
Heavy Quark-Antiquark Threshold at NNLL

[hep-ph/0611292]
[1102.0269 [hep-ph]]

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Outline

- Heavy quark-antiquark threshold
- The effective theory v NRQCD
- Renormalization
 - Currents
 - Potentials
- Recent Results (QCD)
- Summary/Outlook

Heavy quark-antiquark threshold

Heavy $q \bar{q}$ production near threshold:

$t \bar{t}$



$e^+ e^- \rightarrow t \bar{t}$
threshold scan



$m_t, y_t, \alpha_s, \Gamma_t$

- perturbative
- nonrelativistic

$b \bar{b}$



$$M_n = \int_0^\infty \frac{ds}{s^{n+1}} R_{b\bar{b}}(s)$$

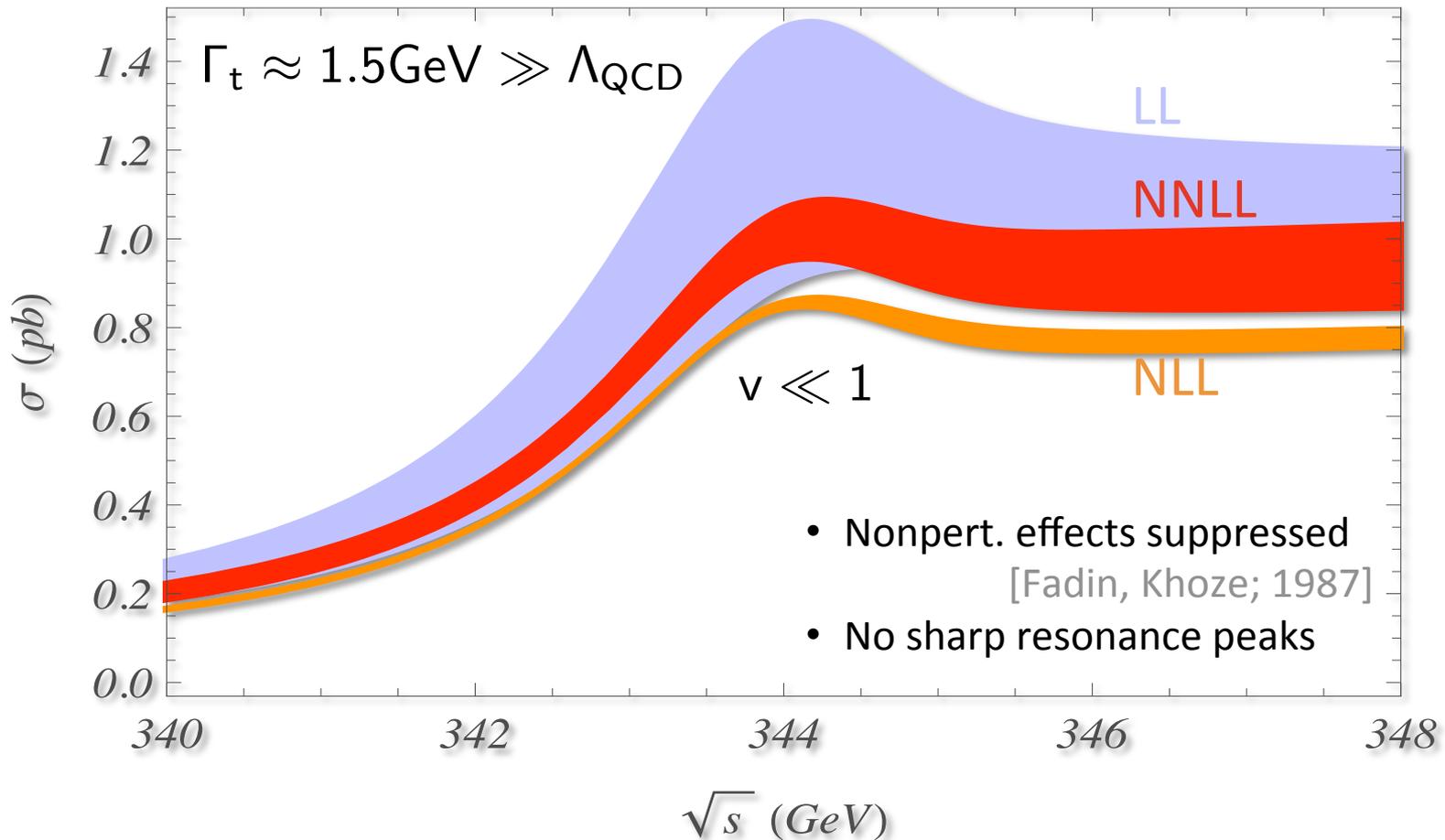
$$4 \leq n \leq 10$$



m_b

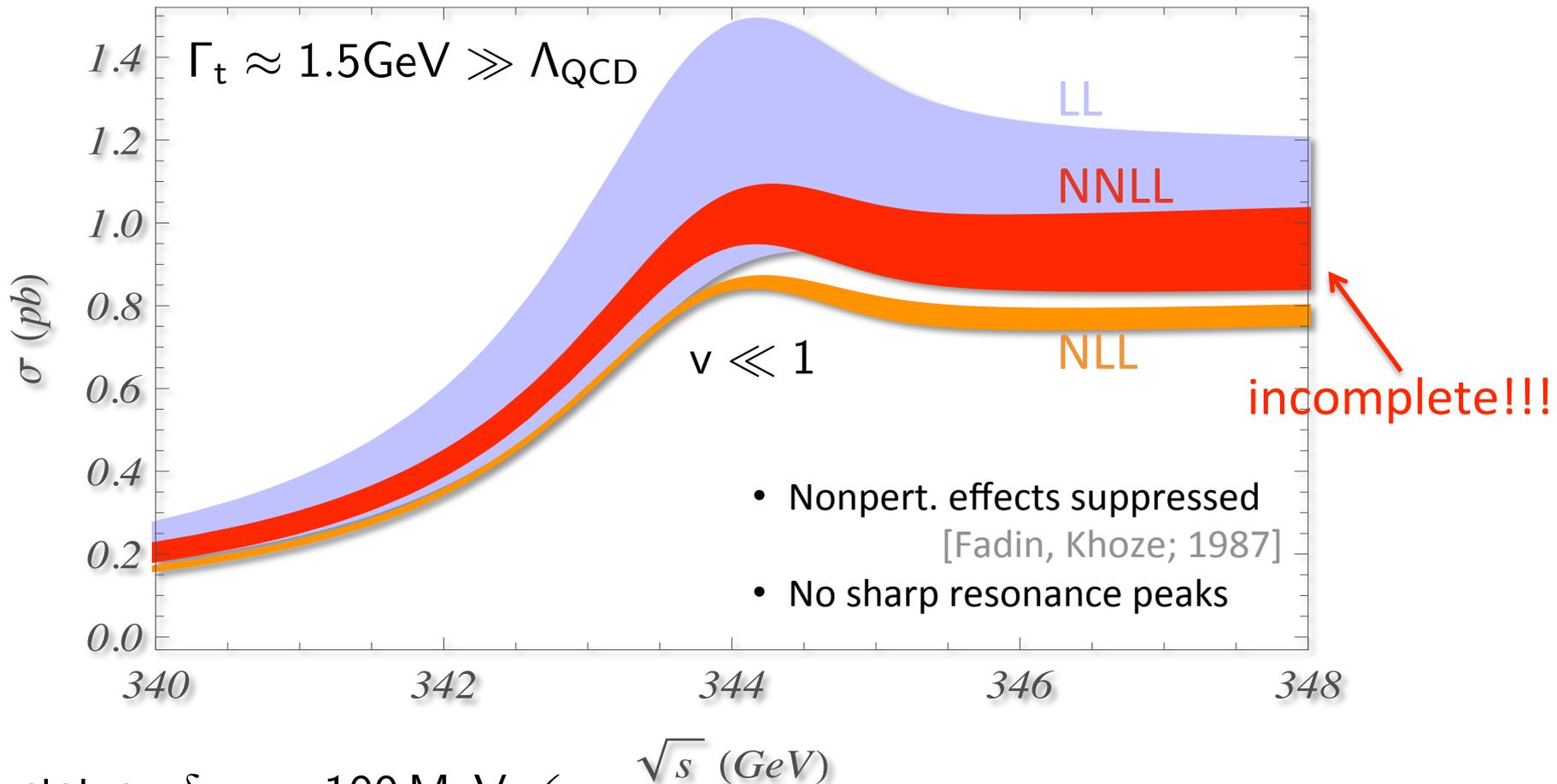
Heavy quark-antiquark threshold

$t\bar{t}$ resonance @ Linear Collider:



Heavy quark-antiquark threshold

$t\bar{t}$ resonance @ Linear Collider:



status: $\delta m_t \sim 100 \text{ MeV} \checkmark$

(theory)

$\delta \sigma_{\text{tot}} / \sigma_{\text{tot}} \sim 6\%$, < 3% needed for precise y_t , α_s , Γ_t

Heavy quark-antiquark threshold

QCD near threshold:

$$v \sim \alpha_s \ll 1$$

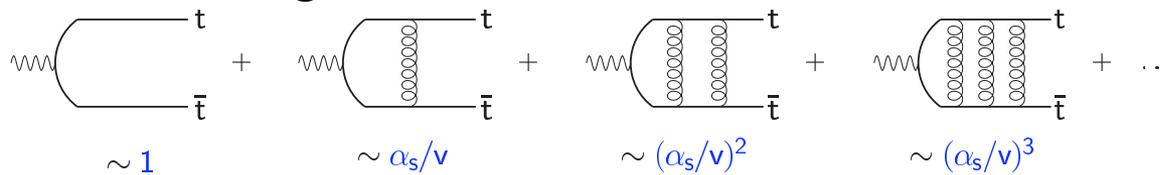
3 scales: $m_t \gg \vec{p} \sim m_t v \gg E_{\text{kin}} \sim m_t v^2$ ($\sim \Gamma_t \gg \Lambda_{\text{QCD}}$)

