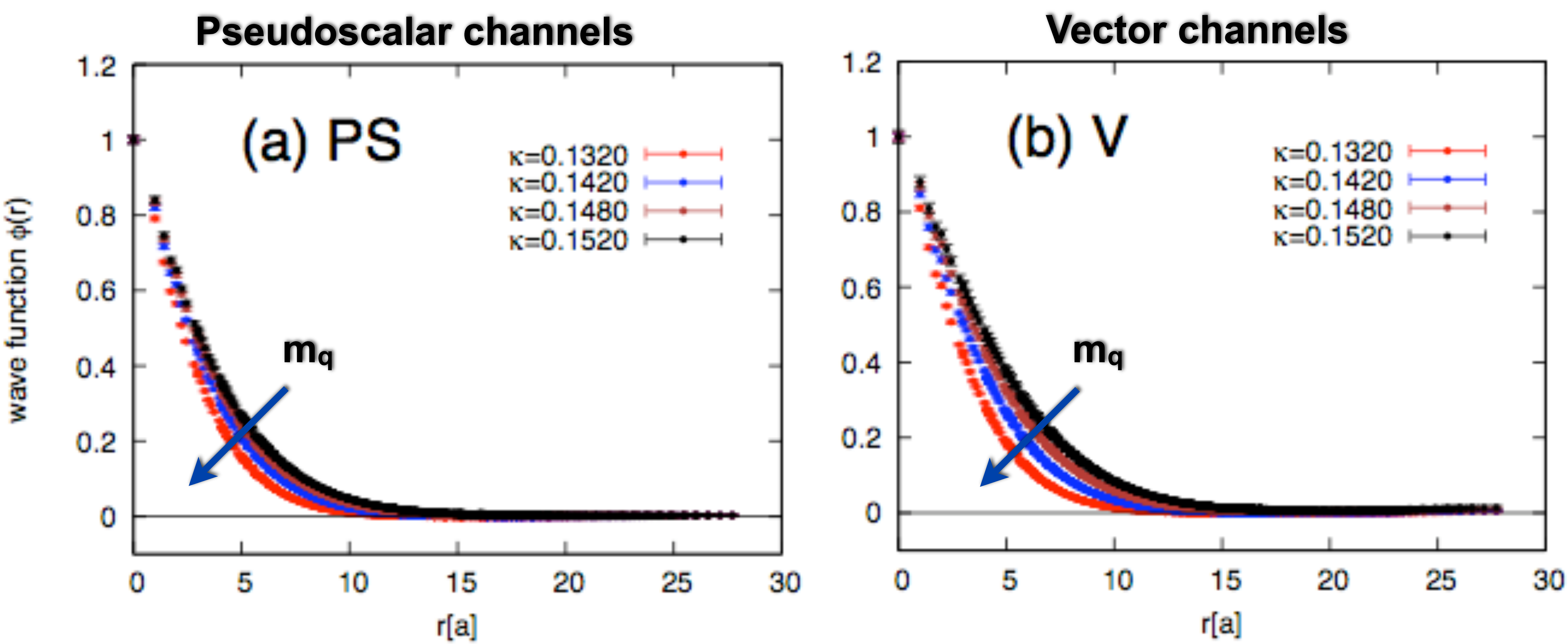
- ✓ NBS wave function is normalized at r=0
- ✓ Wave functions are localized within 1.5 fm (box size is enough large)
- ✓ There is little channel dependence

Q^{bar}-Q wave function

Quark mass dependence of NBS wave functions

M_{PS}=2.53, 1.77, 1.27, 0.94 (GeV), M_V=2.55, 1.81,1.35, 1.04 (GeV)

$$\boxed{\phi_E(\mathbf{r}) = \sum_{\mathbf{x}} \langle 0|\bar{q}(\mathbf{x})\Gamma q(\mathbf{x}+\mathbf{r})|E; J^{PC}\rangle}$$



Size of wave function becomes smaller as increasing m_q

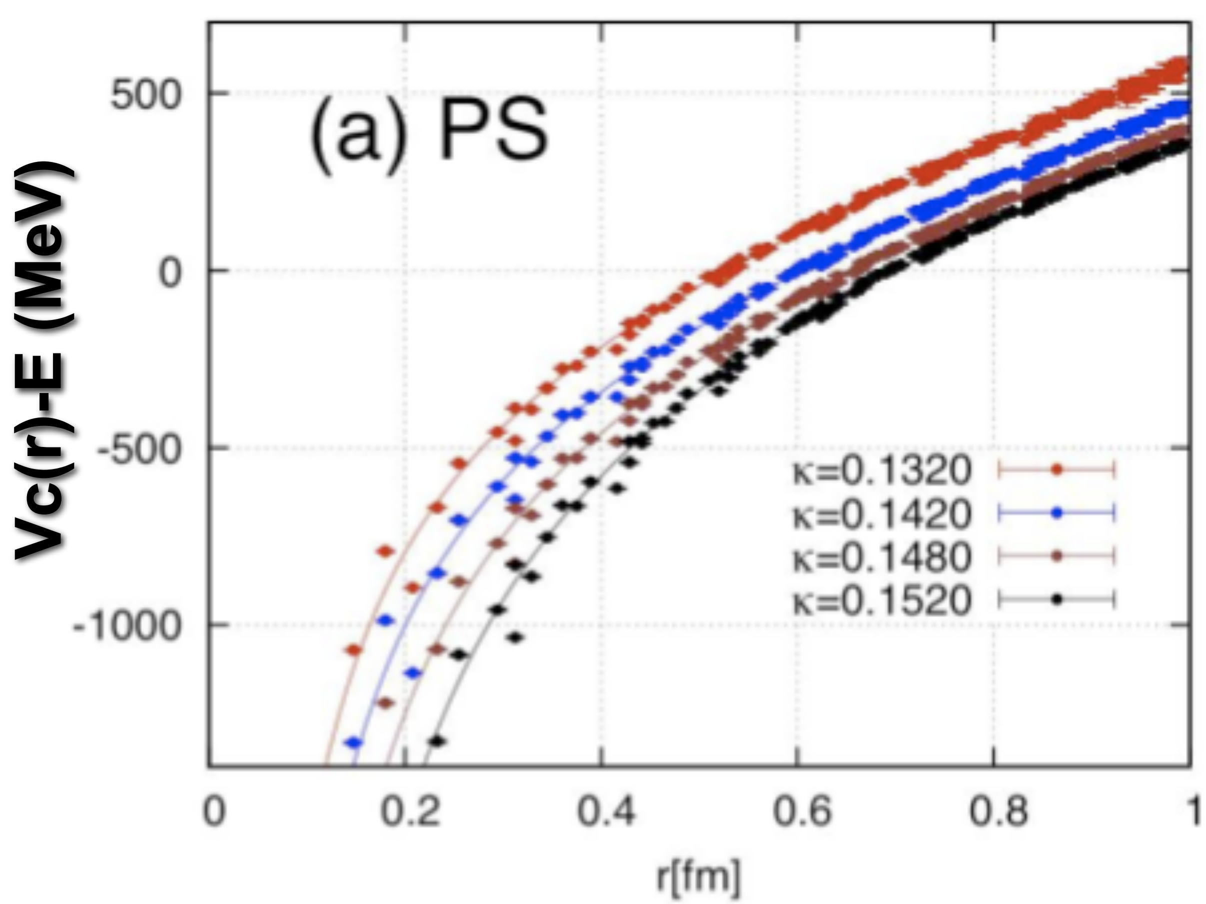
Q^{bar}-Q potential from NBS wave function

