Dipion and tripion invariant mass spectra in $X(3872) \to J/\psi\pi\pi(\pi)$ within chiral lagrangian approach

Pyungwon Ko
(Work in preparation with Chaehyun Yu)

School of Physics
Korea Inst. for Advanced Study (KIAS)

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Outline

Motivations

- What is $I(X)$?
  $-$ $I = 0$, if it is a pure charmonium, e.g. $\chi_{c1}(2P)$ or $^1D_2(\eta_c^{\prime}(2D))$
  $-$ (Equal) Mixture of $I = 0$ and $I = 1$, if a $D^{*0}D^0$ molecule
- Knowns : $C(X) = +1$ from radiative transitions

$$\frac{\Gamma(X \to J/\psi\pi\pi\pi)}{\Gamma(X \to J/\psi\pi\pi)} = 1.0 \pm 0.4 \pm 0.3(BaBar),$$

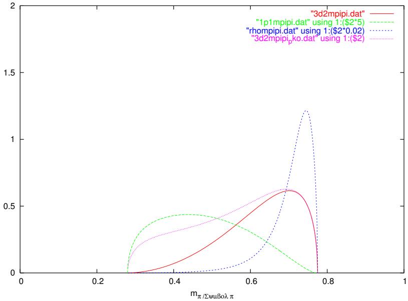
$$\frac{\Gamma(X \to J/\psi\gamma)}{\Gamma(X \to J/\psi\pi\pi)} = \begin{cases} 0.14 \pm 0.05 & \text{Belle} \\ 0.33 \pm 0.12 & \text{BaBar} \end{cases}$$

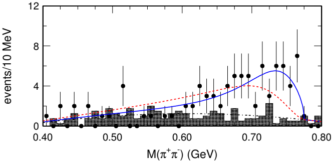
$$\frac{\Gamma(X \to \psi^{\prime}\gamma)}{\Gamma(X \to J/\psi\gamma)} = 3.4 \pm 1.4$$

- What is J^P of $X(3872)$?

$m_{\pi\pi}$ spectrum in $X(3872) \to J/\psi\pi\pi$ for $I(X) = 0$ and $C = -1$, or $I(X) = 1$ and $C = +1$

[Kim and Ko, hep-ph/0405265, PRD (2005),Erratum-ibid.(2005)]



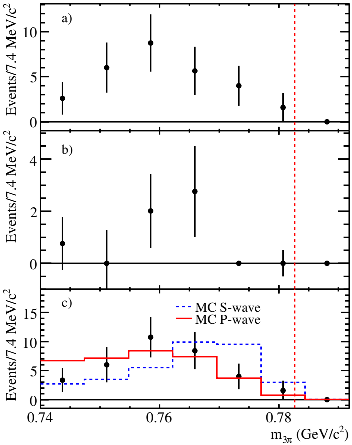


Green : ${}^1P_1(1^{+-})$, Red : ${}^3D_2(2^{--}) \to J/\psi(\pi\pi)_{\mathrm{D-wave}}$
${}^3D_2(2^{--}) \to J/\psi(\pi\pi)$ Blue : ${}^3P_1(1^{++})$ (with $I(X) = 0$) or $D\bar{D}^*$ molecule

Question : 1^{++} or 2^{-+} ? (Belle, CDF from $\pi\pi$ decay channel)

Is $J^{PC}(X) = 1^{++}$ or 2^{-+} ?

- New data from BaBar from $X(3872) \to J/\psi\omega \to J/\psi\pi\pi\pi$ seem to prefer 2^{-+} instead of 1^{++}



Is $J^{PC}(X) = 1^{++}$ or 2^{-+} ?

- New data from BaBar from $X(3872) \to J/\psi\omega \to J/\psi\pi\pi\pi$ seem to prefer 2^{-+} instead of 1^{++} $\to$ Really ?
- No theoretical calculations on 3π and 2π invariant mass spectra for $C = +1$ as of today
- Work by Kim and Ko (PRD in 2004) was mostly for $C = -1$, and does not apply for $C = +1$ case
- Our goal is to provide such predictions with chiral lagrangian with light vector mesons, implemented for transitions between heavy quarkonia
- We calculate the $m_{3\pi}$ spectrum and compare with the data
- And reconsider $m_{\pi\pi}$ spectrum in $X(3872) \to J/\psi\pi\pi$ for $I(X) = 0$ and $C = +1$, where isospin symmetry breaking effects are important

Chiral lagrangian

- QCD with 3 light quarks has spontaneously broken global chiral symmetry: $SU(3)_L \times SU(3)_R \to SU(3)_V$
  $q_L \to Lq_L, \quad q_R \to Rq_R$ (L, R : $SU(3)$ matrices)
- Pion field $\Sigma(x) \equiv exp(2i\pi(x)/f_\pi) = \xi^2(x)$ transforms as

$$\Sigma(x) \to L\Sigma(x)R^\dagger, \quad \xi(x) \to L\xi(x)U^\dagger(x) = U(x)\xi(x)R^\dagger$$