“hard”

“soft”

“ultrasoft”

- “Coulomb singularities”



vNRQCD

- Large logs: $\ln(E^2/m^2)$, $\ln(\mathbf{p}^2/m^2)$, $\ln(E^2/\mathbf{p}^2)$

Heavy quark-antiquark threshold

QCD near threshold:

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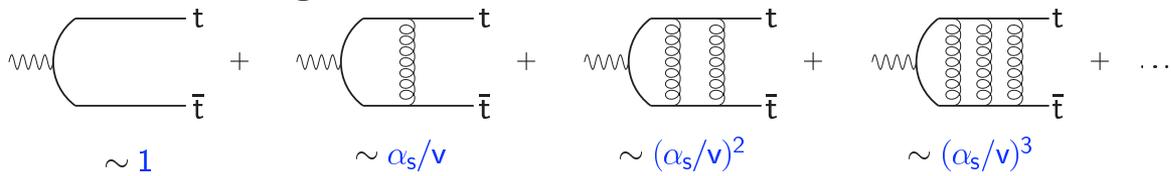
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→ **vNRQCD**

- Large logs: $\ln(E^2/m^2)$, $\ln(\mathbf{p}^2/m^2)$, $\ln(E^2/\mathbf{p}^2)$



Two RG scales:

$$\mu_s = m\nu, \quad \mu_u = m\nu^2$$

ν “subtraction velocity”

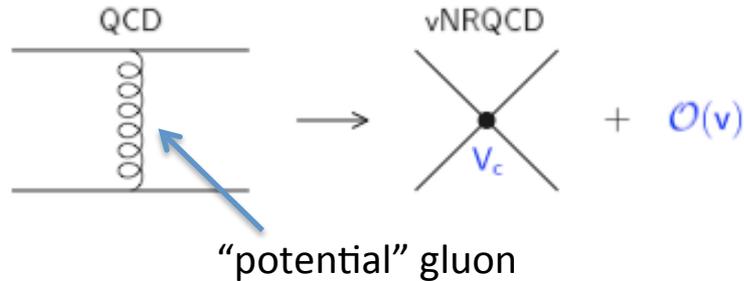
[Luke, Manohar, Rothstein; 2000]

→ RGE’s resum logs: $[\alpha_s \ln v]^n$, $\alpha_s [\alpha_s \ln v]^n$, $\alpha_s^2 [\alpha_s \ln v]^n \dots$

LL
NLL
N²LL

vNRQCD

- Nonresonant dof's integrated out, e.g.:



- Resonant dof's \rightarrow fields in the vNRQCD Lagrangian:

nonrel. quark:	$(E, \mathbf{p}) \sim (mv^2, mv)$	$\psi_{\mathbf{p}}(x)$	
soft gluon:	$(q_0, \mathbf{q}) \sim (mv, mv)$	$A_{\mathbf{q}}(x)$	
ultrasoft gluon:	$(q_0, \mathbf{q}) \sim (mv^2, mv^2)$	$A(x)$	

- Systematic expansion in $v \Rightarrow$ consistent power counting in $v \sim \alpha_s$

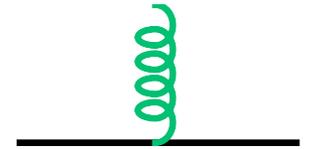
vNRQCD

[Luke, Manohar, Rothstein; 2000]

$$\mathcal{L}_{\text{vNRQCD}} = \mathcal{L}_{\text{usoft}} + \mathcal{L}_{\text{pot}} + \mathcal{L}_{\text{soft}}$$

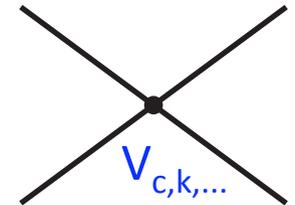
$$D^\mu = \partial^\mu + igA^\mu(x)$$

$$\mathcal{L}_{\text{usoft}} : \psi_{\mathbf{p}}^\dagger(x) \left[iD^0 - \frac{(\mathbf{p} - i\mathbf{D})^2}{2m} + \dots \right] \psi_{\mathbf{p}}(x) + \dots$$

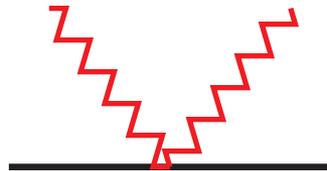


$$\mathcal{L}_{\text{pot}} : -V \psi_{\mathbf{p}'}^\dagger \psi_{\mathbf{p}} \chi_{-\mathbf{p}'}^\dagger \chi_{-\mathbf{p}} + \dots$$

$$V \sim \frac{V_c}{k^2} + \frac{V_k \pi^2}{mk} + \frac{V_r (\mathbf{p}^2 + \mathbf{p}'^2)}{2m^2 k^2} + \frac{V_2}{m^2} + \frac{V_s}{m^2} \mathbf{S}^2 + \dots$$



$$\mathcal{L}_{\text{soft}} :$$



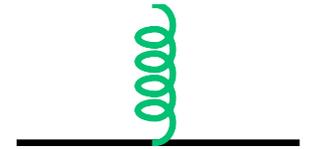
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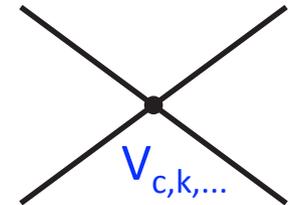
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$$\mathcal{L}_{\text{pot}} : -V \psi_{\mathbf{p}'}^\dagger \psi_{\mathbf{p}} \chi_{-\mathbf{p}'}^\dagger \chi_{-\mathbf{p}} + \dots$$

$$V \sim \frac{V_c}{k^2} + \frac{V_k \pi^2}{mk} + \frac{V_r (\mathbf{p}^2 + \mathbf{p}'^2)}{2m^2 k^2} + \frac{V_2}{m^2} + \frac{V_s}{m^2} \mathbf{S}^2 + \dots$$



Production/annihilation current (3S_1):

$$\begin{array}{c} \otimes \\ \diagup \\ \diagdown \end{array} \sim c_1(\nu) \cdot \underbrace{\vec{j}_1^{\text{eff}}(x)}_{\psi_{\mathbf{p}}^\dagger \vec{\sigma} (i\sigma_2) \chi_{-\mathbf{p}}^*} + \dots \quad (\text{CMS})$$

Renormalization

$$\begin{aligned}
 \sigma_{\text{tot}} &\sim \text{Im} \left[\text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \dots \right] \\
 &\sim |c_1(\nu)|^2 \cdot \text{Im} \left[-i \int d^4x e^{i\hat{q}x} \langle 0 | T \vec{j}_1^{\text{eff}*}(x) \vec{j}_1^{\text{eff}}(0) | 0 \rangle \right] + \dots \\
 &\sim |c_1(\nu)|^2 \cdot \text{Im} \left[G(0, 0, E, \nu) \right] + \dots
 \end{aligned}$$

$\text{LO SG: } \left[-\frac{\nabla_{\vec{r}}^2}{m} + V_c(\vec{r}) - E \right] G(\vec{r}, \vec{r}', E) = \delta^{(3)}(\vec{r} - \vec{r}')$

G^{NNLL} known ✓ [Hoang, Manohar, Stewart, Teubner; 2002]
 [Pineda, Signer; 2006]

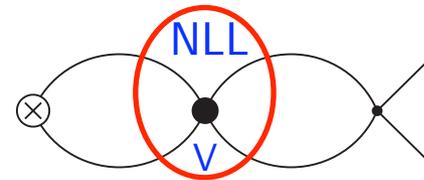
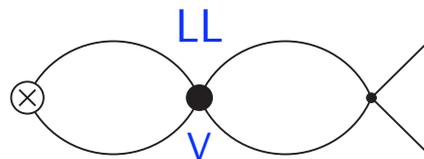
G^{NNNLO} known ✓ [Beneke, Kiyo, Schuller; 2007]

Renormalization

$$\sigma_{\text{tot}} \sim |c_1(\nu)|^2 \cdot \text{Im} [G(0, 0, E, \nu)] + \dots$$

G^{NNLL} known ✓

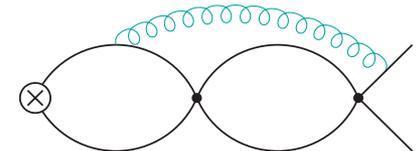
current
renormalization



$$\ln \left[\frac{c_1(\nu)}{c_1(1)} \right] = \underbrace{\xi^{\text{LL}}}_0 + \xi^{\text{NLL}} + \xi^{\text{NNLL}}_{\text{mix}} + \xi^{\text{NNLL}}_{\text{nonmix}}$$

[Luke, Manohar, Rothstein; 2000]

[Pineda; 2002] [Hoang, Stewart; 2003]