Inter-quark potential with various finite quark masses

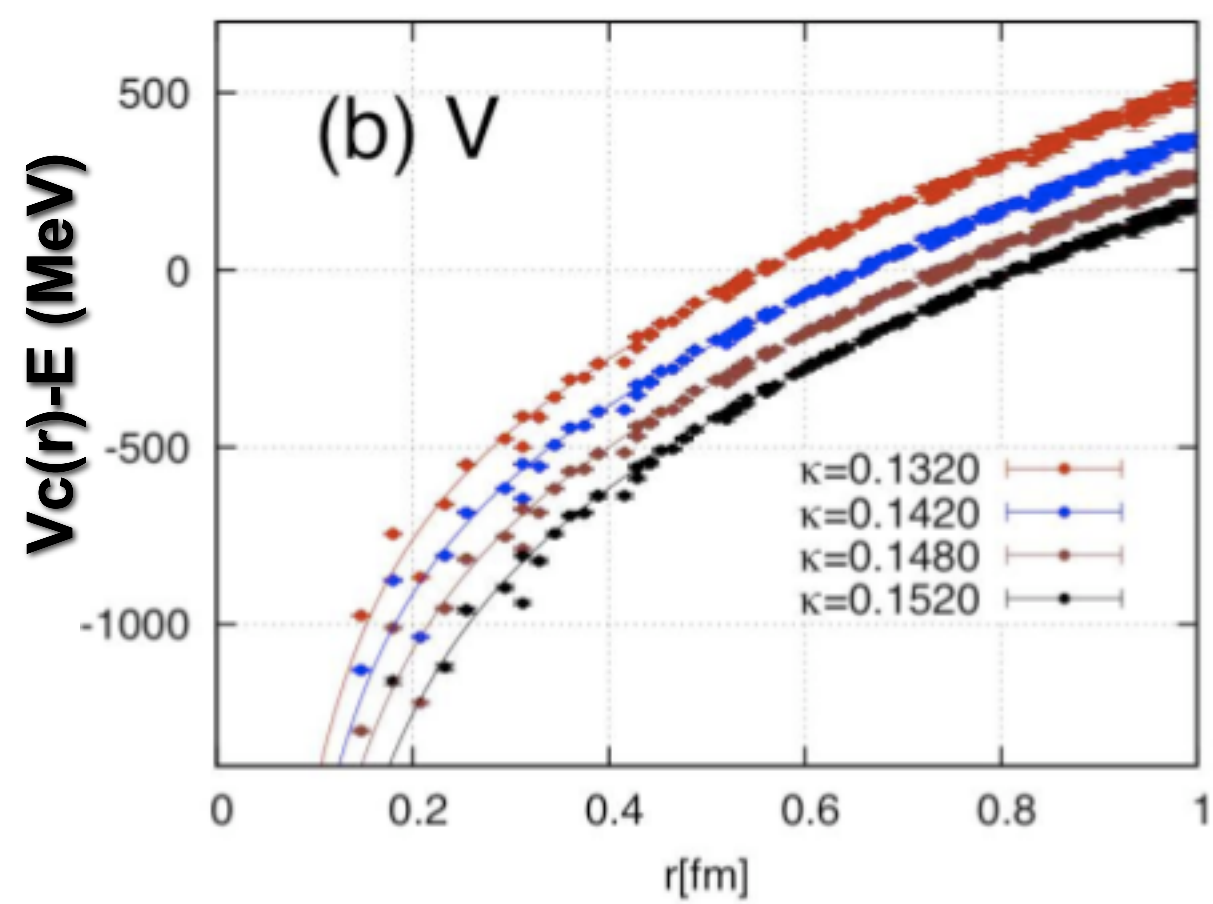
M_{PS}=2.53, 1.77, 1.27, 0.94 (GeV), M_V=2.55, 1.81,1.35, 1.04 (GeV)

$$\boxed{V_C^{\mathrm{eff}}(\mathbf{r}) - E = \frac{1}{m_q}\frac{\nabla^2 \phi_E(\mathbf{r})}{\phi_E(\mathbf{r})}} \qquad m_q = M_V/2$$

