Quarkonium Physics in CASCADE

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PLAN OF THE TALK

1. **Preface on k_t-factorization**
2. **Implementing the Quarkonium physics**
3. **Numerical results**
4. **Conclusions**

1. PREFACE ON k_t-FACTORIZATION

1.1 Parton Off-Shellness

QED

Weizsäcker-Williams approximation (collinear on-shell photons)

$$F_\gamma(x) = \frac{\alpha}{2\pi}\left[1 + (1-x^2)\right]\log\frac{s}{4m^2}$$

Equivalent Photon approximation

$$F_\gamma(x, Q^2) = \frac{\alpha}{2\pi}\frac{1}{Q^2}\left[1 + (1-x^2)\right]$$

$$Q^2 \approx k_t^2/(1-x)$$

Photon spin density matrix

$$L^{\mu\nu} \approx p^\mu p^\nu$$

use $k = xp + k_t$, then do gauge shift $\epsilon \to \epsilon - k/x$

QCD

Conventional Parton Model (collinear gluon density)

$$x\,G(x, \mu^2)$$

Unintegrated gluon density

$$\mathcal{F}(x, k_t^2, \mu^2)$$

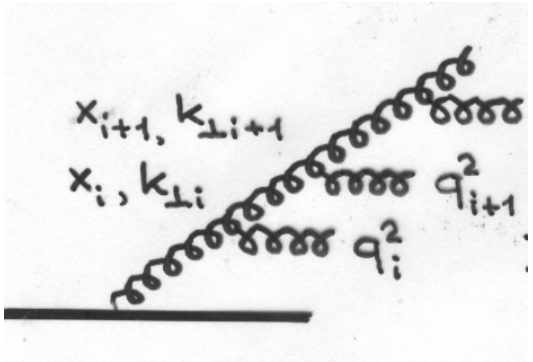
$$\int \mathcal{F}(x, k_t^2, \mu^2) dk_t^2 = x\,G(x, \mu^2)$$

Gluon spin density matrix

$$\epsilon^\mu \epsilon^{\nu *} = k_t^\mu k_t^\nu / |k_T|^2$$

so called nonsense polarization with longitudinal components

1.2 Initial State Radiation Cascade



Every elementary emission gives $\alpha_s \cdot 1/x \cdot 1/q^2$
x **= longitudinal momentum fraction**
q^2 **= gluon virtuality**

Integration over the phase space yields
$\alpha_s \cdot \ln x \cdot \ln q^2$

Random walk in the k_T**-plane:** ... $\langle k_{T\,i-1} \rangle < \langle k_{T\,i} \rangle < \langle k_{T\,i+1} \rangle$ ...

Technical method of $\alpha_s^n [\ln(1/x)]^n$ **resummation: the integral equations, BFKL**

E.A. Kuraev, L.N. Lipatov, V.S. Fadin, Sov. Phys. JETP 45, 199 (1977);
Ya. Balitsky, L.N. Lipatov, Sov. J. Nucl. Phys. 28, 822 (1978);

or CCFM

S.Catani, F.Fiorani, G.Marchesini, Phys.Lett.B 234, 339 (1990); Nucl.Phys. B336, 18 (1990);
G.Marchesini, Nucl.Phys. B445, 49 (1995); M.Ciafaloni, Nucl.Phys. B296, 49 (1998);

CCFM is more convenient for programming because of strict angular ordering: $...\theta_{i-1} < \theta_{i-1} < \theta_{i+1}...$

1.3 CCFM in the Backward Evolution Scheme

CCFM equation

$$\frac{d}{d\ln(\bar{q}^2)}\,\frac{x\mathcal{F}(x,k_t,\bar{q})}{\Delta_s(\bar{q},Q_0)} = \int \frac{\bar{P}(z,\bar{q}/z,k_t)}{\Delta_s(\bar{q},Q_0)}\,x'\mathcal{F}(x',k_t,\bar{q})\,\frac{d\phi}{2\pi}dz, \qquad z = x/x'$$

Sudakov form-factor

$$\Delta_s(\bar{q},Q_0) = \exp\left\{\int\limits_{Q_0^2}^{\bar{q}^2}\frac{dq^2}{q^2}\int\limits_0^{1-Q_0/q}\frac{\bar{\alpha}_s(q^2(1-z)^2)}{1-z}\,dz\right\}, \qquad \bar{\alpha}_s = \frac{3\alpha_s}{\pi}$$