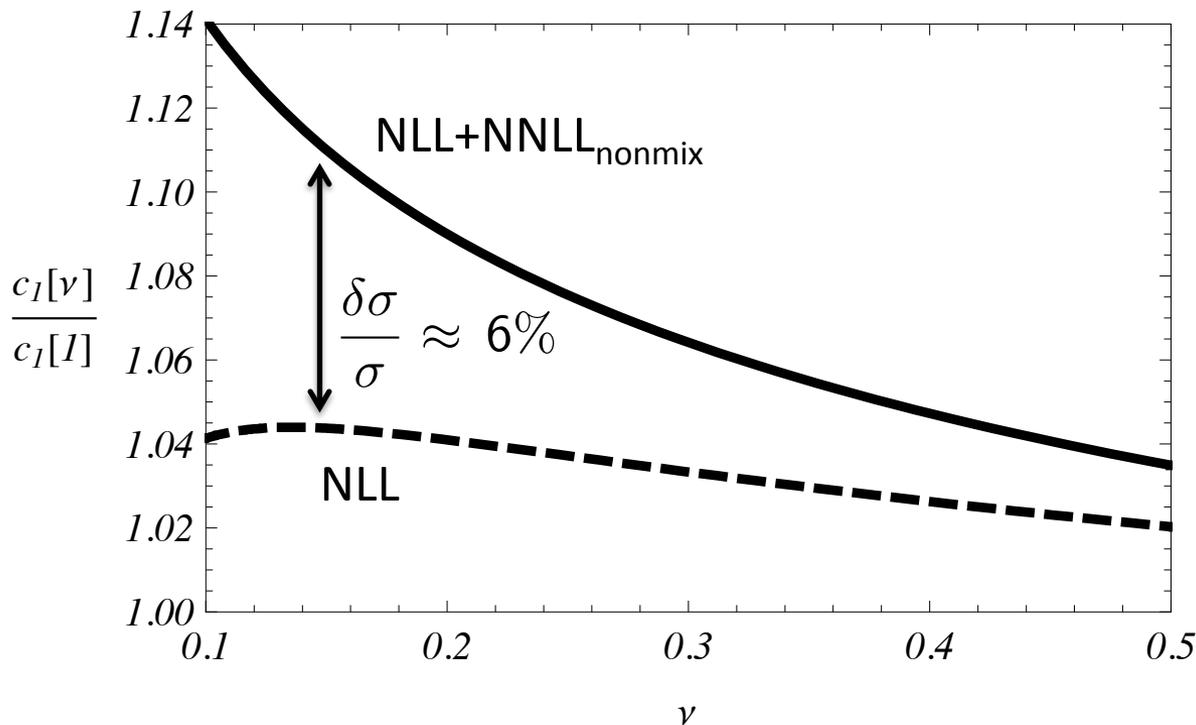
[Hoang; 2003]

missing

Renormalization

$$\sigma_{\text{tot}} \sim |c_1(\nu)|^2 \cdot \text{Im} [G(0, 0, E, \nu)] + \dots$$

G^{NLL} known ✓



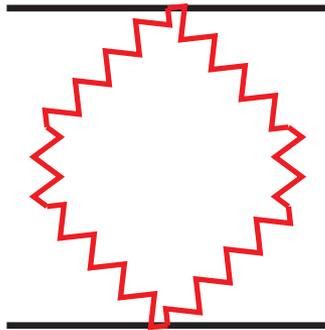
Missing NNLL_{mix} contribution may reduce th. error of σ_{tot} ?

⇒ $V^{\text{NLL}}(\nu)$ needed !

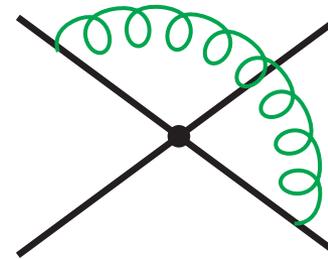
Renormalization

$$\text{NLL: } \nu \frac{\partial}{\partial \nu} \ln[c_1(\nu)] = -\frac{\mathcal{V}_c(\nu)}{16\pi^2} \left[\frac{\mathcal{V}_c(\nu)}{4} + \mathcal{V}_2(\nu) + \mathcal{V}_r(\nu) + \mathbf{S}^2 \mathcal{V}_s(\nu) \right] + \frac{1}{2} \mathcal{V}_k(\nu)$$

soft



ultrasoft

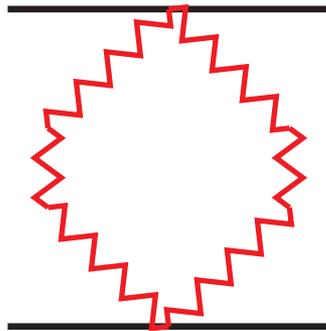


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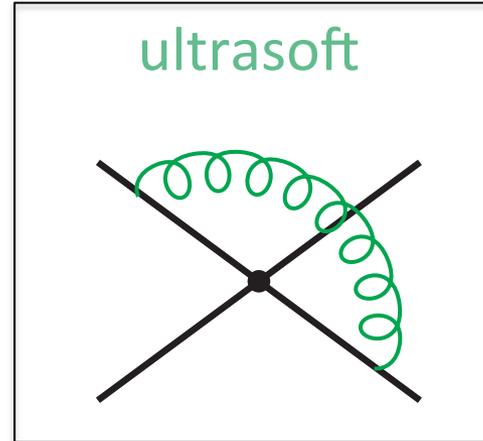


soft

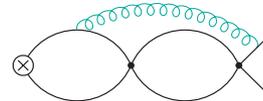


- known soft contributions small

ultrasoft



- dominant: $\alpha_s(mv^2) > \alpha_s(mv)$
- large contribution to $\xi_{\text{nonmix}}^{\text{NNLL}}$

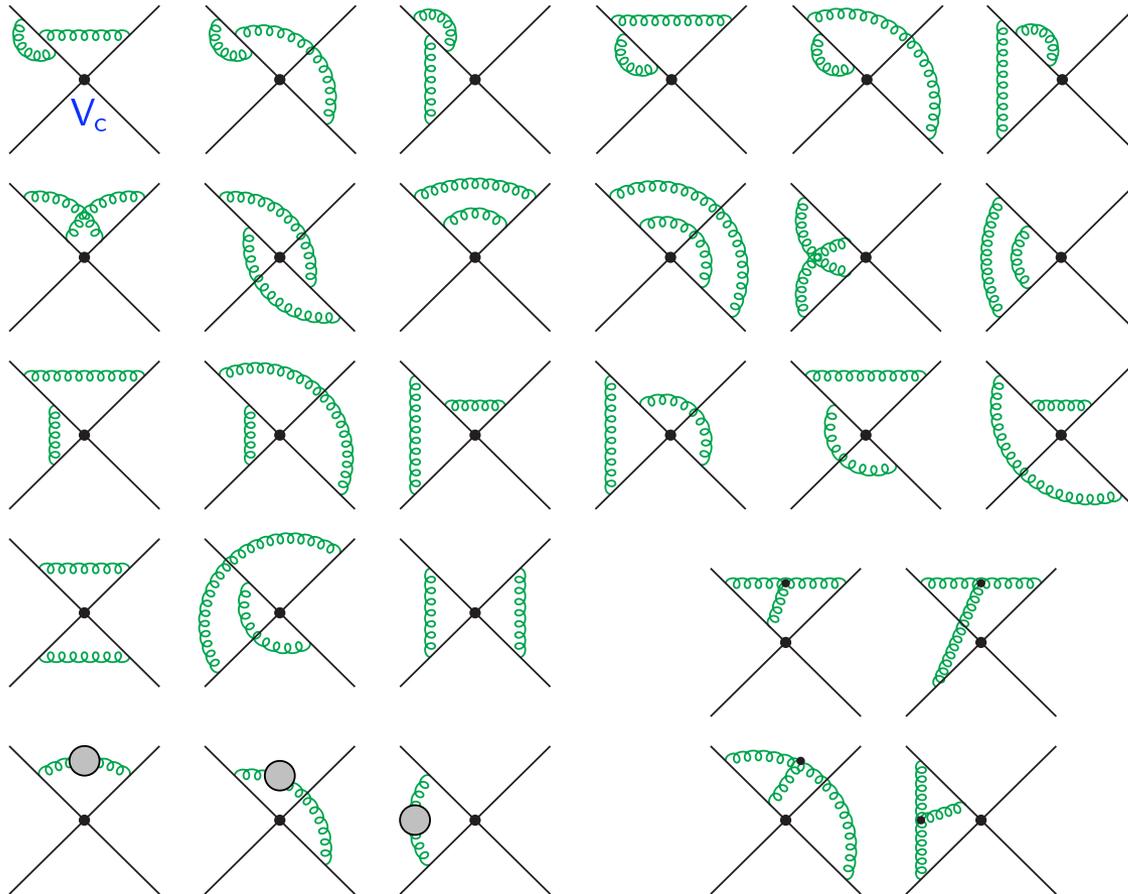


[Hoang; 2003]

Renormalization

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Two usoft loops:



renormalize

- Feynman gauge
- $\overline{\text{MS}}$, dim. reg.
- $\mathcal{O}(10^3)$ diagrams