- $U(x)$ is a local $SU(3)_V$

$$\sqrt{2}\pi(x) = \begin{pmatrix} \pi^0/\sqrt{2} + \eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta/\sqrt{6} & K^0 \\ K^- & \overline{K}^0 & -2\eta/\sqrt{6} \end{pmatrix}$$

Chiral lagrangian with light vector mesons

- Convenient to consider two vector fields:

$$V_\mu = \frac{1}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger), \qquad V_\mu \to U V_\mu U^\dagger + U \partial_\mu U^\dagger,$$
$$A_\mu = \frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger), \qquad A_\mu \to U A_\mu U^\dagger.$$

- V_μ: vector and A_μ: axial vector
- Can use A_μ and V_μ to construct chiral lagrangian e.g., the usual nonlinear σ model larangian can be written as

$$\mathrm{Tr}(A_\mu A^\mu) \propto \mathrm{Tr}\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right)$$

- $\partial_\mu \to D_\mu$ with V_μ as a gauge field

Explicit chiral sym breaking (isospin breaking)

- Chiral symmetry is explicitly broken by current-quark masses (and electromagnetic interactions, which we ignore here)
- Consider $m = \mathrm{diag}\,(m_u, m_d, m_s)$ as a spurion with transformation property

$$m \to LmR^\dagger = RmL^\dagger$$

- Convenient to use $(\xi m \xi + \xi^\dagger m \xi^\dagger)$, $\sim \mathrm{SU}(3)$ octet:

$$(\xi m \xi + \xi^\dagger m \xi^\dagger) \to U(x)(\xi m \xi + \xi^\dagger m \xi^\dagger)U^\dagger(x)$$

Including ρ meson

- Can introduce ρ as hidden local gauge field

$$\rho_\mu(x) \to U(x)\rho_\mu(x)U^\dagger(x) + U(x)\partial_\mu U^\dagger(x),$$

under global chiral transformations:

$$\mathcal{L}_\rho = -\frac{1}{2}\,\mathrm{Tr}(\rho_{\mu\nu}\rho^{\mu\nu}) + \frac{1}{2}\,m_\rho^2\,\mathrm{Tr}(\rho_\mu - V_\mu)^2$$

- ρ gets chirally invariant mass
- ρ_μ and V_μ : $P-$ and $C-$ odd

Chiral tr. property of a heavy quarkonium

- The final quarkonium moves very slowly in the rest frame of the initial quarkonium
- Use the heavy particle effective theory approach, by introducing a velocity dependent field $X_v(x) \equiv X e^{i m_X v \cdot x}$, and similarly for J/ψ field $\psi_v(x)$
[e.g. Casalbuoni et al. Phys.Rep.]
- If X_v is a isosinglet, then

$$X_v(x) \to X_v(x).$$

- If X_v is an isovector $I(X_v) = 1$, then

$$X_v(x) \to U(x) X_v(x) U^\dagger(x).$$

Construction of chiral lagrangian

- Then we can construct chiral invariant lagrangian using X_v, ψ_v, v_μ, $g_{\mu\nu}$, $\epsilon_{\mu\nu\alpha\beta}$ and the pion field $\Sigma(x)$ or V_μ and A_μ
- Symmetry : Lorentz tr., Chiral tr., Parity and Charge Conjugation
- Coeff. of each operator : unknown, to be fixed by experimental data
(EFT for strongly interacting underlying theory)
- Transformation properties of X_v, ψ_v and v under parity and charge conjugation are given in the following Table.

Heavy quarkonium EFT

Convenient to organize the heavy quarkonia states in a multiplet where spin symmetry is manifest
[e.g. Casalbuoni et al. Phys.Rep. (1997)]

- $S-$wave charmonium:

$$J = \left(\frac{1+ \not{v}}{2}\right) \left[\psi_\mu \gamma^\mu - \eta_c \gamma_5\right] \left(\frac{1- \not{v}}{2}\right),$$

- $P-$wave charmonium:

$$\begin{aligned} J^\mu &= \left(\frac{1+ \not{v}}{2}\right) \left[H_2^{\mu\alpha}\gamma_\alpha + \frac{1}{\sqrt{2}}\, \epsilon^{\mu\alpha\beta\gamma} v_\alpha \gamma_\beta H_{1\gamma} \right. \\ &+ \left. \frac{1}{\sqrt{3}}(\gamma^\mu - v^\mu) H_0 + K_1^\mu \gamma_5 \right] \left(\frac{1- \not{v}}{2}\right) \end{aligned}$$

