

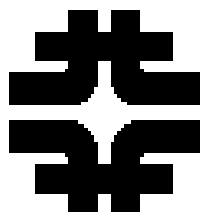
# The Status of Pion Transitions in Quarkonium

Estia Eichten (Fermilab)

## Outline:

- Update of Known Transitions
- New Issues Above Threshold
- Revisiting the QCDME assumptions
- Summary and Wish List





# Present Status

- NRQCD

Kinetic

Potential

relativistic  
corrections

$$\mathcal{H} = Q^\dagger \left[ \delta m_Q - \frac{\mathbf{D}^2}{2m_Q} \right] Q + \int d^3x j_a^0(x) \mathcal{G}^{ab} j_b^0(0) \\ - Q^\dagger \left[ \frac{c_4}{8m_Q^3} (\mathbf{D}^2)^2 + \frac{c_D}{8m_q^2} (\mathbf{D} \cdot g\mathbf{E} - g\mathbf{E} \cdot \mathbf{D}) \right] Q \\ - Q^\dagger \left[ \frac{c_s}{8m_q^2} i\sigma(\mathbf{D} \times g\mathbf{E} + g\mathbf{E} \times \mathbf{D}) + \frac{c_f}{2m_q} \sigma \cdot g\mathbf{B} \right] Q + \dots$$

$$\frac{\Lambda_{QCD}}{m_Q} \text{ (HQET); } v \text{ (NRQCD)}$$

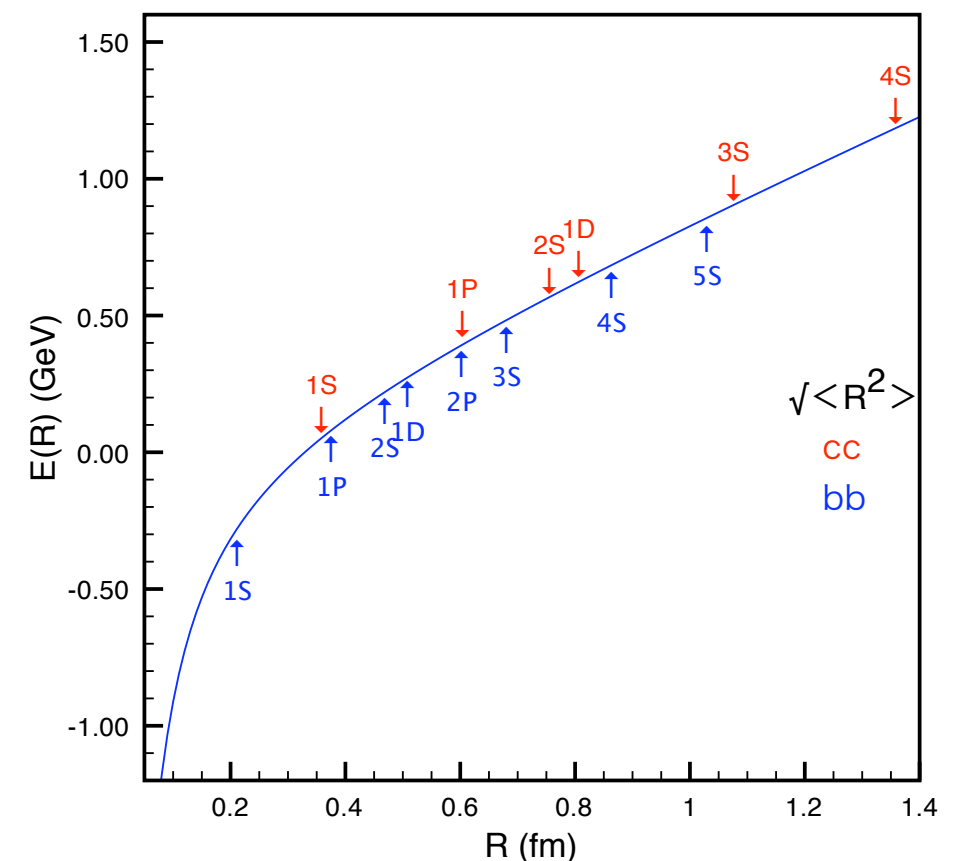
Static Energy

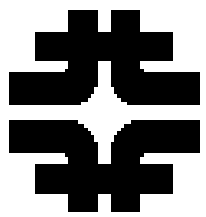
where  $j_a^0 = Q^\dagger g t_a Q + g^2 f^{abc} \mathbf{E}_b \cdot \mathbf{A}_c + \dots$   
and  $\mathcal{G}^{ab} = \frac{1}{\nabla^2} \nabla^2 \frac{1}{\nabla^2}$

- Below threshold

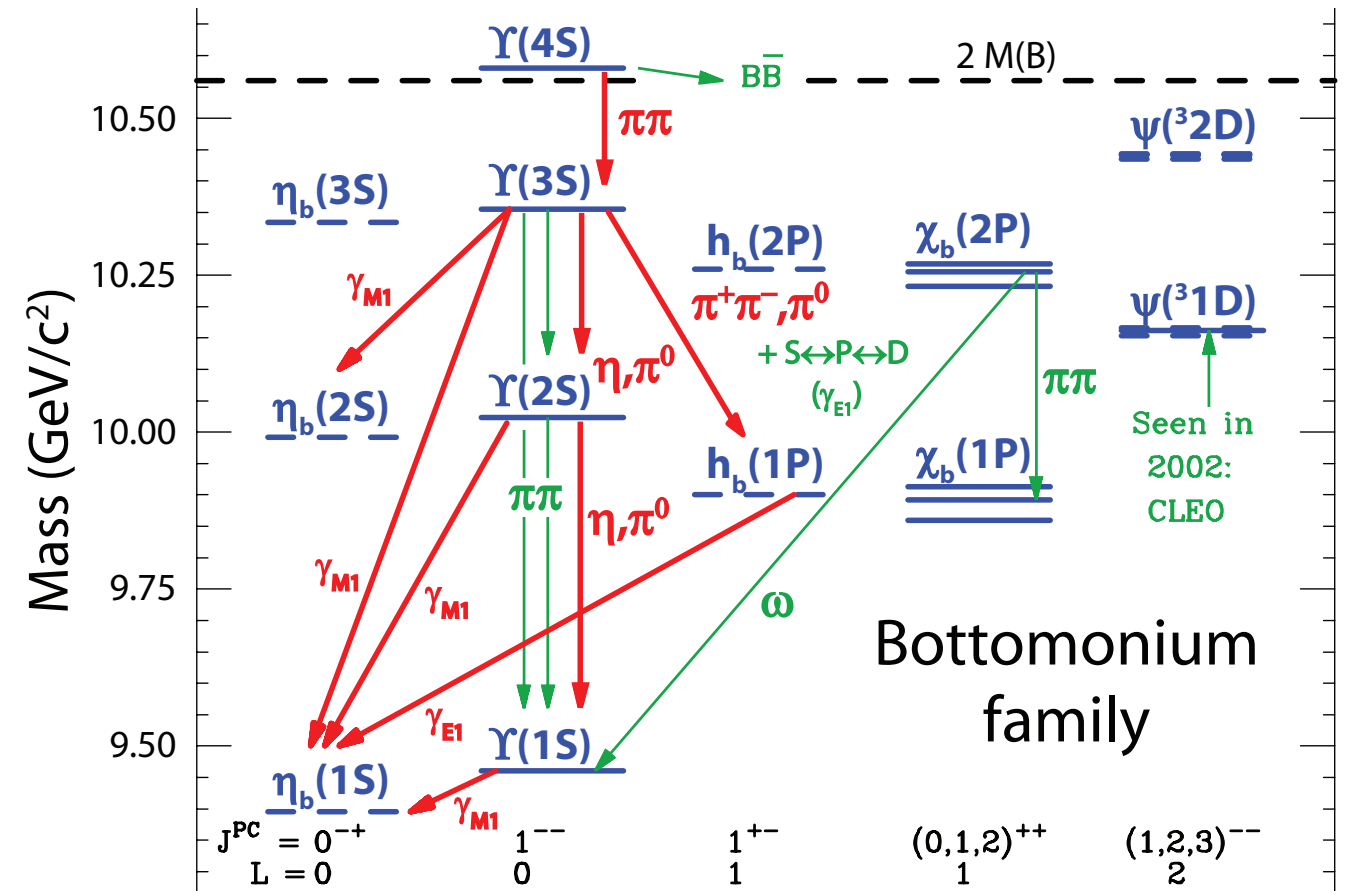
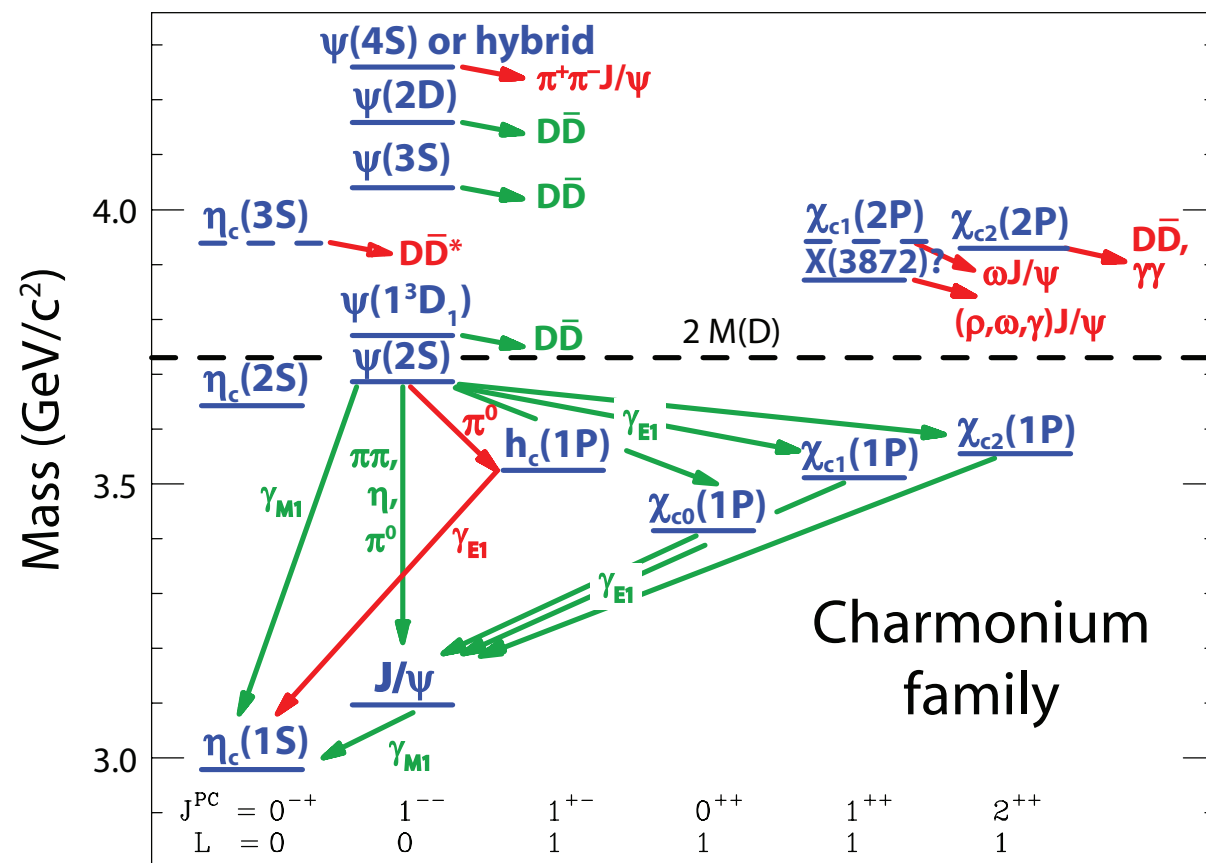
- Narrow states allow precise experimental probes of the subtle nature of QCD.
- Consistency between  $(b\bar{b})$  and  $(c\bar{c})$  systems validates NRQCD approach. At LHCb the  $(b\bar{c})$  system can also be studied.
- NRQCD approach is a spectacular success
  - masses and spin splittings (pot  $\rightarrow$  LQCD)
  - direct decays (pQCD)
  - EM transitions (ME)
  - hadronic transitions (QCDME)
- Lattice QCD can provide nonperturbative elements

Potential Model

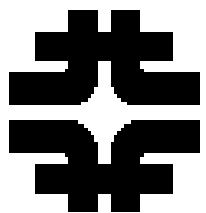




# Present Status



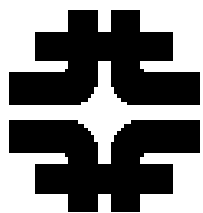
Stephen Godfrey, Hanna Mahlke, Jonathan L. Rosner and E.E.  
[Rev. Mod. Phys. 80, 1161 (2008)]



# Present Status

| Transition                                | $\Gamma_{\text{partial}}$ (keV)<br>(Experiment) | $\Gamma_{\text{partial}}$ (keV)<br>(KY Model) |
|-------------------------------------------|-------------------------------------------------|-----------------------------------------------|
| $\psi(2S)$                                |                                                 |                                               |
| $\rightarrow J/\psi + \pi^+ \pi^-$        | $102.3 \pm 3.4$                                 | input ( $ C_1 $ )                             |
| $\rightarrow J/\psi + \eta$               | $10.0 \pm 0.4$                                  | input ( $C_3/C_1$ )                           |
| $\rightarrow J/\psi + \pi^0$              | $0.411 \pm 0.030$ [446]                         | 0.64 [522]                                    |
| $\rightarrow h_c(1P) + \pi^0$             | $0.26 \pm 0.05$ [47]                            | 0.12-0.40 [527]                               |
| $\psi(3770)$                              |                                                 |                                               |
| $\rightarrow J/\psi + \pi^+ \pi^-$        | $52.7 \pm 7.9$                                  | input ( $C_2/C_1$ )                           |
| $\rightarrow J/\psi + \eta$               | $24 \pm 11$                                     |                                               |
| $\Upsilon(2S)$                            |                                                 |                                               |
| $\rightarrow \Upsilon(1S) + \pi^+ \pi^-$  | $5.79 \pm 0.49$                                 | 8.7 [528]                                     |
| $\rightarrow \Upsilon(1S) + \eta$         | $(6.7 \pm 2.4) \times 10^{-3}$                  | 0.025 [521]                                   |
| $\Upsilon(1^3D_2)$                        |                                                 |                                               |
| $\rightarrow \Upsilon(1S) + \pi^+ \pi^-$  | $0.188 \pm 0.046$ [63]                          | 0.07 [529]                                    |
| $\chi_{b1}(2P)$                           |                                                 |                                               |
| $\rightarrow \chi_{b1}(1P) + \pi^+ \pi^-$ | $0.83 \pm 0.33$ [523]                           | 0.54 [530]                                    |
| $\rightarrow \Upsilon(1S) + \omega$       | $1.56 \pm 0.46$                                 |                                               |
| $\chi_{b2}(2P)$                           |                                                 |                                               |
| $\rightarrow \chi_{b2}(1P) + \pi^+ \pi^-$ | $0.83 \pm 0.31$ [523]                           | 0.54 [530]                                    |
| $\rightarrow \Upsilon(1S) + \omega$       | $1.52 \pm 0.49$                                 |                                               |
| $\Upsilon(3S)$                            |                                                 |                                               |
| $\rightarrow \Upsilon(1S) + \pi^+ \pi^-$  | $0.894 \pm 0.084$                               | 1.85 [528]                                    |
| $\rightarrow \Upsilon(1S) + \eta$         | $< 3.7 \times 10^{-3}$                          | 0.012 [521]                                   |
| $\rightarrow \Upsilon(2S) + \pi^+ \pi^-$  | $0.498 \pm 0.065$                               | 0.86 [528]                                    |
| $\Upsilon(4S)$                            |                                                 |                                               |
| $\rightarrow \Upsilon(1S) + \pi^+ \pi^-$  | $1.64 \pm 0.25$                                 | 4.1 [528]                                     |
| $\rightarrow \Upsilon(1S) + \eta$         | $4.02 \pm 0.54$                                 |                                               |
| $\rightarrow \Upsilon(2S) + \pi^+ \pi^-$  | $1.76 \pm 0.34$                                 | 1.4 [528]                                     |

Heavy quarkonium: progress, puzzles,  
and opportunities  
N. Brambilla et.al. [arXiv:1010.5827]



# Some Puzzles

## - $\eta$ transitions

- Ratio of  $\eta$  to  $\pi\pi$  transitions: same initial and final quarkonium states at ( $M_{\pi\pi} = M_\eta$ )

$$R_{Q\bar{Q}}(n \rightarrow m) \equiv \frac{\Gamma(n^3S_1 \rightarrow m^3S_1 + \eta)}{\Gamma(n^3S_1 \rightarrow m^3S_1 + \pi^+\pi^-)} = \frac{8\pi^2}{27} \frac{1}{m_Q^2} \left(\frac{C_3}{C_1}\right)^2 \left[ \frac{[(M_i + M_f)^2 - M_\eta^2][(M_i - M_f)^2 - M_\eta^2]}{G} \right]^{3/2}$$

is independent of the details of the intermediate states.

[kinematic factor]

- Comparing theory (KY) and experiment.

| Ratio                           | theory                | experiment              |
|---------------------------------|-----------------------|-------------------------|
| $R^{c\bar{c}}(2 \rightarrow 1)$ | $3.29 \times 10^{-3}$ | $9.78 \times 10^{-2}$   |
| $R^{b\bar{b}}(2 \rightarrow 1)$ | $1.16 \times 10^{-3}$ | $1.16 \times 10^{-3}$   |
| $R^{b\bar{b}}(3 \rightarrow 1)$ | $4.57 \times 10^{-3}$ | $< 4.13 \times 10^{-3}$ |
| $R^{b\bar{b}}(4 \rightarrow 1)$ | $2.23 \times 10^{-3}$ | 2.45                    |
| $R^{b\bar{b}}(4 \rightarrow 2)$ | $5.28 \times 10^{-4}$ |                         |

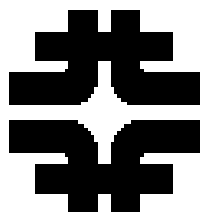
$\sim 30 > \text{theory}$

sets  $C_3/C_1 = 0.143 \pm 0.024$

suppressed?

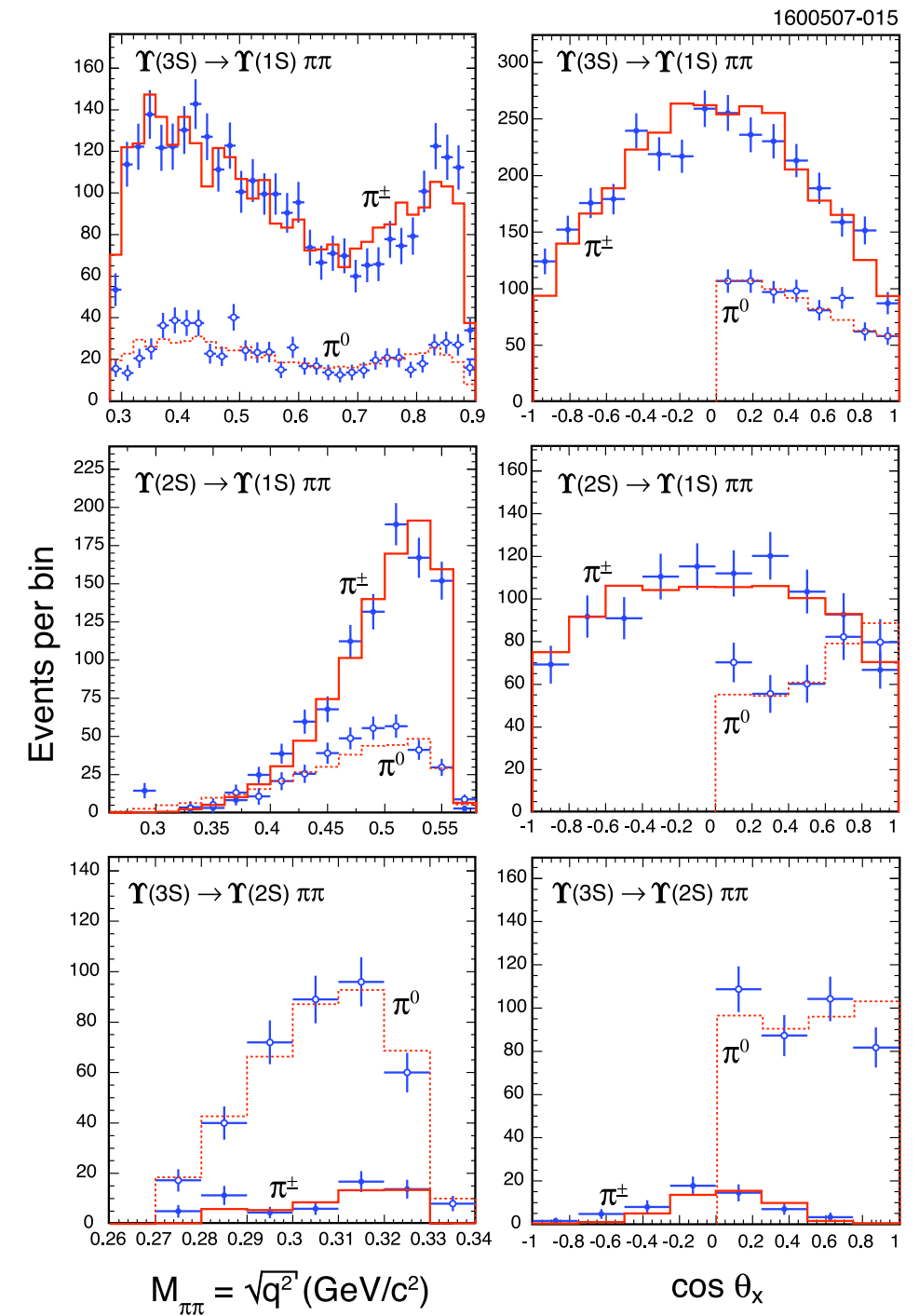
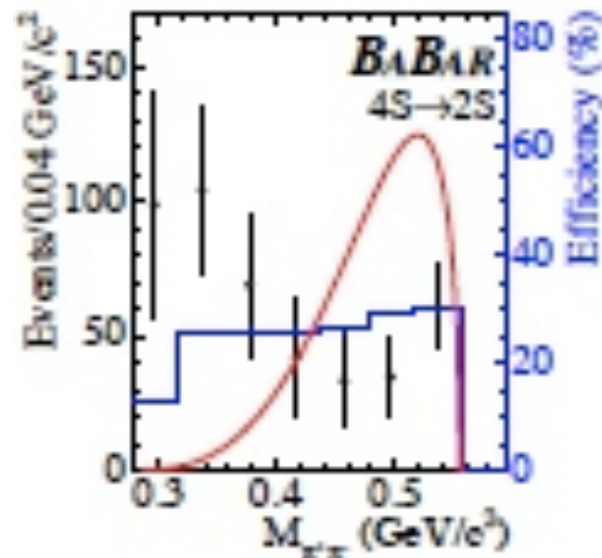
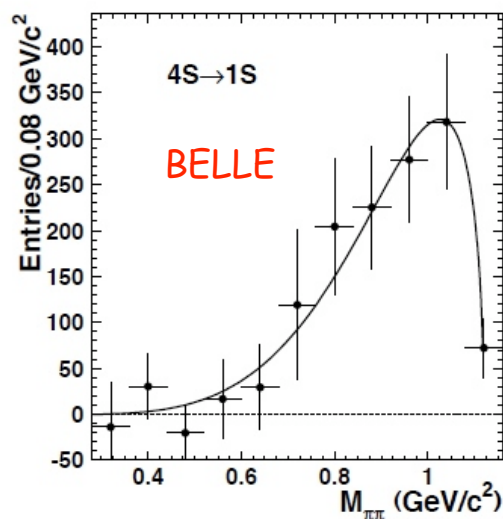
$\sim 1000 > \text{theory}$

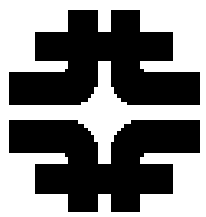
- These transitions are very poorly understood.



# Some Puzzles

- $\Upsilon(3S) \rightarrow \Upsilon(1S) \pi\pi$  and  $\Upsilon(4S) \rightarrow \Upsilon(2S) \pi\pi$  transitions
  - $M_{\pi\pi}$  distributions **NOT** the expected S-wave behaviour
  - Likely explanation - same as overlap dynamically suppressed in  $\Upsilon(3S) \rightarrow \chi_b(1P) \gamma$  EM transitions
  - CLEO detailed study [arXiv:0706.2317]
    - Hindered M1-M1 term  $\Rightarrow C \approx 0$ . Consistent with CLEO results.
    - Small D-wave contributions
  - Further study would be useful. Look at polarization. Dubynskik & Voloshin [hep-ph/0707.1272]





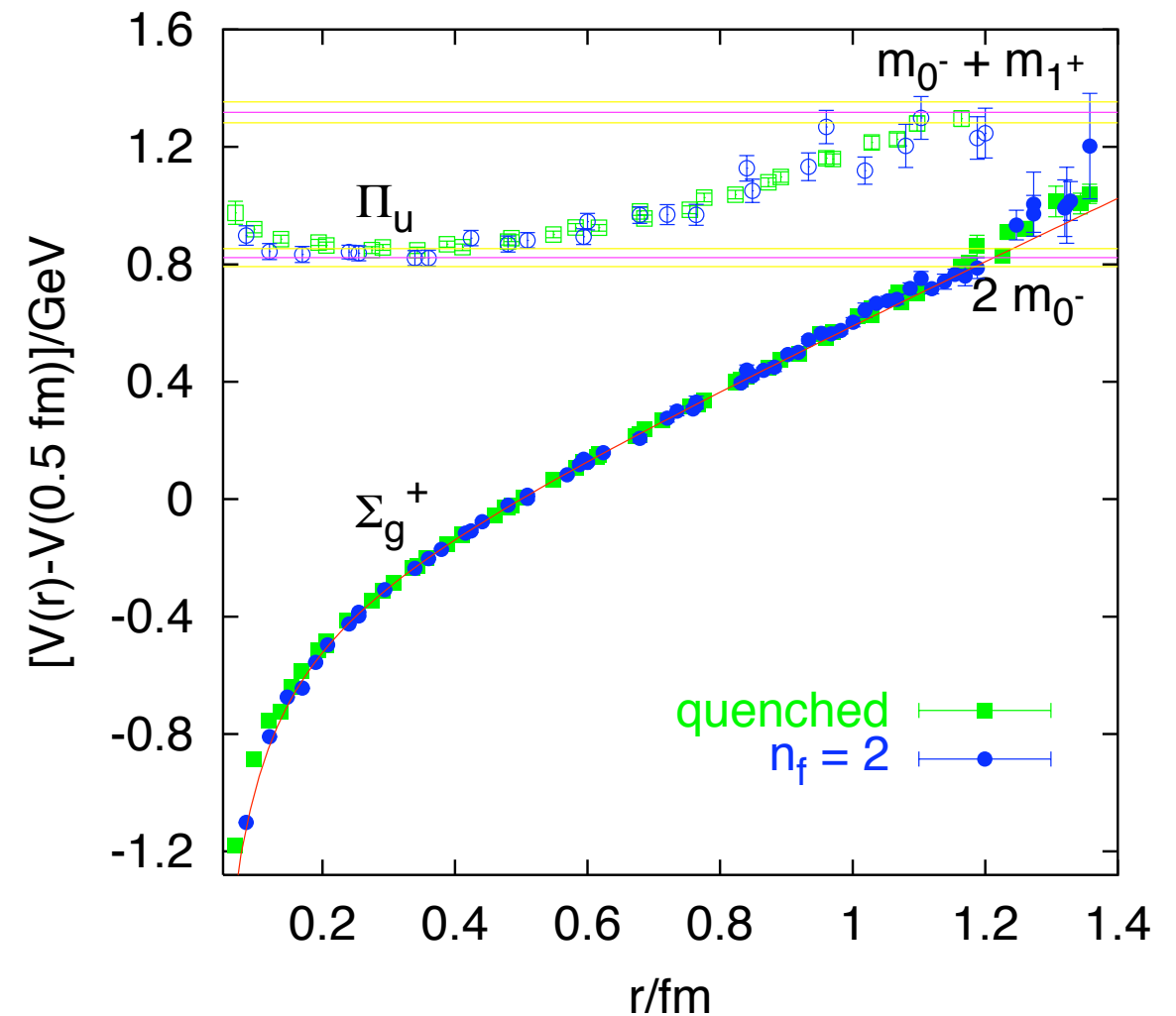
# Why it works so well

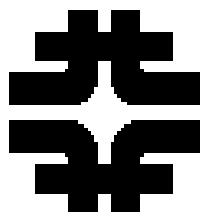
- Lattice calculation  $V(r)$ , then SE

$$-\frac{1}{2\mu} \frac{d^2 u(r)}{dr^2} + \left\{ \frac{\langle \mathbf{L}_{Q\bar{Q}}^2 \rangle}{2\mu r^2} + V_{Q\bar{Q}}(r) \right\} u(r) = E u(r)$$

- What about the gluon and light quark degrees of freedom in QCD?
- Two thresholds
  - Usual  $(Qq) + (qQ)$  decay thresholds
  - Exciting the string - hybrids
- Hybrid states will appear in the spectrum associated with the potentials  $\Pi_u, \dots$
- In the static limit this occurs at separation  $r \approx 1.2$  fm.
  - Between the 3S and 4S in (cc) system
  - Just above the 5S in the (bb) system

LQCD calculation of static energy

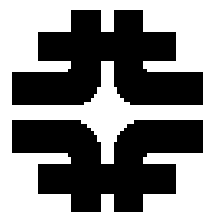




# Crossing the Threshold

- Many new degrees of freedom influence transitions
  - Normal strong decay channels - strong coupled channel effects
  - New four quark states possible:
    - molecules  $(Q\bar{q})(q\bar{Q})$
    - diquark-antidiquark  $(Qq)(\bar{q}\bar{Q})$
    - hadrocharmonium  $(Q\bar{Q})(\bar{q}q)$
  - Hybrid states:
    - exciting the gluon degrees of freedom
    - valence gluons picture
    - string picture
- Not hopeless. Two handles to understand systematics:
  - lattice QCD
  - known scaling from  $(c\bar{c})$  to  $(b\bar{b})$  systems



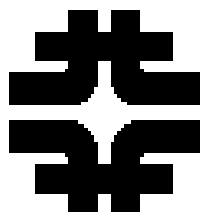


# Transitions - States Above Threshold

| State            | EXP                   | $M + i \Gamma$ (MeV)                                                   | $J^{PC}$             | Decay Modes                                                                              | Production Modes                    |
|------------------|-----------------------|------------------------------------------------------------------------|----------------------|------------------------------------------------------------------------------------------|-------------------------------------|
| X(3872)          | Belle, CDF, DO, BaBar | $3871.67 \pm 0.17 + i(<3)$                                             | $1^{++}$ or $2^{-+}$ | $\pi^+\pi^-J/\psi$ , $\omega J/\psi$ , $\Upsilon J/\psi$ , $\Upsilon\psi'$ , $D^0D^{*0}$ | B decays, ppbar                     |
| $\chi_c(2^3P_2)$ | Belle, BaBar          | $3927.2 \pm 2.6 + i(24 \pm 6)$                                         | $2^{++}$             | $D^0D^0$ , $D^+D^-$ , $\omega J/\psi$                                                    | $\Upsilon\Upsilon$ , B decays       |
| X(3940)          | Belle                 | $3942^{+7}_{-6} \pm 6 + i(37^{+26}_{-15} \pm 8)$                       | $J^{P+}$             | $DD^*$                                                                                   | $e^+e^-$ (recoil against $J/\psi$ ) |
| Y(4008)          | Belle<br>BaBar        | $4008 \pm 40^{+72}_{-28} + i(226 \pm 44^{+87}_{-79})$<br>(not seen)    | $1^{--}$             | $\pi^+\pi^-J/\psi$                                                                       | $e^+e^-$ (ISR)                      |
| Y(4140)          | CDF<br>LHCb           | $4143.0 \pm 2.9 \pm 1.2 + i(11.7^{+8.3}_{-5.0} \pm 3.7)$<br>(not seen) | $J^{P+}$             | $\phi J/\psi$                                                                            | ppbar<br>pp                         |
| ★ $\psi(4160)$   | CLEO                  | $4153 \pm 3 + i(103 \pm 8)$                                            | $1^{--}$             | $\pi^+\pi^-h_c(1P)$                                                                      | $e^+e^-$                            |
| X(4160)          | Belle                 | $4156^{+25}_{-20} \pm 15 + i(139^{+111}_{-61} \pm 21)$                 | $J^{P+}$             | $D^*D^*$                                                                                 | $e^+e^-$ (recoil against $J/\psi$ ) |
| ★ Y(4260)        | BaBar, CLEO, Belle    | $4263^{+8}_{-9} + i(95 \pm 14)$                                        | $1^{--}$             | $\pi^+\pi^-J/\psi$ , $\pi^0\pi^0J/\psi$ , $K^+K^-J/\psi$                                 | $e^+e^-$ (ISR), $e^+e^-$            |
| Y(4360)          | BaBar, Belle          | $4361 \pm 9 \pm 9 + i(74 \pm 18 \pm 10)$                               | $1^{--}$             | $\pi^+\pi^-\psi(2S)$                                                                     | $e^+e^-$ (ISR)                      |
| Y(4660)          | Belle                 | $4664 \pm 11 \pm 8 + i(48 \pm 15 \pm 3)$                               | $1^{--}$             | $\pi^+\pi^-\psi(2S)$ , $\Lambda_c\Lambda_c\text{bar}$                                    | $e^+e^-$ (ISR)                      |
| ★ Y(5S)          | Belle, BaBar          | $10,876 \pm 11 + i(55 \pm 28)$                                         | $1^{--}$             | $\pi^+\pi^-\psi(nS)$ $n=1,2,3$<br>$\pi^+\pi^-h_b(nP)$ $n=1,2$                            | $e^+e^-$                            |

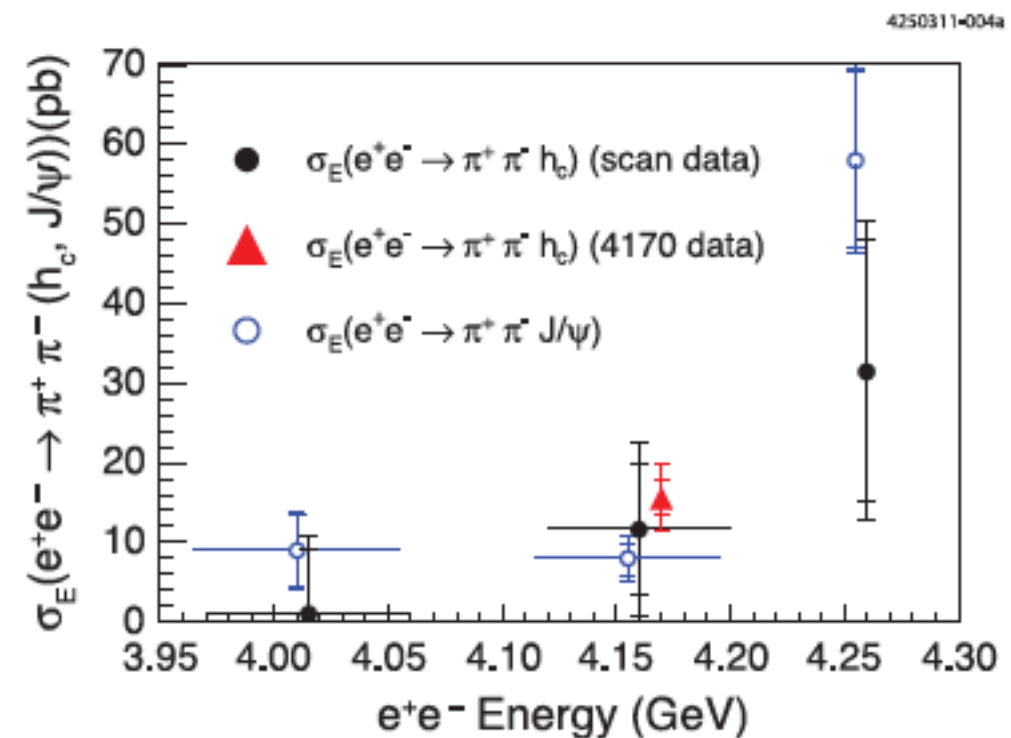
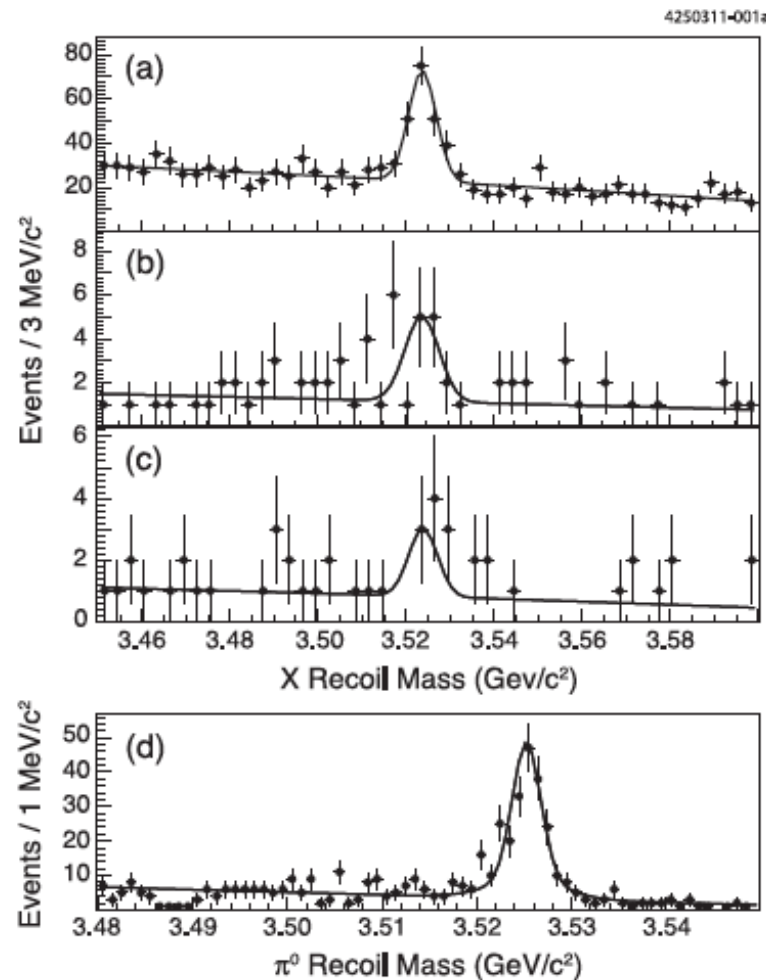
Charged states:

$X^\pm(4250) \rightarrow \pi^\pm \chi_{c1}(1P)$ ,  $X^\pm(4430) \rightarrow \pi^\pm \psi(2S)$ ,  $Z_b^\pm(10,610) \rightarrow \pi^\pm h_b(nP)$  and  $Z_b^\pm(10650) \rightarrow \pi^\pm h_b$

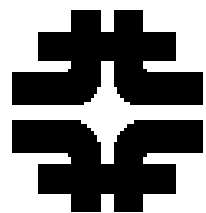


# $\psi(4160)$ Transitions

- The  $\psi(4160) \rightarrow h_c(1P) + \pi^+\pi^-$  transition observed by CLEO (Ryan's talk)



- Find unexpectedly large transition rate.
- Spin flip transition: E1 M1



# $\Upsilon(4260)$ Transitions

- The  $\Upsilon(4260) \rightarrow J/\psi + \pi^+\pi^-$  transition observed by Belle, BaBar and CLEO

Babar

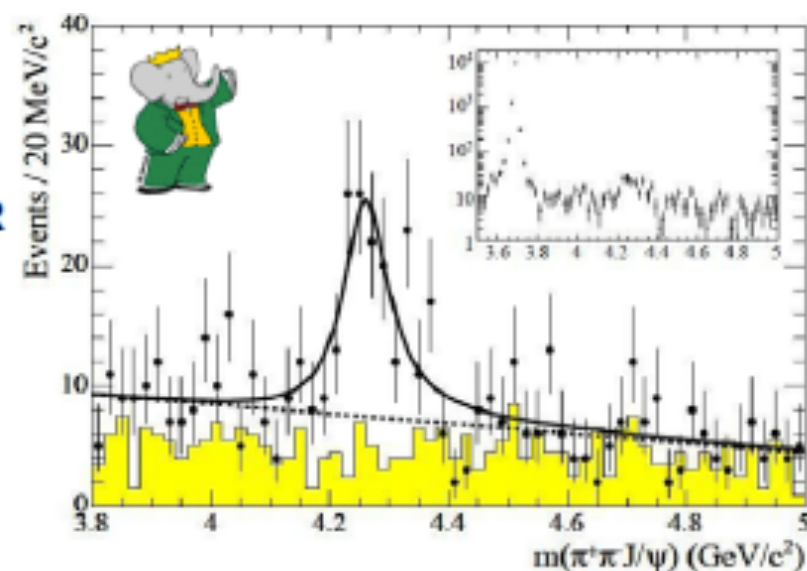
$M: 4259 \pm 8^{+2}_{-6} \text{ MeV}$

$\Gamma: 88 \pm 23^{+6}_{-8} \text{ MeV}$

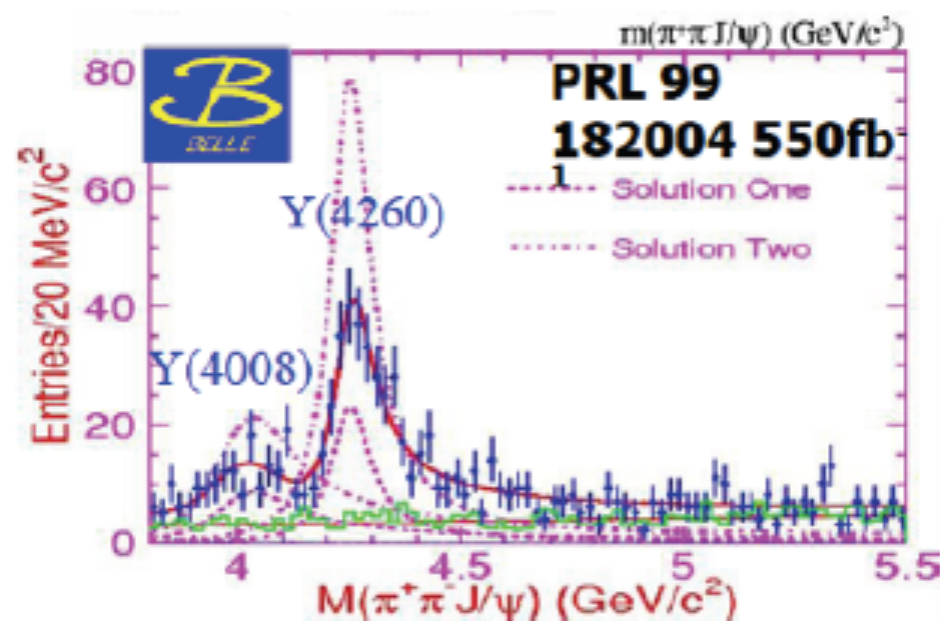
Belle

$4247 \pm 12^{+17}_{-32} \text{ MeV}$

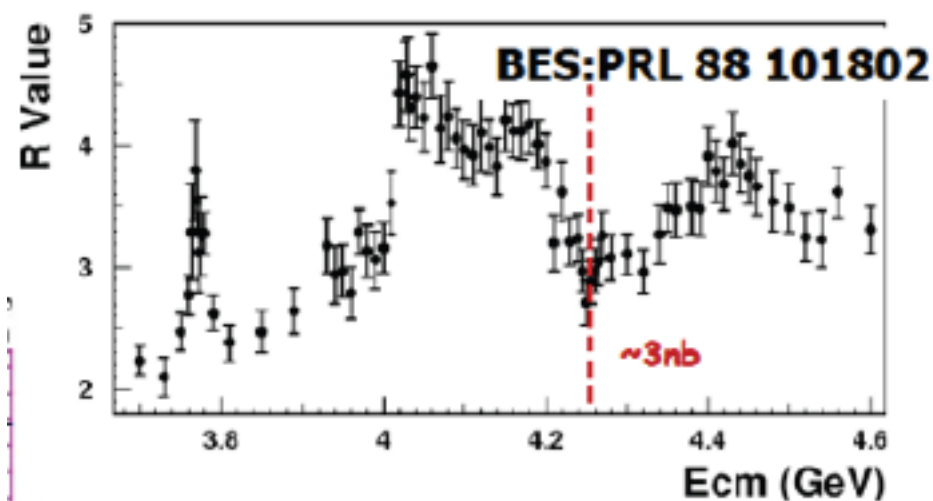
$108 \pm 19 \pm 10 \text{ MeV}$



**BABAR: PRL 95 142001**

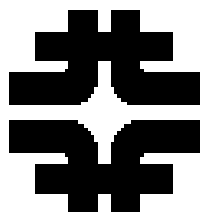


- The  $\Upsilon(4260)$  not seen in  $D^{(*)}D^{(*)}$  final states. At dip in R. Large rate.



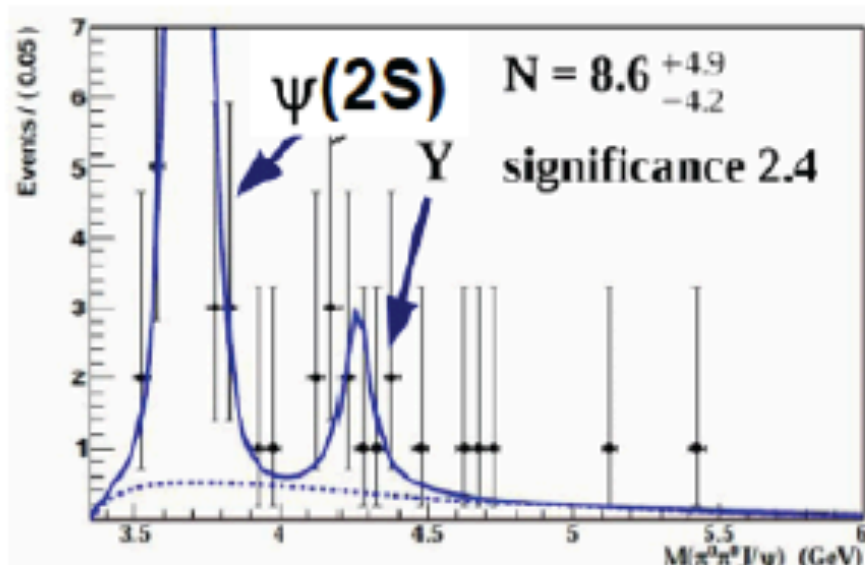
PL B640, 182 (2006)

$\Gamma(\Upsilon_{4260} \rightarrow \pi^+\pi^-J/\psi) > 0.508 \text{ MeV @ 90\%}$



# Y(4260) Transitions

- Belle and CLEO observe  $Y(4260) \rightarrow J/\psi + \pi^0\pi^0$  consistent with  $I=0$

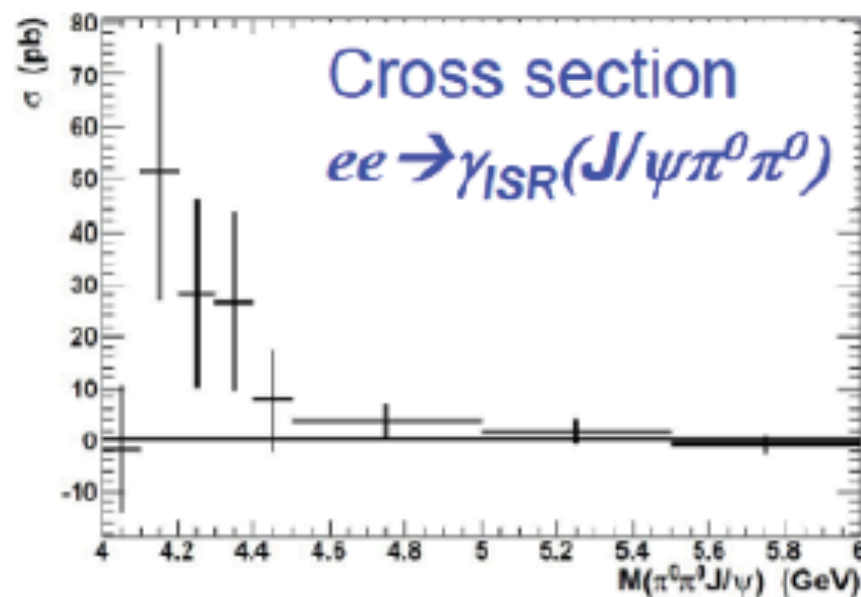


A. Vinokurova EPS 2011,

$$\Gamma_{ee} BF(J/\psi \pi^0 \pi^0) = 3.19^{+1.82+0.64}_{-1.53-0.35} \text{ eV}$$

$$PDG: \Gamma_{ee} BF(J/\psi \pi^+ \pi^-) = (5.9^{+1.2}_{-0.9}) \text{ eV}$$

It implies that the Y(4260) has  $I=0$ , as expected for a  $c\bar{c}$  state.



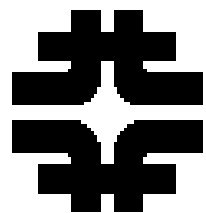
CLEO: PRL 96 162003 (2006)

From  $e^+e^-$  collision

$$BF(J/\psi \pi^0 \pi^0) / BF(J/\psi \pi^+ \pi^-) \sim 0.5$$

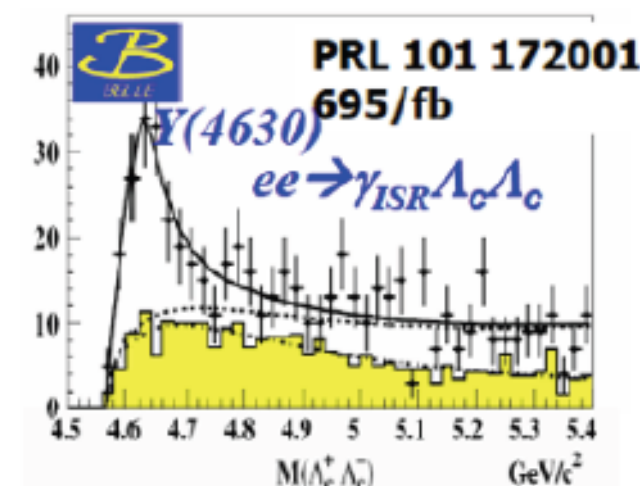
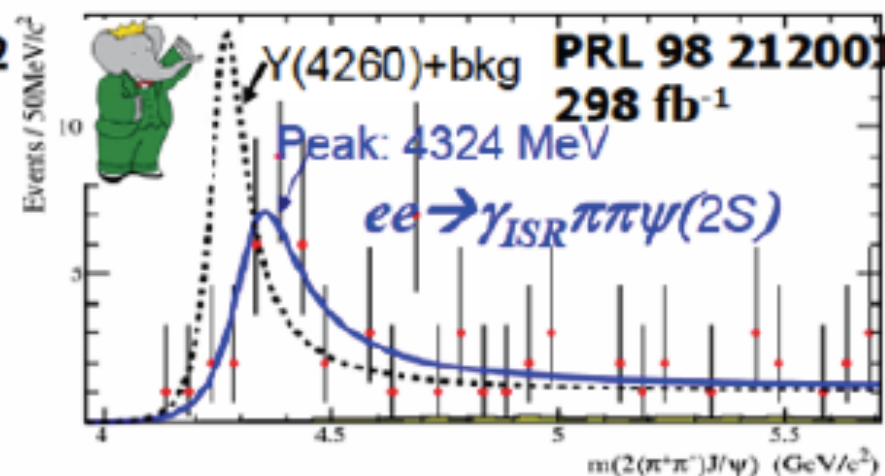
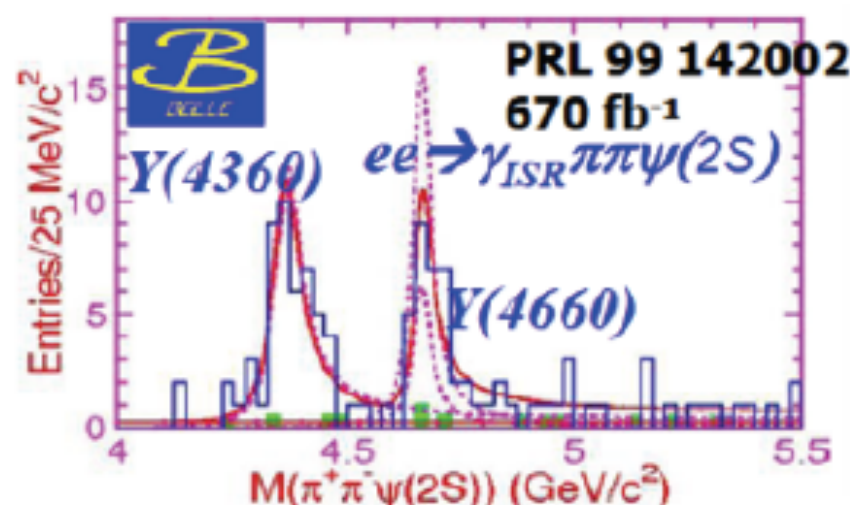
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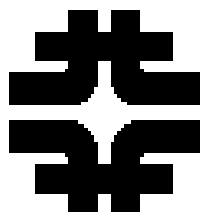


# $\Upsilon(4260)$ Transitions

- Additional  $1^-$  states:  $\Upsilon(4360)$ ,  $\Upsilon(4660)$  with transitions  $\psi(2S) + \pi^+\pi^-$



- Clear evidence for large transition rates. But these  $\Upsilon$  states are not conventional charmonium states. No available  $1^-$  states.



# $\Upsilon(5S)$ Transitions

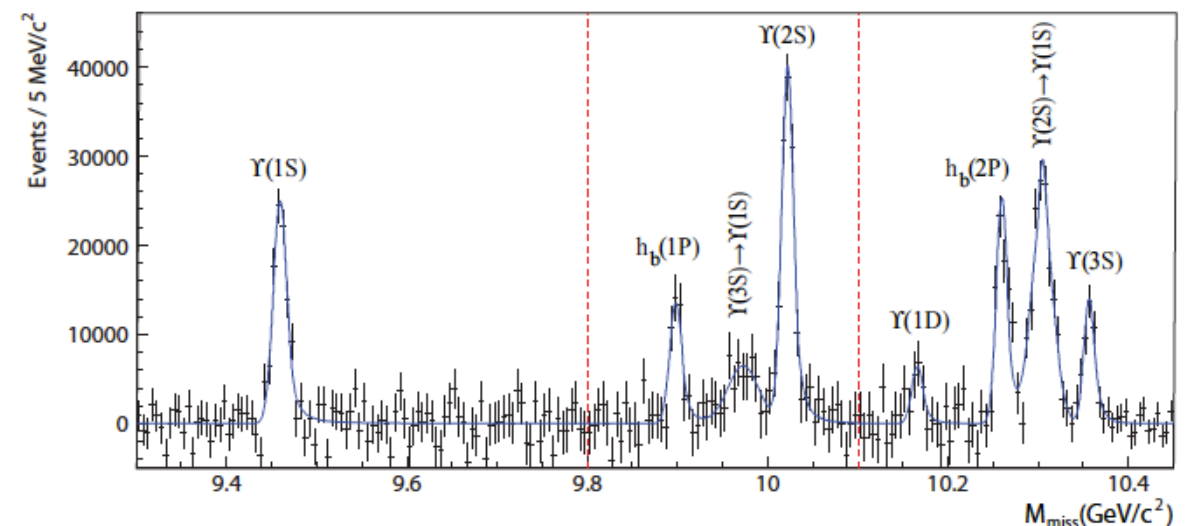
- Large rates
  - $\Upsilon(5S)$ :  $m=10,876 \pm 11 \text{ MeV}$  and  $\Gamma = 55 \pm 23 \text{ MeV}$
  - $\text{BR}(\Upsilon(5S) \rightarrow \Upsilon(2S) + \pi^+\pi^-) = (0.78 \pm 0.13) \%$

$$\bar{\sigma}[\Upsilon(1S)\pi^+\pi^-] = 0.638 \pm 0.065^{+0.037}_{-0.056}$$

$$\bar{\sigma}[\Upsilon(3S)\pi^+\pi^-] = 0.517 \pm 0.082 \pm 0.070$$

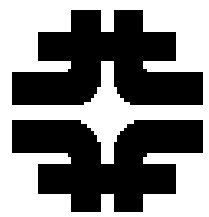
$$\bar{\sigma}[h_b(1P)\pi^+\pi^-] = 0.407 \pm 0.07^{+0.043}_{-0.076}$$

$$\bar{\sigma}[h_b(2P)\pi^+\pi^-] = 0.78 \pm 0.09^{+0.22}_{-0.10}$$



BELLE [arXiv:1103.3419]

- $\pi^+\pi^-$  system  $I=0$
- total branching ratio for known hadronic transitions  $(3.9 \pm 0.7)\% \Rightarrow \Gamma = 2.1 \pm 0.9 \text{ MeV}$
- Clear violation of QCDME expectations:
  - the transitions  $\Upsilon(5S) \rightarrow h_b(1P,2P) + \pi^+\pi^-$  requires a heavy quark spin flip (M1)(E1)
- The usual formulation of QCDME needs modification, Structure in the transition amplitudes not found in the usual (KY) model.



# Revisiting the QCDME Assumptions

- QCD multipole expansion (QCDME) in a nutshell
  - Analogous to the QED multipole expansion with gluons replacing photons.

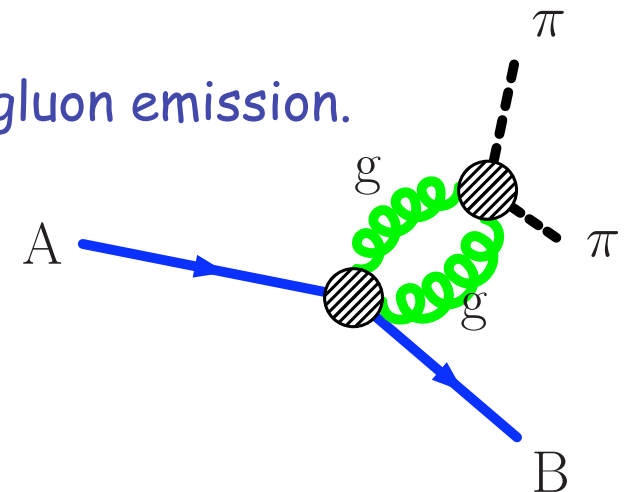
$$H_{\text{QCD}}^{\text{eff}} = H_{\text{QCD}}^{(0)} + H_{\text{QCD}}^{(1)} + H_{\text{QCD}}^{(2)} \quad H_{\text{QCD}}^{(1)} \equiv Q_a A_0^a(\mathbf{X}, t)$$

zero for color singlet

$$H_{\text{QCD}}^{(2)} \equiv -\mathbf{d}_a \cdot \mathbf{E}^a(\mathbf{X}, t) - \mathbf{m}_a \cdot \mathbf{B}^a(\mathbf{X}, t) + \dots$$

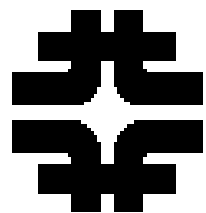
E1 M1 ...

- color singlet physical states means lowest order terms involve two gluon emission.  
So lowest multipoles E1 E1, E1 M1, E1 E2, ...
- factorize the heavy quark and light quark dynamics



$$\mathcal{M}(\Phi_i \rightarrow \Phi_f + h) = \frac{1}{24} \sum_{KL} \frac{\langle f | d_m^{ia} | KL \rangle \langle KL | d_{ma}^j | i \rangle}{E_i - E_{KL}} \langle h | \mathbf{E}^{ai} \mathbf{E}_a^j | 0 \rangle + \text{higher order multipole terms.}$$

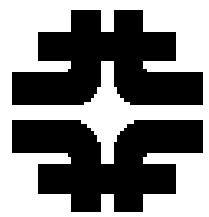
- assume a model for the heavy quarkonium states  $\Phi_i$ ,  $\Phi_f$  and a model for the intermediate states  $|KL\rangle$  hybrid states.
- use chiral effective lagrangians to parameterize the light hadronic system.



# Revisiting the QCDME Assumptions

- Four options exist for breakdown of the QCDME
  1. Because states above threshold are not compact the expansion becomes unreliable.
  2. The model of hybrid intermediate states is insufficient as hybrid thresholds are crossed.
  3. The coupling to decay channels adds new contributions. Transitions in the two meson channels (breaks the factorization assumption)
  4. There are new exotic states (that are not hybrids) which appear in the intermediate state. (Again breaks the factorization assumption)
- These options are ranked from least surprising (1) to most extreme (4). I will discuss (2) below. But Belle has observed new states that if confirmed will show that at least in the  $\Upsilon(5S)$  transitions the breakdown is caused by option (4).





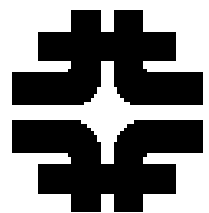
# $Z_b^\pm(10,610)$ and $Z_b^\pm(10,650)$

- BELLE has observed two new charged states in the  $\Upsilon(5S) \rightarrow \Upsilon(nS) + \pi^+\pi^-$  ( $n=1,2,3$ ) and the  $\Upsilon(5S) \rightarrow h_b(nP) + \pi^+\pi^-$  ( $n=1,2$ ) transitions [\[arXiv:1105.4583\]](#)

**TABLE 1.** Masses, widths, and relative phases of peaks observed in  $h_b\pi$  and  $\Upsilon\pi$  channels, from fits described in text.

|                     | $h_b(1P)\pi^\pm\pi^\mp$         | $h_b(2P)\pi^\pm\pi^\mp$  | $\Upsilon(1S)\pi^\pm\pi^\mp$ | $\Upsilon(2S)\pi^\pm\pi^\mp$ | $\Upsilon(3S)\pi^\pm\pi^\mp$ | Average         |
|---------------------|---------------------------------|--------------------------|------------------------------|------------------------------|------------------------------|-----------------|
| $M_1$ (MeV/ $c^2$ ) | $10605.1 \pm 2.2^{+3.0}_{-1.0}$ | $10596 \pm 7^{+5}_{-2}$  | $10609 \pm 3 \pm 2$          | $10616 \pm 2^{+3}_{-4}$      | $10608 \pm 2^{+5}_{-2}$      | $10608 \pm 2.0$ |
| $\Gamma_1$ (MeV)    | $11.4^{+4.5+2.1}_{-3.9-1.2}$    | $16^{+16+13}_{-10-14}$   | $22.9 \pm 7.3 \pm 2$         | $21.1 \pm 4^{+2}_{-3}$       | $12.2 \pm 1.7 \pm 4$         | $15.6 \pm 2.5$  |
| $M_2$ (MeV/ $c^2$ ) | $10654.5 \pm 2.5^{+1.0}_{-1.9}$ | $10651 \pm 4 \pm 2$      | $10660 \pm 6 \pm 2$          | $10653 \pm 2 \pm 2$          | $1 - 652 \pm 2 \pm 2$        | $10653 \pm 1.5$ |
| $\Gamma_2$ (MeV)    | $20.9^{+5.4+2.1}_{-1.7-5.7}$    | $12^{+11+8}_{-9-2}$      | $12 \pm 10 \pm 3$            | $16.4 \pm 3.6^{+4}_{-6}$     | $10.9 \pm 2.6^{+4}_{-2}$     | $14.4 \pm 3.2$  |
| $\phi$ ( $^\circ$ ) | $188^{+44+4}_{-58-9}$           | $255^{+56+12}_{-72-183}$ | $53 \pm 61^{+5}_{-50}$       | $-20 \pm 18^{+14}_{-9}$      | $6 \pm 24^{+23}_{-59}$       | —               |

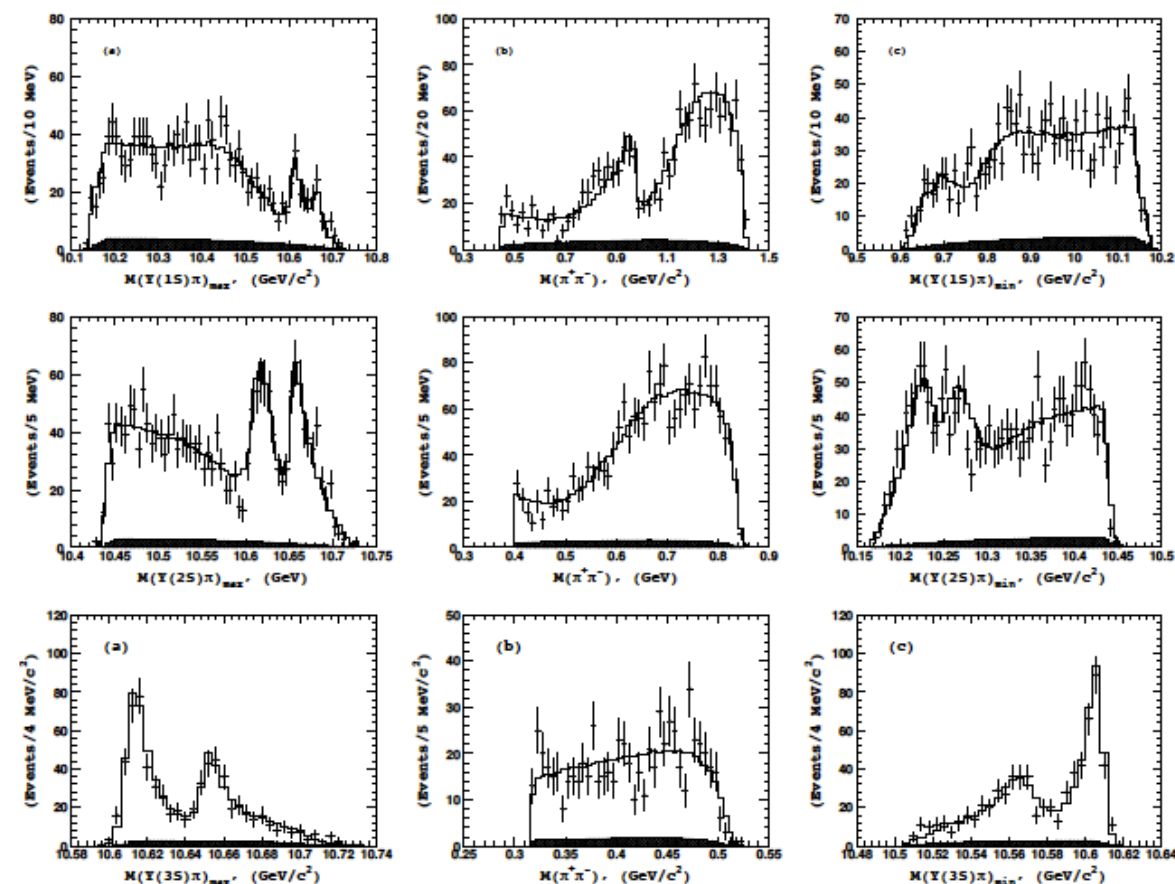
- $\Upsilon(5S) \rightarrow Z_b^{++} \pi^-$  and  $Z_b^- \rightarrow h_b(nP) + \pi^+$ .
- Explicitly violates the factorization assumption.



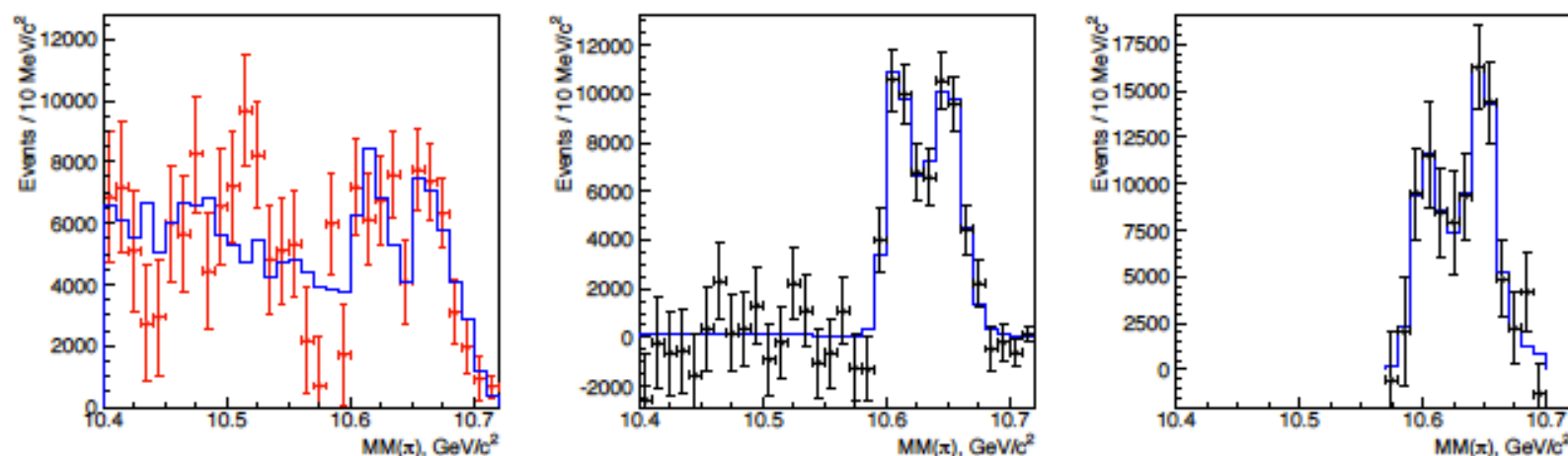
# $Z_b^\pm(10,610)$ and $Z_b^\pm(10,650)$

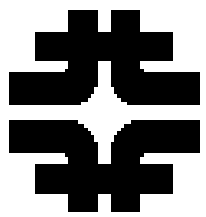
- $\Upsilon(5S) \rightarrow \Upsilon(nS) + \pi^+\pi^-$  ( $n=1,2,3$ )

Mizuk's talk  
BELLE [arXiv:1105.4583]



- $Z_b$  in  $\Upsilon(2S)$ ,  $h_b(1P)$  and  $h_b(2P)$  pion transitions





# Spectrum of Low-Lying Hybrid States

- Born-Oppenheimer Approximation

$$\Psi_{Q\bar{Q}}(\vec{r}) = \frac{u_{nl}(r)}{r} Y_{lm}(\theta, \phi)$$

$$-\frac{1}{2\mu} \frac{d^2 u(r)}{dr^2} + \left\{ \frac{\langle \mathbf{L}_{Q\bar{Q}}^2 \rangle}{2\mu r^2} + V_{Q\bar{Q}}(r) \right\} u(r) = E u(r)$$

Spectroscopic notation of diatomic molecules

- Put the correct short (pNRQCD) and long distance (NG string) behaviour together using lattice QCD can determine the hybrid potentials

- Toy model - minimal parameters

$$V_n(R) = \frac{\alpha_s}{6R} + \sigma R \sqrt{1 + \frac{2\pi}{\sigma R^2} (n(R) - \frac{1}{24}(d-2))} + V_0 \quad (n > 0)$$

$$V_{\Sigma_g^+}(R) = -\frac{4\alpha_s}{3R} + \sigma R + V_0 \quad (n = 0)$$

Fixes  $M_c = 1.84 \text{ GeV}$ ,  $\sqrt{\sigma} = .427 \text{ GeV}$ ,  $\alpha_s = 0.39$

$n(R) = [n]$  (string level) if no level crossing  
 $[n - 2 \tanh(R_0/R)]$  for  $\Sigma_u^-$  potential ( $n=3$ )

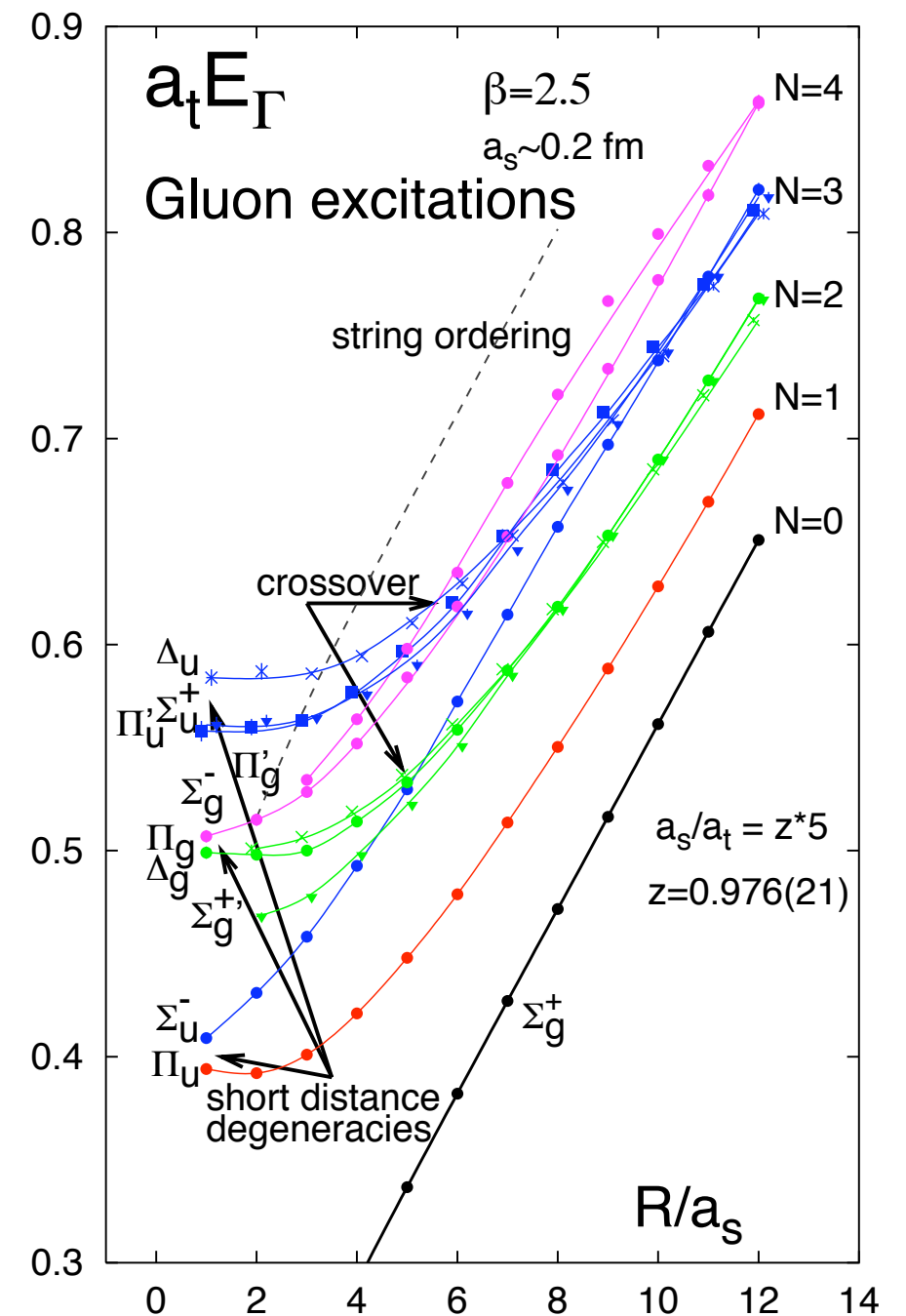
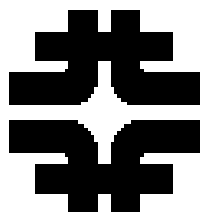
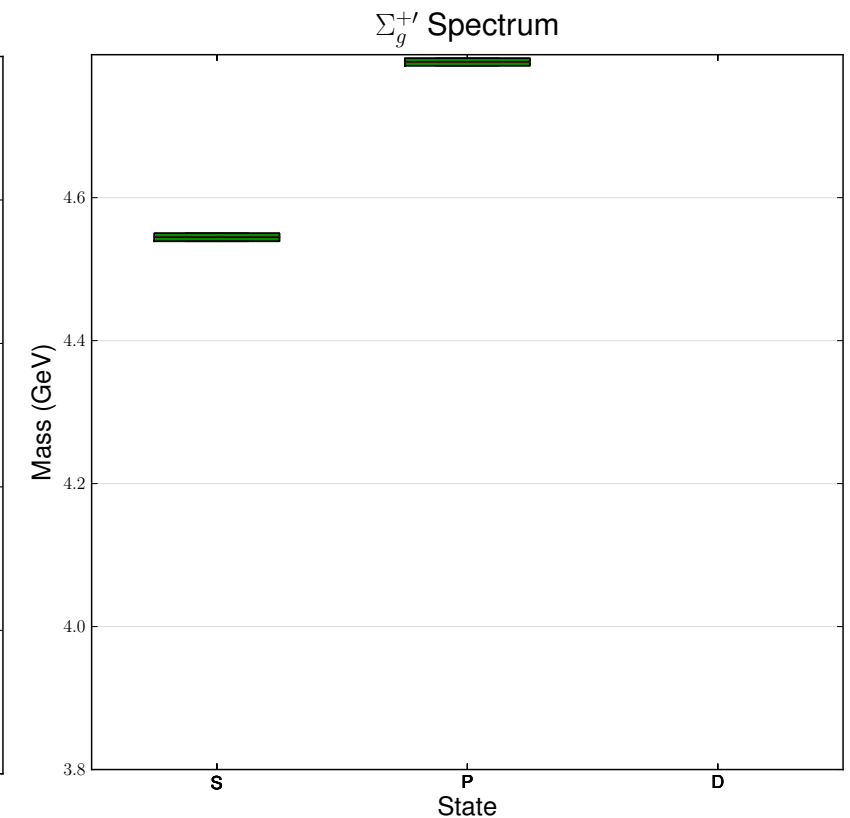
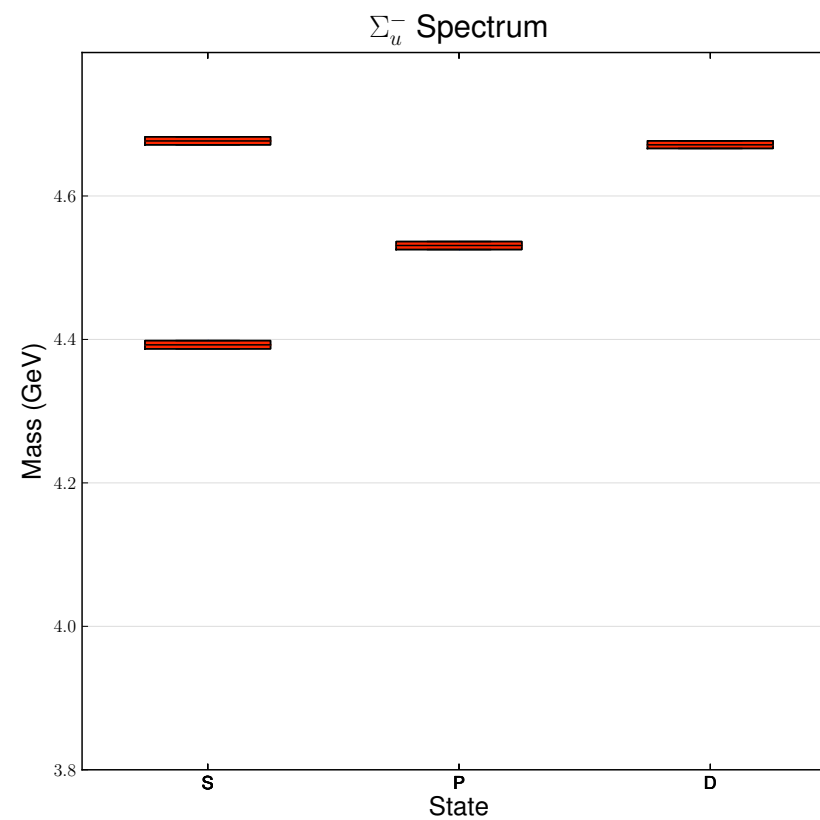
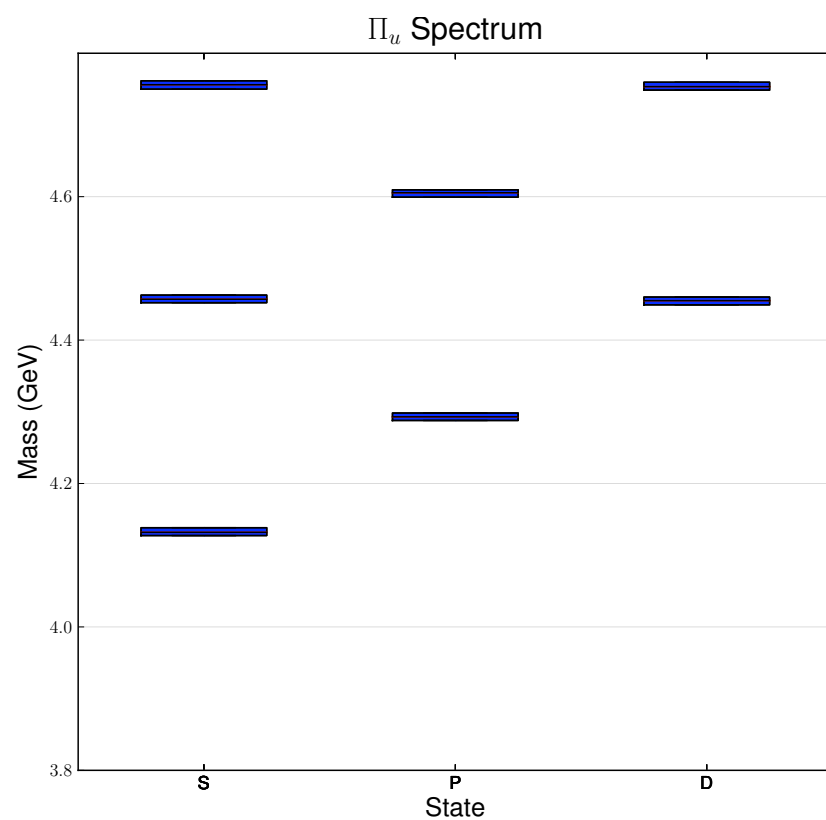


FIG. 2: Short-distance degeneracies and crossover in the spectrum. The solid curves are only shown for visualization. The dashed line marks a lower bound for the onset of mixing effects with glueball states which requires careful interpretation.

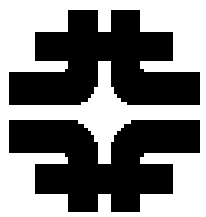


# Spectrum of Low-Lying Hybrid States

- Only interested in states below 4.8 GeV for cc system.  
Unlikely higher states will be narrow (DD, glueball+J/ψ, etc)

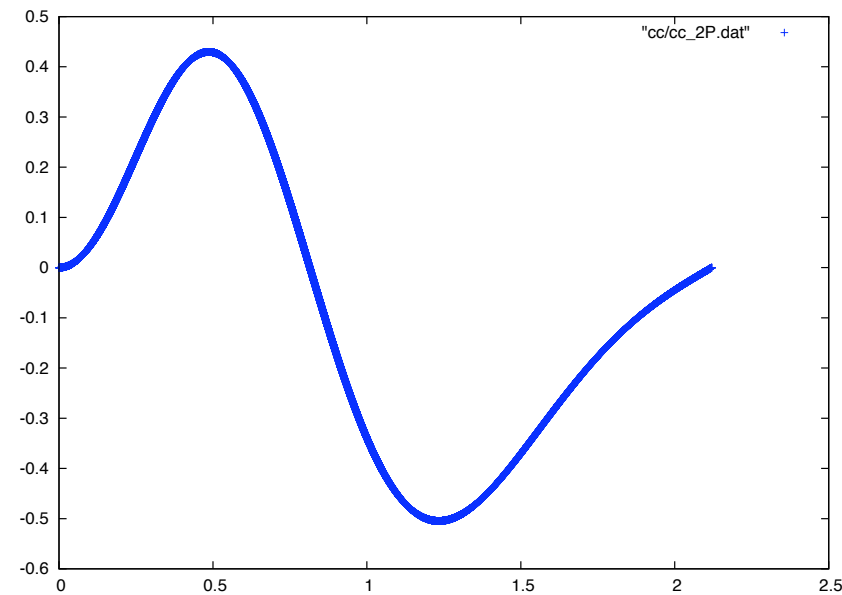
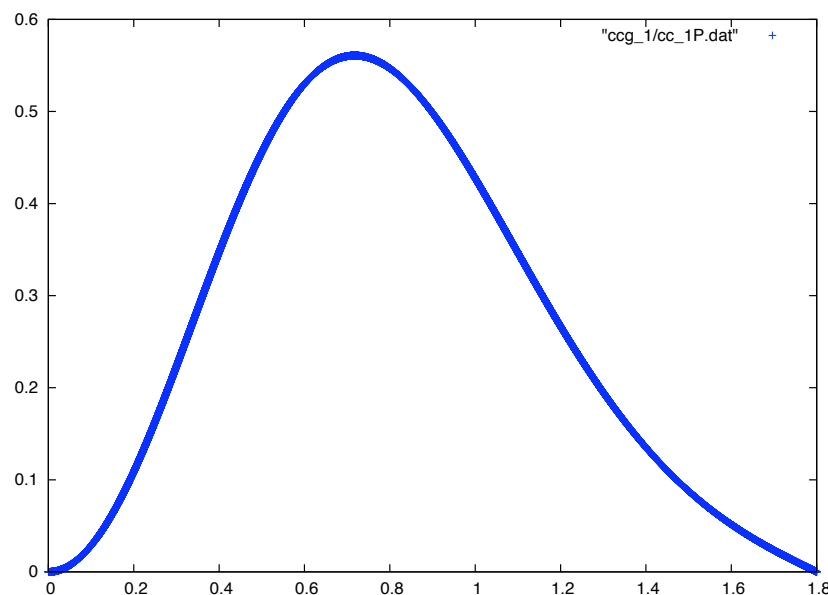


- Only  $\Pi_u$ ,  $\Sigma_u^-$ , and  $\Sigma_g^{+'}$  systems have sufficiently light states.

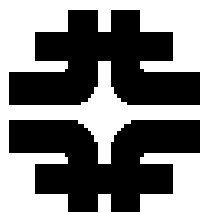


# Spectrum of Low-Lying Hybrid States

- $\Pi_u(1S)$   $m = 4.132 \text{ GeV}$      $\Pi_u(2S)$   $m = 4.465 \text{ GeV}$      $J^{PC} = 0^{++}, 0^{--}, 1^{+-}, 1^{-+}$   
 $\Pi_u(1P)$   $m = 4.445 \text{ GeV}$      $\Pi_u(2P)$   $m = 4.773 \text{ GeV}$      $J^{PC} = 1^{--}, 1^{++}, 0^{-+}, 0^{+-}, 1^{+-}, 1^{-+}, 2^{+-}, 2^{-+}$



- $\Sigma_g^{+'}(1S)$   $m = 4.547 \text{ GeV}$      $J^{PC} = 0^{-+}, 1^{--}$
- The  $\Pi_u(1P)$ ,  $\Pi_u(2P)$  and  $\Sigma_g^{+'}(1S)$  have  $1^{--}$  states with spacing seen in the  $\Upsilon(4260)$  system
- $\Sigma_u^{-(1S)}$   $m = 4.292 \text{ GeV}$      $\Sigma_u^{-(1P)}$   $m = 4.537 \text{ GeV}$      $\Sigma_u^{-(2S)}$   $m = 4.772 \text{ GeV}$
- Numerous states with  $C=+$  in the  $4.2 \text{ GeV}$  region.



# Spectrum of Low-Lying Hybrid States

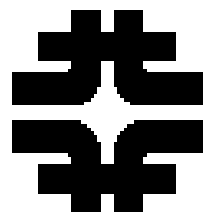
- The spectrum of bottomonium hybrids is completely predicted as well
- For the  $\Pi_u$  states

(cc) L n mass(GeV)

|   |   |   |          |
|---|---|---|----------|
|   | 0 | 1 | 4.132580 |
|   | 0 | 2 | 4.454556 |
|   | 0 | 3 | 4.752947 |
|   | 0 | 4 | 5.032962 |
|   | 0 | 5 | 5.298250 |
|   | 0 | 6 | 5.551412 |
| ✓ | 1 | 1 | 4.293717 |
|   | 1 | 2 | 4.604123 |
|   | 1 | 3 | 4.893249 |
|   | 1 | 4 | 5.165793 |
|   | 1 | 5 | 5.424925 |
|   | 2 | 1 | 4.454768 |
|   | 2 | 2 | 4.753368 |
|   | 2 | 3 | 5.033384 |

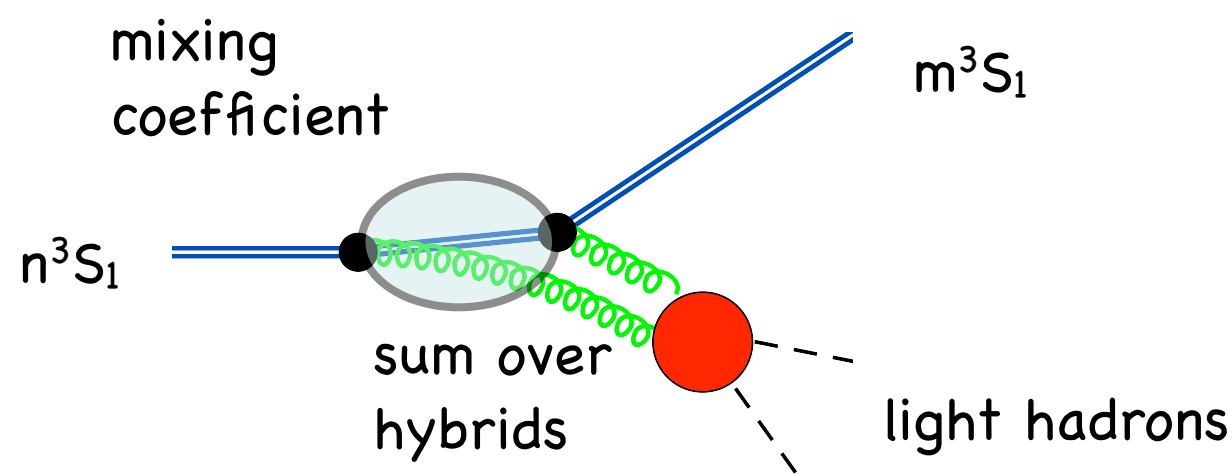
(bb) L n mass(GeV)

|   |   |   |           |
|---|---|---|-----------|
|   | 0 | 1 | 10.783900 |
|   | 0 | 2 | 10.982855 |
|   | 0 | 3 | 11.172408 |
|   | 0 | 4 | 11.353469 |
|   | 0 | 5 | 11.527274 |
|   | 0 | 6 | 11.694851 |
|   | 0 | 7 | 11.856977 |
|   | 0 | 8 | 12.014256 |
| ✓ | 1 | 1 | 10.877928 |
|   | 1 | 2 | 11.073672 |
|   | 1 | 3 | 11.259766 |
|   | 1 | 4 | 11.437735 |
|   | 1 | 5 | 11.608810 |
|   | 1 | 6 | 11.773931 |
|   | 1 | 7 | 11.933823 |
|   | 2 | 1 | 10.976071 |
|   | 2 | 2 | 11.167070 |
|   | 2 | 3 | 11.349124 |
|   | 2 | 4 | 11.523652 |
|   | 2 | 5 | 11.691737 |
|   | 2 | 6 | 11.854216 |

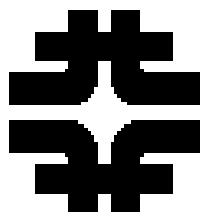


# Hybrid Decays and Hadronic Transitions

- Information from hadronic transitions might be used to estimate decay rates for a hybrid  $1^{--}$  state (H) to a  $(Q\bar{Q})$  state + light hadrons.



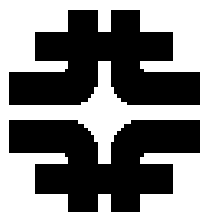
- If one hybrid state dominates, branching ratios calculable. :  
e.g.  $BR(H \rightarrow \psi' + \pi^+\pi^-) / BR(H \rightarrow J/\psi + \pi^+\pi^-)$ .
- Mixing between  $(Q\bar{Q})$  states and hybrid  $(Q\bar{Q}g)$  states can be calculated using Lattice QCD.



# Summary

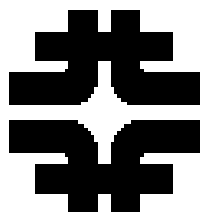
- The wealth of precision data brings the QCDME approach for hadronic transitions into sharp focus.
- Below threshold many successes but some puzzles:  
 $\Upsilon(nS) \rightarrow \Upsilon(mS) + \pi\pi$  (3:1), (4:2) and  $\eta$  transitions
- We see new states and possibly a new spectroscopy:  
 $X(3872)$ ,  $Y(4140)$ ,  $Y(4350)$ ,  $Y(4260)$ ,  $Y(4360)$ ,  $Y(4660)$ ,  
 $Z_c^+(4430)$ ,  $Z_b^+(10610)$ ,  $Z_b^+(10650)$ , ...
- Above threshold QCDME is inadequate as formulated. Incorporation of strong thresholds and possible new degrees of freedom required.
- Systematic inclusion of hybrid spectrum is possible.
- Future prospects bright:
  - NRQCD and HQET allows scaling from c to b systems. This will eventually provide critical tests of our understanding of hadronic transitions.
  - Lattice QCD will provide needed insight into theoretical issues.
  - Answers will require the new generation of heavy flavor experiments - BES III, LHCb and Super-B factories.



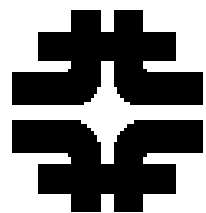


# Wish List

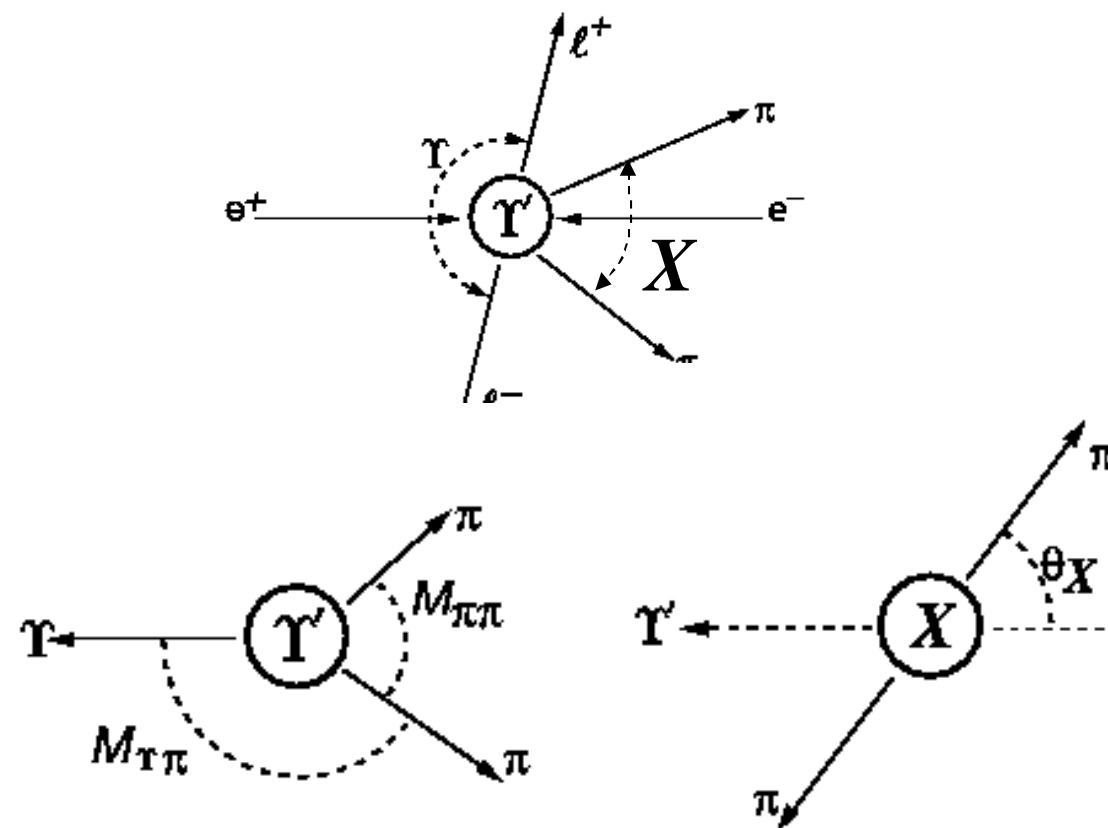
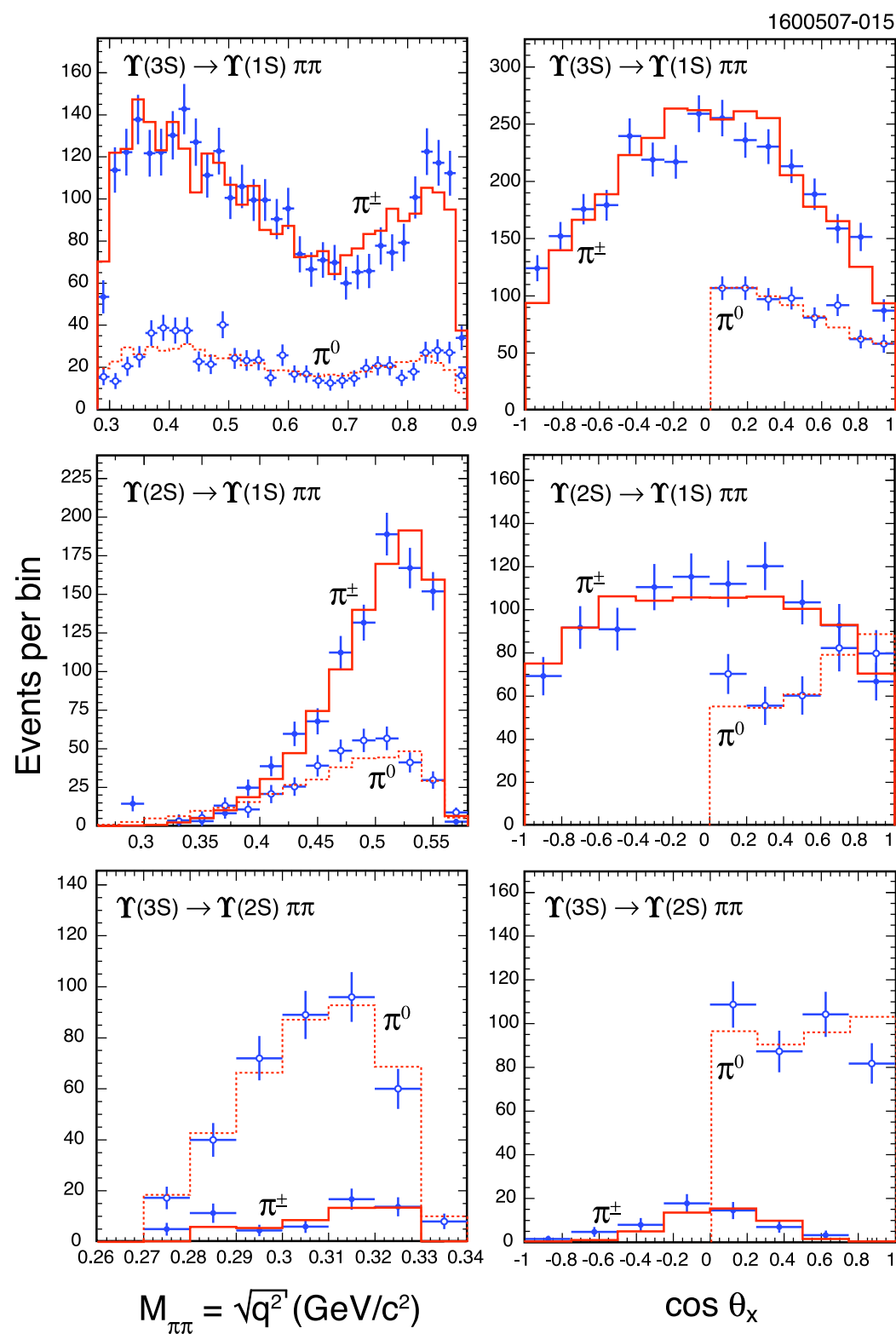
- Study all the above threshold resonances for all allowed hadronic transitions
  - $\psi(4040)$ ,  $\psi(4160)$ ,  $\psi(4260)$ ,  $\psi(4350)$ ,  $\psi(4415)$  (BESIII, LHCb)
  - $\Upsilon(5S)$  and  $\Upsilon(6S)$
- Further studies of the  $\Upsilon(3S) \rightarrow \Upsilon(1S) + \pi\pi$  including polarization
- Theory of  $\eta$  transitions.
- Observation of  $^3D_2$  and/or  $^3D_3$  in transitions to  $J/\psi + \pi\pi$  at LHCb



# Backup Slides



# $\Upsilon(3S) \rightarrow \Upsilon(1S) + \pi\pi$

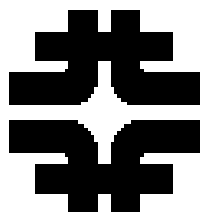


$$q^2 = M_{\pi\pi}^2$$

$$r^2 = M_{\Upsilon\pi}^2$$

$$\cos \theta_X = \frac{M_{\Upsilon(2S)}^2 + M_{\Upsilon(1S)}^2 + 2m_\pi^2 - q^2 - 2r^2}{\sqrt{\Lambda_3(M_{\Upsilon(2S)}^2, M_{\Upsilon(1S)}^2, q^2)} \frac{q^2 - 4m_\pi^2}{q^2}}$$

CLEO



# Detailed study

$$\mathcal{M} = S (\epsilon_1 \cdot \epsilon_2) + D_1 \ell_{\mu\nu} \frac{P^\mu P^\nu}{P^2} (\epsilon_1 \cdot \epsilon_2) + D_2 q_\mu q_\nu \epsilon^{\mu\nu} + D_3 \ell_{\mu\nu} \epsilon^{\mu\nu} .$$

Voloshin [PR D74:054022(2006)]

## S-wave

$$S(\psi_2 \rightarrow \pi^+ \pi^- \psi_1) = -\frac{4\pi^2}{b} \alpha_0^{(12)} \left[ (1 - \chi_M) (q^2 + m^2) - (1 + \chi_M) \kappa \left( 1 + \frac{2m^2}{q^2} \right) \left( \frac{(q \cdot P)^2}{P^2} - \frac{1}{2} q^2 \right) \right] (\psi_1 \cdot \psi_2) , \quad (25)$$

$$P_\mu = M_A \delta_\mu^0$$

$$r_\mu = (k_{1\mu} - k_{2\mu})$$

## and three D-waves

$$D_1(\psi_2 \rightarrow \pi^+ \pi^- \psi_1) = -\frac{4\pi^2}{b} \alpha_0^{(12)} (1 + \chi_M) \frac{3\kappa}{2} \frac{\ell_{\mu\nu} P^\mu P^\nu}{P^2} (\psi_1 \cdot \psi_2) , \quad \text{spin independent}$$

$$D_2(\psi_2 \rightarrow \pi^+ \pi^- \psi_1) = \frac{4\pi^2}{b} \alpha_0^{(12)} \left( \chi_2 + \frac{3}{2} \chi_M \right) \frac{\kappa}{2} \left( 1 + \frac{2m^2}{q^2} \right) q_\mu q_\nu \psi^{\mu\nu}$$

spin dependent

$$D_3(\psi_2 \rightarrow \pi^+ \pi^- \psi_1) = \frac{4\pi^2}{b} \alpha_0^{(12)} \left( \chi_2 + \frac{3}{2} \chi_M \right) \frac{3\kappa}{4} \ell_{\mu\nu} \psi^{\mu\nu}$$

$$\psi^{\mu\nu} = \psi_1^\mu \psi_2^\nu + \psi_1^\nu \psi_2^\mu - (2/3) (\psi_1 \cdot \psi_2) (P^\mu P^\nu / P^2 - g^{\mu\nu})$$

$$\ell_{\mu\nu} = r_\mu r_\nu + \frac{1}{3} \left( 1 - \frac{4m^2}{q^2} \right) (q^2 g_{\mu\nu} - q_\mu q_\nu)$$

magnetic

S-D mixing

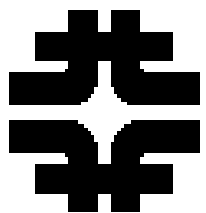
$$\chi_M = \frac{\alpha_M}{\alpha_0} , \quad \chi_2 = \frac{\alpha_2}{\alpha_0}$$

$O(v^2)$

$O(v^2)$

If <M1-M1> term significant,

expect noticeable presence of D2 and D3 in  $\Upsilon(3S) \rightarrow \Upsilon + \pi\pi$



$$M = \mathbf{A}(\varepsilon' \cdot \varepsilon)(q^2 - 2m_\pi^2) + \mathbf{B}(\varepsilon' \cdot \varepsilon)E_1E_2 + \mathbf{C}[(\varepsilon' \cdot q_1)(\varepsilon \cdot q_2) + (\varepsilon' \cdot q_2)(\varepsilon \cdot q_1)]$$

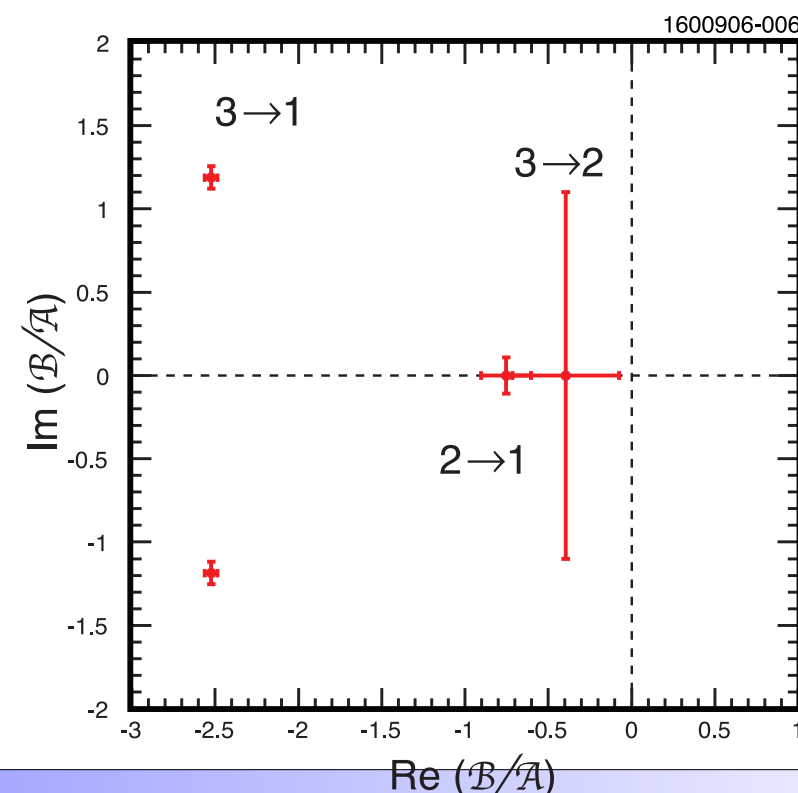
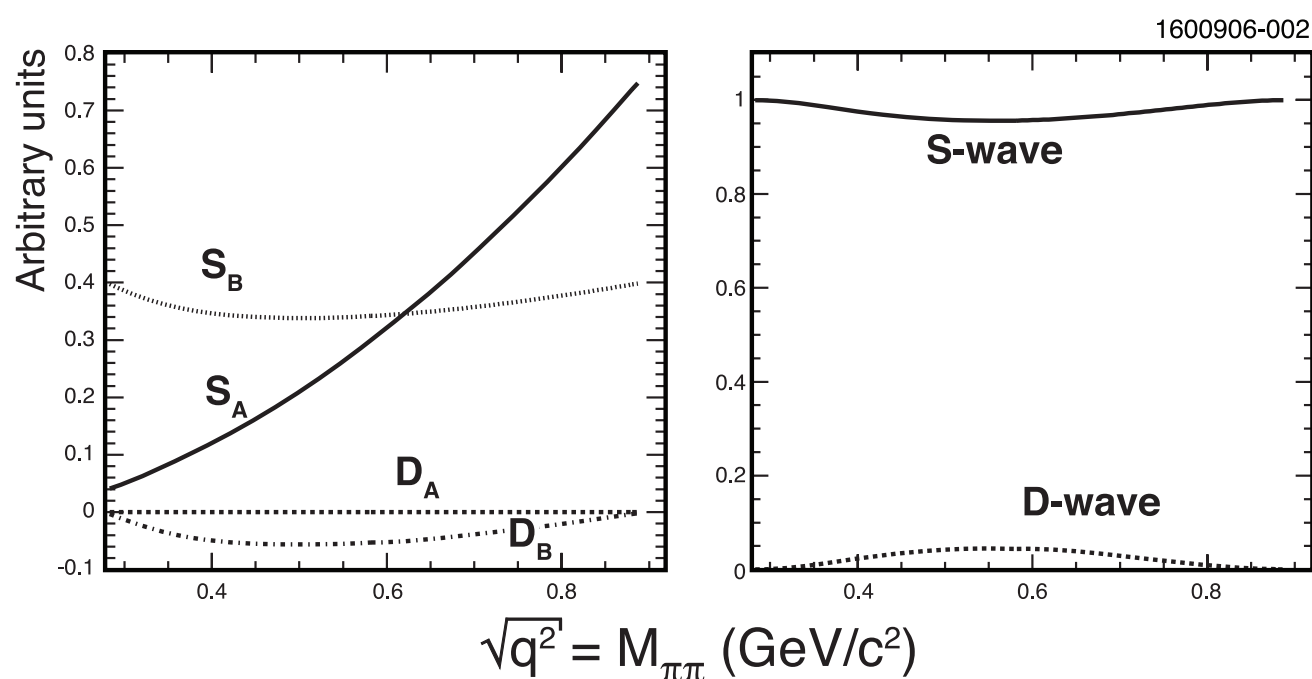
- Hindered M1-M1 term  $\Rightarrow C \approx 0$ . Consistent with CLEO results.
- Small D-wave contributions
- Useful to look at polarization info.

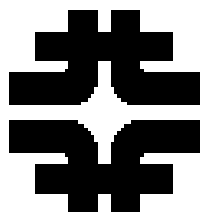
Dubynskiy & Voloshin [hep-ph/0707.1272]

CLEO

| Fit, No $\mathcal{C}$                         |                                |             | stat.       | effcy. ( $\pi^\pm$ ) | effcy. ( $\pi^0$ ) | bg. sub.    |
|-----------------------------------------------|--------------------------------|-------------|-------------|----------------------|--------------------|-------------|
| $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi$ | $\Re(\mathcal{B}/\mathcal{A})$ | -2.523      | $\pm 0.031$ | $\pm 0.019$          | $\pm 0.011$        | $\pm 0.001$ |
|                                               | $\Im(\mathcal{B}/\mathcal{A})$ | $\pm 1.189$ | $\pm 0.051$ | $\pm 0.026$          | $\pm 0.018$        | $\pm 0.015$ |
| $\Upsilon(2S) \rightarrow \Upsilon(1S)\pi\pi$ | $\Re(\mathcal{B}/\mathcal{A})$ | -0.753      | $\pm 0.064$ | $\pm 0.059$          | $\pm 0.035$        | $\pm 0.112$ |
|                                               | $\Im(\mathcal{B}/\mathcal{A})$ | 0.000       | $\pm 0.108$ | $\pm 0.036$          | $\pm 0.012$        | $\pm 0.001$ |
| $\Upsilon(3S) \rightarrow \Upsilon(2S)\pi\pi$ | $\Re(\mathcal{B}/\mathcal{A})$ | -0.395      | $\pm 0.295$ |                      | $\pm 0.025$        | $\pm 0.120$ |
|                                               | $\Im(\mathcal{B}/\mathcal{A})$ | $\pm 0.001$ | $\pm 1.053$ |                      | $\pm 0.180$        | $\pm 0.001$ |
| Fit, float $\mathcal{C}$                      |                                |             | stat.       | effcy. ( $\pi^\pm$ ) | effcy. ( $\pi^0$ ) | bg. sub.    |
| $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi$ | $ \mathcal{B}/\mathcal{A} $    | 2.89        | $\pm 0.11$  | $\pm 0.19$           | $\pm 0.11$         | $\pm 0.027$ |
|                                               | $ \mathcal{C}/\mathcal{A} $    | 0.45        | $\pm 0.18$  | $\pm 0.28$           | $\pm 0.20$         | $\pm 0.093$ |

3S $\rightarrow$ 1S





# Some Puzzles

- Reducing model dependence

- transitions well below the first string excitation ( $E_{\text{TH}}$ ), so expand

$$\begin{aligned}\mathcal{G}(E) &= \sum_{KL} |KL\rangle \frac{1}{E - E_{KL}} \langle KL| \\ &= \frac{1}{E - E_{\text{TH}}} + \sum_{KL} \left( \frac{E_{KL} - E_{\text{TH}}}{E - E_{\text{TH}}} \right) |KL\rangle \frac{1}{E - E_{KL}} \langle KL|\end{aligned}$$

model dependence  
suppressed

(a)  $E \ll E_{\text{TH}}$

(b) small overlap of low-lying QQ states with high  $|KL\rangle$  states.

- for E1-E1 transitions

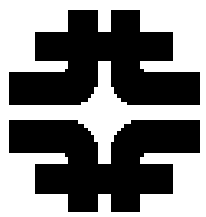
$$\langle B | \mathbf{r}^i \chi^a \mathcal{G}(E_i) \mathbf{r}^j \chi_b | A \rangle = \frac{\delta^{ij} \delta_b^a}{E_A - E_{\text{TH}}} \langle B | \mathbf{r}^2 | A \rangle + \dots$$

- compare results with known transitions

$E_{\text{TH}}^{c\bar{c}} = 4.5 \text{ GeV}$  and  $E_{\text{TH}}^{b\bar{b}} = 11.25 \text{ GeV}$  assumed

| Transition                                            | G (GeV) <sup>7</sup>  | $\langle f   r^2   i \rangle > (\text{GeV})^{-2}$ | $\Gamma(\text{exp})$ (keV) | $\Gamma(\text{overlap})$ (keV) |
|-------------------------------------------------------|-----------------------|---------------------------------------------------|----------------------------|--------------------------------|
| $\psi(2S) \rightarrow J/\psi + \pi^+ \pi^-$           | $3.56 \times 10^{-2}$ | 3.36                                              | $102.3 \pm 3.4$            | input( $ C_1 $ )               |
| $\Upsilon(2S) \rightarrow \Upsilon(1S) + \pi^+ \pi^-$ | $2.87 \times 10^{-2}$ | 1.19                                              | $5.79 \pm 0.49$            | 5.9                            |
| $\Upsilon(3S) \rightarrow \Upsilon(1S) + \pi^+ \pi^-$ | 1.09                  | $2.37 \times 10^{-1}$                             | $0.894 \pm 0.084$          | 12.9                           |
| $\Upsilon(3S) \rightarrow \Upsilon(2S) + \pi^+ \pi^-$ | $9.09 \times 10^{-5}$ | 3.70                                              | $0.498 \pm 0.065$          | 0.26                           |
| $\Upsilon(4S) \rightarrow \Upsilon(1S) + \pi^+ \pi^-$ | 5.58                  | $9.74 \times 10^{-2}$                             | $1.64 \pm 0.25$            | 19.9                           |
| $\Upsilon(4S) \rightarrow \Upsilon(2S) + \pi^+ \pi^-$ | $2.61 \times 10^{-2}$ | $4.64 \times 10^{-1}$                             | $1.76 \pm 0.34$            | 2.1                            |

OK only if overlap is sizable



| Transition Ratio | Belle                    |
|------------------|--------------------------|
| $R(2, 1)$        | $1.47 \pm 0.15 \pm 0.20$ |
| $R(3, 1)$        | $0.91 \pm 0.35 \pm 0.15$ |

$$R(n, m) \equiv \frac{\Gamma(\Upsilon(5S) \rightarrow \pi^+ \pi^- + \Upsilon(nS))}{\Gamma(\Upsilon(5S) \rightarrow \pi^+ \pi^- + \Upsilon(mS))}$$

phase space ( $\text{GeV}^{-7}$ )

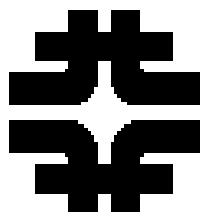
$$\Gamma(\Upsilon(5S) \rightarrow \pi^+ \pi^- + \Upsilon(nS)) \propto G(n) |f(n)|^2$$

$$\text{with } f(n) = \sum_l \frac{< \Upsilon(5S) | r | \Sigma_g^{+'}(lP) > < \Sigma_g^{+'}(lP) | r | \Upsilon(nS) >}{M_{\Upsilon(5S)} - E_l(\Sigma) + i\Gamma_l(\Sigma)}|^2$$

$$G(n) = 28.7, 0.729, 1.33 \times 10^{-2} \text{ for } n = 1, 2, 3$$

## theory – hadronic transition rates

- If lowest hybrid mass near  $\Upsilon(5S)$  a few states dominate sum. Results sensitive to mass value.
- If hybrid mass  $10.75 + i(0.1)$  (GeV), obtain  $R(2,1) \approx 1.1$  and  $R(3,1) \approx 0.08$ .
- Overall scale of transitions nearly two orders of magnitude larger than low-lying transitions.



# Hybrid States and Lattice QCD

- Heavy quark limit: Born-Oppenheimer approximation

$$-\frac{1}{2\mu} \frac{d^2 u(r)}{dr^2} + \left\{ \frac{\langle \mathbf{L}_{Q\bar{Q}}^2 \rangle}{2\mu r^2} + V_{Q\bar{Q}}(r) \right\} u(r) = E u(r) \quad \Psi_{Q\bar{Q}}(\vec{r}) = \frac{u_{nl}(r)}{r} Y_{lm}(\theta, \phi)$$

Spectroscopic notation of diatomic molecules

$$\mathbf{J} = \mathbf{L} + \mathbf{S}, \quad \mathbf{S} = \mathbf{s}_Q + \mathbf{s}_{\bar{Q}}, \quad \mathbf{L} = \mathbf{L}_{Q\bar{Q}} + \mathbf{J}_g$$

$$\langle \mathbf{L}_r \mathbf{J}_{gr} \rangle = \langle \mathbf{J}_{gr}^2 \rangle = \Lambda^2 \quad \langle \mathbf{L}_{Q\bar{Q}}^2 \rangle = L(L+1) - 2\Lambda^2 + \langle \mathbf{J}_g^2 \rangle. \quad \langle \mathbf{J}_g^2 \rangle = 0, 2, 6, \dots$$

$\Lambda = 0, 1, 2, \dots$  denoted  $\Sigma, \Pi, \Delta, \dots$

naively 0, 1, 2, ... valence gluons

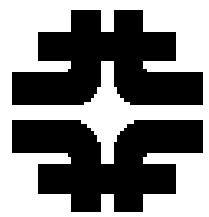
$$P = \varepsilon (-1)^{L+\Lambda+1}, \quad C = \eta \varepsilon (-1)^{L+S+\Lambda}.$$

$\eta = \pm 1$  (symmetry under combined charge conjugation and spatial inversion)  
denoted g(+1) or u(-1)

$$|LSJM; \lambda \eta\rangle + \varepsilon |LSJM; -\lambda \eta\rangle$$

with  $\varepsilon = +1$  for  $\Sigma^+$  and  $\varepsilon = -1$  for  $\Sigma^-$  both signs for  $\Lambda > 0$ .





# Determining the Hybrid Potentials

- Short distance ( $R < 0.25$  fm)

The short distance behavior of pNRQCD is confirmed by lattice studies of hybrid potentials and the relation to gluelumps is computed.

G. S. Bali and A. Pineda, Phys. Rev. D 69, 094001 (2004)

A. Pineda [hep-lat/0702019]

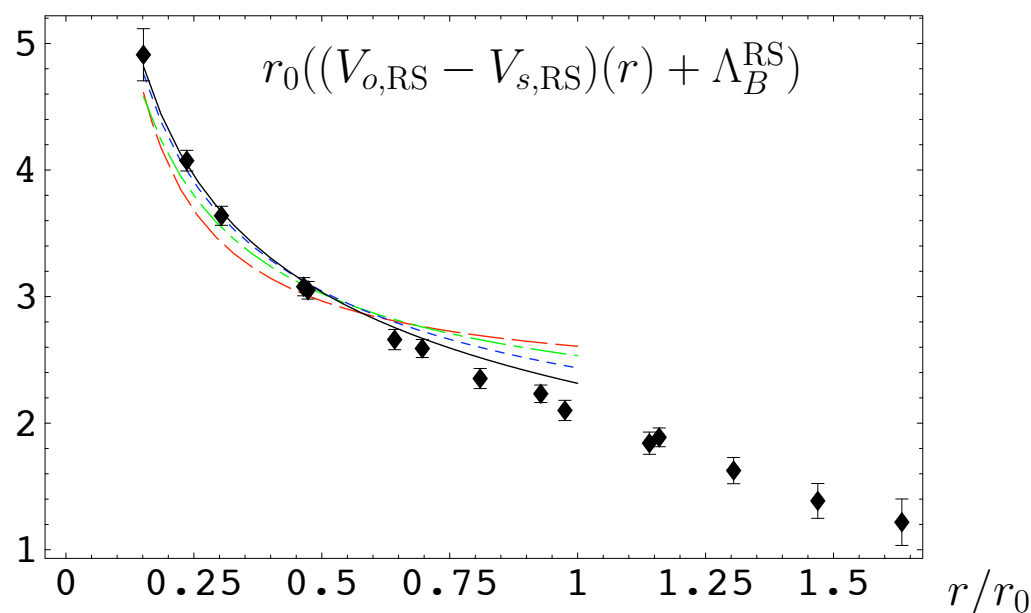
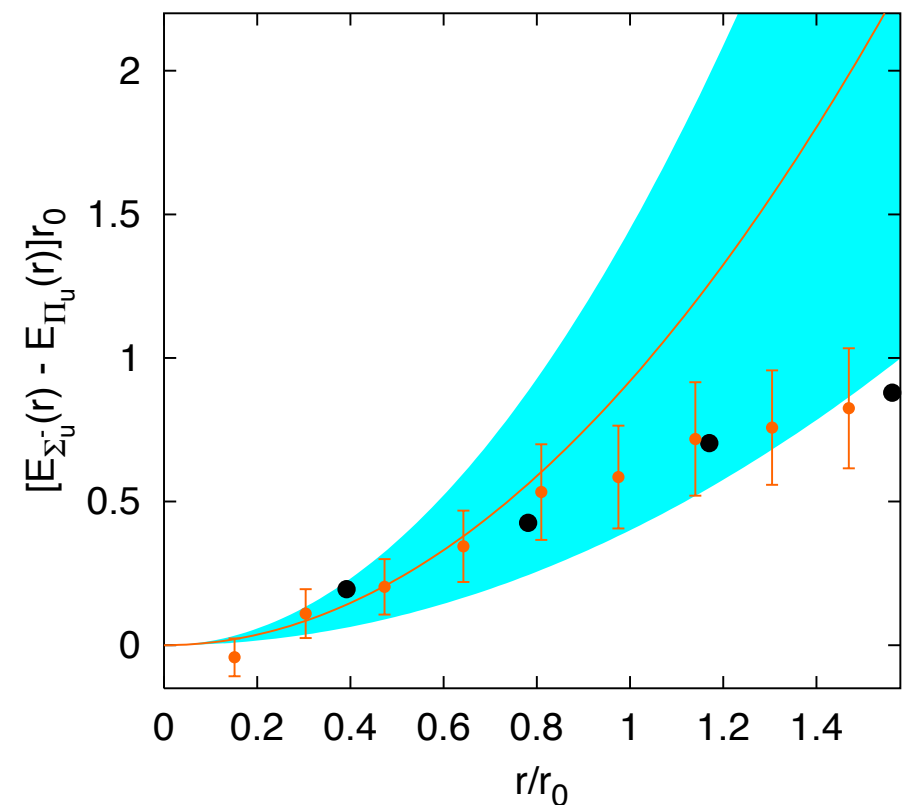
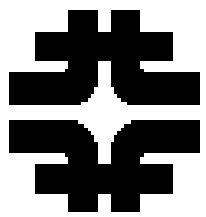


Figure 12: Splitting between the  $\Pi_u$  and the  $\Sigma_g^+$  potentials and the comparison with Eq. (65) for  $\nu = \nu_i$  [see Eq. (16)] at  $\nu_f = 2.5 r_0^{-1}$ .  $r_0[(V_{o,RS} - V_{s,RS})(r) + \Lambda_B^{RS}]$  is plotted at tree level (dashed line), one-loop (dashed-dotted line), two loops (dotted line) and three loops (estimate) plus the leading single ultrasoft log (solid line).

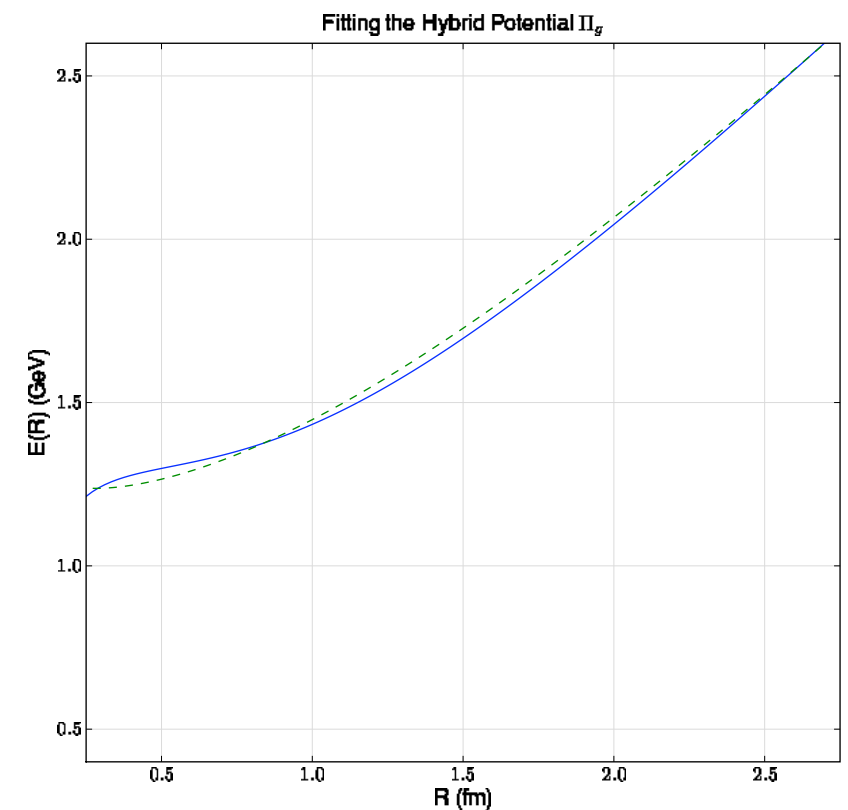
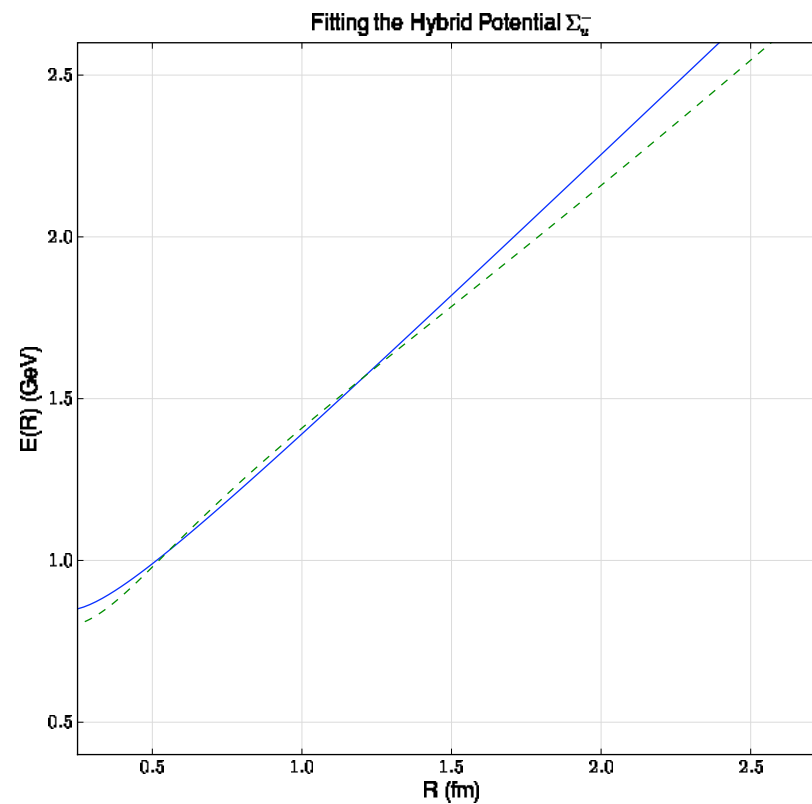
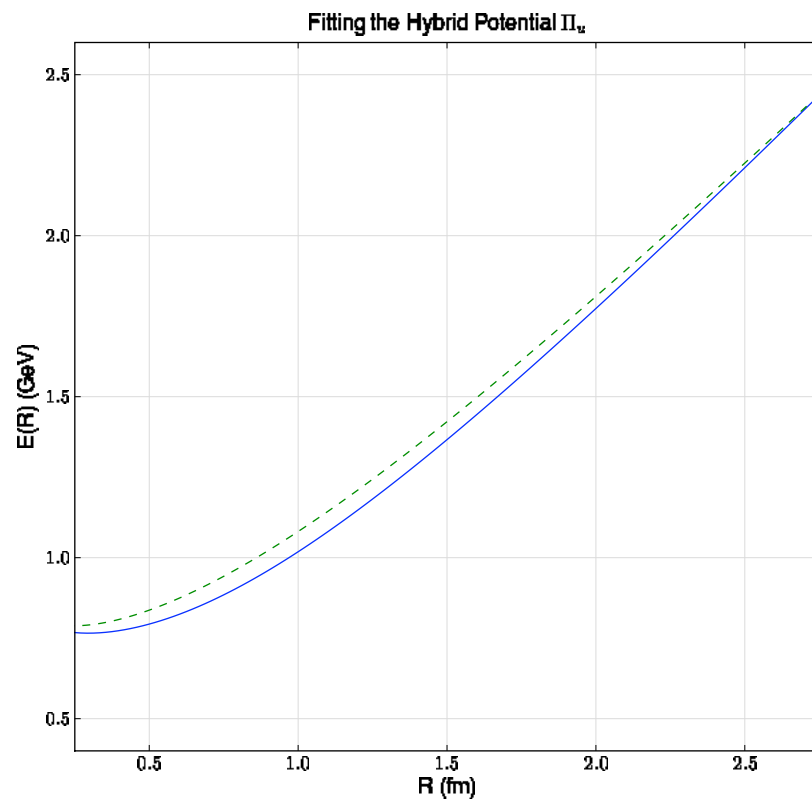


The corrections of order  $R^2$  split the gluelump degeneracies:

Roughly speaking  $V(R) = 1/6 \alpha(R)/R + C_0(\text{gluelump state}) + C_2(R)R^2 + \dots$



# Comparing Toy Model to Lattice Results



Comparing this model (dashed lines) to the parameterization of The fits to Juge, Kuti and Morningstar lattice results (thanks to Juge) (solid lines) one finds fairly good agreement in the region ( $0.25 \text{ fm} < R < 2 \text{ fm}$ )