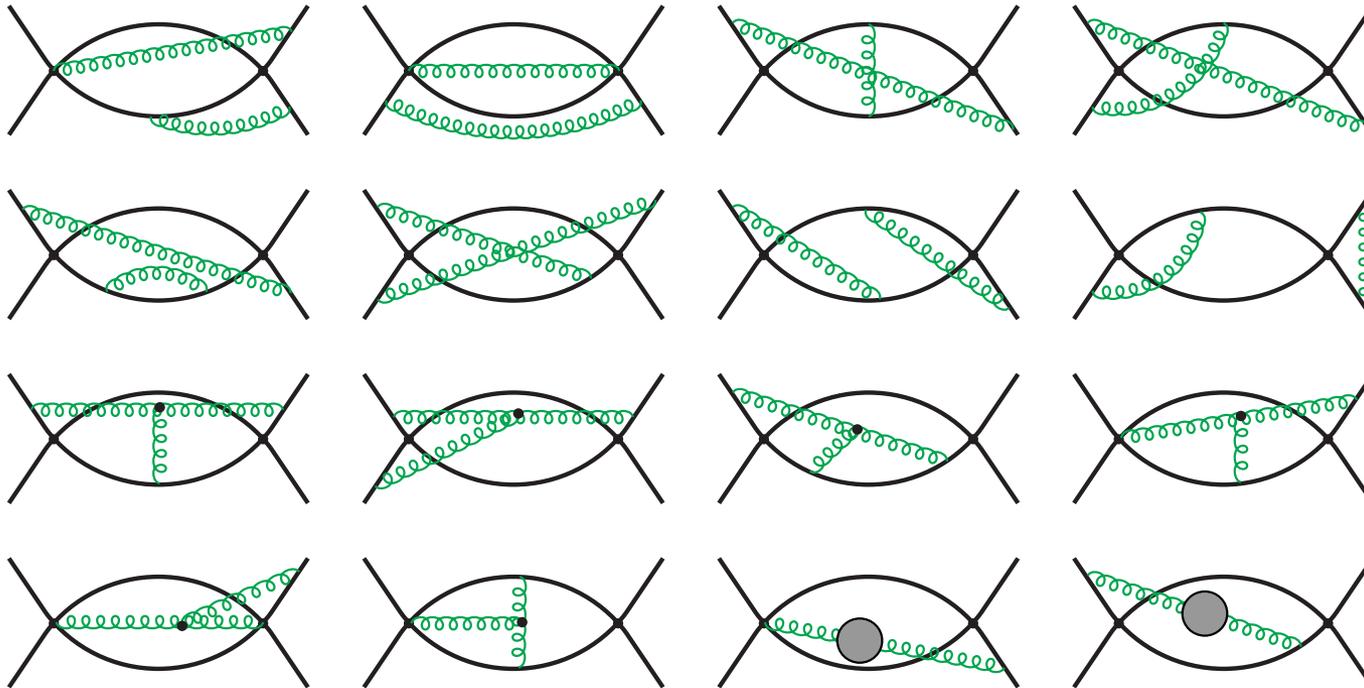
$$\Rightarrow \delta \mathcal{V}_{r,2}^{2\text{ loop}} \xrightarrow{\text{RGE}} \mathcal{V}_{r,2}^{\text{NLL}}(\nu)$$

[MS, Hoang; hep-ph/0611292]

Renormalization

$$\text{NLL: } \nu \frac{\partial}{\partial \nu} \ln[c_1(\nu)] = -\frac{\mathcal{V}_c(\nu)}{16\pi^2} \left[\frac{\mathcal{V}_c(\nu)}{4} + \mathcal{V}_2(\nu) + \mathcal{V}_r(\nu) + \mathbf{S}^2 \mathcal{V}_s(\nu) \right] + \frac{1}{2} \mathcal{V}_k(\nu)$$

Two usoft loops:



renormalize

- 3 loops:
2 x usoft
1 x potential (finite)
- Feynman gauge
- $\overline{\text{MS}}$, dim. reg.
- $O(10^4)$ diagrams
- Generation:
own **Mathematica** code
- Integrals:
IBP & partial frac.

$$\Rightarrow \boxed{\delta \mathcal{V}_k^{2\text{ loop}} \xrightarrow{\text{RGE}} \mathcal{V}_k^{\text{NLL}}(\nu)} \quad [\overline{\text{MS}}, \text{Hoang}, 1102.0269 [\text{hep-ph}]]$$

Results

RGE's + matching at hard scale ($\nu = 1$) give:

	LL	NLL
$[\mathcal{V}_2(\nu)]_{\text{usoft}}^{\text{NLL}}$	$= 4\pi\alpha_s(m\nu) \left[-\frac{4\pi}{\beta_0} A_2 \ln \frac{\alpha_s(m\nu^2)}{\alpha_s(m\nu)} + \left(\frac{\beta_1}{\beta_0^2} A_2 - [\alpha_s(m\nu^2) - \alpha_s(m\nu)] \frac{8\pi}{\beta_0} B_2 \right) \right]$	
$[\mathcal{V}_r(\nu)]_{\text{usoft}}^{\text{NLL}}$	$= 8\pi\alpha_s(m\nu) \left[-\frac{4\pi}{\beta_0} A_r \ln \frac{\alpha_s(m\nu^2)}{\alpha_s(m\nu)} + \left(\frac{\beta_1}{\beta_0^2} A_r - [\alpha_s(m\nu^2) - \alpha_s(m\nu)] \frac{8\pi}{\beta_0} B_r \right) \right]$	
$[\mathcal{V}_k(\nu)]_{\text{usoft}}^{\text{NLL}}$	$= 2\alpha_s^2(m\nu) \left[-\frac{4\pi}{\beta_0} A_k \ln \frac{\alpha_s(m\nu^2)}{\alpha_s(m\nu)} + \left(\frac{\beta_1}{\beta_0^2} A_k - [\alpha_s(m\nu^2) - \alpha_s(m\nu)] \frac{8\pi}{\beta_0} B_k \right) \right]$	

$$\begin{bmatrix} A_2 \\ B_2 \end{bmatrix} = C_F(C_A - 2C_F) \begin{bmatrix} A \\ B \end{bmatrix}$$

$$\begin{bmatrix} A_r \\ B_r \end{bmatrix} = -C_A C_F \begin{bmatrix} A \\ B \end{bmatrix}$$

$$\begin{bmatrix} A_k \\ B_k \end{bmatrix} = -C_A C_F(C_A - 2C_F) \begin{bmatrix} A \\ B \end{bmatrix}$$

$$A = \frac{1}{3\pi}$$

$$B = \frac{C_A(47 + 6\pi^2) - 10n_f T}{108\pi^2}$$

[MS, Hoang; 2011]

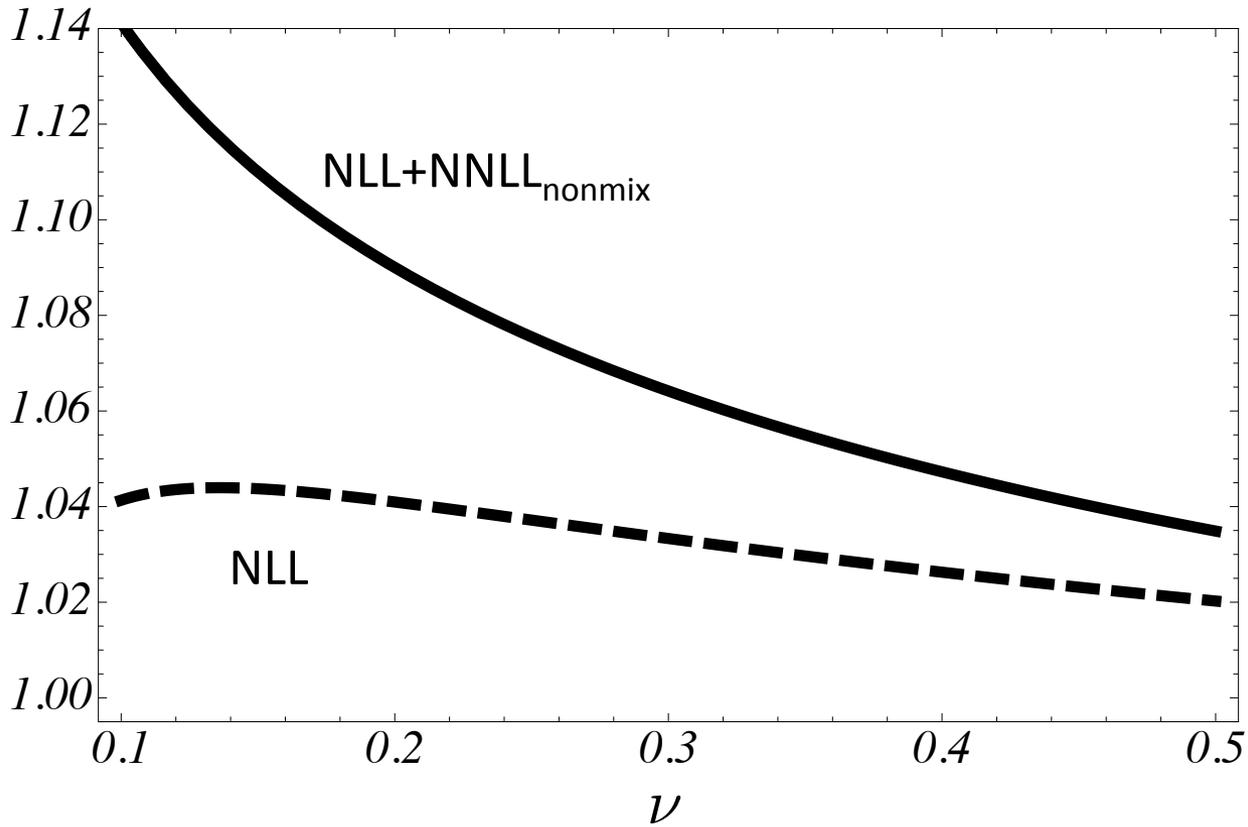
[Pineda; 2011]