Heavy quarkonium EFT

- $D-$wave charmonium:

$$
\begin{aligned}
J^{\mu\nu} &= \left(\frac{1+\not{v}}{2}\right)\left[H_3^{\mu\nu\alpha}\gamma_\alpha + \frac{1}{\sqrt{6}}\left\{\epsilon^{\mu\alpha\beta\gamma}v_\alpha\gamma_\beta H_{2\gamma}^{\nu} + (\mu \leftrightarrow \nu)\right\}\right. \\
&+ \left.\frac{1}{2}\sqrt{\frac{3}{5}}\left\{(\gamma^\mu - v^\mu)H_1^\nu + (\mu \leftrightarrow \nu)\right\} + K_2^{\mu\nu}\gamma_5\right]\left(\frac{1-\not{v}}{2}\right).
\end{aligned}
$$

- v: the velocity of heavy quarkonium
- $J^{\mu\nu} = J^{\nu\mu}$ and $v_\mu J^{\mu\nu\cdots} = 0$

Heavy quarkonium EFT

- Transformation under P and C:

$$\begin{aligned} J^{\mu\nu\ldots} &\to \gamma^0 J_{\mu\nu\ldots}\gamma^0 \quad (\text{under } P) \\ &\to (-1)^l C^{-1}\left(J^{\mu\nu\ldots}\right)^T C \quad (\text{under } C) \end{aligned}$$

- l is the orbital angular momentum between Q and $\overline{Q}$
- $C = i\gamma^0\gamma^2$ is the charge conjugation matrix for Dirac fields, satisfying the following relations:

$$\begin{aligned} -C = C^T = C^\dagger = C^{-1} \\ C\gamma_\mu C^{-1} = -\gamma_\mu^T, C\sigma_{\mu\nu}C^{-1} = -\sigma_{\mu\nu}^T, \\ C\gamma_5 C^{-1} = \gamma_5^T, C\gamma_5\gamma_\mu C^{-1} = (\gamma_5\gamma_\mu)^T, \end{aligned}$$

Transformation laws under P and C

Fields	P	C
v^μ	$v_\mu = (v^0, -\vec{v})$	$v^\mu = (v^0, \vec{v})$
$\mathcal{A}^\mu$	$-\mathcal{A}_\mu$	$\mathcal{A}^{T\mu}$
$(\mathcal{V} + ig\rho)^\mu$	$(\mathcal{V} + ig\rho)_\mu$	$-(\mathcal{V} + ig\rho)^{T\mu}$
$\langle \bar{J} J^\nu \rangle$	$\langle \bar{J} J_\nu \rangle$	$-\langle \bar{J} J^\nu \rangle$
$\langle \bar{J} J^{\alpha\beta} \rangle$	$\langle \bar{J} J_{\alpha\beta} \rangle$	$+\langle \bar{J} J^{\alpha\beta} \rangle$
$\langle \bar{J} \sigma^{\mu\nu} J^{\alpha\beta} \rangle$	$\langle \bar{J} \sigma_{\mu\nu} J_{\alpha\beta} \rangle$	$-\langle \bar{J} J^{\alpha\beta} \sigma^{\mu\nu} \rangle$

SO, HOW TO CONSTRUCT LAGRANGIAN ?

Combine $\langle \bar{J} ... J^{\mu(\nu)} \rangle$ and $\mathcal{A}_\mu$ $(\mathcal{V} + ig\rho)_\nu$, v_μ, etc. to make a lagrangian with the following symmetries:
Lorentz sym, global chiral sym, P, C, heavy quark spin sym

Well established examples

- Our approach reproduces the old calculations for $\psi' \to J/\psi\pi\pi$, $\Upsilon(2S) \to \Upsilon(1S)\pi\pi$, etc. based on QCD multipole expansion
- Can explain the double peaks in $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$ using

$$\mathcal{M} \sim \left(q^2 + bE_1E_2 + cm_\pi^2\right) \epsilon_i \cdot \epsilon_f + D\left(\epsilon_i p_1 \epsilon_f \cdot p_2 + \epsilon_i p_2 \epsilon_f \cdot p_1\right)$$

(3 papers in PRD by Ko et al, 1993-1994)

- B, C and D are fixed from the χ^2 fit to the $m_{\pi\pi}$ spectrum

Chiral lagrangian with light vector mesons

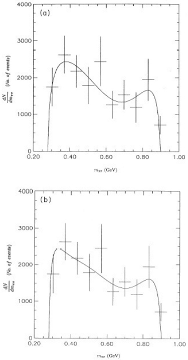


FIG. 1. The measured $\pi\pi$ spectrum in $\Upsilon(3S)\to\Upsilon(1S)\pi\pi$ [where the final $\Upsilon(1S)$ is not reconstructed] and the best fits to it using Eq. (2). (a) for $P1$ and $P2$, and (b) for $P0$. The data are taken from Ref. [12].

