NLO calculation of heavy quarkonia production at hadron colliders

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- NLO results.
- Uncertainties (in detail for J/Ψ, similar for other heavy quarkonia).
- Predictions v.s. LHC and RHIC data.
- Summary.

Ψ' puzzle

- About twenty year ago, CDF collaboration found a surprising large production rate of Ψ' at high p_T.
- As shown on the right Fig, the yield is larger than the theoretic prediction by a factor of 30, even though the fragmentation contribution is included.

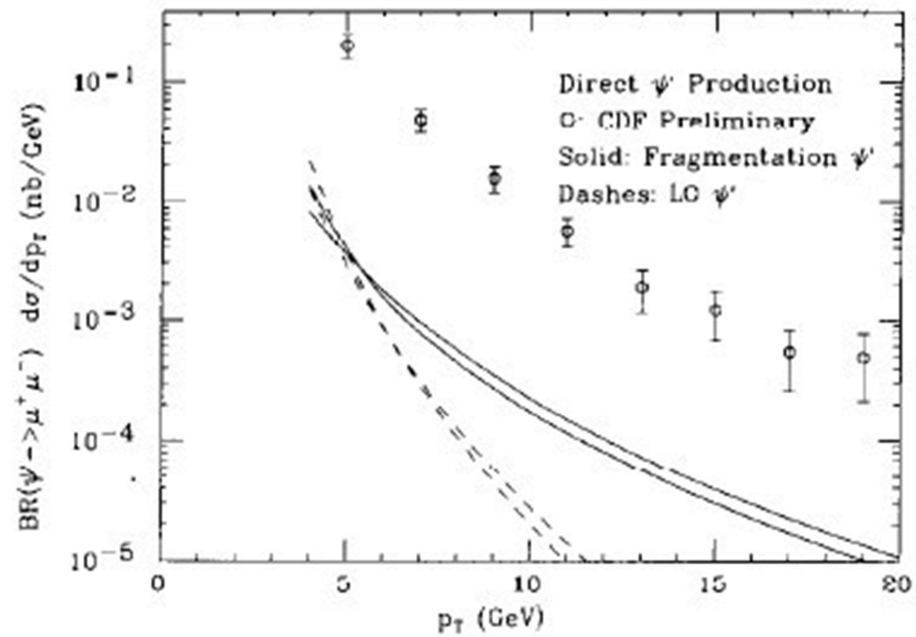


Fig. 4. Preliminary CDF data for prompt ψ' production (O) compared with theoretical predictions of the total fragmentation contribution (solid curves) and the total leading-order contribution (dashed curves).

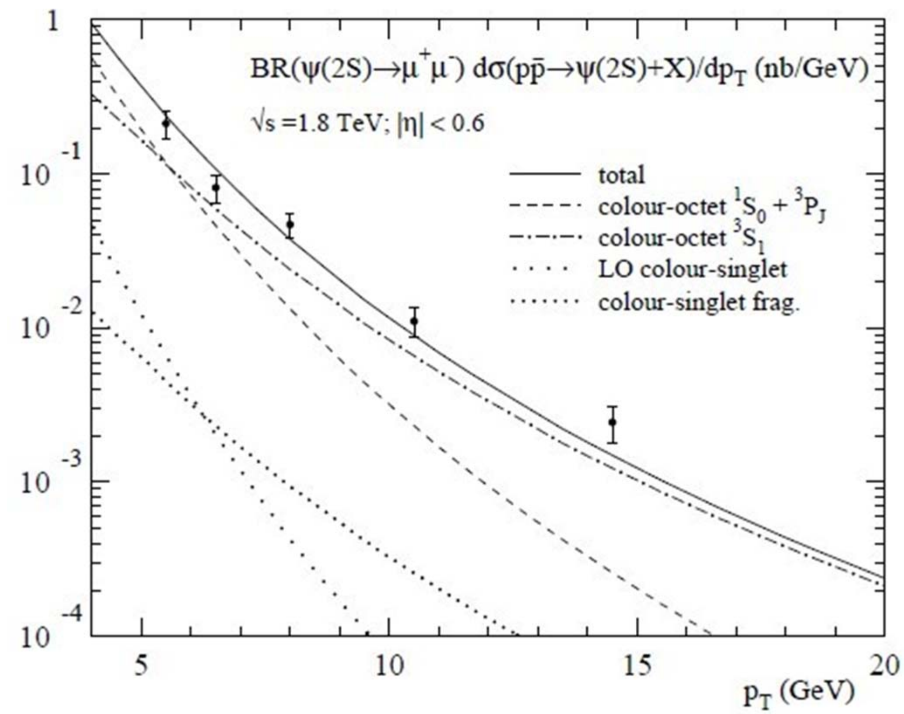
E. Braaten et al. (1994)

Color-Octet mechanism

States	p_T behavior at LO
$^3S_1^{[1]}$	p_T^{-8}
$^3S_1^{[8]}$	p_T^{-4}
$^1S_0^{[1,8]}$	p_T^{-6}
$^3P_J^{[1,8]}$	p_T^{-6}

- To solve the Ψ' puzzle, a color-octet(CO) mechanism was proposed by Braaten and Fleming based on the NRQCD factorization.
- The CO states decline much slower compared to the p_T^{-8} scaling of color-singlet(CS) state, and give an natural explanation of the observed experiment data.

M.Kramer, arXiv:hep-ph/0106120



Polarization puzzle

- Although it seems to successfully explain the differential cross sections, CO encounters difficulties when the polarization is also taken into consideration.
- Dominated by gluon fragmentation to $^3S_1^{[8]}$ at large p_T, LO NRQCD predicts a sizable transverse polarization, while the measurement gives almost unpolarized.

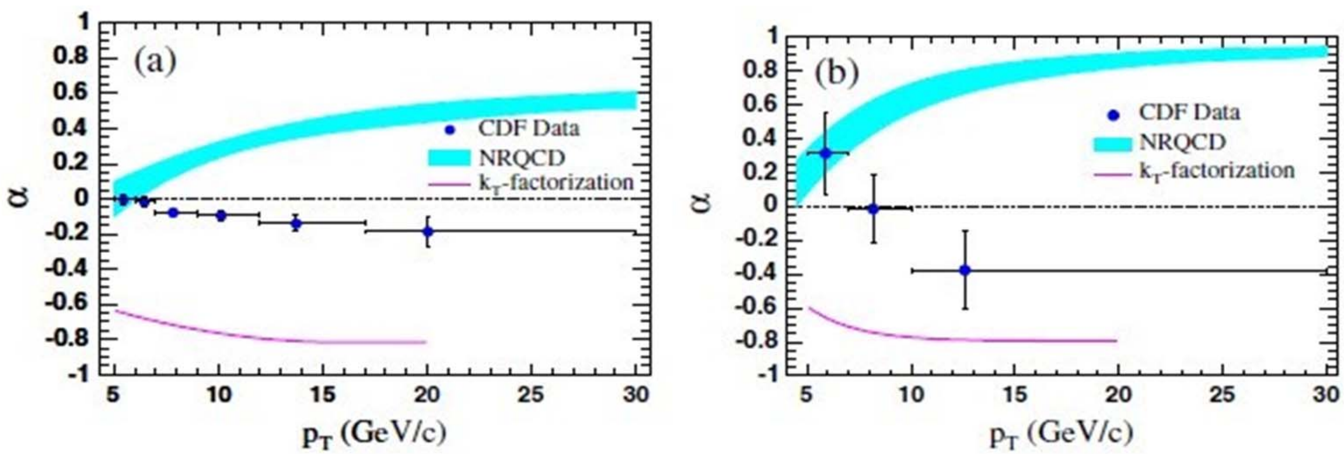


FIG. 4 (color online). Prompt polarizations as functions of p_T: (a) J/ψ and (b) $\psi(2S)$. The band (line) is the prediction from NRQCD [4] (the k_T-factorization model [9]).

A. Abulencia et al. (2007)

NLO calculation

- To solve the polarization puzzle, a lot of effort has been made.
- NLO QCD correction to ${}^3S_1^{[1]}$ channel.
- Differential cross section is enhanced by 2 order relative to LO ${}^3S_1^{[1]}$ result at high p_T. Polarization changed.