→ Pineda's talk

Results

$t\bar{t}$

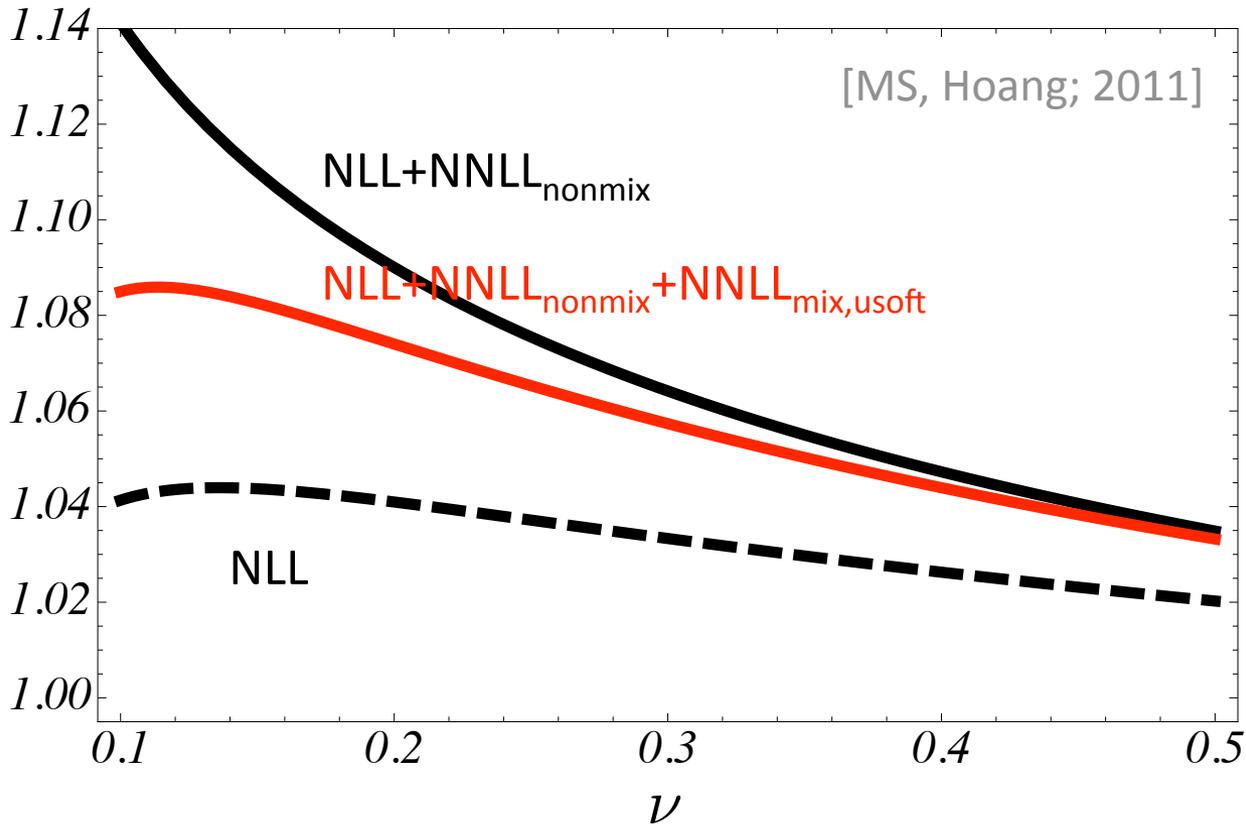
$$\frac{c_1(\nu)}{c_1(1)}$$



Results

$t\bar{t}$

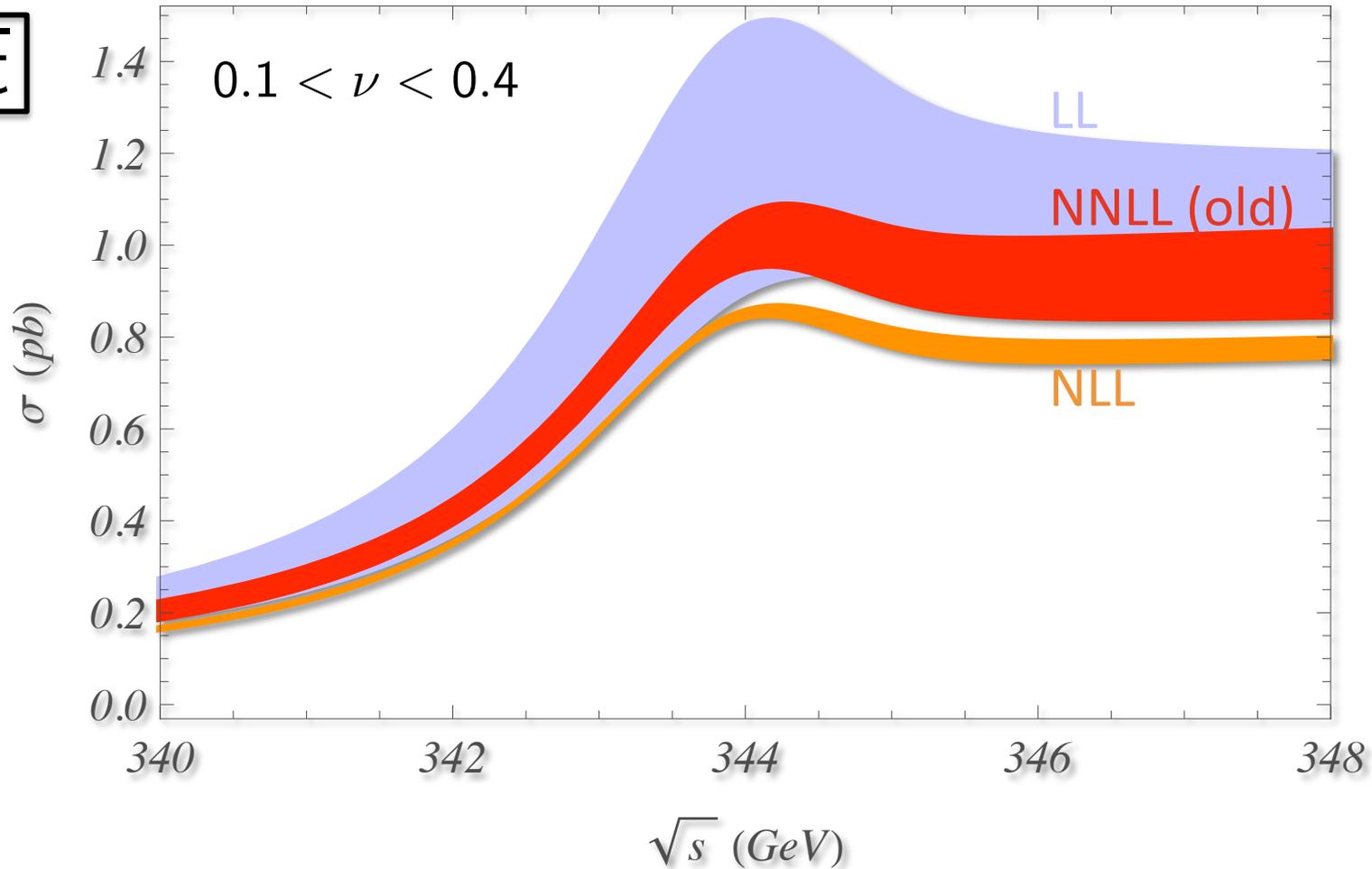
$$\frac{c_1(\nu)}{c_1(1)}$$



- large usoft NNLL contributions compensate each other!
- good convergence: $[c_1(\nu = 0.15)]^2 = 1 - 0.096|_{\text{NLL}} - 0.029|_{\text{NNLL}}$