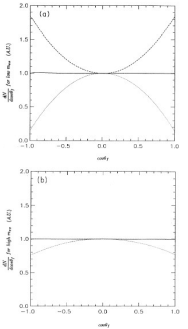


FIG. 2. The polar angle distribution of the final $\Upsilon(1S)$: (a) for low $m_{\pi\pi}$ (<0.64 GeV) and (b) for high $m_{\pi\pi}$ (>0.64 GeV). The angle θ_f is the angle between the direction of $\Upsilon(1S)$ and the beam direction in the rest frame of the initial $\Upsilon(3S)$. The solid, the dotted, and dashed curves correspond to $P0$, $P1$, and $P2$, respectively. $P0$ and $P2$ give the same distributions in (b).

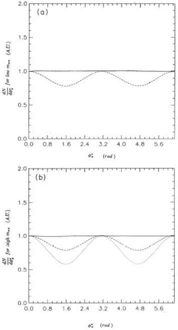


FIG. 4. The azimuthal angle (ϕ^*_π) distribution of π^+ with respect to the production plane in the rest frame of the $\pi\pi$ system: (a) for low $m_{\pi\pi}$ and (b) for high $m_{\pi\pi}$. ($\pi/2-\phi^*_\pi$) is the angle between the production plane and the plane formed by the two recoiling pions. The solid, dotted, and dashed curves correspond to $P0$, $P1$, and $P2$, respectively. $P0$ and $P1$ give the same distributions in (a).

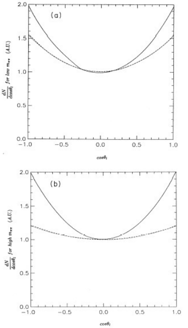
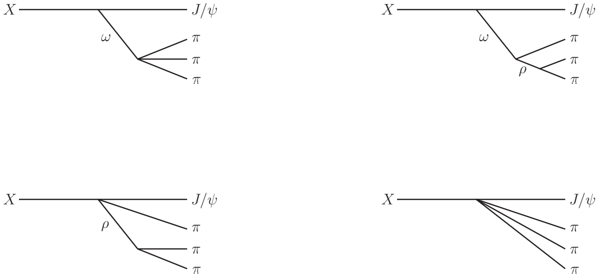


FIG. 5. The polar angle distribution of μ^+ emerging from the final $\Upsilon(1S)$: (a) for low $m_{\pi\pi}$ and (b) for high $m_{\pi\pi}$. θ_l is the angle between the direction of μ^+ and the beam direction in the rest frame of the initial $\Upsilon(3S)$. The solid, dotted, and dashed curves correspond to $P0$, $P1$, and $P2$, respectively. $P1$ and $P2$ give the same distributions.

Feynman diagrams for $X(3872) \to J/\psi\pi\pi\pi$

Now let us consider $X(3872) \to J/\psi\pi\pi\pi$ for $I(X) = 0$ and $C = +1$



$J^{PC}(X) = 1^{++}$: P-wave charmonium

- Interaction lagrangian: ω pole dominance

$$\begin{aligned}\mathcal{L}_{\rm int} &= A\langle \bar{J} J^{\mu}\rangle \, \langle (\mathcal{V}_{\mu} + ig\rho_{\mu})\rangle \\ &= A\epsilon^{\mu\nu\alpha\beta} \, v_{\mu}{\rm Tr}[(\mathcal{V} + ig\rho)_{\nu}] \, X_{v\alpha} \, \psi_{v\beta}.\end{aligned}$$

- Amplitude:

$$\begin{aligned}\mathcal{M} &= i\frac{2Ag^2 h_{\tilde{\omega}}}{f_{\pi}^3}\epsilon^{\mu\nu\alpha\beta} v_{\nu}\epsilon^{*}_{\psi\alpha}\epsilon_{X\beta}\epsilon_{\mu\nu'\alpha'\beta'} p_1^{\nu'} p_2^{\alpha'} p_3^{\beta'} \\ &\times \frac{1}{(p_X - p_{\psi})^2 - m_{\omega}^2 + im_{\omega}\Gamma_{\omega}} \\ &\times \left[3(-c_1 + c_2 + c_3) + c_3 \sum_{i=1}^{3} \frac{4gg_{\tilde{\rho}\pi\pi} f_{\pi}^2}{(p_X - p_{\psi} - p_i)^2 - m_{\rho}^2 + im_{\rho}\Gamma_{\rho}} \right]\end{aligned}$$

$J^{PC}(X) = 2^{-+}$: D-wave charmonium

- If $X(3872)$ is a spin-singlet $D-$wave with $J^{PC} = 2^{-+}$, the transition from X to J/ψ occurs through the gluonic E1 – M1 transition. Therefore the amplitude will involve the heavy quark spin flip factor, $\sim \Lambda_{\mathrm{QCD}}/m_c$.
- In order to contract all the Lorentz indices, we could introduce a number of v's, and contract them. Antisymmetry property of Levi-Civita tensor and $v_\mu X^{\mu\nu} = 0$ make these contractions zero.