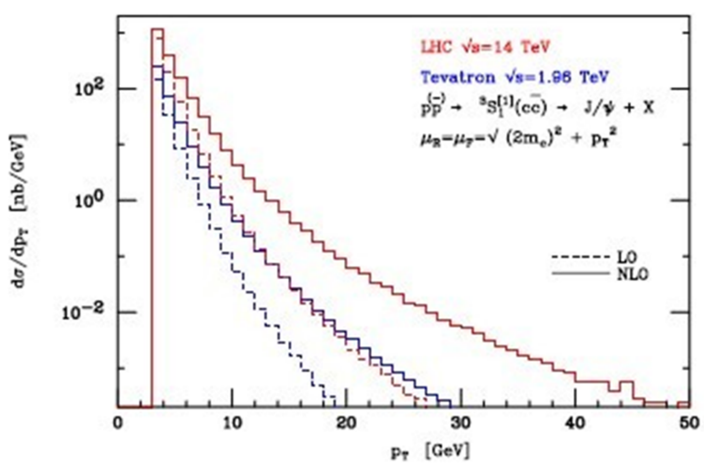
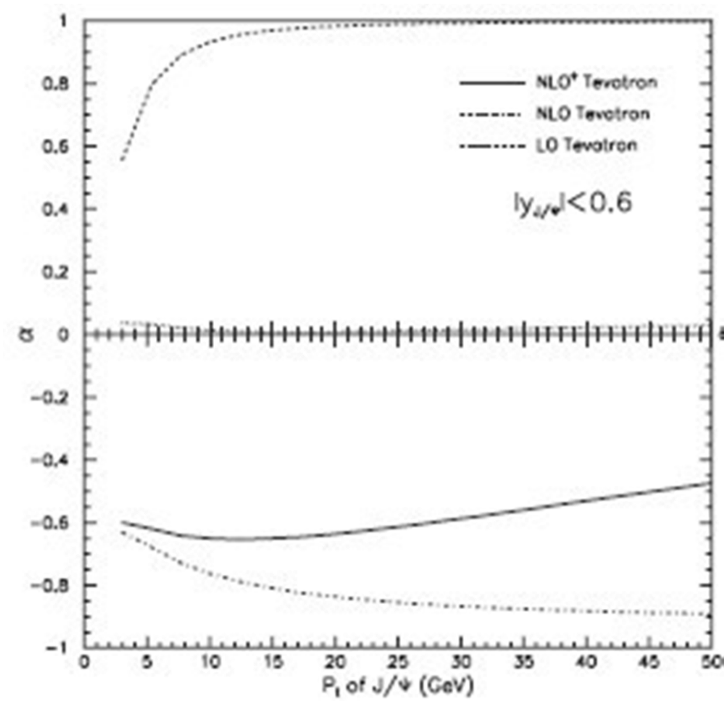


FIG. 5 (color online). Differential cross sections for direct J/ψ production via a ${}^3S_1^{[1]}$ intermediate state, at the Tevatron (lower histograms) and LHC (upper histograms), at LO (dashed line) and NLO (solid line). $p_T^{J/\psi} > 3$ GeV and $|y^{J/\psi}| < 3$. Details on the input parameters are given in the text.

J.M.Campbell et al. (2007)



B.Gong et al. (2008)

p_T enhancement is essential

- NLO correction for $^3S_1^{[1]}$ channel implies: kinematic enhancement is very important for heavy quarkonia production at large p_T.
- So one can conclude nothing definitely until the p_T^{-4} behavior of all channels are opened.

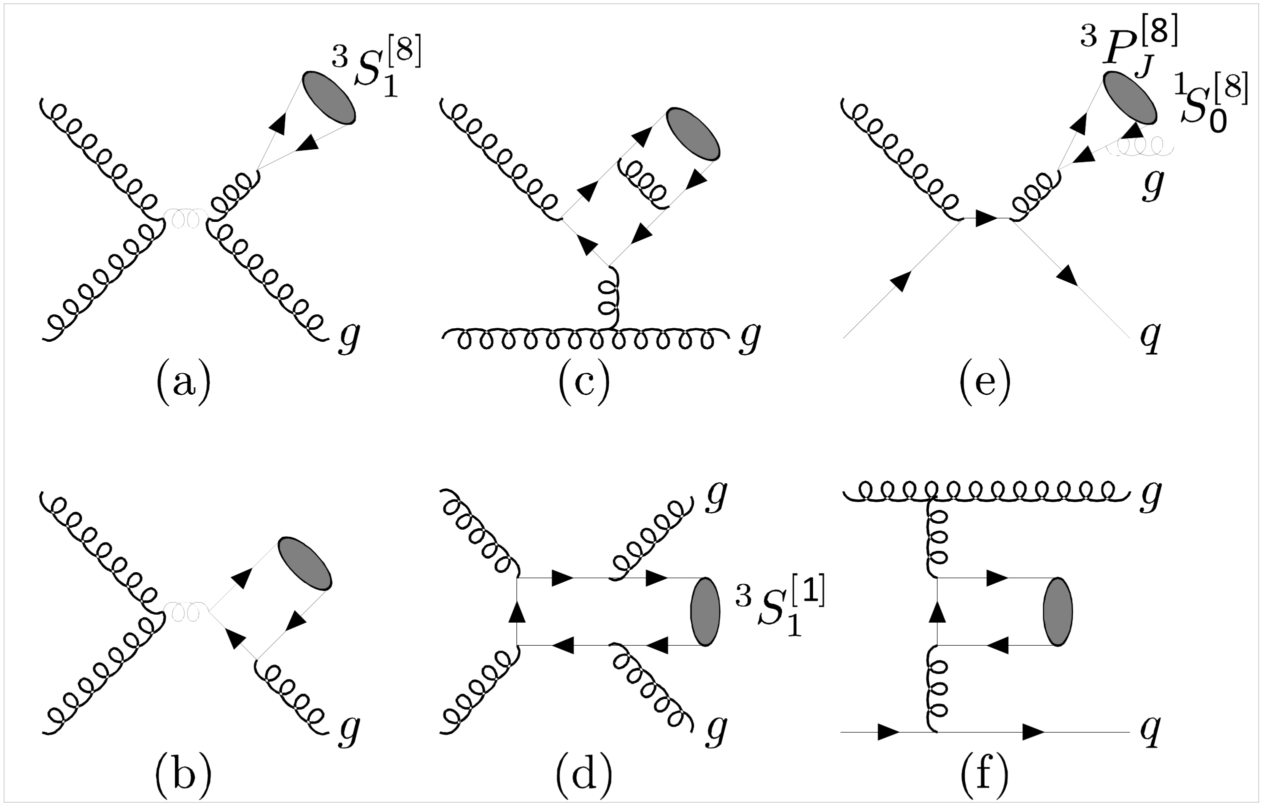
States	Order where p_T^{-4} present
$^3S_1^{[1]}$	NNLO
$^3S_1^{[8]}$	LO
$^1S_0^{[1,8]}$	NLO
$^3P_J^{[1,8]}$	NLO

- For the NNLO correction to $^3S_1^{[1]}$ channel is out of current state of the art, we must estimate its contribution:
  - The only new behavior, which scaling as p_T^{-4} , is the gluon fragmentation. Other contributions at this order is suppressed by α_s relative to NLO.
  - The fragmentation contribution has been calculated (*E. Braaten et al.1993*), and they are as small as 1/30 of the experimental data for J/Ψ production.
  - So we can ignore the NNLO $^3S_1^{[1]}$ contributions.