CCFM splitting kernel

$$\bar{P}_g(z,q,k_t) = \frac{\bar{\alpha}_s(q^2(1-z)^2)}{1-z} + \frac{\bar{\alpha}_s(k_t^2)}{z}\Delta_{ns}(z,q^2,k_t^2)$$

Non-Sudakov form-factor

$$\Delta_{ns} = \exp\left\{-\bar{\alpha}_s(k_t^2)\int\limits_0^1\frac{dz'}{z'}\int\frac{dq^2}{q^2}\Theta(k_t-q)\Theta(q-z'q_t)\right\}, \qquad z_{i-1}q_{i-1} < q_i$$

CASCADE manual: H.Jung et al., Eur. Phys. J. C 70, 1237 (2010)

CASCADE is a Monte-Carlo generator using the CCFM equation for the evolution of parton densities (in the backward evolution scheme). On the technical side, similar to other generators.

Point of importance: using the k_T-, not the collinear factorization.

In the collinear scheme, the evolution is only used to calculate the parton densities and has no effect on the hard interaction subprocess.

In the k_t-factorization, the parton evolution changes the character of the hard interaction: both the kinematics (due to the initial parton transverse momentum) and polarization properties (longitudinal component for the off-shell gluons).

The evolution cascade is part of the hard interaction. By means of the evolution equation we resum a subset of Feynman diagrams (up to infinitely high order) representing higher-order contributions: i.e., the ladder diagrams enhanced with $\alpha_s^n[\ln(1/x)]^n$.

THE BENEFIT:

With the LO matrix elements for the hard subprocess we get access to effects requiring complicated next-to-leading order calculations in the collinear scheme. Many important results have been obtained in the k_t-factorization much earlier than in the collinear case.

Upon including more NLO, NNLO,.. corrections, the collinear results become closer to the k_t-factorization predictions.

EXAMPLES:

– **Azimuthal correlations in open Heavy Flavor production;**

– p_t **dependence of the** J/ψ **and** Υ **cross sections** ($1/p_t^8$ **versus** $1/p_t^4$)

– J/ψ **and** Υ **spin alignement (transverse versus longitudinal)**

Now concentrate on the Quarkonium physics, see below

2. IMPLEMENTING THE QUARKONIUM PHYSICS

2.1 Color-Singlet Gluon-Gluon Fusion

Perturbative production of a heavy quark pair within QCD;
Gluon polarization vectors: $\epsilon_g^\mu = k_T^\mu / |k_T|$

E.A. Kuraev, L.N. Lipatov, V.S. Fadin, Sov. Phys. JETP 45, 199 (1977);
Ya. Balitsky, L.N. Lipatov, Sov. J. Nucl. Phys. 28, 822 (1978);
L.V. Gribov, E.M. Levin, M. G. Ryskin, Phys. Rep. 100, 1 (1983).

Spin projection operators to guarantee the proper quantum numbers:

for Spin-triplet states $\mathcal{P}(^3S_1) = \not{\epsilon}_V(\not{p}_Q + m_Q)/(2m_Q)$

for Spin-singlet states $\mathcal{P}(^1S_0) = \gamma_5(\not{p}_Q + m_Q)/(2m_Q)$

Probability to form a bound state is determined by the wave function:

for S**-wave states** $|R_S(0)|^2$ **is known from leptonic decay widths;**

for P**-wave states** $|R'_P(0)|^2$ **is taken from potential models.**

E. J. Eichten, C. Quigg, Phys. Rev. D 52, 1726 (1995)

If $L \neq 0$ **and** $S \neq 0$ **we use the Clebsch-Gordan coefficients to reexpress the** $|L, S\rangle$ **states in terms of** $|J, J_z\rangle$ **states, namely, the** χ_0, χ_1, χ_2 **mesons.**

2.2 Accessing the Polarization of vector mesons

Polarization is measured via the angular distributions of the decay products. The most general form for $V \to \mu^+\mu^-$:

$$\frac{d\sigma}{d\cos\theta\, d\phi} \propto 1 + \lambda \cos^2\theta + \mu \sin 2\theta\ \cos\phi + \frac{\nu}{2}\sin^2\theta\ \cos 2\phi$$

Four conventional frame definitions:

– **Recoil** $\vec{z} = -\vec{p}_1 - \vec{p}_2$

– **Gottfried-Jackson** $\vec{z} = \vec{p}_1$

– **Target** $\vec{z} = -\vec{p}_2$

– **Collins-Soper** $\vec{z} = \vec{p}_1/|p_1| - \vec{p}_2/|p_2|$

and always $\vec{y} = [\vec{p}_1 \times \vec{p}_2]$, $\vec{x} = [\vec{y} \times \vec{z}]$

Vector meson $(V = J/\psi,\ \psi',\ \Upsilon,\ \Upsilon',\ \Upsilon'')$ **spin density matrix:**

$$\epsilon_V^\mu \epsilon_V^{*\nu} = 3(l_1^\mu l_2^\nu + l_2^\mu l_1^\nu - m_V^2 g^{\mu\nu}/2)/m_V^2$$

Equivalent to $-g^{\mu\nu} + p_V^\mu p_V^\nu / m_V^2$ **but gives access to the decay variables. Mode d'emloi: generate MC events including decays and apply a three-parametric fit.**

2.3 Feed-down from P-wave states

Assuming the dominance of electric dipole transitions, we have:

Angular distributions in the polarized χ_J decays

$$d\Gamma(\chi_1 \to V\gamma)/d\cos\theta \propto \left[\left(1+\tfrac{1}{2}\rho\right)+\left(1-\tfrac{3}{2}\rho\right)\cos^2\theta\right]$$
$$d\Gamma(\chi_2 \to V\gamma)/d\cos\theta \propto \left[\left(\tfrac{5}{6}-\tfrac{1}{12}\xi-\tfrac{1}{3}\tau\right)-\left(\tfrac{1}{2}-\tfrac{1}{4}\xi-\tau\right)\cos^2\theta\right]$$

where $\rho = d\sigma_{\chi_1(|h|=1)}/d\sigma_{\chi_1}, \quad \xi = d\sigma_{\chi_2(|h|=1)}/d\sigma_{\chi_2}, \quad \tau = d\sigma_{\chi_2(|h|=2)}/d\sigma_{\chi_2}$
(all known from the χ_J production matrix elements)