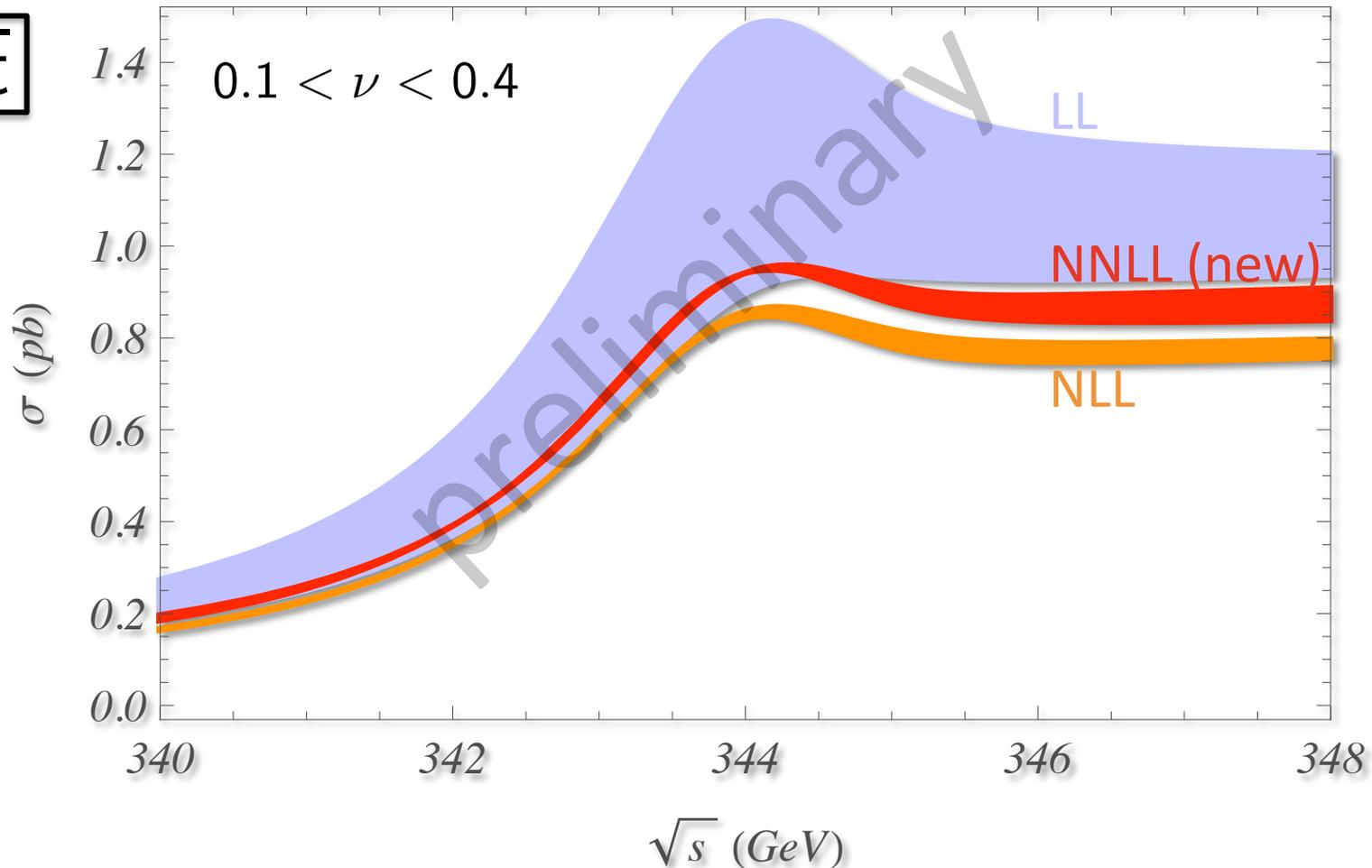
Results

$t\bar{t}$



Results

$t\bar{t}$



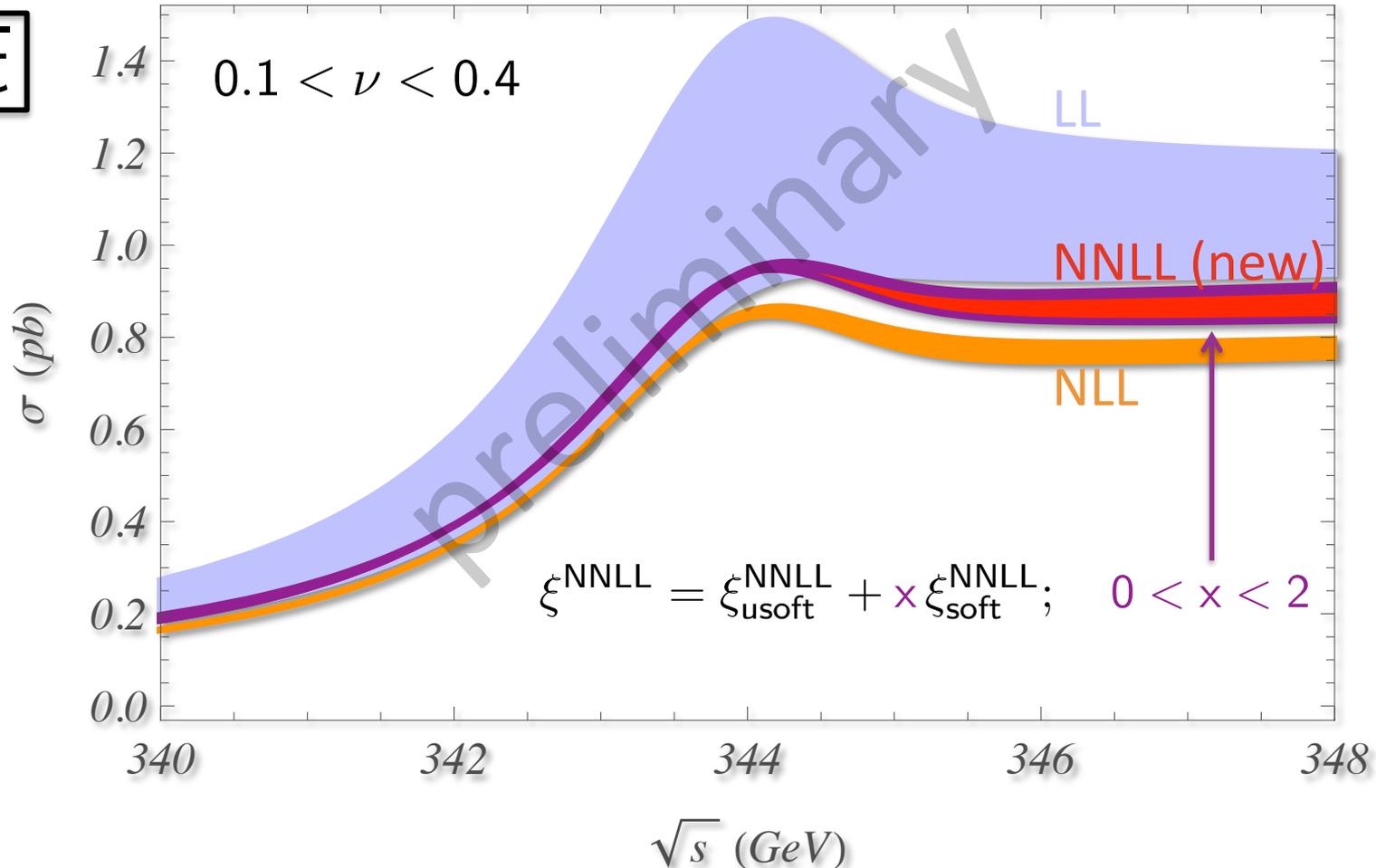
- EW contributions known up to NNLL ✓

[Hoang, Reisser, Ruiz-Femenia; 2010]

[Beneke, Jantzen, Ruiz-Femenia; 2010]

Results

$t\bar{t}$

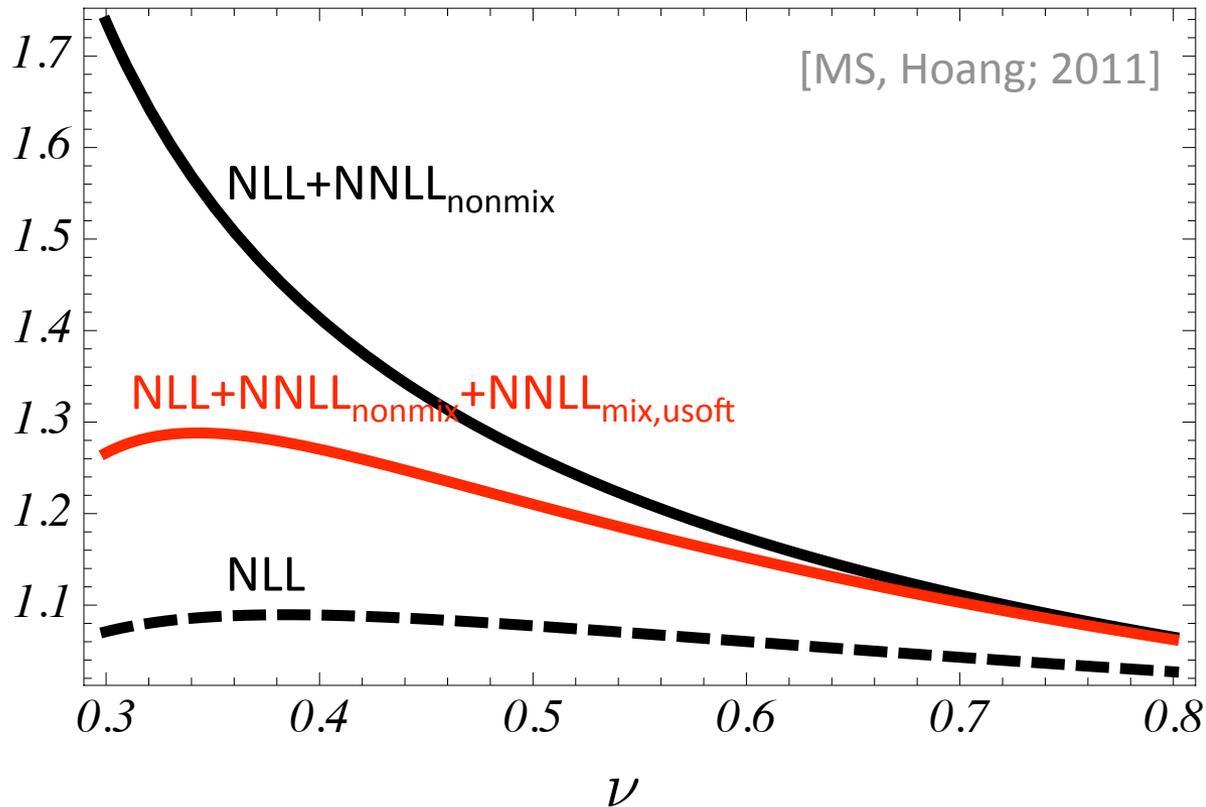


- known soft NNLL contributions small
- detailed error analysis (incl. EW effects, variation of matching scale, ...) → **WIP**

Results

$b\bar{b}$

$$\frac{c_1(\nu)}{c_1(1)}$$



- large usoft NNLL contributions compensate each other!
- poor convergence: $[c_1(\nu=0.35)]^2 = 1 - 0.21|_{\text{NLL}} - 0.18|_{\text{NNLL}}$