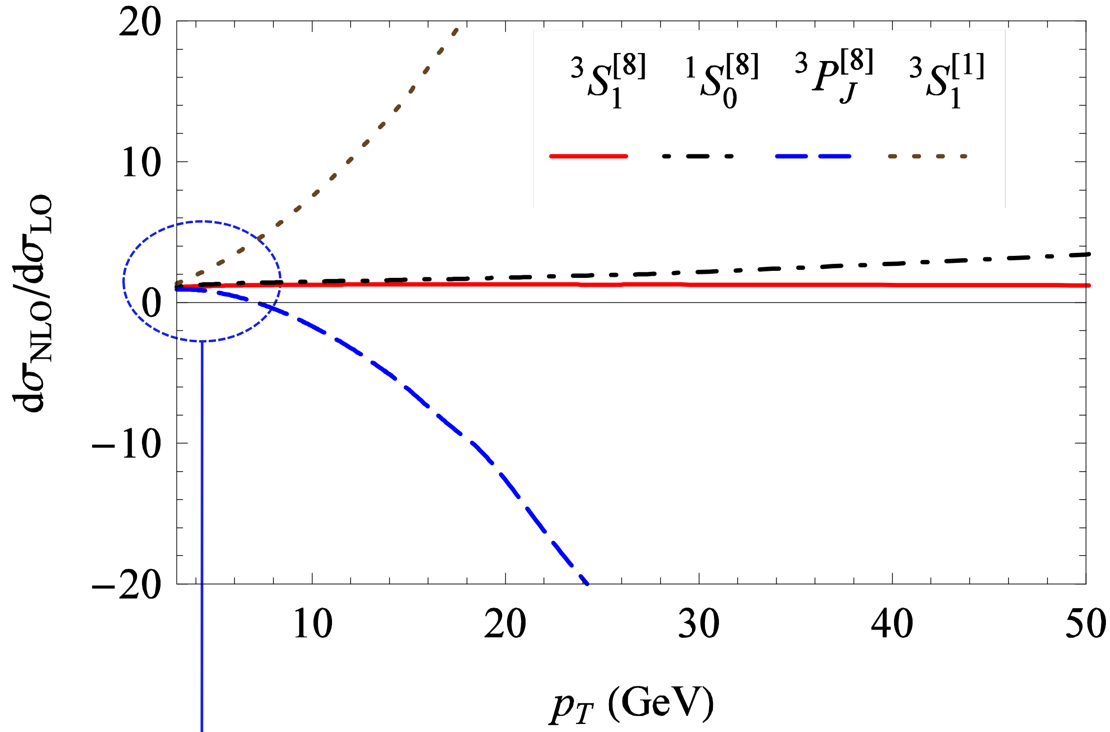
Conclusion: a complete NLO correction to heavy quarkonia production is needed!

Feynman diagrams for J/Ψ production

For J/Ψ, as an example:



K factor



K factor of each channel.

Large but negative corrections for P wave.

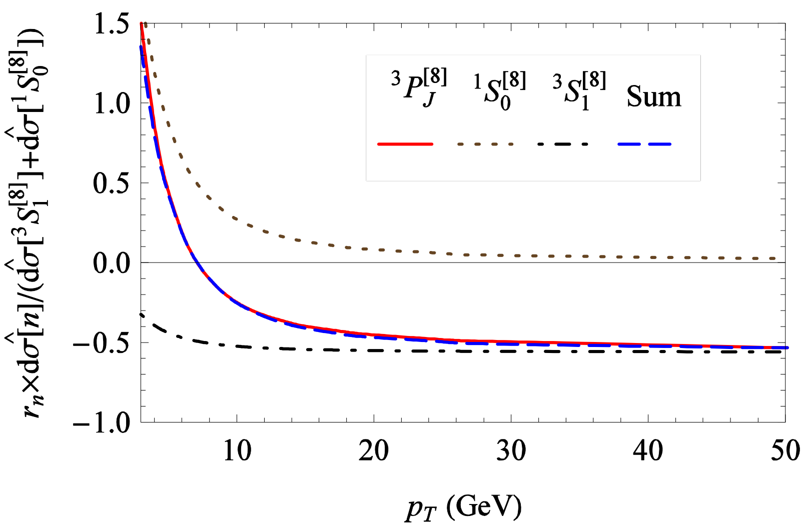
Renormalization scheme and renormalization scale dependent.

Large corrections are originated from $p_T/(2m_c)$

Decomposition of P-wave channel

- Because of the large K factor of P-wave channel, ${}^3S_1^{[8]}$ channel is no longer the unique source at high p_T. We find the following decomposition holds within error of a few percent:

$$\boxed{d\hat{\sigma}[{}^3P_J^{[8]}]=r_0\, d\hat{\sigma}[{}^1S_0^{[8]}]+r_1\, d\hat{\sigma}[{}^3S_1^{[8]}]}$$



NLO short-distance coefficients of $d\hat{\sigma}[{}^3P_J^{[8]}]$, $r_0\, d\hat{\sigma}[{}^1S_0^{[8]}]$, $r_1\, d\hat{\sigma}[{}^3S_1^{[8]}]$ and Sum=$r_0\, d\hat{\sigma}[{}^1S_0^{[8]}]$+$r_1\, d\hat{\sigma}[{}^3S_1^{[8]}]$ in Tevatron.

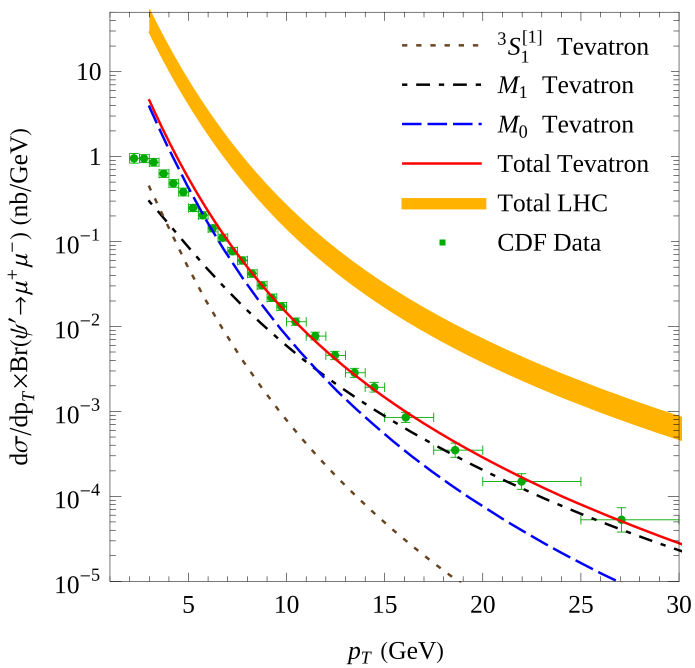
- As a result, we will use two linear combined LDMEs:

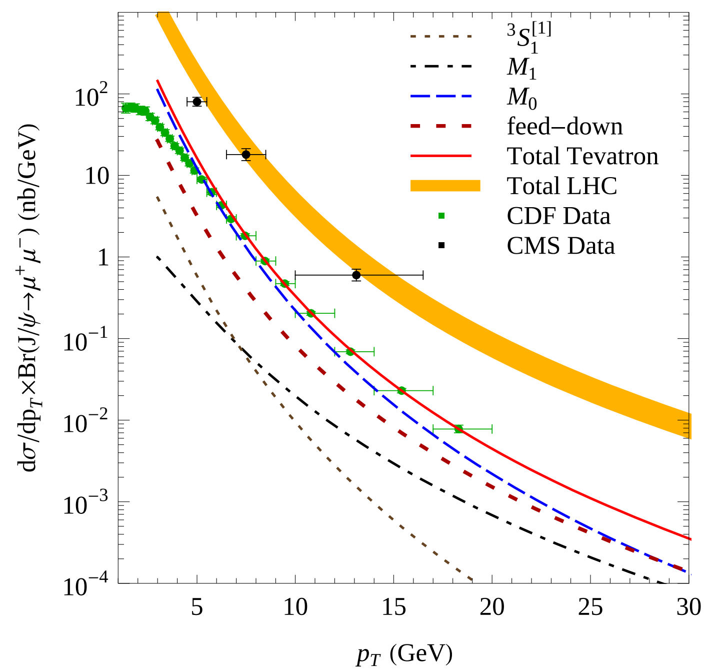
$$M_{0,r_0}^{J/\psi} = \langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle + \frac{r_0}{m_c^2}\langle \mathcal{O}^{J/\psi}({}^3P_0^{[8]})\rangle$$
$$M_{1,r_1}^{J/\psi} = \langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle + \frac{r_1}{m_c^2}\langle \mathcal{O}^{J/\psi}({}^3P_0^{[8]})\rangle$$