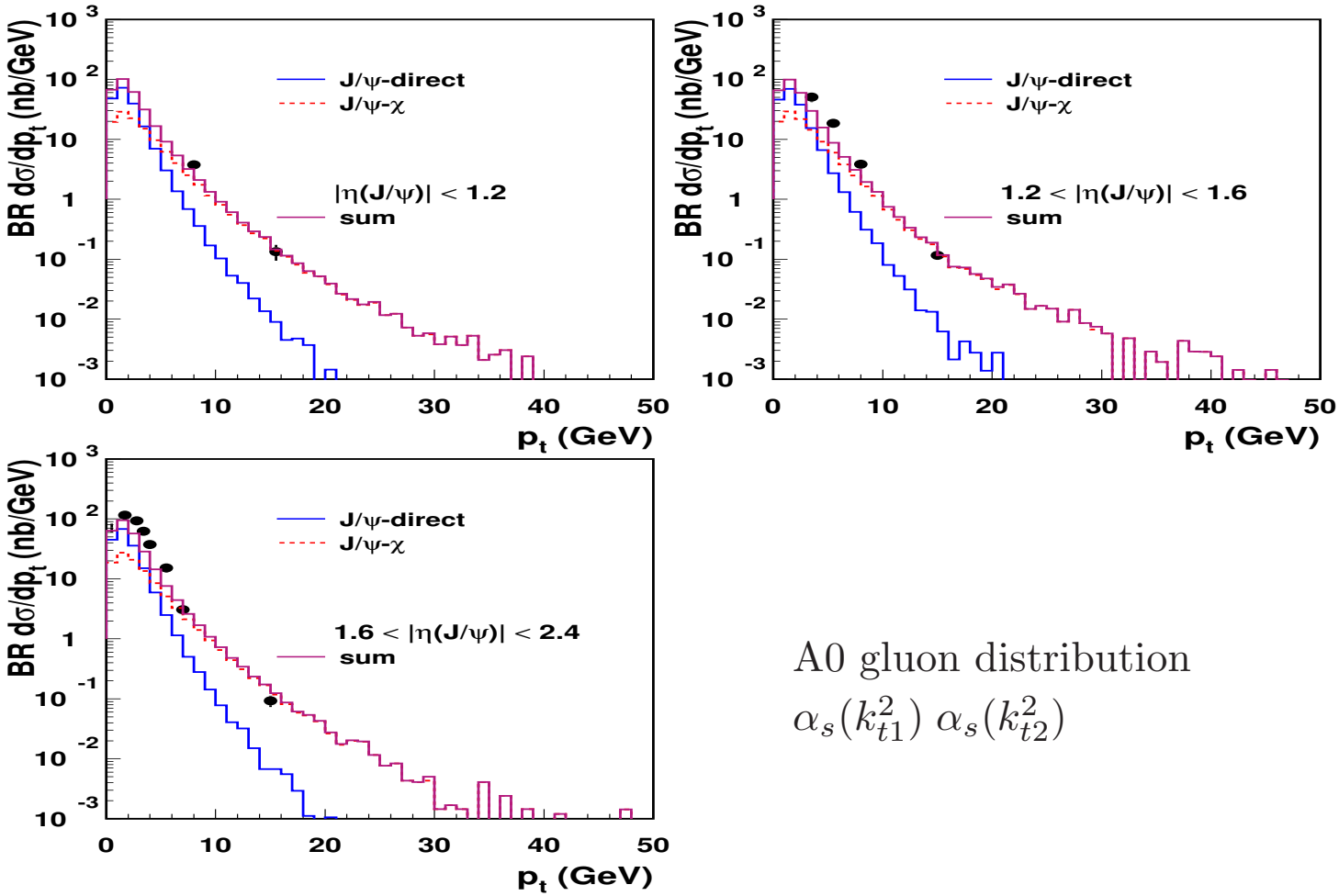
Polarization of the decay products

$$\begin{aligned}
\sigma_{V(h=0)} &= B(\chi_1 \to V\gamma)\left[(1/2)\,\sigma_{\chi_1(|h|=1)}\right] \\
&+ B(\chi_2 \to V\gamma)\left[(2/3)\,\sigma_{\chi_2(h=0)} + (1/2)\,\sigma_{\chi_2(|h|=1)}\right] \\
\sigma_{V(|h|=1)} &= B(\chi_1 \to V\gamma)\left[\sigma_{\chi_1(h=0)} + (1/2)\,\sigma_{\chi_1(|h|=1)}\right] \\
&+ B(\chi_2 \to V\gamma)\left[(1/3)\,\sigma_{\chi_2(h=0)} + (1/2)\,\sigma_{\chi_2(|h|=1)} + \sigma_{\chi_2(|h|=2)}\right].
\end{aligned}$$

P.Cho, M.Wise, S.Trivedi, Phys. Rev. D 51, R2039 (1995)

3. NUMERICAL RESULTS

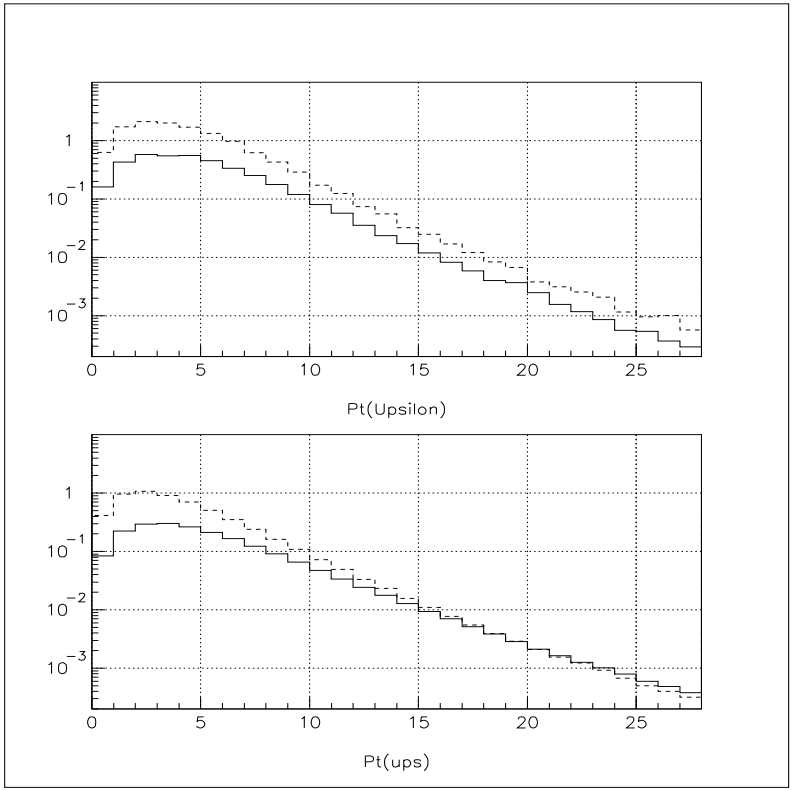
Comparison with LHC data on the J/ψ production

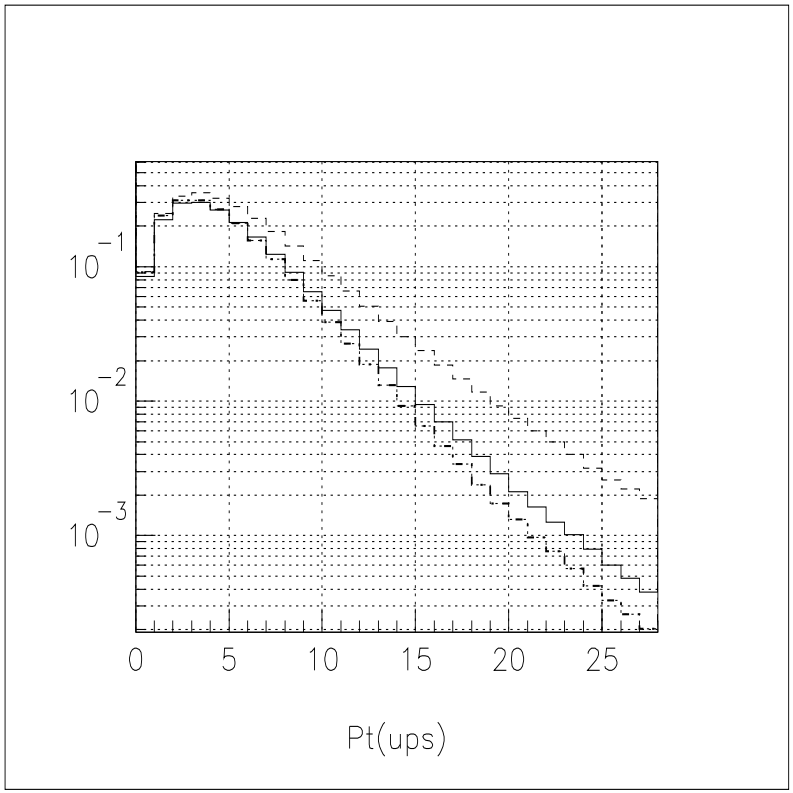


A0 gluon distribution

$\alpha_s(k_{t1}^2)\ \alpha_s(k_{t2}^2)$

More on theoretical uncertainties

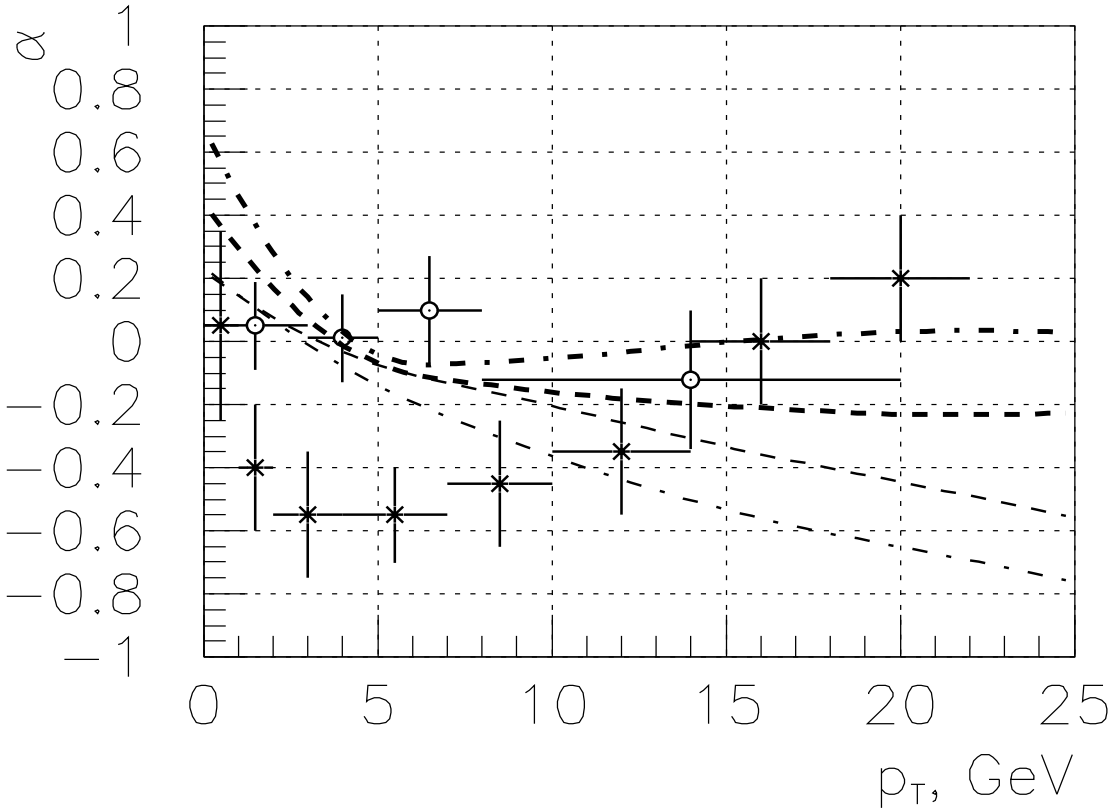




Effect of the scale in the $\alpha_s(\mu^2)$:
Upper (dashed) lines – $\mu^2 = k_t^2$;
lower (solid) lines – $\mu^2 = p_t^2 + m^2$
Upper panel – Υ, lower panel – χ_b

Effect of the flux definition:
Solid lines – $1/\lambda^{1/2}(\hat{s}, k_{t1}^2, k_{t2}^2)$
dashed lines – $1/\hat{s}$
thick dash-dotted – $1/(p_t^2 + m^2)$

$\Upsilon(1S)$ Spin alignement at the TEVATRON



Dash-dotted lines – JB gluons; dashed – dGRV gluons;
Thin lines – direct Υ only; thick lines – with χ_b decays added.
$\circ$ D.Acosta et al.(CDF), Phys. Rev. Lett. **88**, 161802 (2002);
$\times$ V.M.Abazov et al.(DO), Phys. Rev. Lett. **101**, 182004 (2008)

4. CONCLUSIONS

For the Quarkonium Physics tasks CASCADE is equipped with:

– **Off-shell** $g^* + g^* \to V + g$ **matrix elements**
for $V = J/\psi,\ \ \psi(2S),\ \ \Upsilon(1S),\ \ \Upsilon(2S),\ \ \Upsilon(3S)$

– **Full spin density matrix for leptonic decays** $V \to l^+ + l^-$

– **Off-shell** $g^* + g^* \to \chi_J$ **matrix elements**
for $\chi_J = \chi_{cJ}(1S),\ \ \chi_{bJ}(1S),\ \ \chi_{bJ}(2S)$ **with** $J = 0, 1, 2$

– **Full information on the** χ_J **spin alignement parameters**
to generate the decays $\chi_J \to V + \gamma$ **followed by** $V \to l^+ + l^-$

ALL IS READY FOR USE

YOU ARE WELCOME!

Backward evolution scheme

Parton unresolution probability

$$S_b(x, t^{max}, t) = \exp\left\{ - \int_t^{t^{max}} dt' \frac{\alpha_s(t')}{2\pi} \int_0^1 dz\, \mathcal{P}_{a\to bc}(z) \frac{x' f_a(x',t')}{x f_b(x,t)} \right\}$$

Take r at random, $r \in [0,1]$, and find the new resolution scale t from the equation $S_b(x, t^{max}, t) = r$

The evolution equation

$$\frac{\partial \mathcal{F}}{\partial \ln(1/x)} = \int K(q,q')\mathcal{F}(x,q',b') \exp\left[\frac{(b-b')^2 q'^2}{4}\right] q'^2 \, d^2b' \frac{1}{4\pi^3} N_c \, \alpha_s(q') \left(1 - \frac{\alpha_s(q')\mathcal{F}(x,q,b)}{\mathcal{F}_0}\right) d^2q'$$

with the kernel

$$K(q,q')\mathcal{F}(x,q',b') = \frac{\mathcal{F}(q')}{(q-q')_t^2} - \frac{q_t^2 \mathcal{F}(q)}{(q-q')_t^2 \left[q'^2_t + (q-q')_t^2\right]}$$