Interplay of quark and meson degrees of freedom in a near-threshold resonance

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Based on

V.Baru, C.Hanhart, Yu.Kalashnikova, A.Kudryavtsev, A.N.
Eur.Phys.J. **A44**, 93 (2010)

C.Hanhart, Yu.Kalashnikova, A.N.,
Eur.Phys.J. **A47**, 101 (2011)

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- Many newly observed resonances reside near two-body mesonic thresholds
- so that both quark and meson dynamics affect their properties
- and the interplay of quark and meson degrees of freedom may result in quite peculiar properties of such near-threshold resonances

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- Interplay of quark and meson degrees of freedom takes place $\Longrightarrow$ **???**

The problem

What is the generalisation of the simple Flatté formulae to account for the interplay of quark and meson degrees of freedom in a near-threshold resonance?

Essentials of the formalism

$$\hat{\mathcal{H}}|X\rangle = E|X\rangle$$

$$|X\rangle = \begin{pmatrix} c|\psi_0\rangle \\ \chi_1(\mathbf{p})|M_{11}M_{12}\rangle \\ \chi_2(\mathbf{p})|M_{21}M_{22}\rangle \\ \cdots \end{pmatrix} \quad \hat{\mathcal{H}} = \begin{pmatrix} E_0 & f_1 & f_2 & \cdots \\ f_1 & H_{h_1} & V_{12} & \cdots \\ f_2 & V_{21} & H_{h_2} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

$$H_{h_i}(\mathbf{p},\mathbf{p}') = \left(\Delta_i + \frac{p^2}{2\mu_i}\right)\delta^{(3)}(\mathbf{p}-\mathbf{p}') + V_{ii}(\mathbf{p},\mathbf{p}')$$

$V_{ij}(\mathbf{p},\mathbf{p}')$ — direct mesonic interactions

$f_i(\mathbf{p})$ — transition between quark state and i-th mesonic state

$\Delta_i = (m_{i1} + m_{i2}) - (m_{11} + m_{12})$ — splitting between thresholds

Lippmann–Schwinger equation for the t-matrix

$$\begin{cases} t_{00} = -\sum_k \int f_k \; S_k \; t_{k0} \; d^3q \\ t_{i0} = f_i - f_i \; S_0 \; t_{00} - \sum_k \int V_{ik} \; S_k \; t_{k0} \; d^3q \\ t_{0i} = f_i - \sum_k \int f_k \; S_k \; t_{ki} \; d^3q \\ t_{ij} = V_{ij} - f_i \; S_0 \; t_{0j} - \sum_k \int V_{ik} \; S_k \; t_{kj} \; d^3q \end{cases}$$

$$S_0 = \frac{1}{E_0 - E - i0} \qquad S_i = \frac{1}{\Delta_i + \frac{p^2}{2\mu_i} - E - i0}$$

Exact analytic solution for the t-matrix

$$t_{00} = -\frac{(E - E_0)\mathcal{G}}{E - E_0 + \mathcal{G}}$$

$$t_{0i} = \frac{(E - E_0)\bar{\phi}_i}{E - E_0 + \mathcal{G}}$$

$$t_{i0} = \frac{(E - E_0)\phi_i}{E - E_0 + \mathcal{G}}$$

$$t_{ij} = t_{ij}^V + \frac{\phi_i \bar{\phi}_j}{E - E_0 + \mathcal{G}}$$

$$\phi_i = f_i - \sum_j \int t_{ij}^V \; S_j \; f_j \; d^3q \qquad \bar{\phi}_i = f_i - \sum_j \int S_j \; f_j \; t_{ji}^V \; d^3q$$

$$\mathcal{G} = \sum_i \int f_i^2 \; S_i \; d^3q - \sum_{i,j} \int f_i \; S_i \; t_{ij}^V \; S_j \; f_j \; d^3k \; d^3q$$