$\sqrt{S}$(TeV)	region of y	r_0	r_1
1.96	(0 ,0.6)	3.9	-0.56
7	(0 ,0.75)	4.0	-0.55
7	(0.75,1.50)	3.9	-0.56
7	(1.50,2.25)	3.9	-0.59
7	(0 ,2.4)	4.1	-0.56
7	(0 ,1.2)	4.1	-0.55
7	(1.2,1.6)	3.9	-0.57
7	(1.6,2.4)	3.9	-0.59
7	(2.5, 4)	3.9	-0.66
7	(2 ,2.5)	4.0	-0.61
7	(2.5, 3)	4.0	-0.65
7	(3 ,3.5)	4.0	-0.68
7	(3.5, 4)	4.0	-0.74
7	(4 ,4.5)	4.2	-0.81
14	(0 , 3)	3.9	-0.57

Fit LDMEs using Tevatron Data

p_T^{cut} GeV	H	$\langle\mathcal{O}^H\rangle$ GeV^3	$M^H_{1,r1}$ 10^{-2} GeV^3	$M^H_{0,r0}$ 10^{-2} GeV^3	$\chi^2/d.o.f.$
7	J/ψ	1.16	0.05 ± 0.02	7.4 ± 1.9	0.33
	ψ'	0.76	0.12 ± 0.03	2.0 ± 0.6	0.56
5	J/ψ	1.16	0.16 ± 0.05	5.2 ± 1.3	3.5
	ψ'	0.76	0.17 ± 0.04	1.1 ± 0.3	2.2





Uncertainty: p_T^{cut}

- In the fit procedure, we abandon data with p_T<7GeV, **which is essential in our work**. There are various reasons for this p_T cut:

✓ Theoretically:

(1) Small p_T region is dominated by non-perturbative effect because of initial state gluon showers (*E.L. Berger et.al. 2005*).

(2) NRQCD factorization is still not proven. But for large p_T region, it was found the factorization holds up to $O(m^4/p_T^4)$ correction (*Z.B. Kang et.al. 2011*). So data in large p_T are more confident to describe.

✓ Experimentally: In the plot, the curvature of data curve is positive at large p_T but negative at small p_T, with a turning point at $p_T \approx 6\ GeV$. Thus data below 7GeV cannot be described by perturbative factorization theory.

✓ Fit: We perform a χ^2 analysis for J/ψ, and find χ^2/d.o.f. decreases rapidly as the cut increases from 3GeV to 7GeV, and then it almost unchanges when the cut becomes larger:

lower p_T cut (Gev)	χ^2/dof	$<O^3S_1^{[8]}>_{J/\psi}$	$<O^1S_0^{[8]}>_{J/\psi}$
3	236.269/16=14.7668	0.360089	1.78736
4	92.9272/12=7.74393	0.250964	3.49161
5	27.8681/8=3.48351	0.157748	5.1679
6	9.07871/6=1.51312	0.101501	6.28956
7	1.31256/4=0.328141	0.0492096	7.43362
8	0.817308/3=0.272436	0.037283	7.71245
9	0.434183/2=0.217091	0.0226552	8.07939
10	0.424269/1=0.424269	0.0192824	8.17001

Uncertainty: decomposition (1)

$d\hat{\sigma}[^3P_J^{[8]}]=r_0\, d\hat{\sigma}[^1S_0^{[8]}]+r_1\, d\hat{\sigma}[^3S_1^{[8]}]$

- Two errors induced by decomposing the P-wave channel:
  1) The decomposition has an error of a few percent;
  2) The resulted r_0 and r_1 vary for different experimental condition.
- The above uncertainties can be determined by using three unconstrained LDMEs to fit data (*we thank G. Bodwin for pointing out this*). Choosing p_T^{cut}=7GeV and by minimizing χ^2 we get:

$$O_1 \equiv \langle \mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle = 15.7\times 10^{-2}GeV^3(\pm 129\%)$$
$$O_2 \equiv \langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle = -1.18\times 10^{-2}GeV^3(\pm 249\%)$$
$$O_3 \equiv \frac{\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]})\rangle}{m_c^2} = -2.28\times 10^{-2}GeV^3(\pm 239\%)$$

- These LDMEs are obviously unphysically determined. Physical variables are eigenvectors of correlation matrix, which corresponds to linear combinations of these LDMEs:

$$\begin{pmatrix}\Lambda_1\\ \Lambda_2\\ \Lambda_3\end{pmatrix} = \begin{pmatrix}0.96 & -0.14 & -0.26\\ 0.29 & 0.31 & 0.91\\ 0.047 & 0.94 & -0.33\end{pmatrix}\begin{pmatrix}O_1\\ O_2\\ O_3\end{pmatrix} = \begin{pmatrix}15.8\pm 134\%\\ 2.11\pm 5.13\%\\ 0.39\pm 2.45\%\end{pmatrix}\times 10^{-2}GeV^3$$

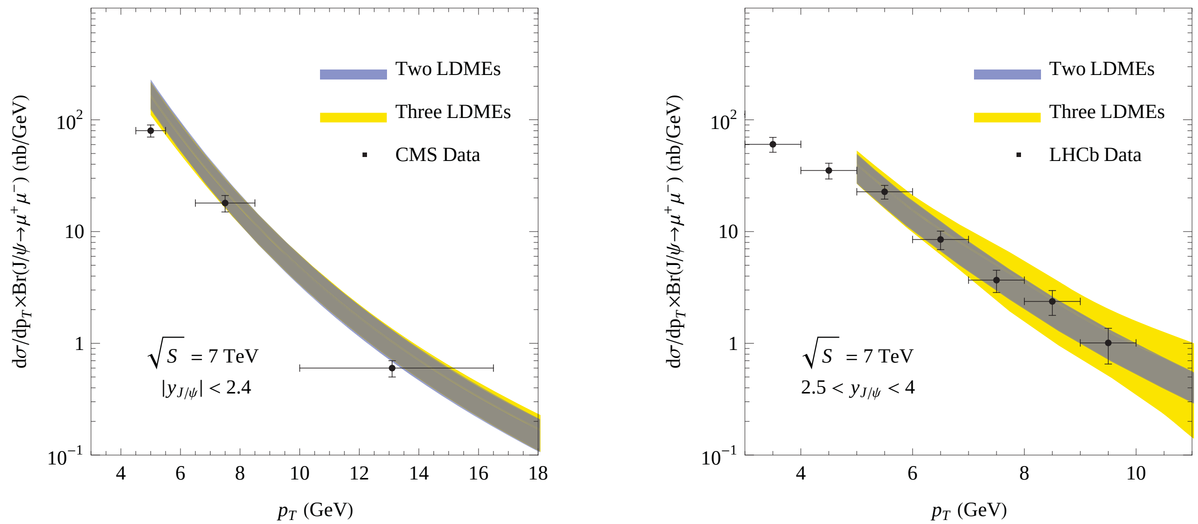
Comparison:

$$\vec{\mathbf{v}}_{M_0} = (0.25 \quad 0 \quad 0.97)$$
$$\vec{\mathbf{v}}_{M_1} = (0 \quad 0.87 \quad -0.48)$$

$M_{0,r_0}^{J/\psi}$ and $M_{1,r_1}^{J/\psi}$ are approximately equivalent to the two well constrained LDMEs:

$$\Lambda_2 \leftrightarrow M_0$$
$$\Lambda_3 \leftrightarrow M_1$$

Uncertainty: decomposition (2)



- Predictions between using two LDMEs method and three LDMEs implies:
  1) In central region, two methods give almost the same error bar;
  2) In forward region, three LDMEs method has larger error bar, which means two LDMEs method may underestimates its theoretical uncertainty. (It is interesting to note that the data still seems to locate within the error bar of two LDMEs method predictions.)
- Reason: r_i have small differences between CMS and CDF, but larger difference between LHCb and CDF.
- **It is possible to determine all three LDMEs when data in forward region are sufficient enough!**

Uncertainty: feeddown

- Feeddown contribution mainly from ψ(2S) and χ_{cJ} , all of which are calculated to NLO and their CO LDMEs are determined by fit Tevatron data.
- The transverse momentum difference is considered and approximated as:

$$p_T^{J/\psi} \approx p_T^H \times \frac{m_{J/\psi}}{m_H}$$

with an very small error $O(\frac{(m_{J/\psi}-m_H)^2}{m_H^2})$. (*We thank J.P. Lansberg and P. Faccioli for helping to test this approximation by simulation*.)