$X(3872) \to J/\psi\pi\pi\pi$ for $J^{PC}(X) = 2^{-+}$

- Therefore, we have to consider one more derivative acting on $\mathcal{V}^\sigma$ or $(\mathcal{V} + ig\rho)^\sigma$:

$$
\begin{aligned}
\rho_{\mu\nu} &= D_\mu \rho_\nu - D_\nu \rho_\mu, \\
\mathcal{V}_{\mu\nu} &= D_\mu \mathcal{V}_\nu - D_\nu \mathcal{V}_\mu
\end{aligned}
$$

- These transforms homogeneously under chiral $SU(3)_L \times SU(3)_R$:

$$
\begin{aligned}
\rho_{\mu\nu} &\to U(x)\rho_{\mu\nu}U^\dagger(x), \\
\mathcal{V}_{\mu\nu} &\to U(x)\mathcal{V}_{\mu\nu}U^\dagger(x),
\end{aligned}
$$

$X(3872) \to J/\psi\pi\pi\pi$ for $J^{PC}(X) = 2^{-+}$

- Lagrangian:

$$\frac{1}{2m_c}\langle \bar{J}(J^{\mu\nu}\sigma^{\alpha\beta} + \sigma^{\alpha\beta}J^{\mu\nu})\rangle$$
$$\langle \mathcal{V}^{\rho\sigma}\rangle, \quad \langle ig\rho^{\rho\sigma}\rangle, \quad \langle \mathcal{A}^{\rho}(\mathcal{V} + ig\rho)^{\sigma}\rangle$$

  with all possible Lorentz contractions with a number of v's, $g_{\gamma\delta}$ and $\epsilon_{\gamma\delta\kappa\tau}$

- This is the most general lagrangin with lowest order in pion or vector meson momentum, and invariant under heavy quark symmetry, chiral symmetry, parity (P) and charge conjugation (C).

There are eight independent interaction terms:

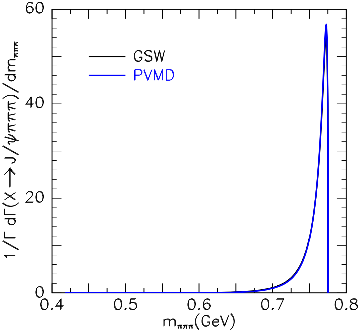
$$
\begin{aligned}
\mathcal{L}_{\rm int} &= d_0 \frac{g}{2m_c} \langle \bar{J}(J^{\mu}{}_{\nu}\sigma^{\alpha\beta} + \sigma^{\alpha\beta} J^{\mu}{}_{\nu})\rangle \langle \rho_{\mu\alpha}\rangle \\
&+ d_1 \frac{1}{2m_c} \epsilon_{\mu\alpha\rho\sigma} \langle \bar{J}(J^{\mu}{}_{\nu}\sigma^{\alpha\nu} + \sigma^{\alpha\nu} J^{\mu}{}_{\nu})\rangle \langle \mathcal{A}^{\rho}(\mathcal{V} + ig\rho)^{\sigma}\rangle \\
&+ d_2 \frac{1}{2m_c} \epsilon_{\mu\alpha\beta\sigma} \langle \bar{J}(J^{\mu}{}_{\nu}\sigma^{\alpha\beta} + \sigma^{\alpha\beta} J^{\mu}{}_{\nu})\rangle \langle \mathcal{A}^{\nu}(\mathcal{V} + ig\rho)^{\sigma}\rangle \\
&+ d_3 \frac{1}{2m_c} \epsilon_{\mu\alpha\beta\rho} \langle \bar{J}(J^{\mu}{}_{\nu}\sigma^{\alpha\beta} + \sigma^{\alpha\beta} J^{\mu}{}_{\nu})\rangle \langle \mathcal{A}^{\rho}(\mathcal{V} + ig\rho)^{\nu}\rangle \\
&+ d_4 \frac{1}{2m_c} \epsilon_{\mu\alpha\rho\lambda} \langle \bar{J}(J^{\mu}{}_{\nu}\sigma^{\alpha\nu} + \sigma^{\alpha\nu} J^{\mu}{}_{\nu})\rangle \langle \mathcal{A}^{\rho}(\mathcal{V} + ig\rho)^{\sigma}\rangle v_{\sigma} v^{\lambda} \\
&+ d_5 \frac{1}{2m_c} \epsilon_{\mu\alpha\sigma\lambda} \langle \bar{J}(J^{\mu}{}_{\nu}\sigma^{\alpha\nu} + \sigma^{\alpha\nu} J^{\mu}{}_{\nu})\rangle \langle \mathcal{A}^{\rho}(\mathcal{V} + ig\rho)^{\sigma}\rangle v_{\rho} v^{\lambda} \\
&+ d_6 \frac{1}{2m_c} \epsilon_{\mu\alpha\beta\lambda} \langle \bar{J}(J^{\mu}{}_{\nu}\sigma^{\alpha\nu} + \sigma^{\alpha\nu} J^{\mu}{}_{\nu})\rangle \langle \mathcal{A}^{\nu}(\mathcal{V} + ig\rho)^{\sigma}\rangle v_{\sigma} v^{\lambda} \\
&+ d_7 \frac{1}{2m_c} \epsilon_{\mu\alpha\beta\lambda} \langle \bar{J}(J^{\mu}{}_{\nu}\sigma^{\alpha\nu} + \sigma^{\alpha\nu} J^{\mu}{}_{\nu})\rangle \langle \mathcal{A}^{\rho}(\mathcal{V} + ig\rho)^{\nu}\rangle v_{\rho} v^{\lambda},
\end{aligned}
$$