Results

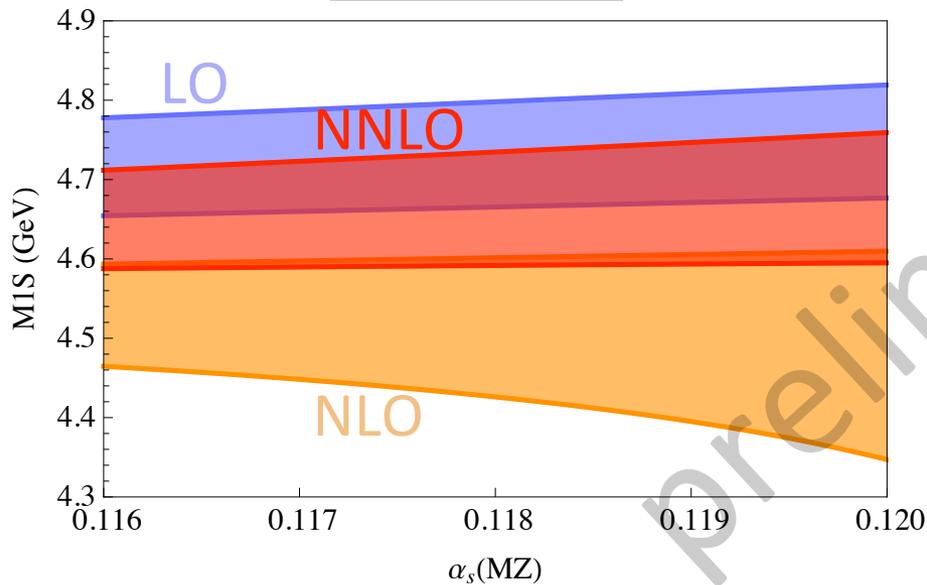
$b\bar{b}$

$$M_n = \int_0^\infty \frac{ds}{s^{n+1}} R_{b\bar{b}}(s)$$

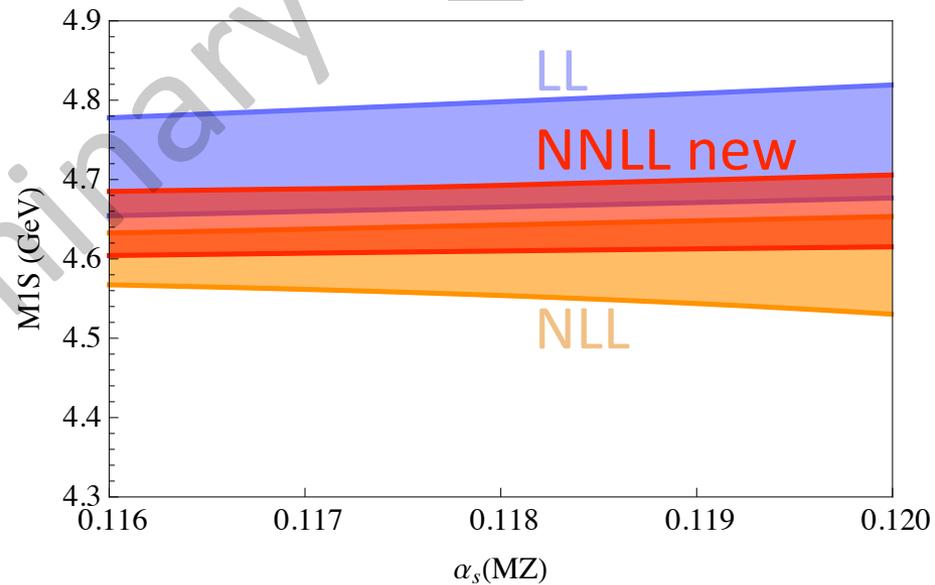
$$v \sim \frac{1}{\sqrt{n}}$$

$n=10$

“Fixed order”



RGI



- fit to single exp. moment from Υ masses and widths + const. continuum

- scales: $\mu_{\text{hard}} = hm$, $\mu_{\text{soft}} = hm\nu$, $\mu_{\text{usoft}} = hm\nu^2$

$$0.3 < \nu < 0.8$$

$$h = 1$$

Results

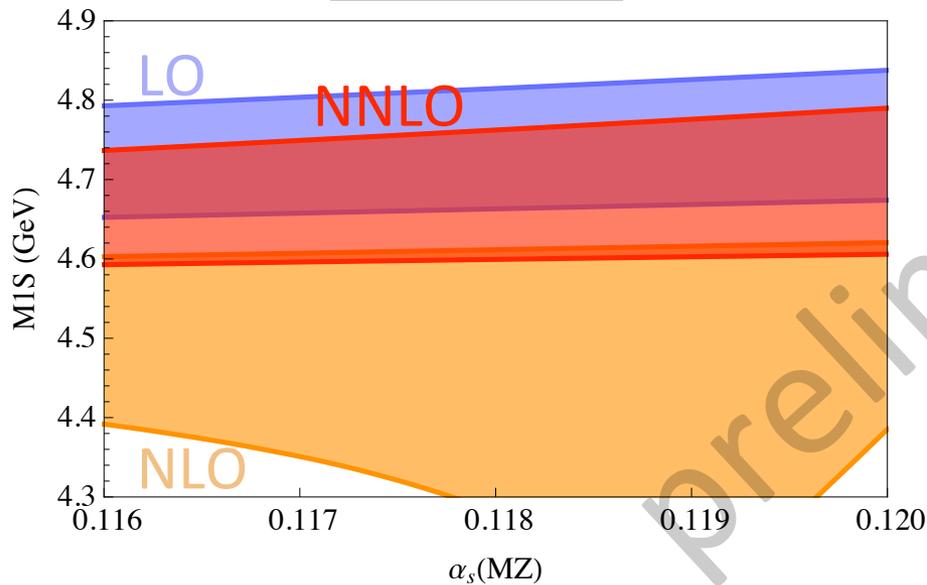
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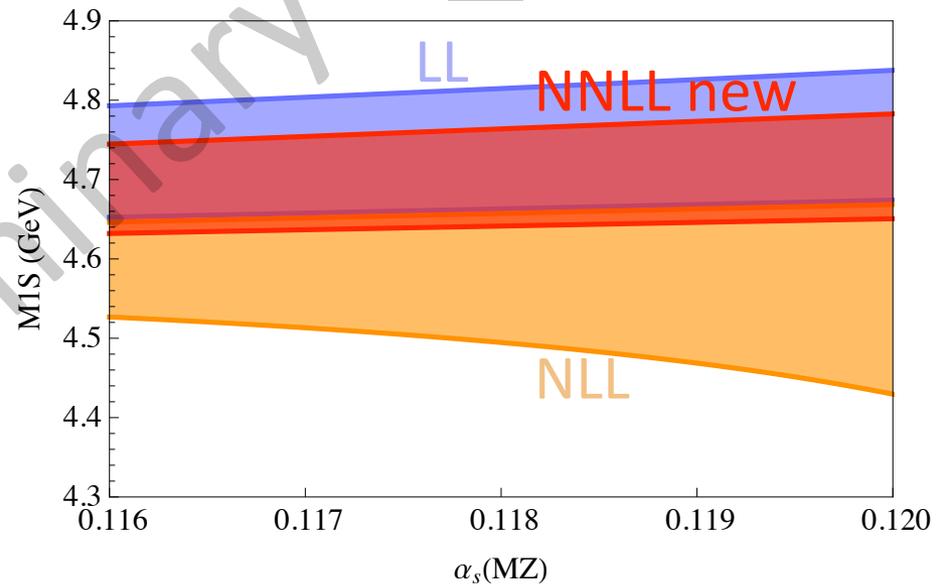
$$\nu \sim \frac{1}{\sqrt{n}}$$

$n=10$

“Fixed order”



RGI



$$\mu_{\text{hard}} = hm, \quad \mu_{\text{soft}} = hm\nu, \quad \mu_{\text{usoft}} = hm\nu^2$$

$$\nu = 0.38$$

$$0.75 < h < 2$$

$$\rightarrow M_b^{1S} = 4.66 \pm 0.7 \text{ GeV} \quad \leftarrow \text{WIP}$$

Summary/Outlook

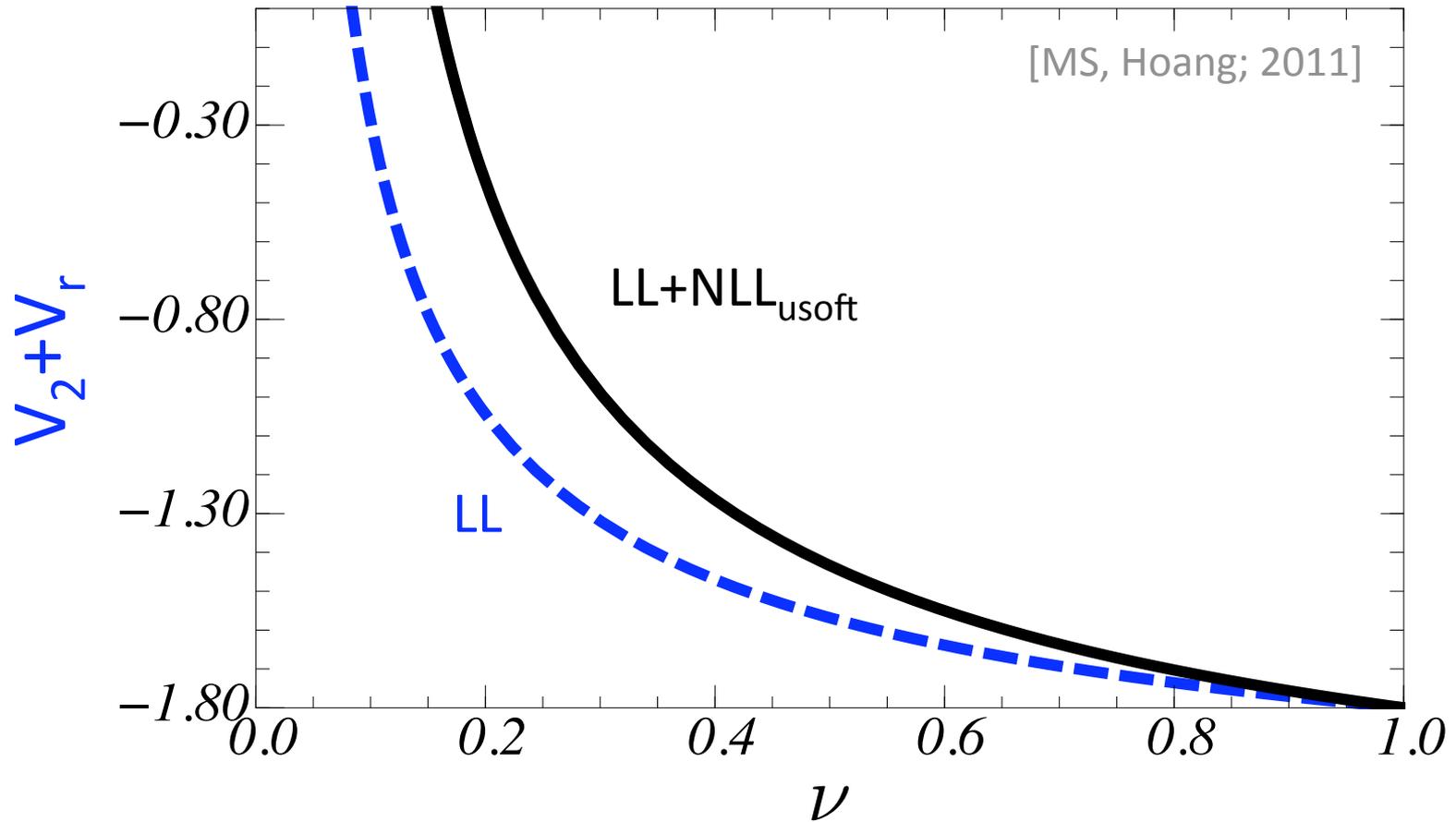
- precise $m_t, y_t, \alpha_s, \Gamma_t$ and m_b from $t\bar{t}$ and $b\bar{b}$ threshold physics
- $\sigma_{\text{tot}} \sim |c_1(\nu)|^2 \cdot \text{Im} [G(0, 0, E, \nu)] + \dots$
- $G(0, 0, E, \nu)$ known up to NNLL ✓ EW contributions up to NNLL ✓
- **New** NNLL_{mix,usoft} compensates for large NNLL_{nonmix} contribution to $c_1(\nu)$

→ $\frac{\delta\sigma_{\text{tot}}^{\text{th}}}{\sigma_{\text{tot}}}$ decreases substantially! → RG Improvement important!