Comparison: If we assume the feeddown from ψ(2S) and χ_{cJ} is a constant for all pT region, e.g. 36%, the results change small:

Using three LDMEs: (36% feeddown)

$$M_{0,r_0}^{J/\psi} = 5.47 \times 10^{-2} GeV^3 (\pm 12\%)$$
$$M_{1,r_1}^{J/\psi} = 0.107 \times 10^{-2} GeV^3 (\pm 63\%)$$

Using two LDMEs: (36% feeddown)

$$M_{0,r_0}^{J/\psi} = 5.71 \times 10^{-2} GeV^3 (\pm 4.4\%)$$
$$M_{1,r_1}^{J/\psi} = 0.08 \times 10^{-2} GeV^3 (\pm 18\%)$$

Using two LDMEs: (default)

$$M_{0,r_0}^{J/\psi} = 7.4 \times 10^{-2} GeV^3 (\pm 5.5\%)$$
$$M_{1,r_1}^{J/\psi} = 0.05 \times 10^{-2} GeV^3 (\pm 45\%)$$

Combining with Butenschön and Kniehl's work

- Butenschön and Kniehl (BK) do a similar work for J/ ψ production (*see Butenschön's talk*), differences from ours include:
  1) BK use both Tevatron data (p_T^{cut}=3GeV) and HERA data (p_T^{cut}=1GeV) ;
  2) BK neglect the feeddown contribution;
  3) BK determine all three CO LDMEs.
- To take advantage of their results, we also neglect feeddown contribution and using three LDMEs to fit (because these choices do not change final results qualitatively, as discussed above), but we still use only Tevatron data with p_T^{cut}=7GeV.

BK's results (errors may be smaller than 10%):

$$M_{0,r_0}^{J/\psi} = 2.47 \times 10^{-2} GeV^3$$
$$M_{1,r_1}^{J/\psi} = 0.594 \times 10^{-2} GeV^3$$

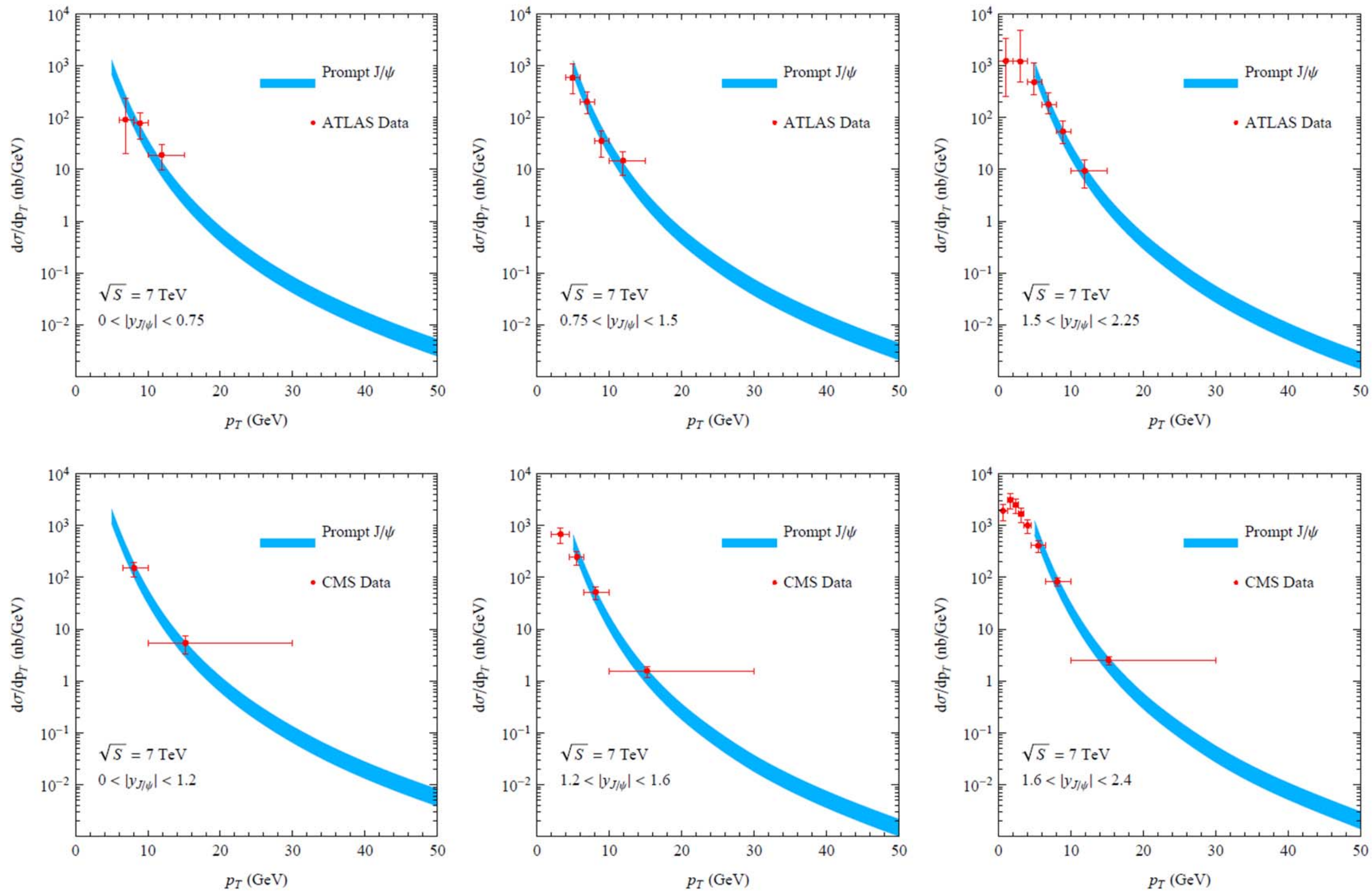
Our results:

$$M_{0,r_0}^{J/\psi} = 8.54 \times 10^{-2} GeV^3 (\pm 12\%)$$
$$M_{1,r_1}^{J/\psi} = 0.167 \times 10^{-2} GeV^3 (\pm 63\%)$$