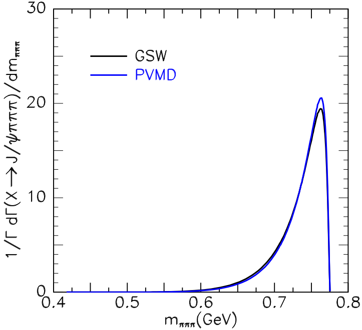
$X(3872) \to J/\psi\pi\pi\pi$ for $J^{PC}(X) = 2^{-+}$

- Amplitude from d_0 only:

$$\begin{aligned}
\mathcal{M} &= i\frac{2Ag^2 h_{\tilde{\omega}}}{f_\pi^3}\epsilon_{X\sigma\nu}\epsilon^{\mu\nu\alpha\beta}\,\epsilon^*_{\psi\alpha}v_\beta(q^\sigma_\omega\epsilon_\mu^{\nu'\alpha'\beta'} - q_{\omega\mu}\epsilon^{\sigma\nu'\alpha'\beta'})p_1^{\nu'}p_2^{\alpha'}p_3^{\beta'} \\
&\times \frac{1}{(p_X - p_\psi)^2 - m_\omega^2 + im_\omega\Gamma_\omega} \\
&\times \left[3(-c_1 + c_2 + c_3) + c_3\sum_{i=1}^{3}\frac{4gg_{\tilde{\rho}\pi\pi}f_\pi^2}{(p_X - p_\psi - p_i)^2 - m_\rho^2 + im_\rho\Gamma_\rho}\right]
\end{aligned}$$

Tripion invariant mass spectra

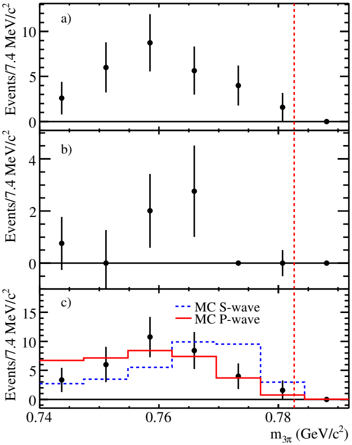


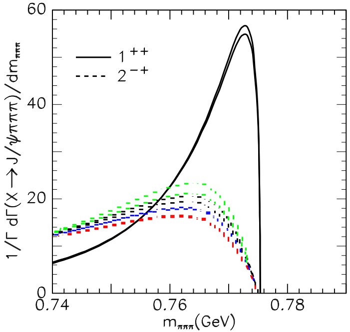


- GSW: $c1 = 5.5c, c2 = 0(i.e. c1 - c2 = 5.5c), c3 = -15c$ where $c = -N_c/(240\pi^2)$
- Another choice: $c1 = 7.5c, c2 = -7.5c, c3 = -15c$

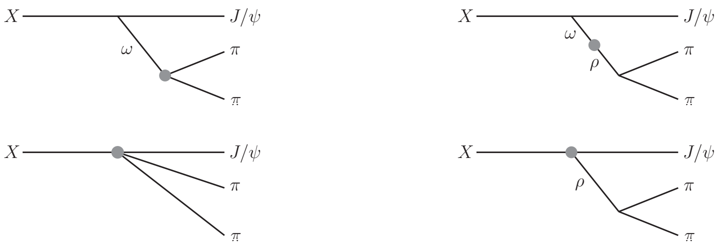
Tripion invariant mass spectra: Comparison with BaBar data

- Can we tell which is favored ?





Feynman diagrams for $X(3872) \to J/\psi\pi\pi$



- For $I(X) = 0$, this decay is isosping violating, which is indicated by blobs
- Isospin violation in the decay vertices is included (New aspect of our approach)
- ρ and $\pi\pi$ always come in together because of chiral symmetry

$J^{PC}(X) = 1^{++}$: P-wave charmonium

- Lagrangian:

$$\mathcal{L}_{\mathrm{int}} = A\frac{c_1}{\Lambda_\chi^2}\langle \bar{J}J^\mu\rangle \,\langle(\mathcal{V}_\mu + ig\rho_\mu)\,\mu m_\xi)\rangle$$

- Amplitude:

$$\mathcal{M}_{X\to J/\psi\pi\pi} = -A\frac{\mu(m_u - m_d)}{2\Lambda_\chi^2}\,\epsilon^{\mu\nu\alpha\beta}v_\mu(p_{1\nu}-p_{2\nu})\epsilon^*_{\psi\alpha}\epsilon_{X\beta}\,F(k,k^{'},p_1,p_2)$$

- The form factor F:

$$F(k,k^{'},p_1,p_2) = c_0 + c_1\frac{m_\rho^2}{q^2 - m_\rho^2 + im_\rho\Gamma_\rho} + c_2\frac{m_\omega^2}{q^2 - m_\omega^2 + im_\omega\Gamma_\omega}$$

- $c_{i=0,1,2}$'s : $\sim O(1)$ parameters
- $c_0 - c_1 - c_2 = 0$ from the soft pion theorem

$J^{PC}(X) = 1^{++}$: P-wave charmonium

- The amplitude squared with the average over the initial spin and the sum of the final spin:

$$\begin{aligned}
\overline{|\mathcal{M}|^2} &= A^2 \frac{(\mu m_u - \mu m_d)^2}{4\Lambda_\chi^4} |F(k,k',p_1,p_2)|^2 \\
&\times \Big[2(m_\pi^4 - tu)(m_X^2 + m_X^2 r_\psi^2 - s) \\
&\quad + m_X^2 \{ 2m_\pi^2 (1 + r_\psi^2)(m_\pi^2 - t - u) + r_\psi^2 (t^2 + u^2) + 2tu \} \\
&\quad - 2m_X^4 r_\psi^2 (t + u + 2m_\pi^2) + 2m_X^6 r_\psi^2 (1 + r_\psi^2) \Big] ,
\end{aligned}$$

where the Mandelstam variables s, t, and u are defined by

$$s = (k - k')^2, \quad t = (k - p_1)^2, \quad u = (k - p_2)^2.$$

$J^{PC}(X) = 2^{-+}$: D-wave charmonium

- The relevant lagrangian is

$$\begin{aligned}\mathcal{L}_{\rm int} &= \frac{1}{\Lambda_\chi}\langle \bar{J}(J^{\mu\nu}\sigma^{\alpha\beta} + \sigma^{\alpha\beta}J^{\mu\nu})\rangle \\ &\times \left[c_0\langle(\mathcal{V}^{\rho\sigma} + ig\rho^{\rho\sigma})\rangle + \frac{1}{\Lambda_\chi^2}\langle(c_1\rho^{\rho\sigma} + c_2\mathcal{V}^{\rho\sigma})\,\mu m_\xi)\rangle\right]\end{aligned}$$

- The amplitude can be written as

$$\begin{aligned}\mathcal{M} &= \frac{(m_u - m_d)}{f_\pi^2}\epsilon_\nu^{X\alpha}(p)\epsilon^{\mu\nu\rho\sigma}\epsilon_\rho^{\psi\dagger}v_\sigma\ \{q_\alpha(p_1 - p_2)_\mu - q_\mu(p_1 - p_2)_\alpha\} \\ &\times\ F(k, k^{'}, p_1, p_2),\end{aligned}$$

$J^{PC}(X) = 2^{-+}$: D-wave charmonium

- The same form factor enter here as before (the case with $J^{PC} = 1^{++}$)

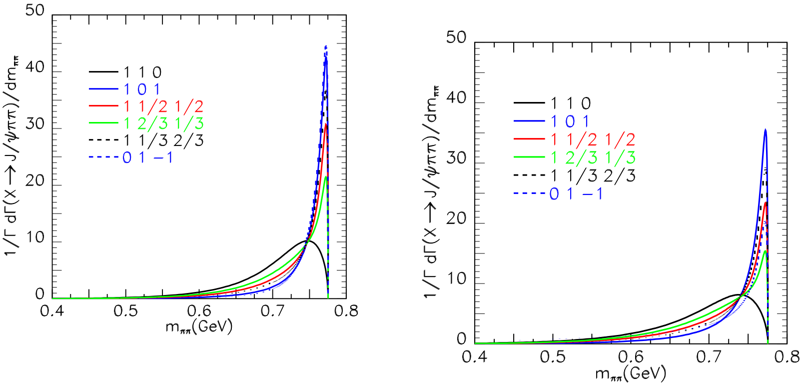
$$F(k, k^{'}, p_1, p_2) = c_0 + c_1 \frac{m_\rho^2}{q^2 - m_\rho^2 + i m_\rho \Gamma_\rho} + c_2 \frac{m_\omega^2}{q^2 - m_\omega^2 + i m_\omega \Gamma_\omega}$$

- Squared amplitude:

$$\begin{aligned}
\overline{|\mathcal{M}|^2} &= \frac{(\mu m_u - \mu m_d)^2}{f_\pi^4} |F(k, k', p_1, p_2)|^2 \\
&\times [m_X^4(3 + 34 r_\psi^2 + 3 r_\psi^4) - 6 m_X^2 s(1 + r_\psi^2) + 3 s^2] \\
&\times [m_X^4 m_\pi^2 (1 - r_\psi^2)^2 + m_X^2 m_\pi^2 s(1 + r_\psi^2) - s(m_X^4 r_\psi^2 + m_\pi^4) +
\end{aligned}$$

Dipion invariant mass spectra in $X(3872) \to J/\psi\pi\pi$

- Difficult to distinguished two cases if $I(X) = 0$
- Need detailed χ^2 fit to the $m_{\pi\pi}$ spectrum



The case for $I(X) = 1$ and $C = +1$

- If $I(X) = 1$, then G-parity will forbid $X_{I=1} \to J/\psi\pi\pi\pi$
- Dipion transition can occur through ρ-dominance

$$F(p_1, p_2) = 1 + \frac{m_\rho^2}{m_\rho^2 - q^2 + i m_\rho \Gamma_\rho}$$

- A special case of the dipion transition form factor with $c_2 = 0$
- Form factor vanishes as $p_\pi \to 0$ in the massless pion limit (consistent with soft pion theorem, namely pions are Nambu-Goldstone boson associated with spontaneously broken chiral symmetry of QCD)

What if $|X\rangle = (|I=0\rangle + |I=1\rangle)/\sqrt{2}$?

- Assume

$$|X(3872)\rangle = C_0|I=0\rangle + C_1|I=1\rangle.$$

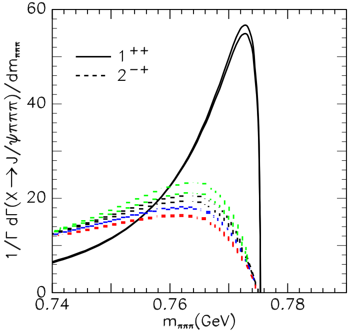
- For example, if $X(3872)$ is an equal admixture of $I=0$ and $I=1$ such as in DD^* molecule picture, we have $C_0 = C_1 = 1/\sqrt{2}$.
- In this case, we can combine the amplitudes for $X_{I=0} \to J/\psi\pi\pi(\pi)$ and $X_{I=1} \to J/\psi\pi\pi(\pi)$
- Since isospin violation is a tiny effect, we can assume that

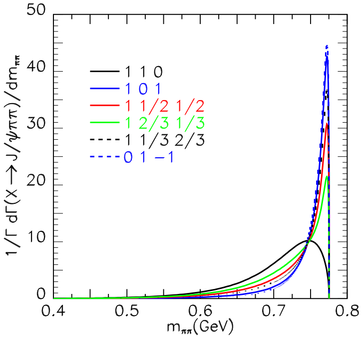
$$\begin{aligned} X_{I=0} &\to J/\psi\pi\pi\pi \\ X_{I=1} &\to J/\psi\pi\pi \end{aligned}$$

- Then there are not much ambiguity in the 2π or 3π invariant mass spectrum $\to$ Reliable predictions

What if $|X\rangle = (|I=0\rangle + |I=1\rangle)/\sqrt{2}$?

- Tripion and dipion invariant mass spectra follow the black curves





Summary

- Tripion invariant mass spectra for $I(X) = 0$ are quite robust (especially for 1^{++}), and we probably have to wait for more data
- It is difficult to understand why $M_X \approx M(D^0) + M(D^*)$, if it is $J^{PC} = 2^{-+}$ with $I(X) = 0$
- Dipion invariant mass spectra for $I(X) = 0, C(X) = +1$ are not so helpful, unlike the case for $I(X) = 1$ or $C(X) = -1$, since both ρ and ω contribute with isospin violation in the mixing and the vertices
- The DD^* molecule picture with $I = 0$ and $I = 1$ is still alive, with

$$X_{I=0} \to 3\pi, \quad X_{I=1} \to 2\pi$$

- Some informations from the rates ...

Outlook

- In most of my talk, I assumed that $X(3872)$ is a pure charmonium with $I = 0$ and $C = +1$, and studied the tripion and dipion invariant mass spectra
- This may not be a good picture for $I = 1$ case, for example, for DD^* molecule picture
- Our approach would be OK as long as J^{PC} and I are concerned
- DD^* molecule picture could be included using EFT for heavy hadrons and heavy quarkonia, and construct another EFT for $X(3872)$, similar to the NRQCD
- A new work in this direction on $X(3872) \to \chi_b \pi\pi$ by Fleming and Mehen