Pseudoscalar channels



Vector channels



Coulomb + linear confinement forces are reproduced with finite quark masses

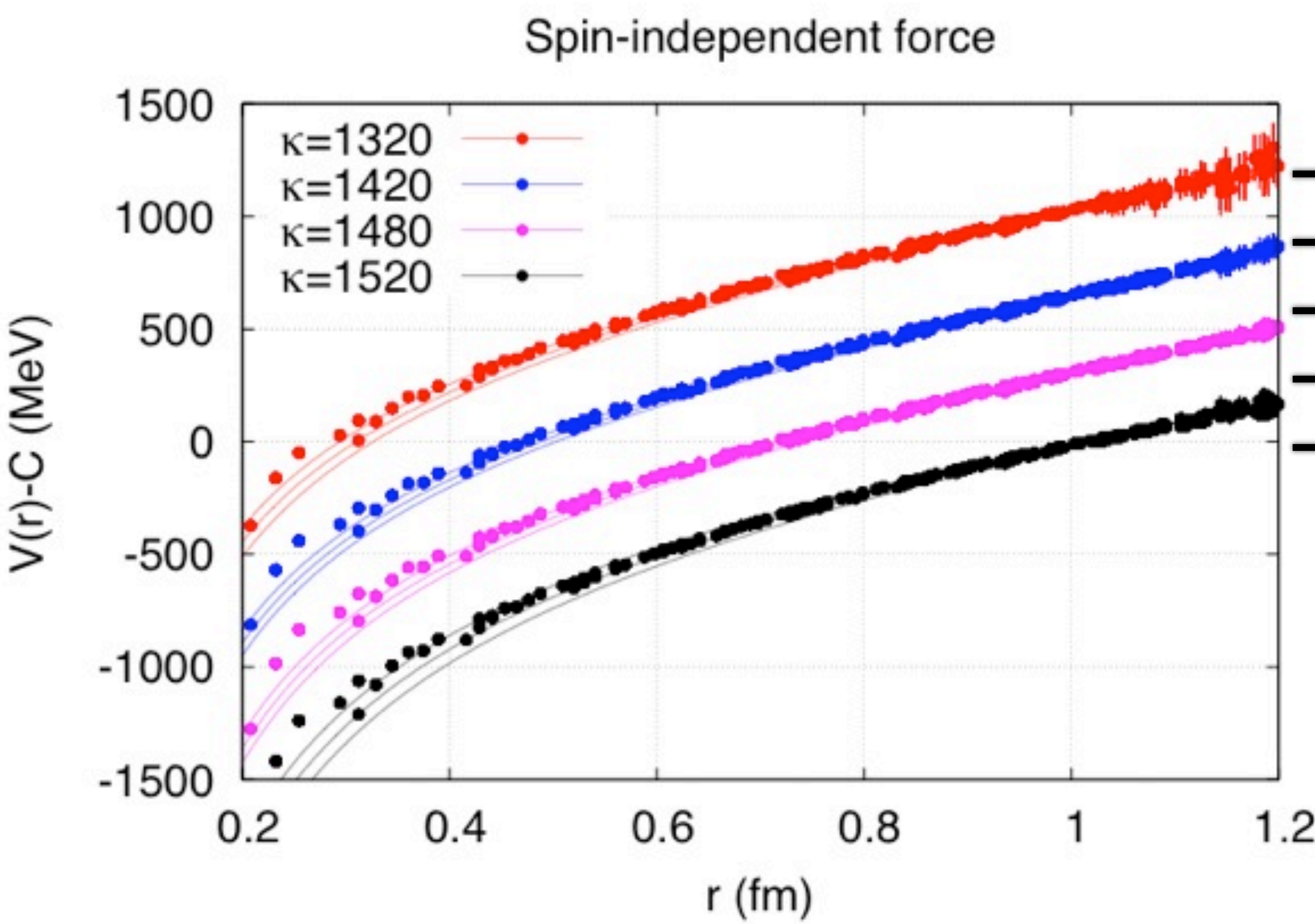
(solid curves representing Coulomb + linear functions)

Fitting results of Q^{bar}-Q potential

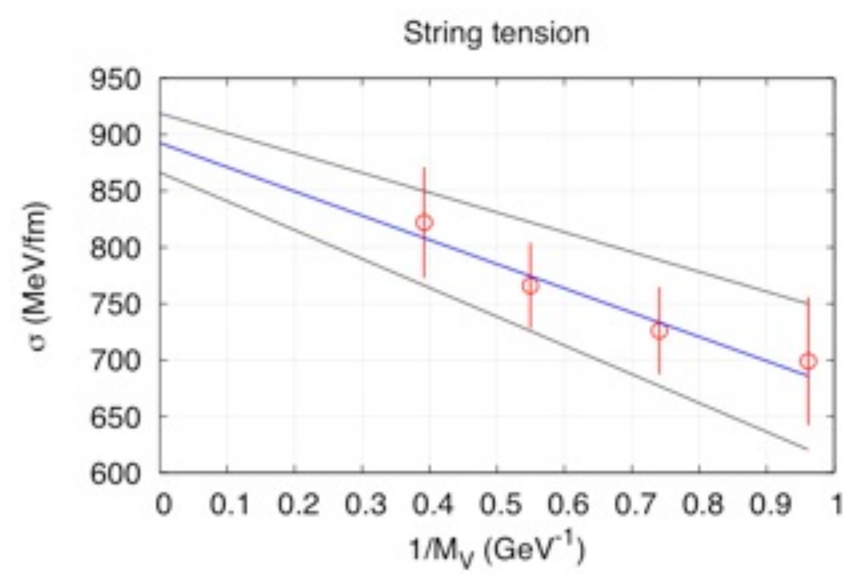
Spin-independent force can be constructed by linear combination of PS & V channels

$$V^{\rm eff}_{\rm spin-indep.}(\mathbf{r}) - E = \frac{1}{m_q}\left[\frac{1}{4}\frac{\nabla^2\phi_{\rm PS}(\mathbf{r})}{\phi_{\rm PS}(\mathbf{r})} + \frac{3}{4}\frac{\nabla^2\phi_{\rm V}(\mathbf{r})}{\phi_{\rm V}(\mathbf{r})}\right]$$

fit function: $V(\mathrm{r}) = \sigma r - \frac{A}{r} + C$



M_V (GeV)	σ (MeV/fm)	A (MeV fm)
2.55	822 (49)	200 (7)
1.87	766 (38)	228 (6)
1.35	726 (39)	269 (7)
1.04	699 (57)	324 (12)



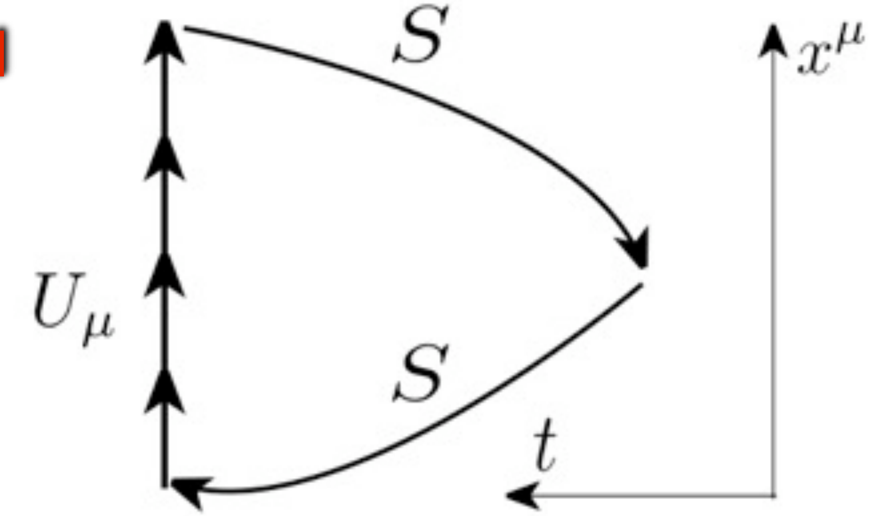
- String tension has moderate m_q dependences
- Naive extrapolation to infinite mass gives comparable value from Wilson loop
- Coulomb coefficients increase as decreasing m_q

see also, Kawanai and Sasaki, PRL 107 (2011).