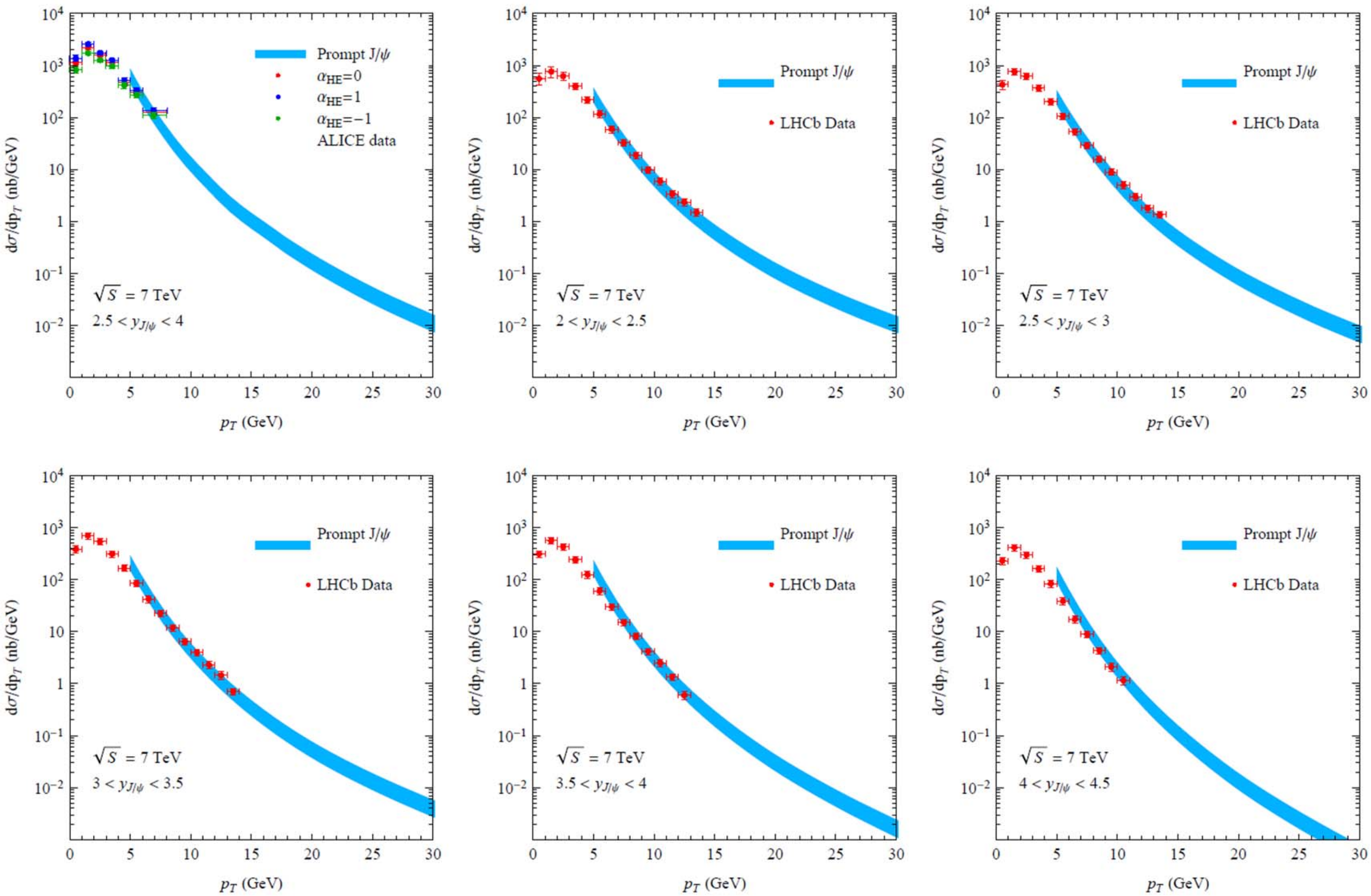
- $M_{0,r_0}^{J/\psi}$ is well constrained in both two groups, but two results are different significantly. Only difference: using different data!!!

☆ Observation: **perturbative NRQCD factorization cannot give a consistent description of both** Tevatron data (7GeV > p_T >3GeV) + HERA data (p_T >1GeV) and Tevatron data (p_T >7GeV).

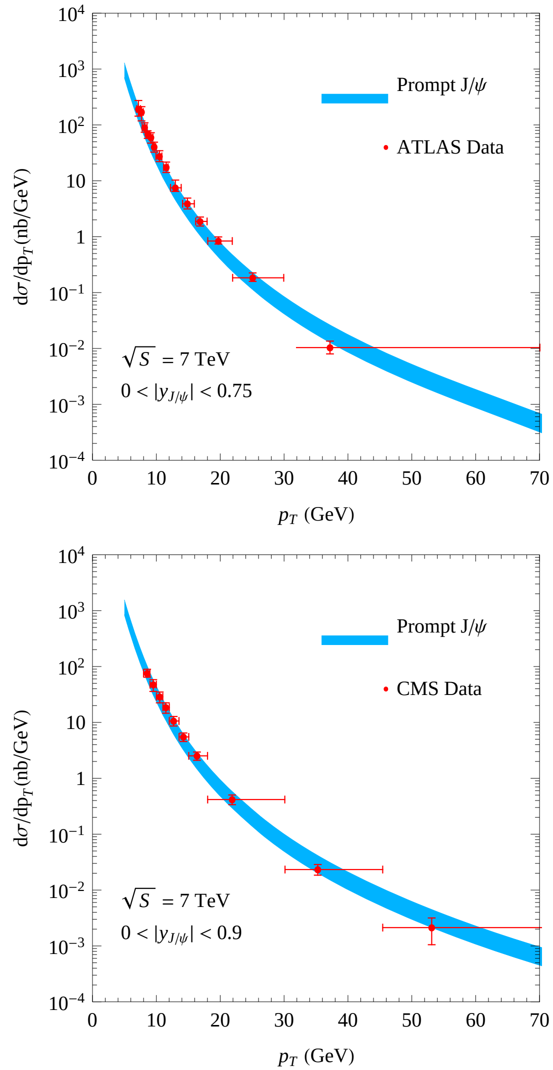
J/Ψ@LHC mid. p_T (1)



J/Ψ@LHC mid. p_T (2)

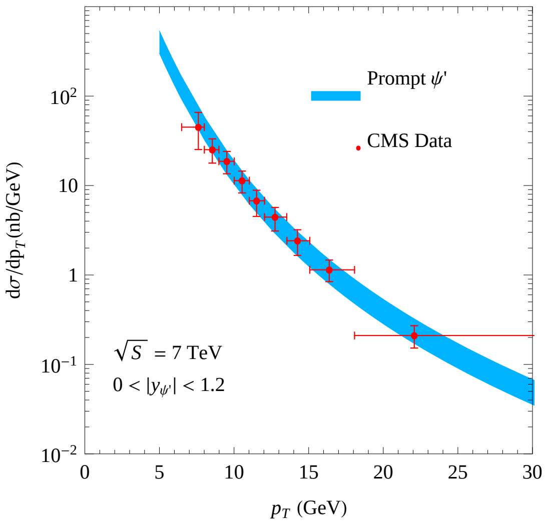


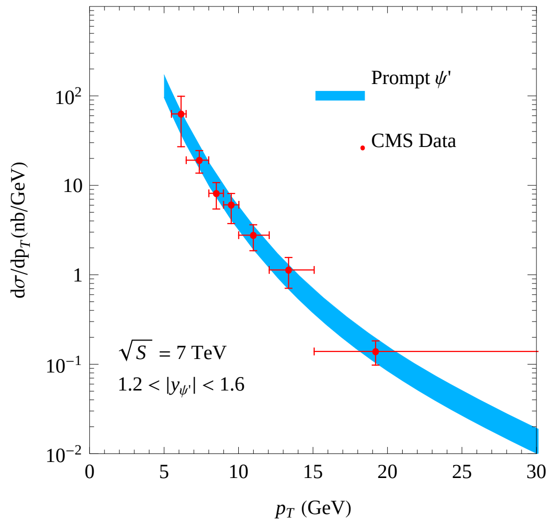
J/Ψ@LHC LARGE p_T

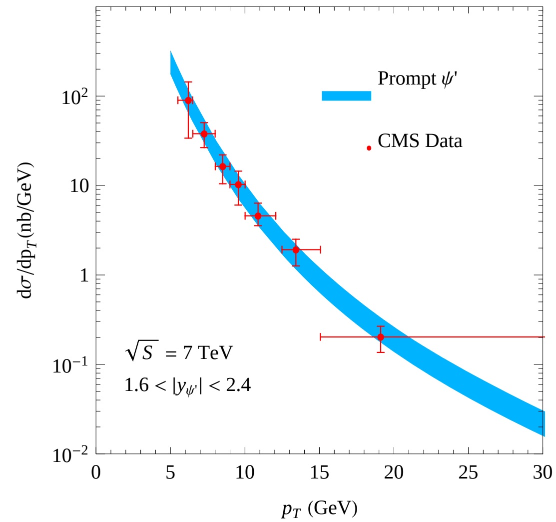


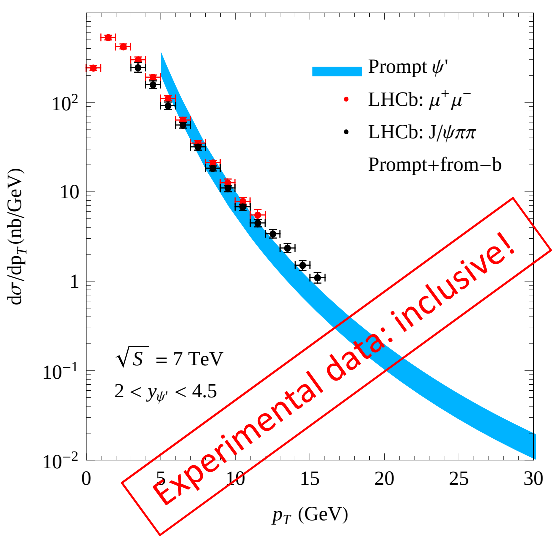
- Data at large p_T: very important to distinguish different models.
- Our predictions agree with large p_T data very well, all data can be described by theory within the uncertainty of a factor of two.
- Fit parameters using Tevatron data with 7< p_T <20GeV, predict LHC up to p_T =70GeV, nontrivial!
- It is needed to determine CO LDMEs from these large p_T data when data are adequate enough.