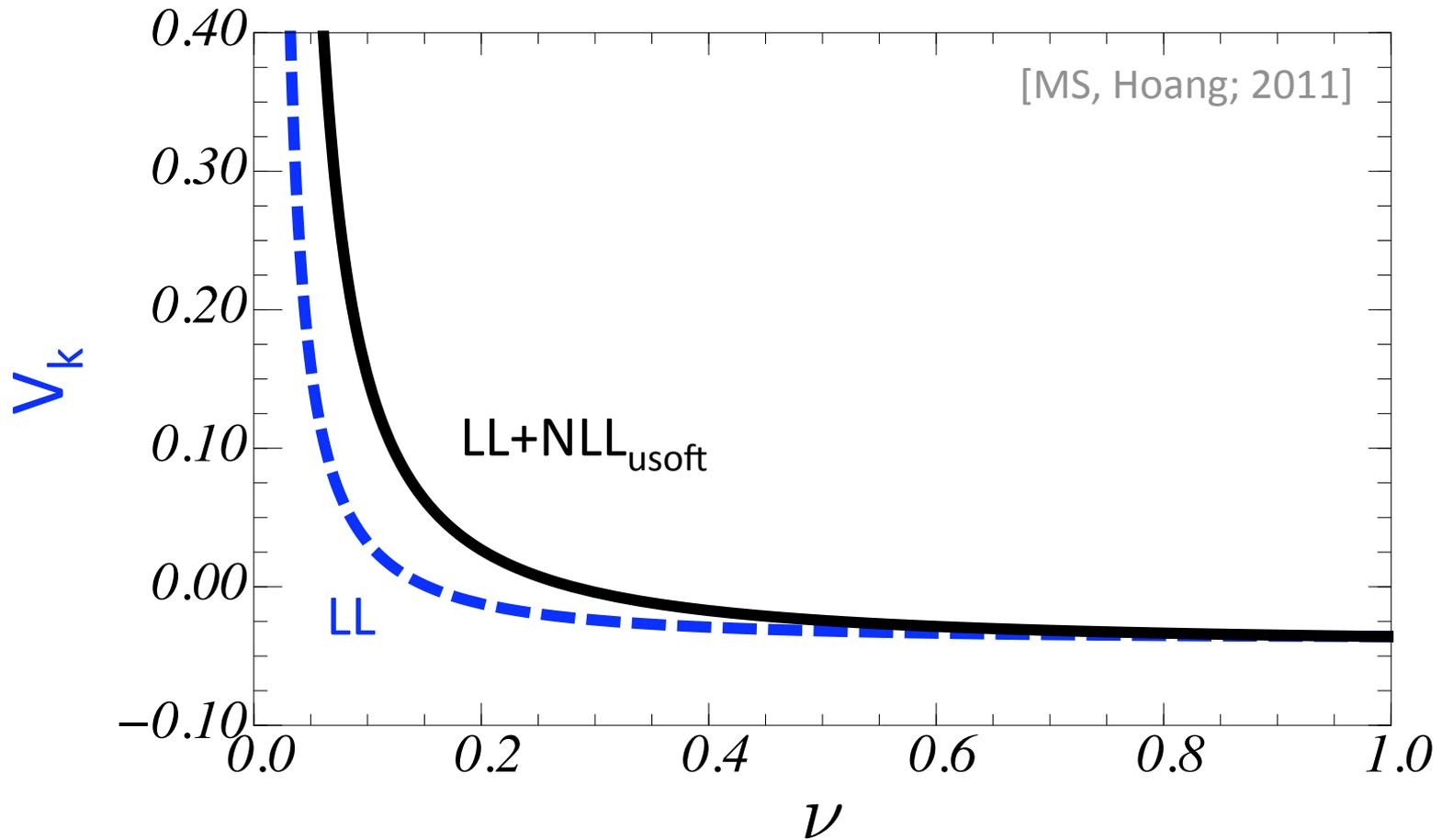
- Outlook:
 - Detailed error analysis for $\sigma_{\text{tot}}(e^+e^- \rightarrow t\bar{t})$ at threshold
 - Determination of bottom mass from nonrel. Υ sum rules

WIP

Backup



Backup



Backup

EW effects:

- LO: Unstable top $\Rightarrow \Gamma_t \approx 1.5 \text{ GeV} \gg \Lambda_{\text{QCD}}$

“IR cutoff”

$$v_{\text{eff}} \equiv \sqrt{\frac{\sqrt{s} - 2m_t}{m_t}} \rightarrow \sqrt{\frac{\sqrt{s} - 2m_t + i\Gamma_t}{m_t}} ;$$

$$|v_{\text{eff}}| \gtrsim 0.1$$

[Fadin, Khoze; 1987]

- Higher orders up to NNLL [Hoang, Reisser, Ruiz-Femenia; 2010]
[Beneke, Jantzen Ruiz-Femenia; 2010]

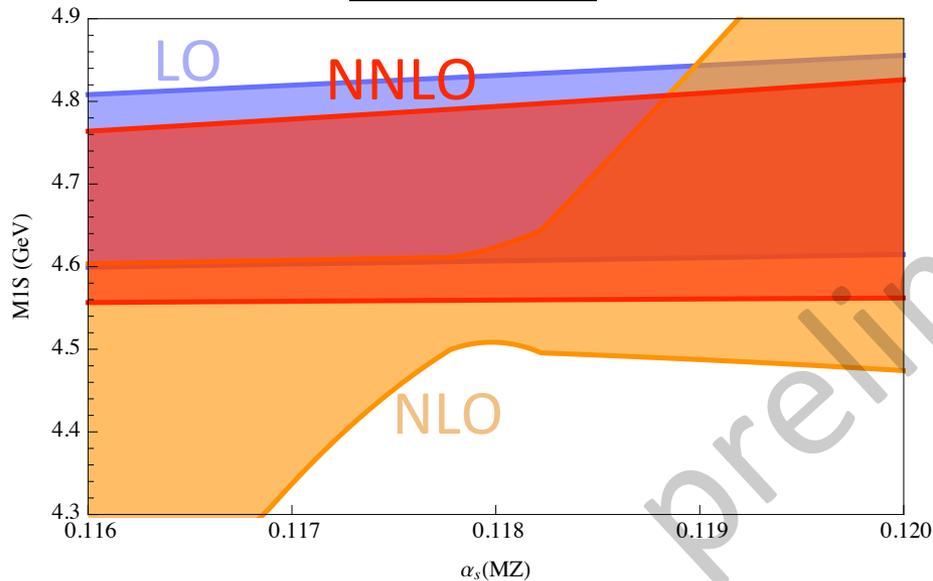
Backup

$b\bar{b}$

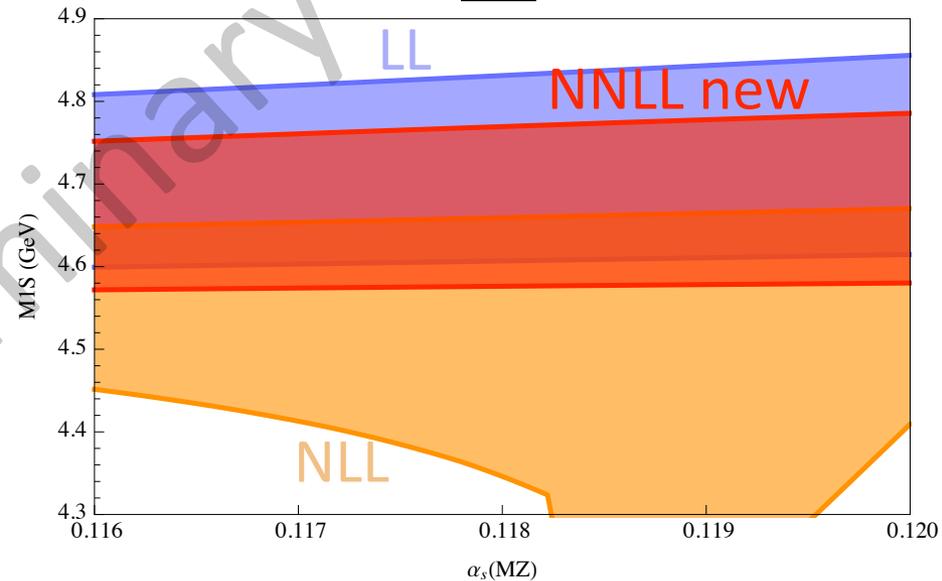
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Fixed order



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- scales: $\mu_{\text{hard}} = hm$, $\mu_{\text{soft}} = hm\nu$, $\mu_{\text{usoft}} = hm\nu^2$

$$0.35 < \nu < 0.75$$

$$0.75 < h < 2$$