Direct interaction t-matrix t^V

$$t^V_{ij} = V_{ij} - \sum_k \int V_{ik}\ S_k\ t^V_{kj}\ d^3q$$

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For two channels ($\Delta_1 = 0,\ \Delta_2 \equiv \Delta > 0,\ \mu_1 \approx \mu_2 = \mu$):

$$t^V = \frac{1}{2\pi^2\mu}\frac{1}{\mathrm{Det}}\begin{pmatrix} \gamma_s + \gamma_t + 2ik_2 & \gamma_t - \gamma_s \\ \gamma_t - \gamma_s & \gamma_s + \gamma_t + 2ik_1 \end{pmatrix}$$

$$\mathrm{Det} = 4(\gamma_s\gamma_t - k_1k_2) + 2i(\gamma_s + \gamma_t)(k_1 + k_2)$$

$$k_1 = \sqrt{2\mu E}\Theta(E) + i\sqrt{-2\mu E}\Theta(-E)$$

$$k_2 = \sqrt{2\mu(E-\Delta)}\Theta(E-\Delta) + i\sqrt{2\mu(\Delta - E)}\Theta(\Delta - E)$$

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3. Range parameters R and R':

$$\int f^2(\mathbf{q}) S_i(\mathbf{q}) d^3q = f_0^2 (R + 4i\pi^2 \mu k_i)$$

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$$\{E_0, f_0, R, R', \gamma_s, \gamma_t\} \Longrightarrow \{E_f, g_f, \gamma_s, \gamma_t\}$$

Solution of the Lippmann–Schwinger equation

$$t_s = \frac{1}{2}(t_{11} + t_{22}) + t_{12} = \frac{(E - E_C)\,(2\gamma_t + i(k_1 + k_2))}{4\pi^2\mu\ D(E)}$$

$$t_t = \frac{1}{2}(t_{11} + t_{22}) - t_{12} = \frac{2\gamma_s(E - E_f) + i(k_1 + k_2)(E - E_C)}{4\pi^2\mu\ D(E)}$$

$$t_{st} = \frac{1}{2}(t_{11} - t_{22}) = \frac{i(k_2 - k_1)(E - E_C)}{4\pi^2\mu\ D(E)}$$

$$D(E) = \gamma_s\Big(2\gamma_t + i(k_1 + k_2)\Big)(E - E_f) - \Big(2k_1k_2 - i\gamma_t(k_1 + k_2)\Big)(E - E_C)$$

$$E_C = E_f - \frac{1}{2}g_f\gamma_s$$

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1. Case (i): $|\gamma_s| \to \infty$ and $|\gamma_t| \to \infty$
2. Case (ii): small γ_s and $|\gamma_t| \to \infty$
3. Case (iii): $|\gamma_s| \to \infty$ and small γ_t
4. Case (iv): both γ_s and γ_t are small

Parameters

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$$\mu = 966.5 \text{ MeV} \quad \Delta = 8.1 \text{ MeV} \quad g_f = 0.25$$

Case	γ_s, MeV	γ_t, MeV	E_f, MeV
(i)	∞	∞	-10.47
(ii)	-30	∞	-3.22
(iii)	∞	-30	-7.77
(iv)	-30	-30	-2.97

E_f is fixed to have a bound state at $E = -0.5$ MeV

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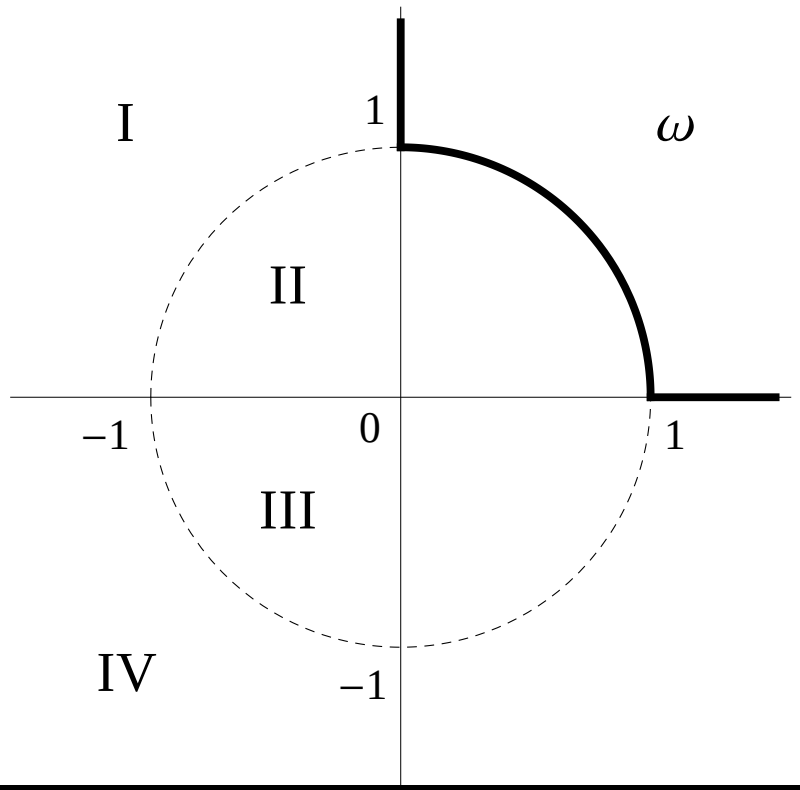
Parameter sets are similar to those describing the $X(3872)$ charmonium, however these are not fits for the $X(3872)$ data

Complex ω-plane

$$k_1 = \sqrt{\frac{\mu_1 \Delta}{2}} \left(\omega + \frac{1}{\omega} \right) \qquad k_2 = \sqrt{\frac{\mu_2 \Delta}{2}} \left(\omega - \frac{1}{\omega} \right),$$

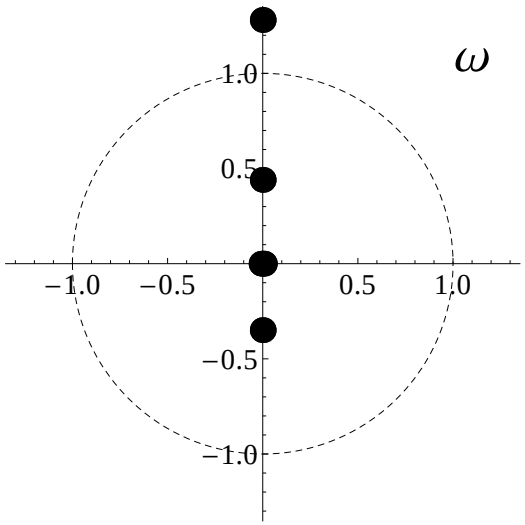
$$E = \frac{k_1^2}{2\mu_1} = \frac{k_2^2}{2\mu_2} + \Delta = \frac{\Delta}{4} \left(\omega^2 + \frac{1}{\omega^2} + 2 \right)$$



$$\begin{aligned} \mathrm{I}: &\quad \mathrm{Im}\ k_1 > 0, \quad \mathrm{Im}\ k_2 > 0 \\ \mathrm{II}: &\quad \mathrm{Im}\ k_1 < 0, \quad \mathrm{Im}\ k_2 > 0 \\ \mathrm{III}: &\quad \mathrm{Im}\ k_1 > 0, \quad \mathrm{Im}\ k_2 < 0 \\ \mathrm{IV}: &\quad \mathrm{Im}\ k_1 < 0, \quad \mathrm{Im}\ k_2 < 0 \end{aligned}$$

Thick solid line corresponds to the real values of the energy E on the first sheet

Pole positions in the complex ω-plane



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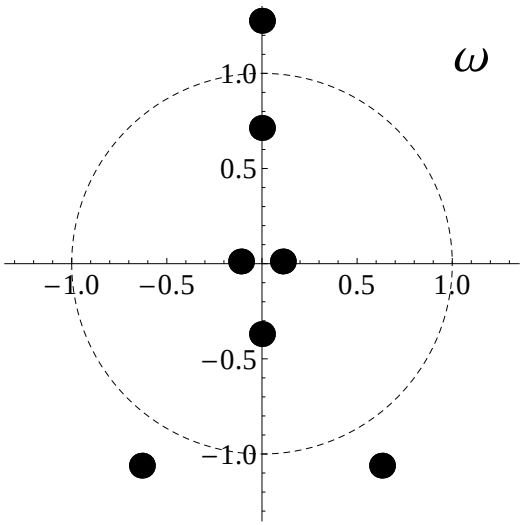
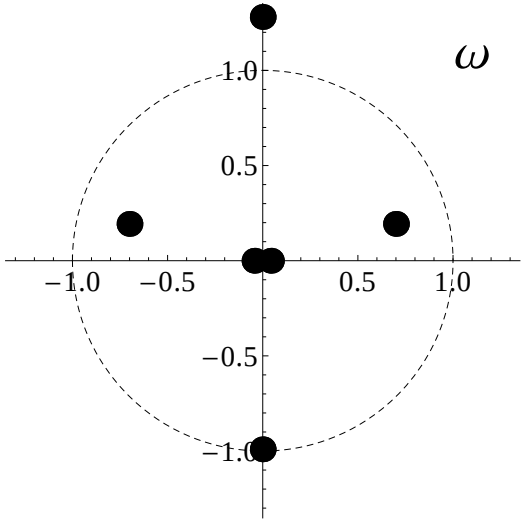


Case (ii)



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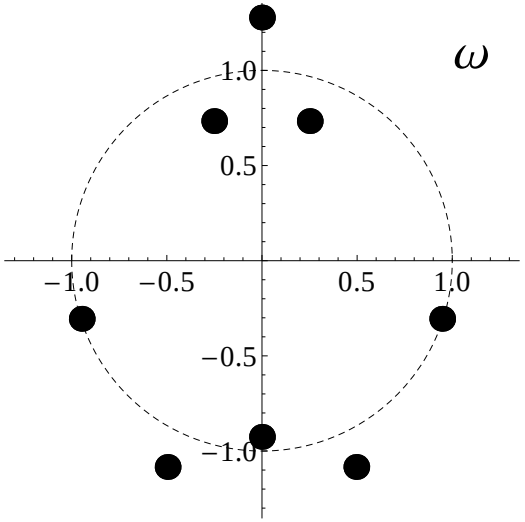

Case (iv)

Production rates

- Production from a point-like source
- Stable constituents (no events below threshold)
- No interference between production mechanisms
- Hadronic channel 1 in the final state
- Normalisation: $\int_0^{10\ \mathrm{MeV}} (dBr/dE)\, dE = 1$

$$\frac{dBr_q}{dE} \propto \left| \frac{1}{E - E_0} t_{01}(E) \right|^2 \Theta(E)\sqrt{E}$$

$$\frac{dBr_{h_1}}{dE} \propto |t_{11}(E)|^2 \Theta(E)\sqrt{E}$$

$$\frac{dBr_{h_2}}{dE} \propto |t_{21}(E)|^2 \Theta(E)\sqrt{E}$$

Production through the quark component

1. Solid line: $|\gamma_s| \to \infty$ and $|\gamma_t| \to \infty$
2. Dashed line: $\gamma_s = -30$ MeV and $|\gamma_t| \to \infty$
3. Dashed-dotted line: $|\gamma_s| \to \infty$ and $\gamma_t = -30$ MeV
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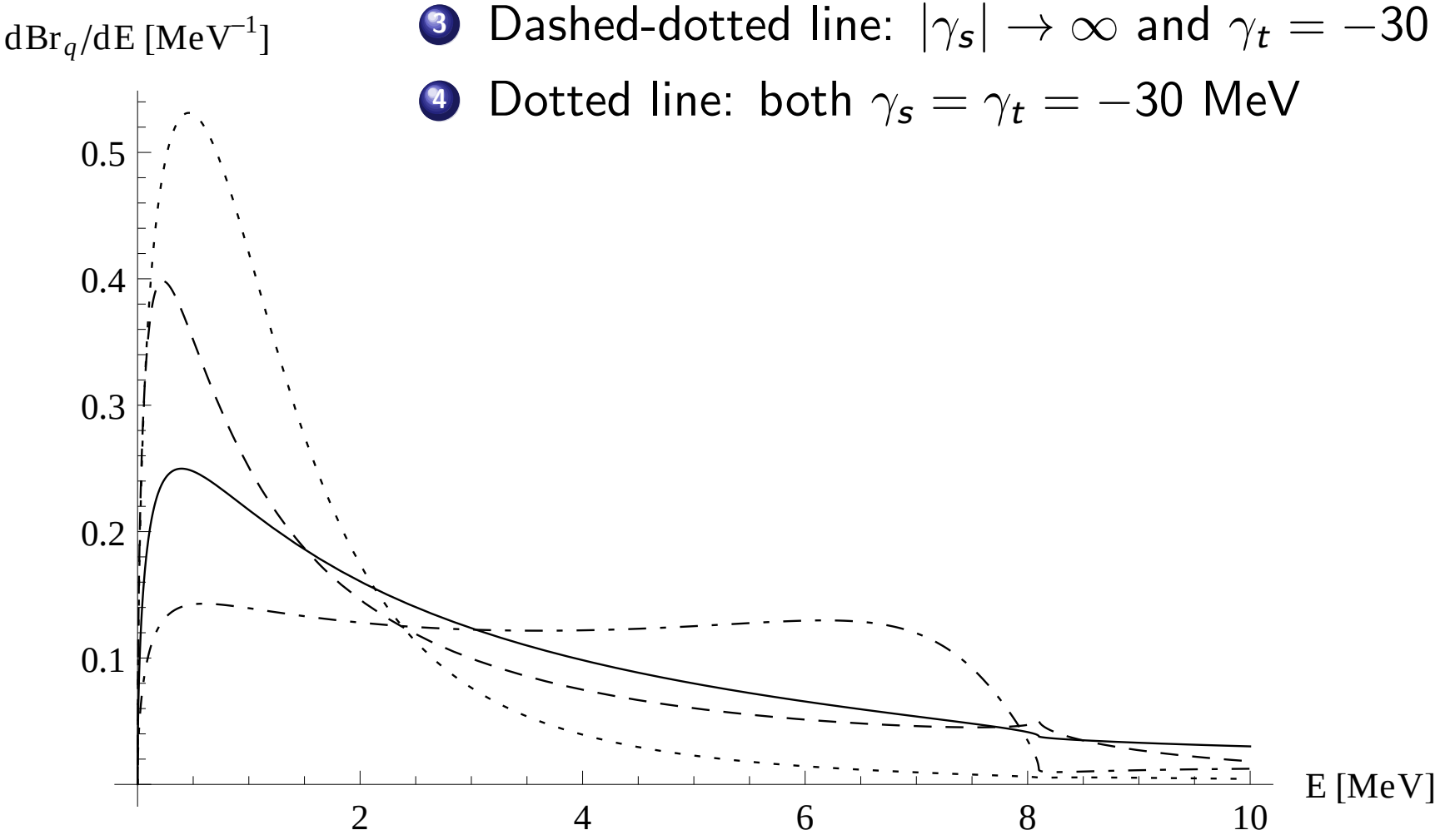

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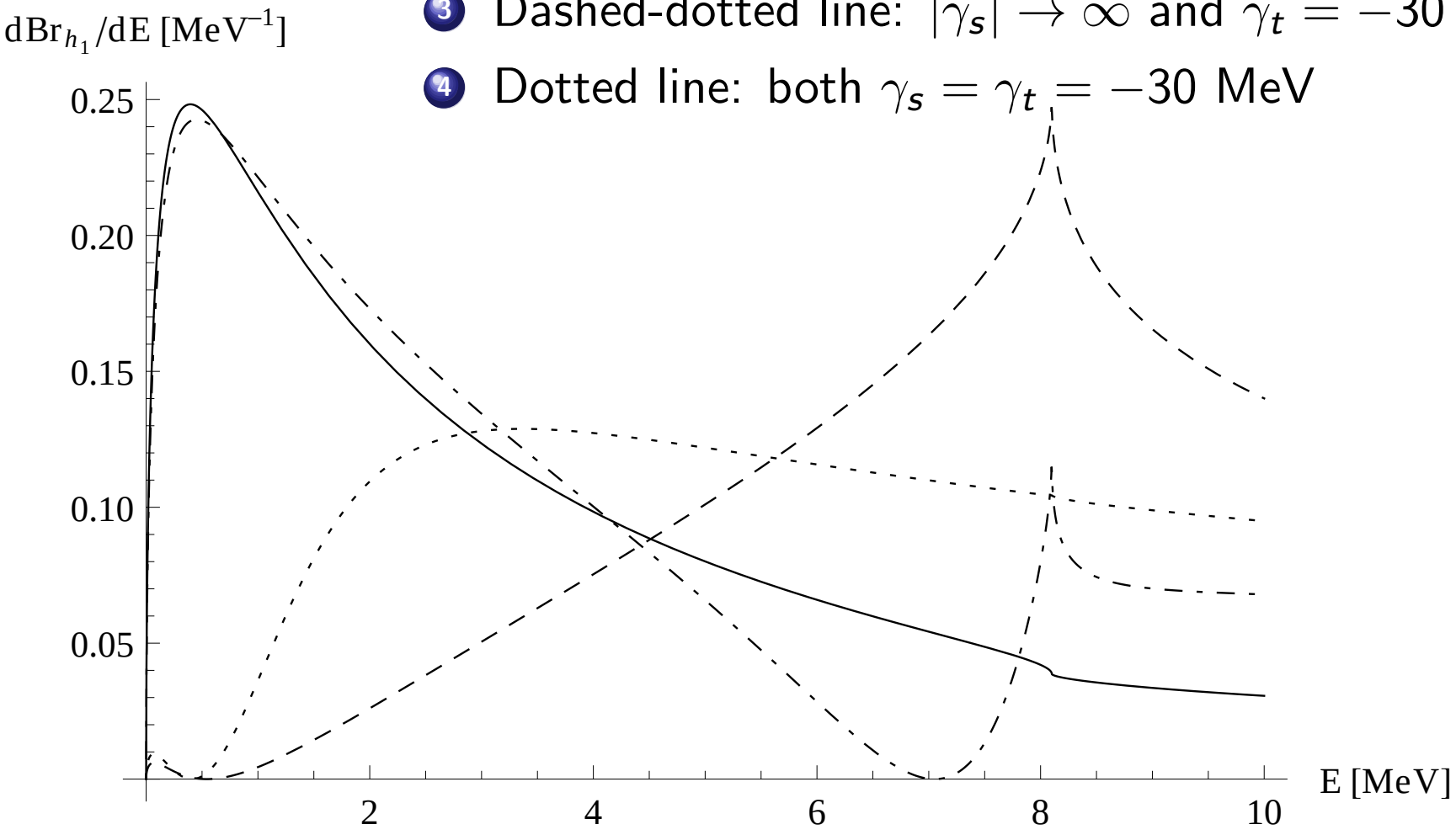

Production through the second hadronic component

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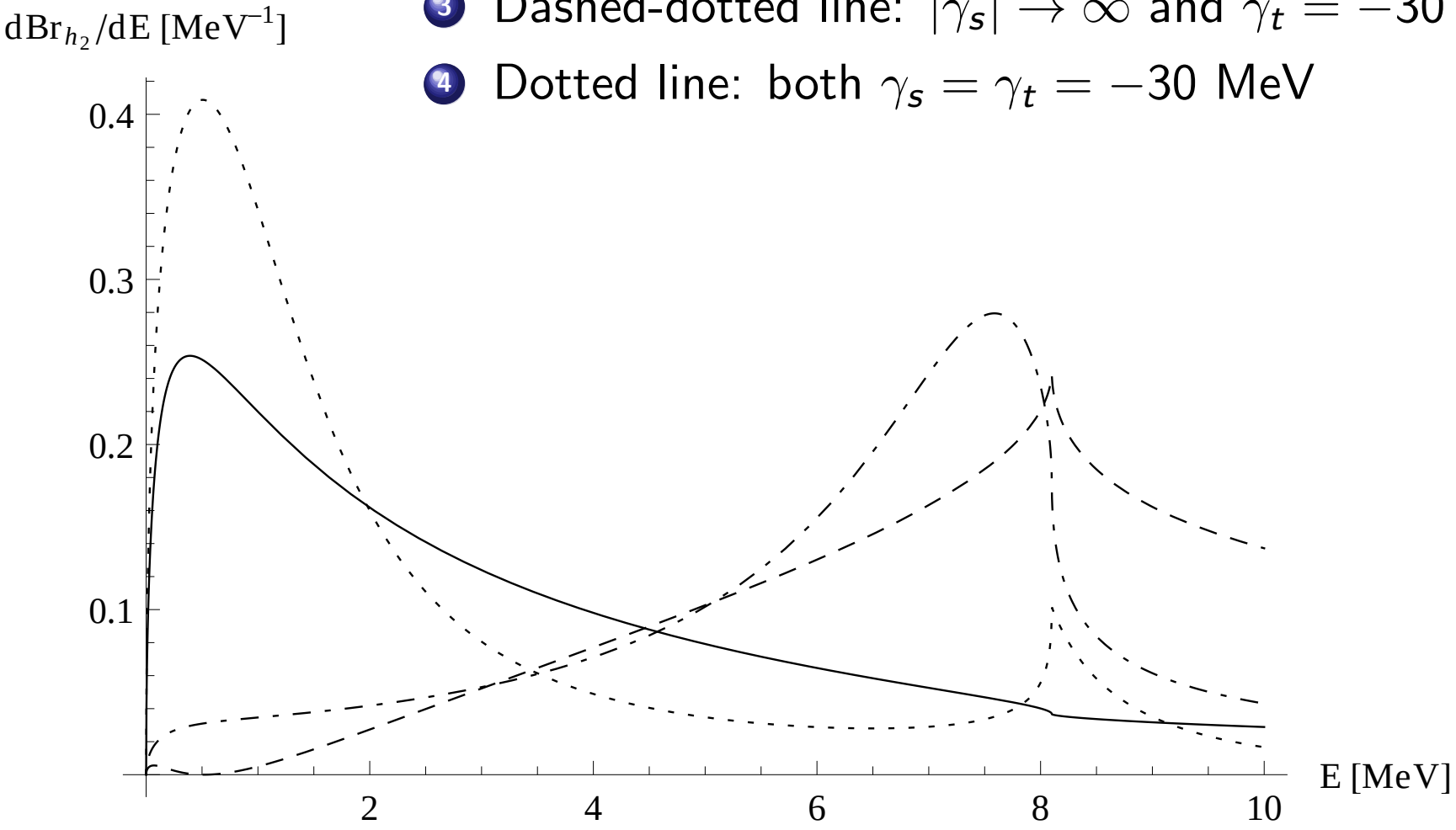

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then

the interplay of quark and meson degrees of freedom can produce line shapes of a very peculiar form

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- With the full expressions derived one can proceed beyond the near-threshold region.
- If data do not exhibit irregular behaviour, this formalism is useful to study to what extent (statistics, resolution, binning procedure, and so on) the data would need to improve to get sensitive to the structures potentially present.