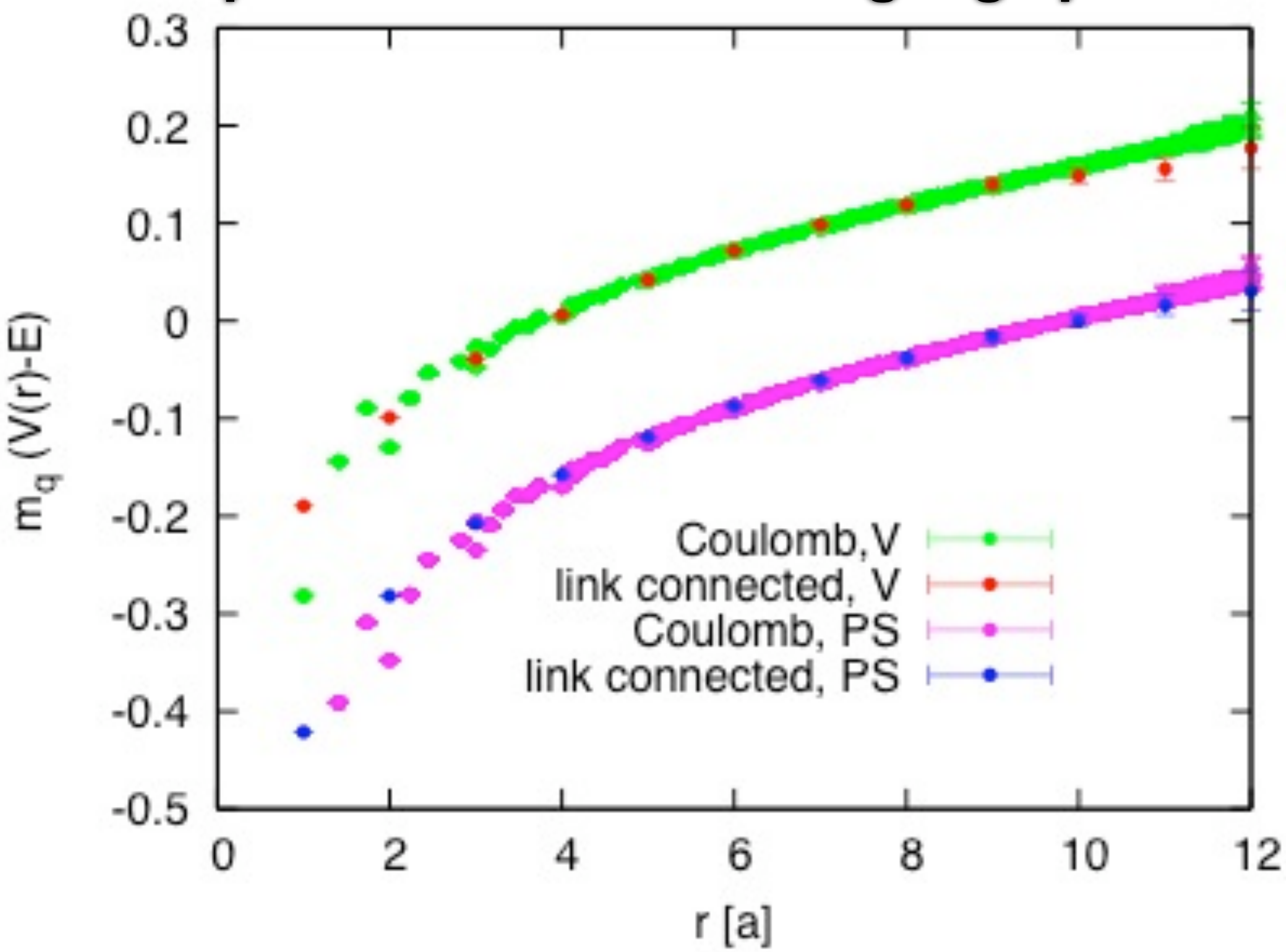
Operator dependence of Q^{bar}-Q potential

Operator dependence of inter-quark potential is studied by using gauge invariant smearing operator

$$\phi_E^{\mathrm{smr.}}(\mathbf{r}) = \sum_{\mathbf{x}} \langle 0|\bar{q}(\mathbf{x})L(\mathbf{x},\mathbf{r})\Gamma q(\mathbf{x}+\mathbf{r})|E;J^{PC}\rangle$$



Comparison with Coulomb gauge potentials



$$m_q(V_{\mathrm{C}}^{\mathrm{eff}}(\mathbf{r}) - E) = \frac{\nabla^2 \phi_E^{\mathrm{smr.}}(\mathbf{r})}{\phi_E^{\mathrm{smr.}}(\mathbf{r})}$$

The potentials obtained from gauge invariant smearing operators are comparable with Coulomb gauge potential

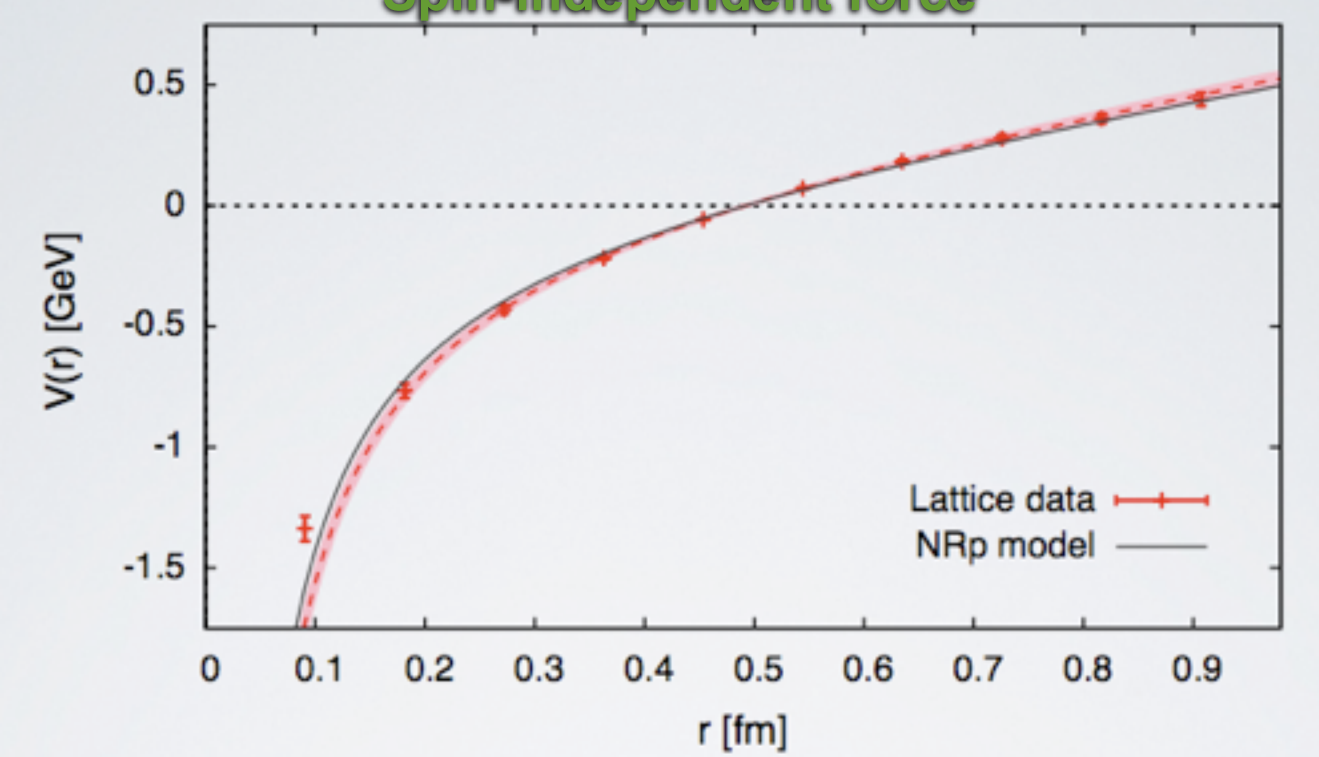
c^{bar}-c potential from full QCD

Kawanai, Sasaki, in preparation.

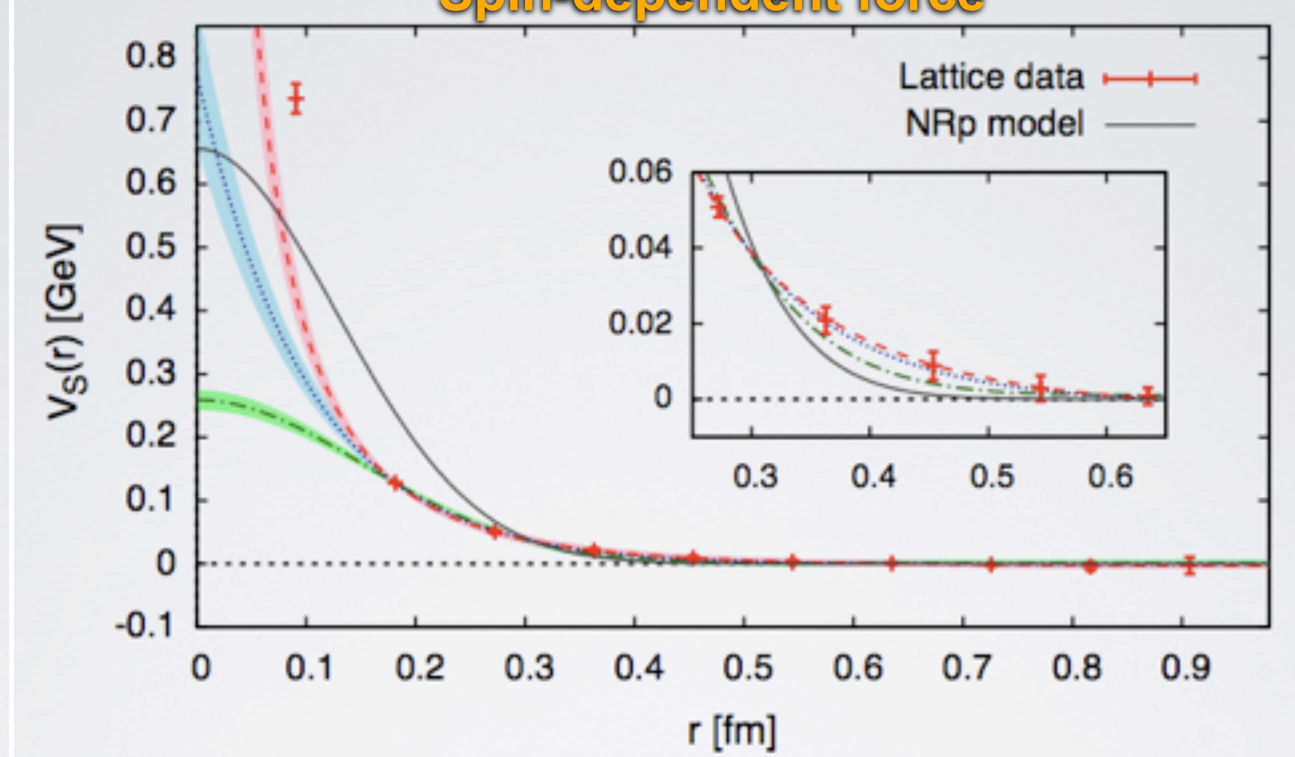
- c^{bar}-c potential from 2+1 flavor FULL QCD simulation at almost PHYSICAL POINT generated by PACS-CS Coll. (m_π=156(7), m_K=553(2) MeV)

- Iwasaki gauge action (β=1.9, a=0.091 fm) + RHQ action
  -> $M_{ave.}$(1S) = 3.069(2) GeV, $M_{hyp.}$=111(2) MeV

Spin-independent force



Spin-dependent force



- Spin-independent force shows Coulomb + linear form

- Lattice QCD potential is consistent with NRp model
  Barnes, Godfrey, Swanson, PRD 72 (2005).

- Spin-dependent force shows short range but not point-like repulsion
  see also, Kawanai and Sasaki, PRL 107 (2011).

Summary

- We study inter-quark interactions with finite quark mass in quenched QCD simulation

- Effective central Q^{bar}-Q potentials from NBS amplitudes reveal Coulomb + linear forms
- Coulomb coefficients become smaller and smaller as increasing m_q
- String tension also has m_q dependence and is comparable with that of Wilson loop analysis with large m_q limit

Future plans : Full QCD @ physical point

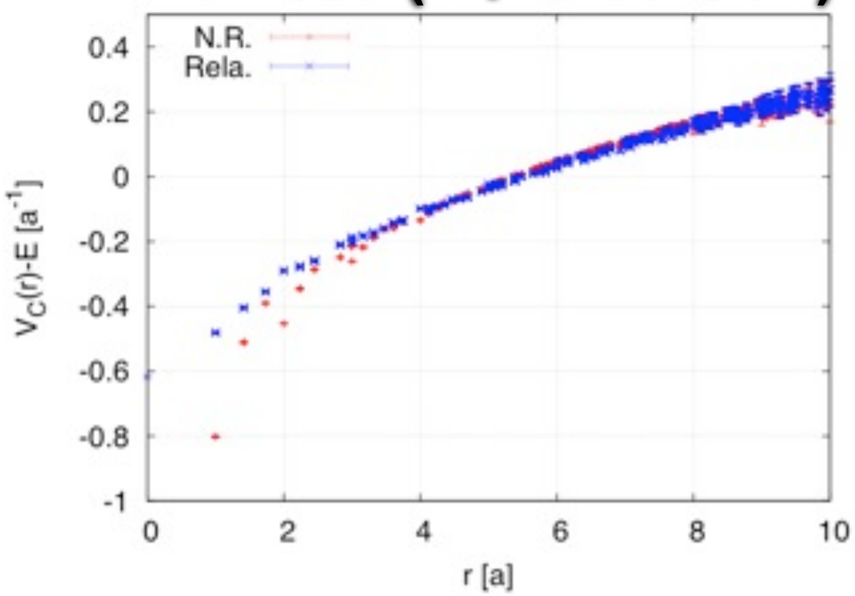
- Studies of tensor, LS, non-locality of inter-quark potential
- Three-quark potentials : ccc, ccs
- Coupled channel analysis toward above open charm threshold
- Investigation of exotic states (X, Y, Z)

Thank you very much for your attention

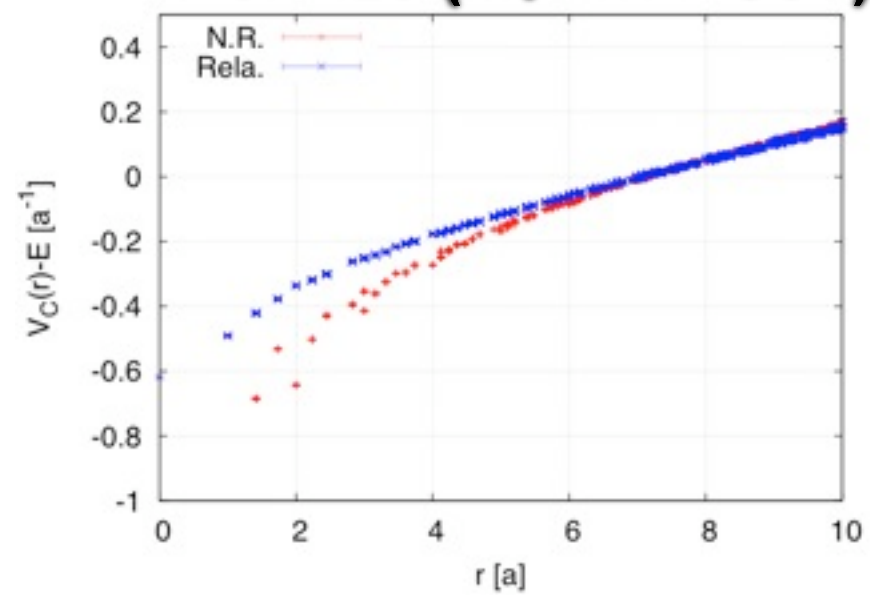
Relativistic kinematics

Inter-quark potentials with relativistic kinematics are studied

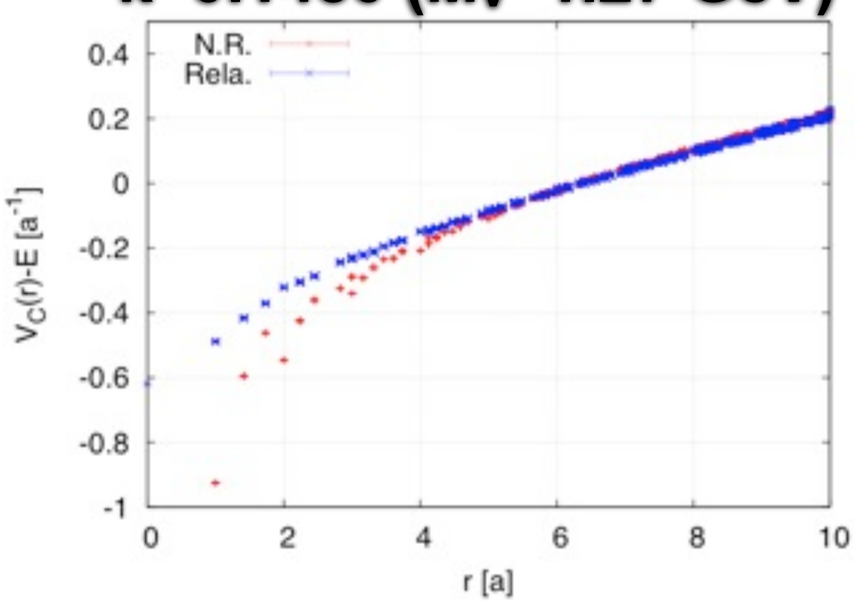
κ=0.1320 (M_V=2.55 GeV)



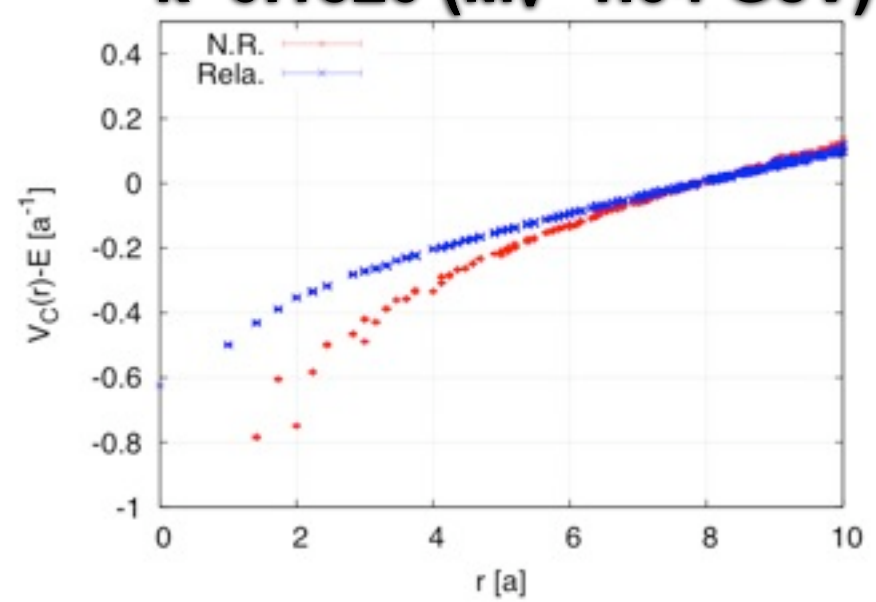
κ=0.1420 (M_V=1.77 GeV)



κ=0.1480 (M_V=1.27 GeV)



κ=0.1520 (M_V=1.04 GeV)

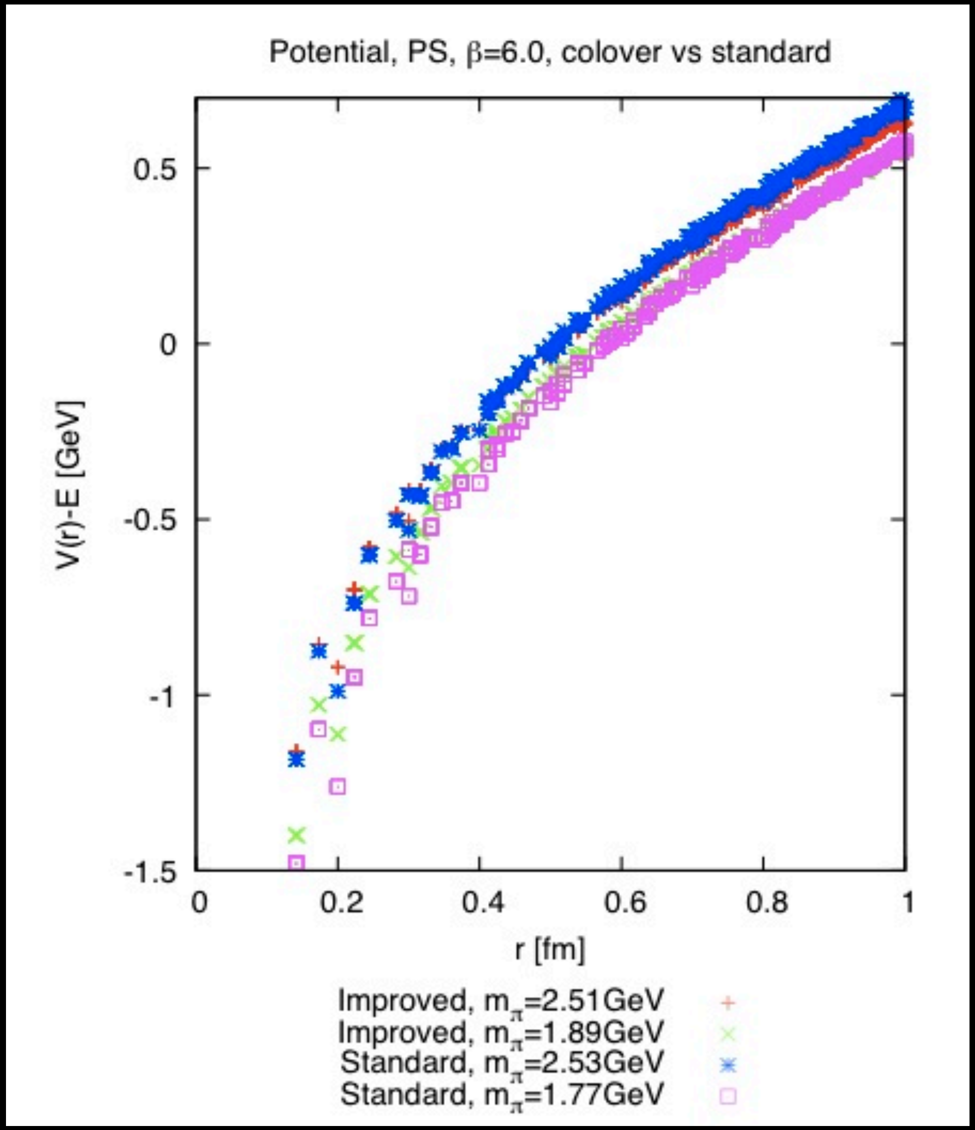


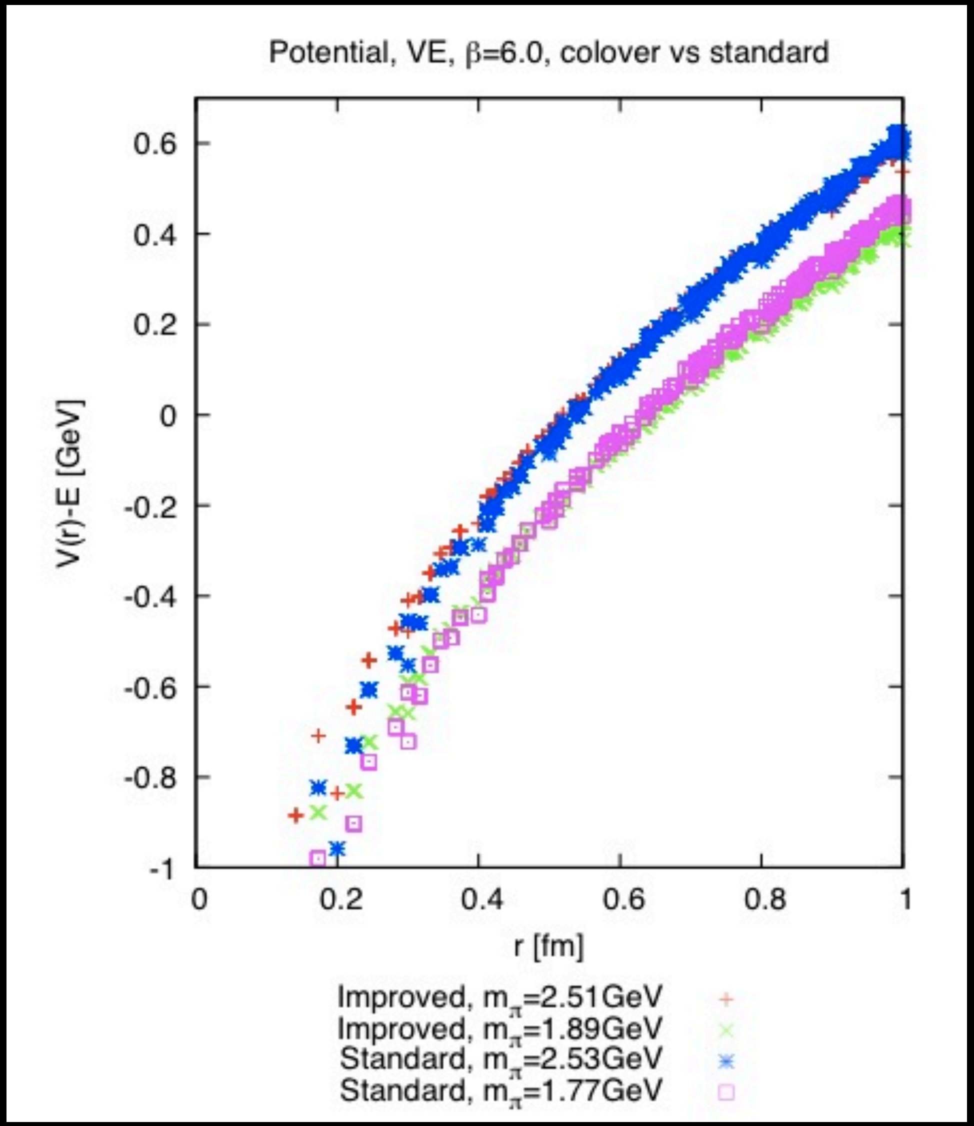
- ⭐ **Even for relativistic kinematics, Coulomb + linear potentials are obtained**
- ⭐ **Long range parts of relativistic potentials are consistent with those of N.R. potentials**
- ⭐ **For charmonium, non-relativistic kinematics is good enough**
- ⭐ **For strangeness sector, non-locality of potentials may get to large, if one employes non-relativistic kinematics**

Check (I) : O(a) improvement

- We study cutoff dependence of the q^{bar}-q potentail by adopting O(a)-improved Wilson-clover quark action

We compare Standard Wilson quark action with O(a) improved action (clover action)



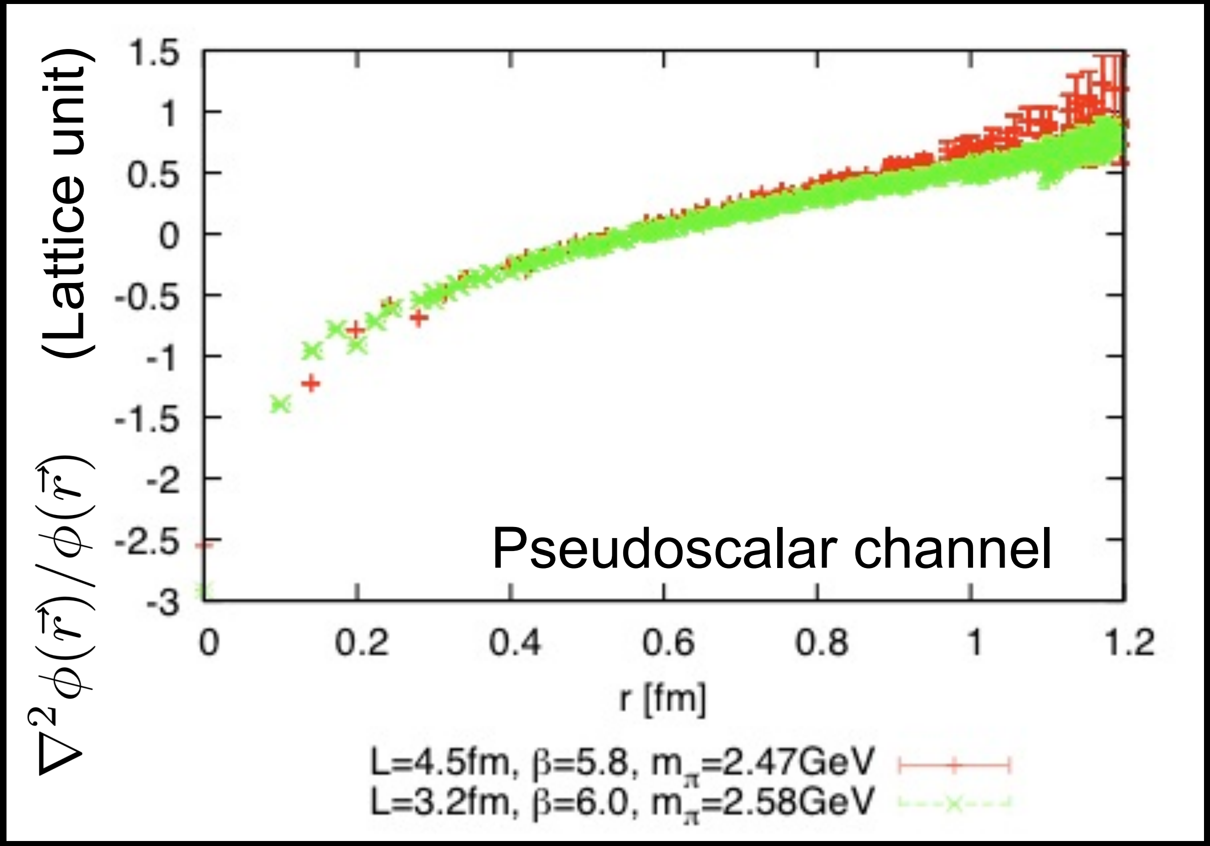


Small difference

Check (II) : Volume dependence

- We study volume dependence of the q^{bar}-q potentail by varying lattice spacing for O(a)-improved Wilson-clover quark action

L=4.5fm (β=5.8, m_{PS}=2.47GeV, clover): red
L=3.2fm (β=6.0, m_{PS}=2.58GeV, standard): green



- Small difference between them … volume is enough

Our setup (β=6.0, a=0.1fm, standard Wilson, $(3.2\text{fm})^3$) seems sufficient for the calculation of q^{bar}-q potential (in quark mass region calculated here)

Finite temperature : trial calcuation

$$\left[-\frac{\partial}{\partial t}-\frac{\nabla^2}{2\mu}+V(\mathrm{r};T)\right]\psi(\mathrm{r},t;T)=0$$

Potential from imaginary-time formalism