Ψ(2S)@LHC



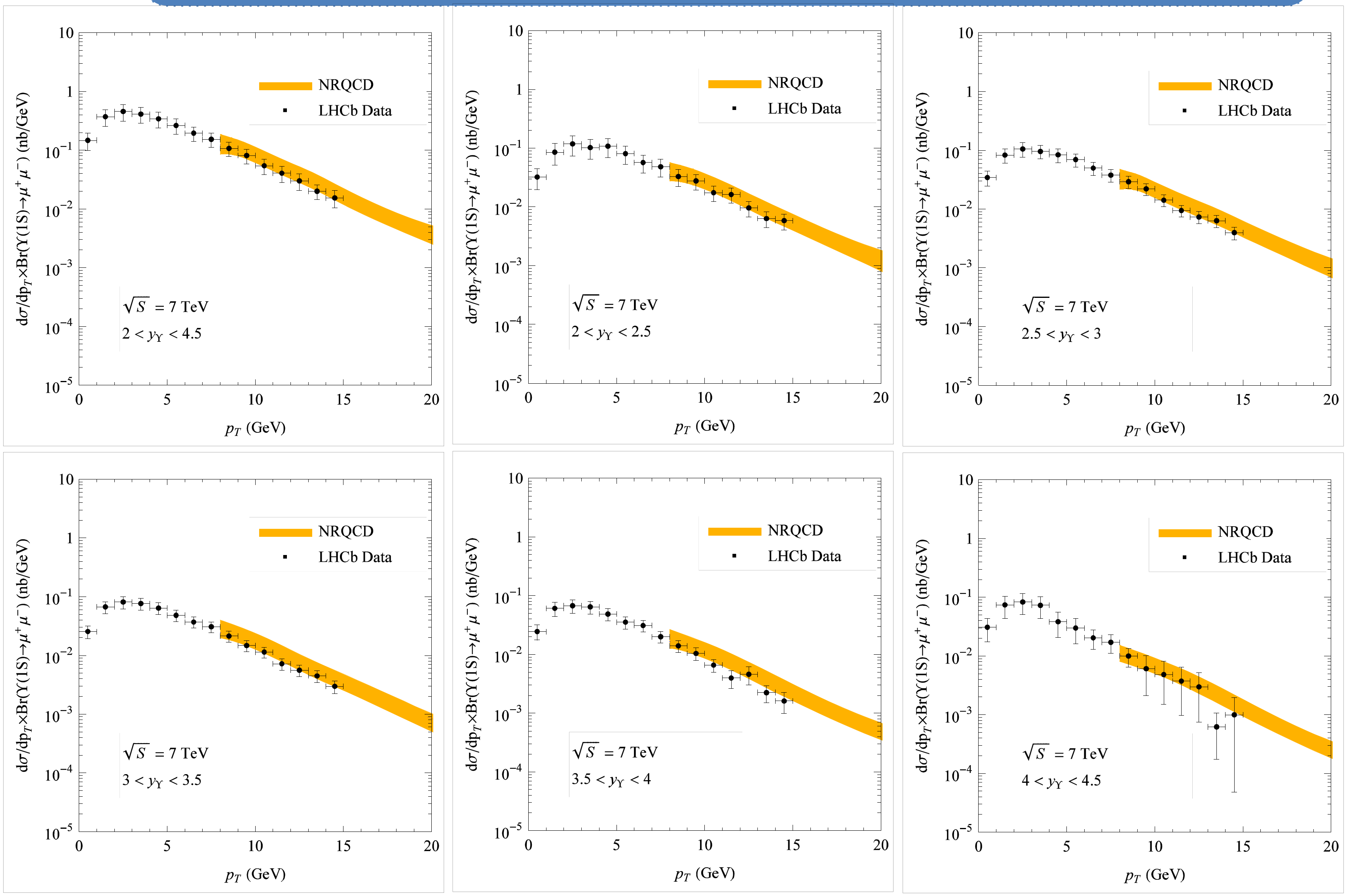




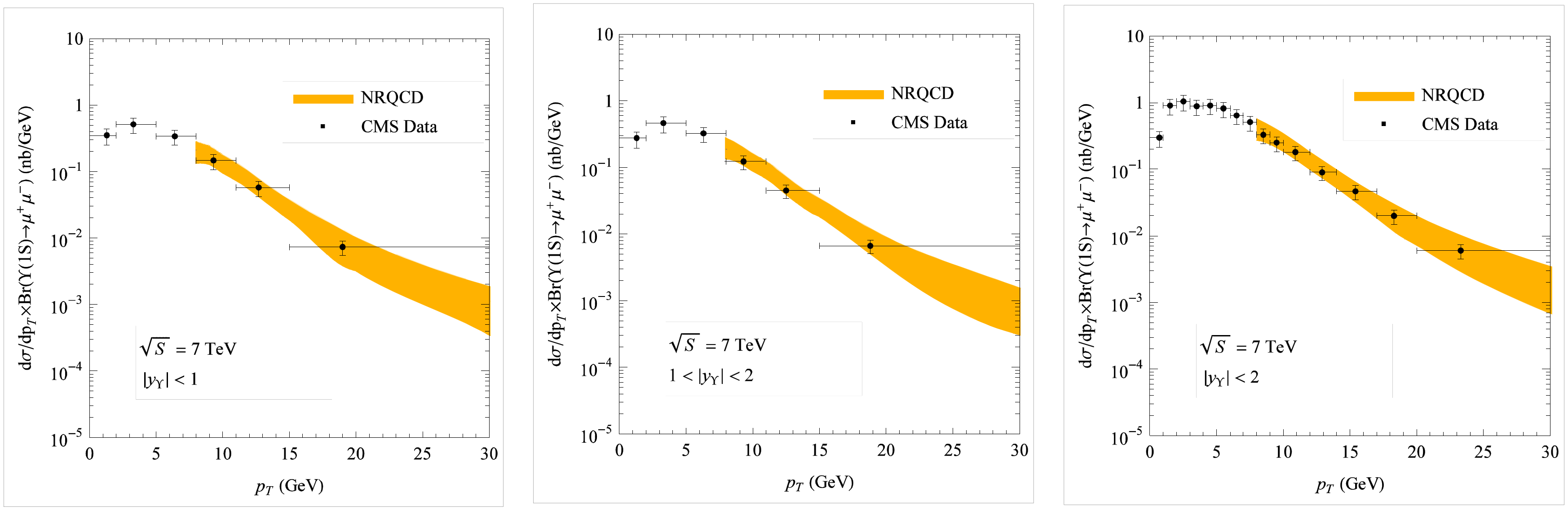


- Fit parameters of $\Psi(2S)$ using Tevatron data.
- Predictions are very good agreement with CMS data.
- We cannot compare with LHCb data directly because b decay contribution is not removed in the data.

