

Electric dipole transitions of heavy quarkonium

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Why should one study EM transitions?

- information about the quarkonium spectrum and the wave-functions
- new experimental data provided in the last and next few years (CLEO, BES, B factories)

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- lattice QCD (quenched):
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- EFT treatment of radiative decays: pNRQCD
→ M1 transitions
Brambilla, Jia, Vairo, Phys.Rev.D 73 (2006)
→ still missing: treatment of E1 transitions

Outline

1 Basic formalism

- Effective Field Theory approach to heavy quarkonium
- Quarkonium states and transitions

2 E1 transitions

- Definition & non-relativistic limit
- Matching of the Lagrangian
- Wave-function corrections
- Results

Basic formalism

EFT for heavy quarkonium
Description of decay processes

Scales in quarkonium

- separation of scales in heavy quarkonium

$$m \gg p \sim mv \gg E \sim mv^2$$

where $v^2 \ll 1$ ($v^2 \approx 0.1$ for $b\bar{b}$, $v^2 \approx 0.3$ for $c\bar{c}$)

- systematic treatment of relativistic corrections in powers of v
- language of effective field theories appropriate

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- weakly coupled quarkonia ($p \gg E \gtrsim \Lambda_{QCD}$)

- perturbative treatment with Coulomb potential at leading order
(valid for the ground states J/ψ , $\Upsilon(1S)$, η_c , η_b)

$$\alpha_s(m) \sim v^2$$

$$\alpha_s(mv) \sim v$$

$$\alpha_s(mv^2) \sim 1$$

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- additional scale: photon energy $k_\gamma \sim mv^2$ (different principal quantum number) or $k_\gamma \sim mv^3$, mv^4 (same principal quantum number)

Effective field theories for quarkonium

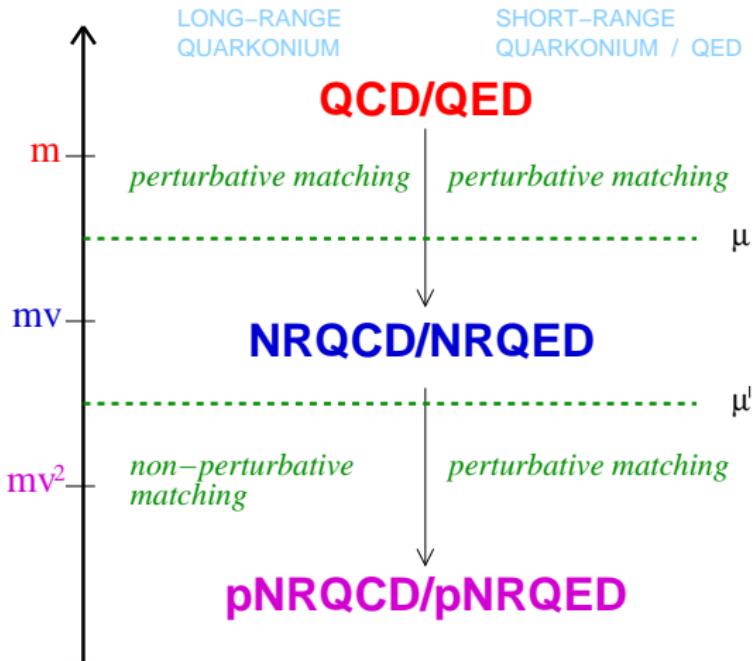


Figure: A.Vairo, arXiv 0902.3346 (2009)

NRQCD

- integrate out energy & momentum modes of order m from QCD
- Lagrangian

$$\begin{aligned}\mathcal{L} = & \psi^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2m} + \frac{\mathbf{D}^4}{8m^3} + \dots \right) \psi \\ & + g\psi^\dagger \left(\frac{c_F}{2m} \boldsymbol{\sigma} \cdot \mathbf{B} + i \frac{c_s}{8m^2} \boldsymbol{\sigma} \cdot [\mathbf{D} \times, \mathbf{E}] + \dots \right) \psi \\ & + ee_Q \psi^\dagger \left(\frac{c_F^{em}}{2m} \boldsymbol{\sigma} \cdot \mathbf{B}^{em} + i \frac{c_s^{em}}{8m^2} \boldsymbol{\sigma} \cdot [\mathbf{D} \times, \mathbf{E}^{em}] + \dots \right) \psi \\ & + c.c. + \mathcal{L}_{\text{light}} + \mathcal{L}_{\text{YM}}\end{aligned}$$

coefficients by matching with QCD

pNRQCD (for weak coupling)

- integrate out
 - quarks with energy & momentum $\sim mv$
 - gluons & photons of energy or momentum $\sim mv$
- new degrees of freedom: $Q\bar{Q}$ color singlet and octet fields

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- new degrees of freedom: $Q\bar{Q}$ color singlet and octet fields
- Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{pNRQCD}} = & \int d^3r \text{Tr} \left\{ S^\dagger \left(i\partial_0 + \frac{\nabla^2}{4m} + \frac{\nabla_r^2}{m} - V_S \right) S \right. \\ & + O^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{4m} + \frac{\nabla_r^2}{m} - V_O \right) O \\ & + gV_A(O^\dagger \mathbf{r} \cdot \mathbf{E} S + S^\dagger \mathbf{r} \cdot \mathbf{E} O) \\ & \left. + gV_B \frac{\{O^\dagger, \mathbf{r} \cdot \mathbf{E}\}}{2} O + \dots \right\} \\ & + \mathcal{L}_{\gamma\text{pNRQCD}} + \mathcal{L}_{\text{light}} + \mathcal{L}_{\text{YM}}\end{aligned}$$

pNRQCD (for weak coupling)

- now: Only relevant degrees of freedom present
- high energy dynamics encoded in Wilson coefficients (obtained by matching with NRQCD at energy mv)
- definite power counting of operators

$$\begin{aligned} r &\sim 1/mv \\ \mathbf{E}, \mathbf{B} &\sim (mv^2)^2 \\ \mathbf{E}^{em}, \mathbf{B}^{em} &\sim k_\gamma^2 \\ \nabla = \partial/\partial \mathbf{R} &\sim mv^2, k_\gamma \end{aligned}$$

Quarkonium states and transitions

- quarkonium state (leading Fock space component):

$$|H(\mathbf{P}, \lambda)\rangle = \int d^3R \int d^3r e^{i\mathbf{P}\cdot\mathbf{R}} \text{Tr} \left\{ \phi_{H(\lambda)}(\mathbf{r}) S^\dagger(\mathbf{r}, \mathbf{R}) |0\rangle \right\}$$

- at leading order:

$$H_S^{(0)} \phi_{H(\lambda)}^{(0)} = \left(-\frac{\nabla_r^2}{m} + V_S^{(0)} \right) \phi_{H(\lambda)}^{(0)} = E_{H(\lambda)}^{(0)} \phi_{H(\lambda)}^{(0)}$$

- at higher orders: wave-function corrections due to higher order potentials and singlet-octet transitions

→ calculation of decay rates for $H \rightarrow H' \gamma$ in CM frame

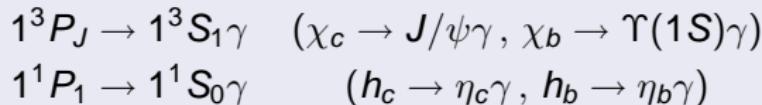
E1 Transitions

Brambilla, Pietrulewicz, Vairo, preprint: TUM-EFT 25/11

General properties

- definition: $\Delta S = 0, |\Delta L| = 1$
- change in parity, no change in C parity

Examples



- for the considered transitions: $k_\gamma \sim mv^2$

Nonrelativistic limit

- leading order operator for E1 transitions

$$\mathcal{L}_{E1} = e e_Q \int d^3r \text{Tr} \{ S^\dagger \mathbf{r} \cdot \mathbf{E}^{em} S \}$$

Nonrelativistic decay rate

$$\Gamma_{n^3P_{J=0,1,2} \rightarrow n'^3S_1\gamma} = \frac{4}{9} \alpha_{em} e_Q^2 k_\gamma^3 I_3^2(n1 \rightarrow n'0) \sim \frac{k_\gamma^3}{m^2 v^2}$$

$$I_3(n1 \rightarrow n'0) = \int_0^\infty dr r^3 R_{n'0}(r) R_{n1}(r)$$

- differences to M1 transitions:
 - leading order amplitude depends on the wave-function
 - enhancement of E1 transitions by factor $1/v^2$
- now: relativistic corrections of $\mathcal{O}(v^2)$

Relevant pNRQCD Lagrangian for decays of order k_γ^3/m^2

$$\begin{aligned} \mathcal{L}_{\gamma\text{pNRQCD}}^{E1} = & ee_Q \int d^3r \text{Tr} \left\{ V^{r \cdot E} S^\dagger \mathbf{r} \cdot \mathbf{E}^{em} S + V_O^{r \cdot E} O^\dagger \mathbf{r} \cdot \mathbf{E}^{em} O \right. \\ & + \frac{1}{24} V^{(r\nabla)^2 r \cdot E} S^\dagger \mathbf{r} \cdot (\mathbf{r}\nabla)^2 \mathbf{E}^{em} S \\ & + i \frac{1}{4m} V^{\nabla \cdot (r \times B)} S^\dagger \{ \nabla \cdot, \mathbf{r} \times \mathbf{B}^{em} \} S \\ & + i \frac{1}{12m} V^{\nabla_r \cdot (r \times (r\nabla)B)} S^\dagger \{ \nabla_r \cdot, \mathbf{r} \times (\mathbf{r}\nabla) \mathbf{B}^{em} \} S \\ & + \frac{1}{4m} V^{(r\nabla)\sigma \cdot B} [S^\dagger, \sigma] \cdot (\mathbf{r}\nabla) \mathbf{B}^{em} S \\ & + \frac{1}{mr} V^{r \cdot E/r} S^\dagger \mathbf{r} \cdot \mathbf{E}^{em} S \\ & \left. - i \frac{1}{4m^2} V^{\sigma \cdot (E \times \nabla_r)} [S^\dagger, \sigma] \cdot (\mathbf{E}^{em} \times \nabla_r) S \right\} \end{aligned}$$

Tree level matching

- project NRQCD Hamiltonian onto the subspace spanned by $\Psi_{\alpha\beta}(\mathbf{x}_1, \mathbf{x}_2, t) \sim \psi_\alpha(\mathbf{x}_1, t)\chi_\beta^\dagger(\mathbf{x}_2, t)$
- decompose $\Psi_{\alpha\beta}(\mathbf{x}_1, \mathbf{x}_2, t)$ into singlet and octet field components
- multipole expand in $r \ll 1/E$

Tree level results

$$\begin{aligned} V_A = V^{r \cdot E} = V_O^{r \cdot E} = V^{(r \nabla)^2 r \cdot E} &= 1 \\ V^{\nabla \cdot (r \times B)} = V^{(r \nabla) \nabla r \cdot (r \times B)} &= 1 \\ V^{(r \nabla) \sigma \cdot B} &= c_F^{em} \\ V^{\sigma \cdot (E \times \nabla_r)} &= c_S^{em} \\ V^{r \cdot E / r} &= 0 \end{aligned}$$

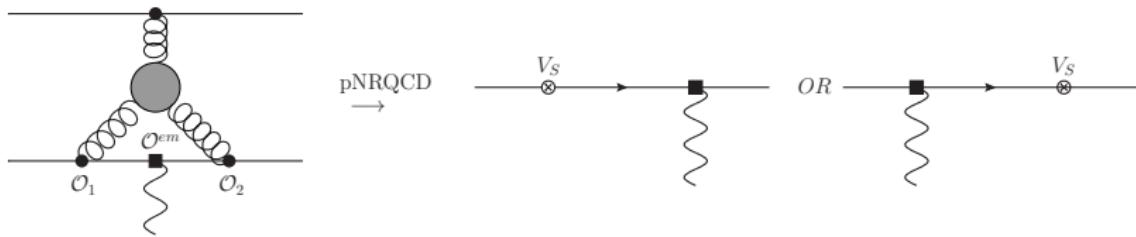
Beyond tree level

- matching of amplitudes order by order in $1/m$
- required for the perturbative matching at $\mathcal{O}(v^2)$:
 - $\mathcal{O}(\alpha_s^2)$ corrections to $V^{r \cdot E}$
 - $\mathcal{O}(\alpha_s)$ corrections to $V^{r \cdot E/r}$
- But: exact relations for all relevant coefficients can be obtained
- crucial argument: factorization of amplitudes into electromagnetic and gluonic terms

General factorization argument

$$[\mathcal{O}^{em}, \mathcal{O}_1] = 0 \text{ OR } [\mathcal{O}^{em}, \mathcal{O}_2] = 0$$

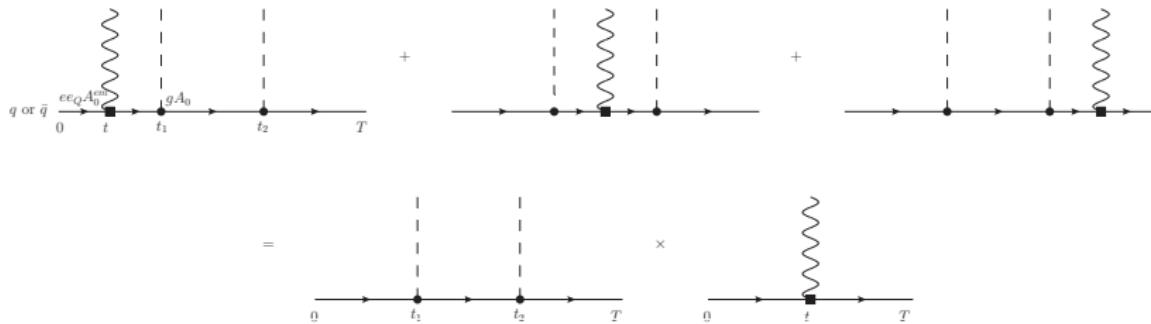
⇒ the amplitude factorizes and gives no contribution to the matching of single operators (valid to all orders in α_s and to first order in α_{em})



Matching of the electric dipole operator

Example: Exact matching of $V^{r \cdot E}$ possible (at order $1/m^0$)

Trivial factorization: $[A_0, A_0^{em}] = 0$



$\rightarrow V^{r \cdot E} = 1$ to all orders in α_s

Similar arguments for all relevant operators
 \Rightarrow tree level results = exact results (for E1)

Wave-function corrections

- corrections at $\mathcal{O}(v^2)$ due to higher order potentials in $1/m$

$$\delta V_S(r) = \frac{V_r^{(1)}(r)}{m} + \frac{V_{SI}^{(2)}(r)}{m^2} + \frac{V_{SD}^{(2)}(r)}{m^2}$$

$$V_{SI}^{(2)}(r) = V_r^{(2)}(r) + \frac{1}{2}\{V_{p^2}^{(2)}(r), \mathbf{p}^2\} + \frac{V_{L^2}^{(2)}(r)}{r^2} \mathbf{L}^2$$

$$V_{SD}^{(2)}(r) = V_{LS}^{(2)}(r) \mathbf{L} \cdot \mathbf{S} + V_{S^2}^{(2)}(r) \mathbf{S}^2 + V_{S_{12}}^{(2)}(r) [3(\hat{\mathbf{r}} \cdot \boldsymbol{\sigma}_1)(\hat{\mathbf{r}} \cdot \boldsymbol{\sigma}_2) - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2]$$

- relativistic kinetic energy correction

$$\delta H_S(r) = -\frac{\mathbf{p}^4}{4m^3}$$

- calculation with QM perturbation theory

Color-octet effects

- higher Fock space components via singlet-octet transitions

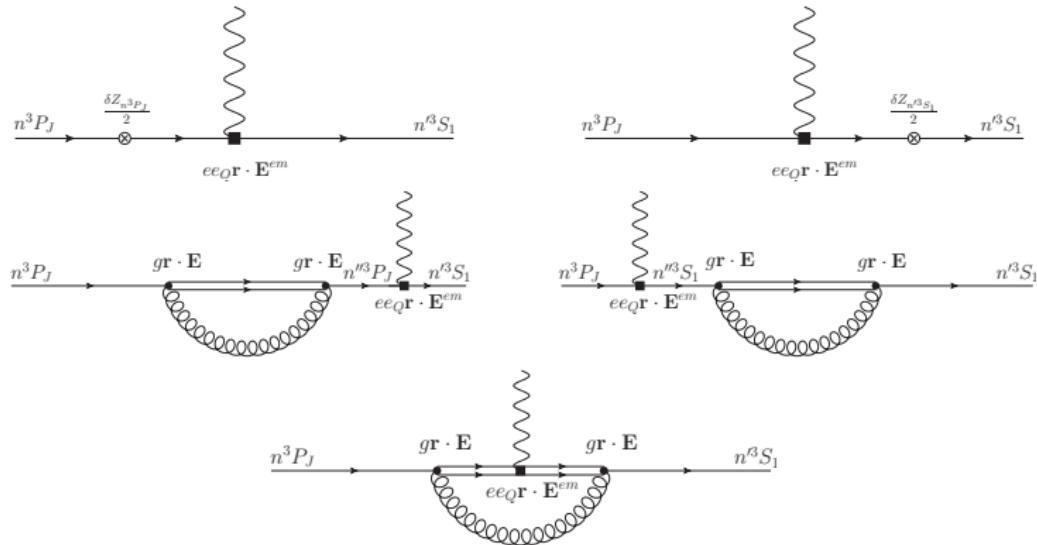
$$\mathcal{L} = \int d^3r \text{Tr} \{ O^\dagger \mathbf{r} \cdot g \mathbf{E} S + S^\dagger \mathbf{r} \cdot g \mathbf{E} O \}$$

- not present in potential model approach
- no cancellation as for M1 transitions
- non-perturbative input (chromoelectric field correlators)

$$\langle 0 | \mathbf{E}^a(\mathbf{R}, t) \phi(t, 0)_{ab}^{\text{adj}} \mathbf{E}^b(\mathbf{R}, 0) | 0 \rangle$$

Color-octet effects

Example: $n^3P_J \rightarrow n'^3S_1\gamma$



Strong coupling case

- strongly coupled quarkonia ($p \gtrsim \Lambda_{QCD} \gg E$ or $p \gg \Lambda_{QCD} \gg E$)
→ non-perturbative treatment with confining potential at leading order (valid for excited states χ_c, χ_b, \dots)
- non-perturbative potentials taken from lattice simulations
- no octet fields
- matching for the relevant operators as before
- for $\Lambda_{QCD} \sim mv$ new operators become relevant (for a conservative non-perturbative power counting)
Ex.: $-\frac{ee_0g^2}{2m} S^\dagger(\mathbf{r} \cdot \mathbf{B})(\mathbf{r} \times \mathbf{B}) \cdot (\mathbf{r} \times \mathbf{B}^{em})S$

Results

Final formula for $n^3P_J \rightarrow n'^3S_1\gamma$

$$\Gamma_{E1} = \Gamma_{E1}^{(0)} \left(1 + R - \frac{k_\gamma^2}{60} \frac{I_5}{I_3} - \frac{k_\gamma}{6m} + \frac{k_\gamma(c_F^{em} - 1)}{2m} \left[\frac{J(J+1)}{2} - 2 \right] \right)$$

$$I_N(n1 \rightarrow n'0) = \int_0^\infty dr r^N R_{n'0}(r) R_{n1}(r)$$

$R \rightarrow$ wave function corrections

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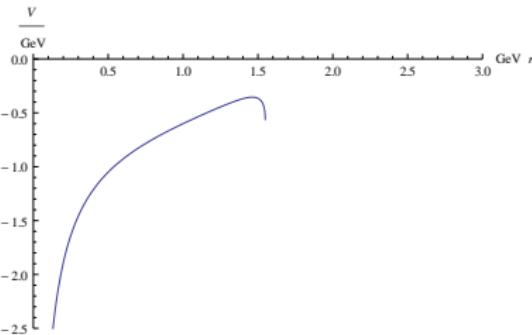
- comparison with potential models (Grotch):
equivalence to the given order, but:
 - range of validity ($E \gtrsim \Lambda_{QCD}$)
 - systematic inclusion of relativistic corrections (including $V_r^{(1)}$)
 - color-octet effects included for weak coupling
- analogous formulas for $n^1P_1 \rightarrow n'^1S_0\gamma$, $n^3S_1 \rightarrow n'^3P_J\gamma$ and $n^1S_0 \rightarrow n'^1P_1\gamma$

Applications

- perturbative potentials for short and non-perturbative ones for long distances

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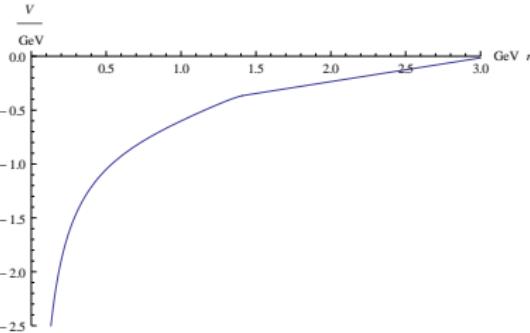
- perturbative potentials for short and non-perturbative ones for long distances
- static potential



→ renormalon subtracted potential at NNLL for short distances
Pineda, J.Phys.G 29 (2003)

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Pineda, J.Phys.G 29 (2003)

→ linear, confining potential at large distances

- subleading potentials: LL for short distances, expressions from string model for large distances

Perez-Nadal, Soto, Phys.Rev.D 79 (2009)

Numerical results

- Fit to spectrum gives string tension σ and the masses m_c , m_b in our scheme
- Preliminary results for charmonium

process	$\Gamma^{\text{pNRQCD}}/\text{keV}$	$\Gamma_{\text{Grotch}}^{\text{mod}}/\text{keV}$	$\Gamma_{\text{Dudek}}^{\text{lat}}/\text{keV}$	$\Gamma_{\text{PDG}}^{\text{exp}}/\text{keV}$
$\chi_{c0}(1P) \rightarrow J/\psi\gamma$	$138 \pm 40 \pm 7$	162-183	232 ± 41	122 ± 11
$\chi_{c1}(1P) \rightarrow J/\psi\gamma$	$256 \pm 85 \pm 13$	340-363	487 ± 122	296 ± 22
$\chi_{c2}(1P) \rightarrow J/\psi\gamma$	$340 \pm 115 \pm 17$	413-464	-	384 ± 27
$h_c \rightarrow \eta_c(1S)\gamma$	$292 \pm 184 \pm 15$	-	601 ± 55	<600

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- Preliminary results for bottomonium

process	$\Gamma^{\text{pNRQCD}}/\text{keV}$	$\Gamma^{\text{mod}}_{\text{Grotch}}/\text{keV}$	$\mathcal{B}_{\text{CLEO}}^{\text{exp}} \times \Gamma^{\text{mod}}_{\text{tot,Rosner}}/\text{keV}$
$\chi_{b0}(1P) \rightarrow \Upsilon(1S)\gamma$	$24.2 \pm 2.1 \pm 0.5$	25.7-27.0	21.1 ± 4.3
$\chi_{b1}(1P) \rightarrow \Upsilon(1S)\gamma$	$27.0 \pm 2.7 \pm 0.5$	29.8-31.2	32.0 ± 2.5
$\chi_{b2}(1P) \rightarrow \Upsilon(1S)\gamma$	$29.7 \pm 3.1 \pm 0.5$	33.0-34.2	41.1 ± 3.1

Conclusion and Outlook

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EFT treatment for E1 transitions up to $\mathcal{O}(\nu^2)$ -corrections
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Thank you for your attention!

Wave-functions

- S-wave states

$$\phi_{n^1S_0}^{(0)}(\mathbf{r}) = \sqrt{\frac{1}{8\pi}} R_{n0}(r)$$

$$\phi_{n^3S_1(\lambda)}^{(0)}(\mathbf{r}) = \sqrt{\frac{1}{8\pi}} R_{n0}(r) \boldsymbol{\sigma} \cdot \hat{\mathbf{e}}_{n^3S_1}(\lambda)$$

- P-wave states

$$\phi_{n^1P_1(\lambda)}^{(0)}(\mathbf{r}) = \sqrt{\frac{3}{8\pi}} R_{n1}(r) \hat{\mathbf{e}}_{n^1P_1}(\lambda) \cdot \hat{\mathbf{r}}$$

$$\phi_{n^3P_0}^{(0)}(\mathbf{r}) = \sqrt{\frac{1}{8\pi}} R_{n1}(r) \boldsymbol{\sigma} \cdot \hat{\mathbf{r}}$$

$$\phi_{n^3P_1(\lambda)}^{(0)}(\mathbf{r}) = \sqrt{\frac{3}{16\pi}} R_{n1}(r) \boldsymbol{\sigma} \cdot (\hat{\mathbf{r}} \times \hat{\mathbf{e}}_{n^3P_1}(\lambda))$$

$$\phi_{n^3P_2(\lambda)}^{(0)}(\mathbf{r}) = \sqrt{\frac{3}{8\pi}} R_{n1}(r) \boldsymbol{\sigma}^i h_{n^3P_2}^{ij}(\lambda) \hat{\mathbf{r}}^j.$$

General non-relativistic formula

$$\Gamma_{n^{2s+1}L_J \rightarrow n'^{2s+1}L'_{J'}\gamma}^{(0)} = \frac{4}{3} \alpha_{em} e_Q^2 (2J' + 1) S^{E1} k_\gamma^3 l_3^2 (nl \rightarrow n'l')$$

$$S^{E1} = \max(l, l') \left\{ \begin{array}{ccc} J & 1 & J' \\ l' & s & l \end{array} \right\}^2$$

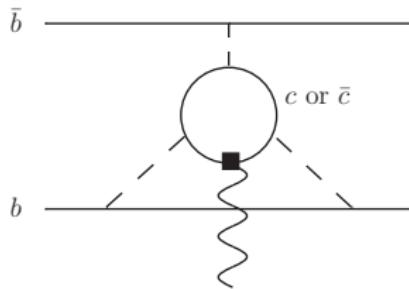
$$l_3(nl \rightarrow n'l') = \int_0^\infty dr r^3 R_{n'l'}(r) R_{nl}(r)$$

Light quark effects

- Loop effects with electromagnetic coupling to u , d and s cancel

$$q_u + q_d + q_s = 0$$

- charm quark effects for bottomonium
→ leading order diagram highly suppressed



→ furthermore: decoupling at typical momentum scale
 Brambilla, Sumino, Vairo, Phys. Rev. D65 (2002)

Lineshape of the h_b

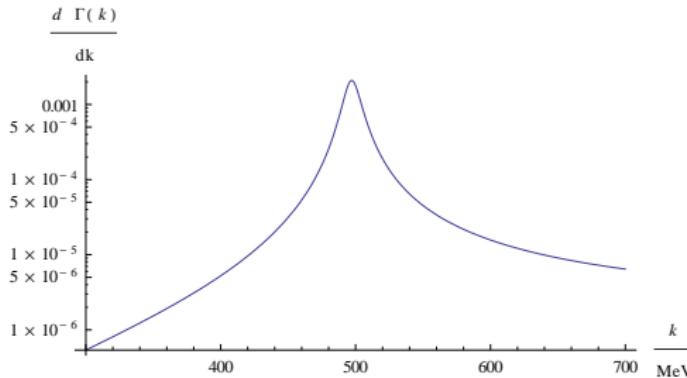
Decay $h_b \rightarrow \eta_b \gamma \rightarrow X \gamma$, resonance in the photon spectrum observable

Lineshape from pNRQCD calculation:

$$\frac{d\Gamma_{h_b}}{dE_\gamma} = \frac{4\alpha_{em}}{81\pi} I_3^2(11 \rightarrow 10) E_\gamma^3 \frac{\Gamma_{\eta_b}/2}{(E_\gamma^{\text{peak}} - E_\gamma)^2 + \Gamma_{\eta_b}^2/4}.$$

with $E_\gamma^{\text{peak}} \approx E_{h_b} - E_{\eta_b}$

→ modified Breit-Wigner curve



Numerical results

- $\sigma = 0.22 \text{ GeV}^2$, $m_c = 1.652 \text{ GeV}$, $m_b = 4.747 \text{ GeV}$ in our scheme
- $\Gamma_{h_b(1P) \rightarrow \eta_b(1S)\gamma}^{\text{pNRQCD}} = (31.5 \pm 4.4 \pm 0.6) \text{ keV}$
- Preliminary results for excited bottomonium transitions

process	$\Gamma^{\text{pNRQCD}}/\text{keV}$	$\Gamma_{\text{Grotch}}^{\text{mod}}/\text{keV}$	$\mathcal{B}_{\text{CLEO}}^{\text{exp}} \times \Gamma_{\text{tot,Rosner}}^{\text{mod}}/\text{keV}$
$\chi_{b0}(2P) \rightarrow \Upsilon(1S)\gamma$	$5.5 \pm 1.7 \pm 1.2$	5.3-6.5	7.8 ± 5.2
$\chi_{b1}(2P) \rightarrow \Upsilon(1S)\gamma$	$11.1 \pm 1.9 \pm 2.5$	11.0-11.8	4.3 ± 0.7
$\chi_{b2}(2P) \rightarrow \Upsilon(1S)\gamma$	$17.6 \pm 2.0 \pm 3.9$	18.2-18.9	10.9 ± 1.5