Υ(1S)@LHC (1)

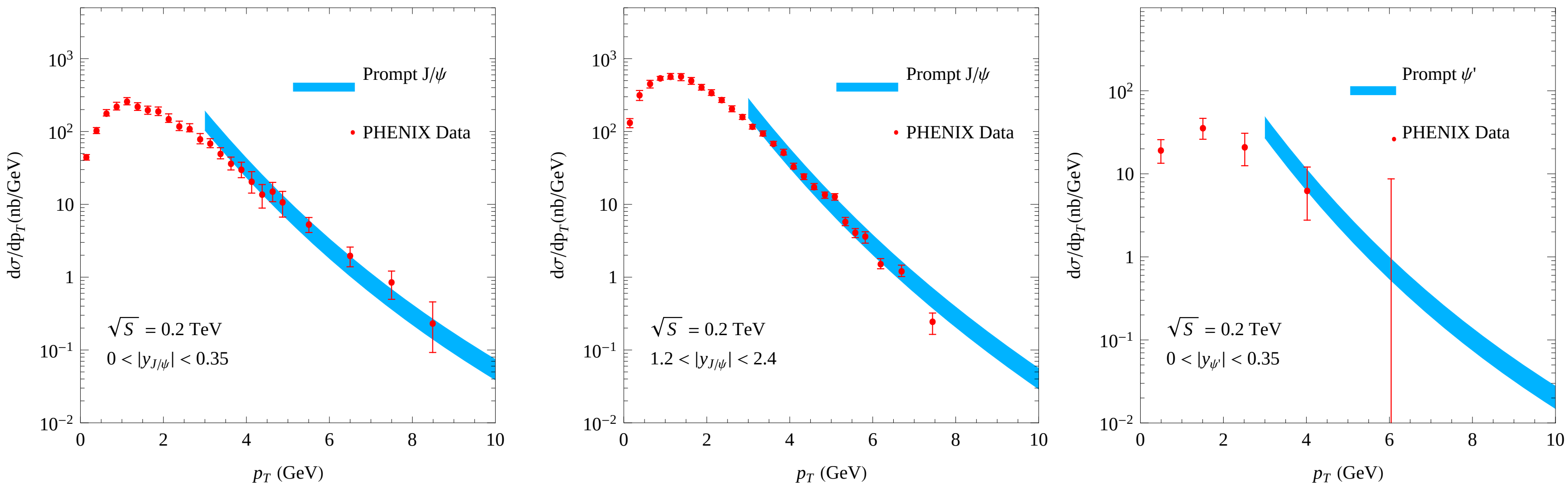

NRQCD
LHCb Data
$\sqrt{S}$ = 7 TeV
$2 < y_\Upsilon < 4.5$
$2 < y_\Upsilon < 2.5$
$2.5 < y_\Upsilon < 3$
$3 < y_\Upsilon < 3.5$
$3.5 < y_\Upsilon < 4$
$4 < y_\Upsilon < 4.5$
$d\sigma/dp_T \times Br(\Upsilon(1S) \to \mu^+\mu^-)$ (nb/GeV)
p_T (GeV)

Υ(1S)@LHC (2)



- Fit parameters of $\Upsilon(1S)$ using Tevatron data.
- Predictions for LHC: a good agreement.
- Large error bar in the prediction: large uncertainty of CO LDMEs because p_T region of Tevatron data is too small.

Heavy Quarkonia@RHIC



- Good agreement with RHIC data for both J/ Ψ and Ψ(2S).
- Curvature turn point in RHIC: about p_T =3GeV.
- Non-perturbative effect may be important at p_T =3GeV even for $\sqrt{S}$=0.2TeV.

Summary

- Based on the very well motivated model, NRQCD factorization, a general method to fit CO LDMEs at NLO is presented. Important uncertainties are discussed.
- Therefore, NLO predictions for all heavy quarkonia (e.g. J/Ψ, Ψ(2S), χ_{cJ}, Υ ···) prompt production in hadron colliders are available now.
- Heavy quarkonia production in hadron colliders are found to be well described by NRQCD factorization, all data can reach the theoretical central line by 1σ shift (for data of $p_T > 7\text{GeV}$ only).
- By taking advantage of BK's results, it was found that, for J/Ψ production, NRQCD factorization cannot give a consistent description of Tevatron data + HERA data if p_T^{cut} are chosen to be 3GeV and 1GeV respectively.

Questions

- Are LDMEs universal? How to understand the constraints of CO LDMEs by B factories V.S. large M_0 needed in hadron colliders?
  1) Results of Belle and Barbar have large difference.
  2) M_0 is a linear combination.
  3) Is experimental cut in B factories reasonable? (*Z.G. He et.al. in preparation*)
  4) Is M_0 just an "effective parameter"?

- Can all three CO LDMEs be extracted?
  1) Only p_T^{-4} and p_T^{-6} terms are proven to be factorized, which may constrain two linear combination of LDMEs in each process. (*Z.B. Kang et.al.2011*)
  2) P-wave can be approximately decomposed, which also results in two combination.
  3) Is global fit possible? (*see Butenschön's talk*)
  4) Combining forward region and central region data in hadron colliders works?

- Have we already understood the production mechanism?
  1) Whether is it just a phenomenological description?
  2) Howe to achieve the underline physics if it is just a successful phenomenological description?

Back up

NRQCD Factorization

$$d\sigma_{\psi} = \sum_{i,j,n} \int \underbrace{dx_1 dx_2 \boxed{G_{i/A} G_{j/B}}}_{\Lambda_{QCD}} \times \underbrace{\boxed{\hat{\sigma}[ij \to c\bar{c}[n] + X]}}_{m_c} \times \underbrace{\boxed{\langle \mathcal{O}_n^{\psi} \rangle}}_{m_c v}$$

PDF

CTEQ6L1, CTEQ6M

Hadronization

Long distance ($\sim 1/(m_c v)$) process: non-perturbative calculations and input from experiments needed.

Production of heavy quarks

Short distance ($\sim 1/m_c$) process: perturbative calculation.

Main task in this work.

Code and packages

Mathematica control code

Self-written Mathematica code

Analyze process with bound state and generate parton-level sub processes

FeynArts

Generate parton-level Feynman amplitudes and Feynman Diagrams

Self-written Mathematica code

Perform tensor integral reduction and analytically simplify

Self-written C++ code

Perform phase space integration and convolution with PDF

LoopTools or QCDOneLoop

Calculate scalar functions

IR singularities

- Collinear singularities:

 Independent of Heavy Quarkonium, cancel as jet production.

- Soft Singularities of S-wave channels:

  Real + Virtual

- Soft Singularities of P-wave channels:

  NRQCD MEs + Real + Virtual

$$\boxed{\mathcal{M}^{\mathrm{R}}|_s = g\,\mu_r^{\epsilon}\,\varepsilon_\mu\, J_f^{a,\mu}\,\mathcal{M}_f^{Born}}$$

$$\boxed{\mathcal{M}^{V}|_s = \frac{1}{2} g^2 \mu_r^{2\epsilon} I_{ff'}\,\mathcal{M}_{ff'}^{Born}}$$

where $J_f^{a,\mu} = \dfrac{p_f^\mu}{p_f \cdot k} T_f^a$ and $I_{ff'} = J_f^{a,\mu} J_{f',\mu}^{a}$

While $\mathcal{M}_f^{Born}$ and $\mathcal{M}_{ff'}^{Born}$ are color connected born level amplitudes.

Divergence cancelation

It can be shown that,

$$\left(T_f^a T_{f'}^a \mathcal{M}_{ff'}^{Born}\right)^\dagger \left(M^{Born}\right) = \left(T_f^a \mathcal{M}_f^{Born}\right)^\dagger \left(T_{f'}^a \mathcal{M}_{f'}^{Born}\right),$$

$$\left(T_f^a T_{f'}^a \mathcal{M}_{ff'}^{Born}\right)\left(M^{Born}\right)^\dagger = \left(T_f^a \mathcal{M}_f^{Born}\right)\left(T_{f'}^a \mathcal{M}_{f'}^{Born}\right)^\dagger, \; f' \neq Q, \overline{Q}$$

So only term that is not canceled between Real and Virtual is the P-wave term:

$$\boxed{-g^2 \mu_r^{2\epsilon} \varepsilon^\alpha \varepsilon^\beta \frac{\partial J_F^{a,\mu}}{\partial q^\alpha} \frac{\partial J_{F',\mu}^{a}}{\partial q^\beta} \left|\mathcal{M}^{Born}\right|^2_{FF'}}$$

Where $F, F' = Q \; or \; \bar{Q}$ and q is the relative momentum of heavy quarks.

Finally, the above divergence can be absorbed by NRQCD MEs: