

Bottomonium spectroscopy from lattice NRQCD with charm in the sea

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in collaboration with:

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Motivation

Recent improvements made in NRQCD and gluon actions

- ▶ $O(\alpha_s)$ improved matching coefficients available for the first time
- ▶ Charm quarks now included in the sea

Part of a wider study of heavy quark physics on the lattice

- ▶ Spectroscopy is a good place to tune parameters
- ▶ Calculate: quark masses, mixing elements, decay constants, etc

Improve upon/complement previous lattice NRQCD spectroscopy

[Gray et al (2005), Meinel (2010), Gregory et al (2010)]

Non-relativistic QCD

- Effective field theory valid for small v , $v^2 \sim 0.1$ for Upsilon
- Hamiltonian:

$$\begin{aligned} aH_0 &= -\frac{\Delta^{(2)}}{2aM_b} \\ a\delta H &= -c_1 \frac{(\Delta^{(2)})^2}{8(aM_b)^3} + c_2 \frac{ig}{8(aM_b)^2} (\nabla \cdot \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \cdot \nabla) \\ &\quad - c_3 \frac{g}{8(aM_b)^2} \sigma \cdot (\tilde{\nabla} \times \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \times \tilde{\nabla}) \\ &\quad - c_4 \frac{g}{2aM_b} \sigma \cdot \tilde{\mathbf{B}} + c_5 \frac{a^2 \Delta^{(4)}}{24aM_b} - c_6 \frac{a(\Delta^{(2)})^2}{16n(aM_b)^2} \end{aligned}$$

- ▶ Wilson coeff. $c_i = 1$ at tree level
- ▶ c_1, c_4, c_5, c_6 are improved through one loop - $O(\alpha_s v^4)$
[T. Hammant et al (2011), E. Müller et al]
- ▶ Non-perturbative tuning of c_3, c_4 studied

Perturbative improvement

- ▶ Matching coefficients in the action have expansion

$$c_i = 1 + \alpha_s c_i^{(1)} + O(\alpha_s^2)$$

- ▶ $c_1^{(1)}, c_5^{(1)}, c_6^{(1)}$ are found by computing the NRQCD quark self-energy and ensuring the correct energy-momentum relation holds
- ▶ $c_4^{(1)}$ from matching one loop effective action using background field method
- ▶ Lattice artefacts introduce tadpole diagrams that complicate matching with perturbation theory
 - Gauge fields are tadpole improved, this keeps $c_i^{(1)}$ small

See Georg von Hippel's talk for more details

Configurations

We use 5 MILC collaboration ensembles [A. Bazavov et al (2010)]

- ▶ u,d,s,c sea quarks included with HISQ action
- ▶ s,c quarks tuned to < 5% accuracy
- ▶ ~ 1000 configurations in each ensemble - high statistics

| β | $a(\text{fm})$ | m_l/m_s | $L^3 \times T$ |
|---------|----------------|-----------|------------------|
| 5.80 | ~ 0.15 | 0.2 | $16^3 \times 48$ |
| 5.80 | ~ 0.15 | 0.1 | $24^3 \times 48$ |
| 6.00 | ~ 0.12 | 0.2 | $24^3 \times 64$ |
| 6.00 | ~ 0.12 | 0.1 | $32^3 \times 64$ |
| 6.30 | ~ 0.09 | 0.2 | $32^3 \times 96$ |

Scale setting is often a major source of uncertainty in lattice calculations

- ▶ Used $\Upsilon(2S - 1S)$ to fix lattice spacings
- ▶ Consistent results obtained using static quark potential parameter r_1

Tuning quark masses

- ▶ b quark tuned using the kinetic mass

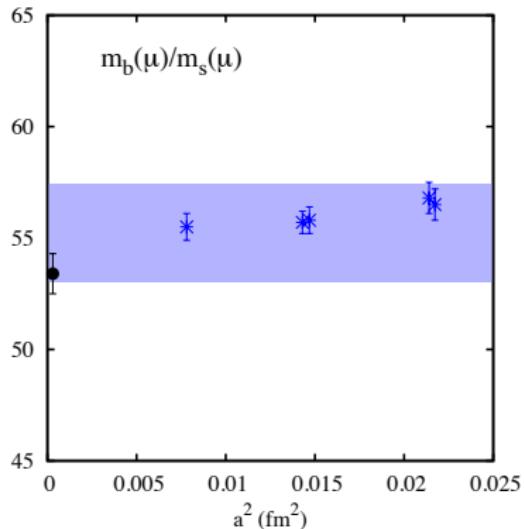
$$M_{\text{Kin}} = \frac{\mathbf{p}^2 - (E(\mathbf{p}) - E(\mathbf{0}))^2}{2(E(\mathbf{p}) - E(\mathbf{0}))}$$

- ▶ Spin average eliminates dominant systematics

$$\overline{M_{b\bar{b}}} = \frac{1}{4} (3M_T - M_{\eta_b})$$

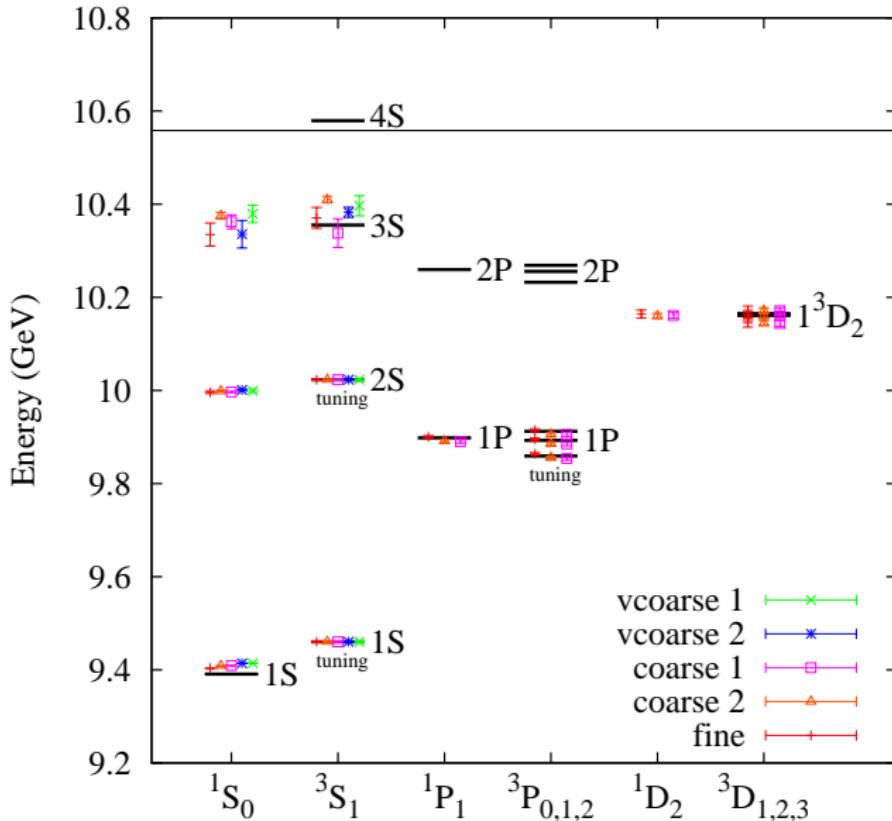
- ▶ Tuning accurate to $\simeq 1\%$ on finer lattices

s and c quarks tuned with M_{η_s}, M_{η_c}



- ▶ Plot of $m_b(\mu)/m_s(\mu)$ in $\overline{\text{MS}}$ from NRQCD b and HISQ s quark
- ▶ Mass renormalisations calculated perturbatively to $O(\alpha_s^2)$
- ▶ Compared to HISQ/HISQ ratio (black), independent results agree

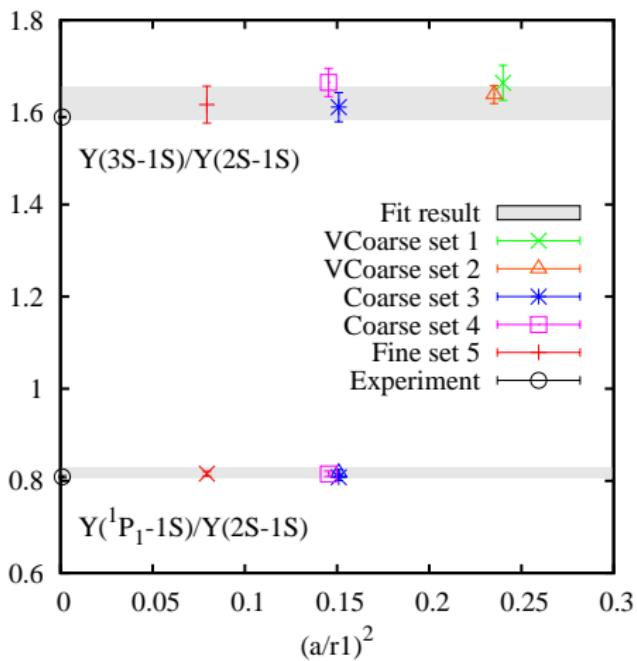
Bottomonium spectrum



Errors are statistical and lattice spacing only.

Splitting Ratios [R. Dowdall et al - in preparation]

- ▶ Several systematic errors cancel in ratio



$$\frac{\Upsilon(3S-1S)}{\Upsilon(2S-1S)} = 1.621(36)$$

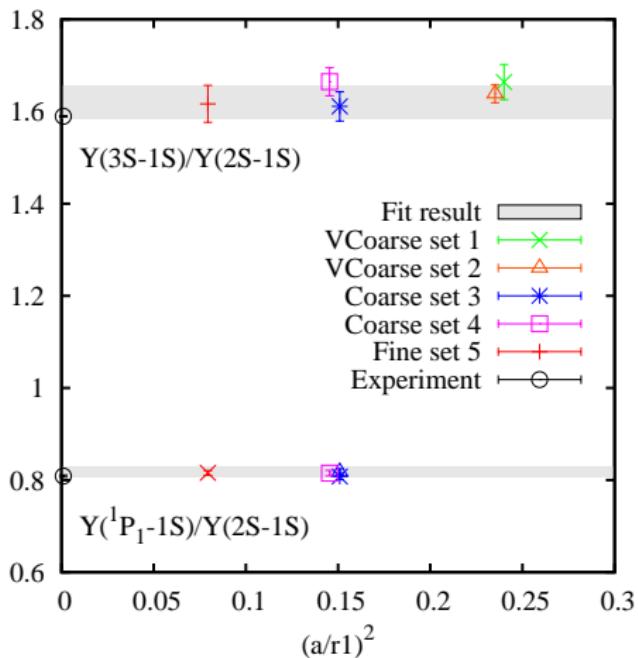
Expt = 1.5896(12)

$$\frac{1^1P_1-1S}{\Upsilon(2S-1S)} = 0.817(11)$$

Expt = 0.8088(23)

Splitting Ratios [R. Dowdall et al - in preparation]

- ▶ Several systematic errors cancel in ratio



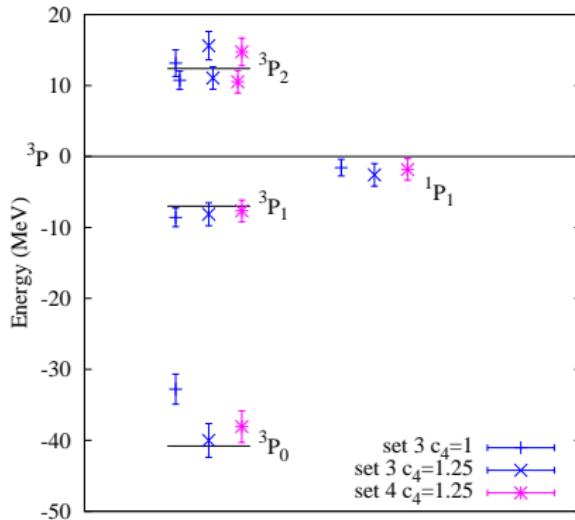
Error budget %:

| | R_P | R_S |
|-------------------|-------|-------|
| stats/fitting | 1.05 | 1.64 |
| a -dependence | 0.62 | 1.13 |
| m_l -dependence | 0.05 | 0.0 |
| m_b -dependence | 0.18 | 0.31 |
| NRQCD syst | 0.44 | 0.58 |
| finite volume | - | - |
| total fit error | 1.32 | 2.25 |

P-wave splittings

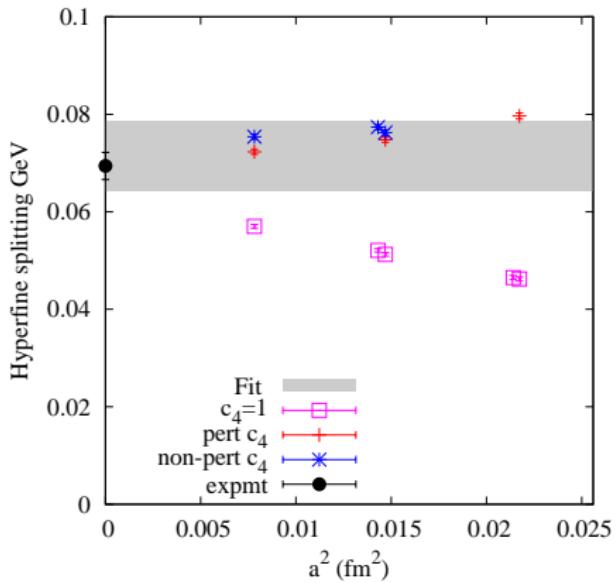
P-wave spectrum is used to non-perturbatively tune c_3, c_4

- ▶ Tree level coefficients give slightly incorrect splittings
- ▶ Tuned c_4 agrees well with perturbative calculation
 $\implies O(\alpha_s^2)$ corrections small



S-wave hyperfine splitting

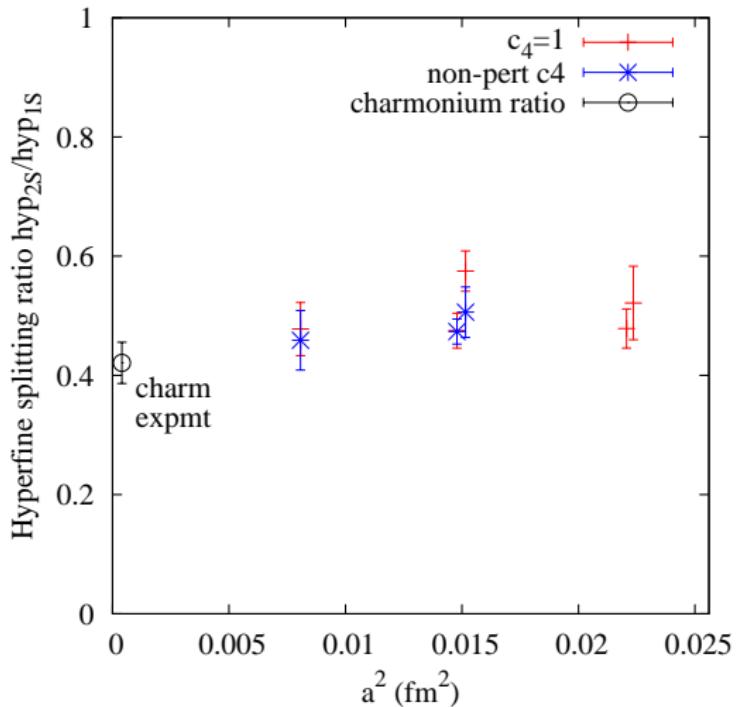
- ▶ Hyperfine splitting proportional to square of $c_4 \frac{g}{2aM_b} \sigma \cdot \tilde{\mathbf{B}}$ term
- ▶ Tree level coefficients underestimate splitting
- ▶ Tuned coeff. give better agreement



- ▶ Effect of 4 quark operators and slight mass mistuning added
- ▶ Previous HPQCD value:
 $M_\Upsilon - M_{\eta_b} = 61(14)$ MeV
- ▶ New result:
 $M_\Upsilon - M_{\eta_b} = 71(7)$ MeV
- ▶ 7 MeV systematic error dominated by missing $O(v^6)$ terms

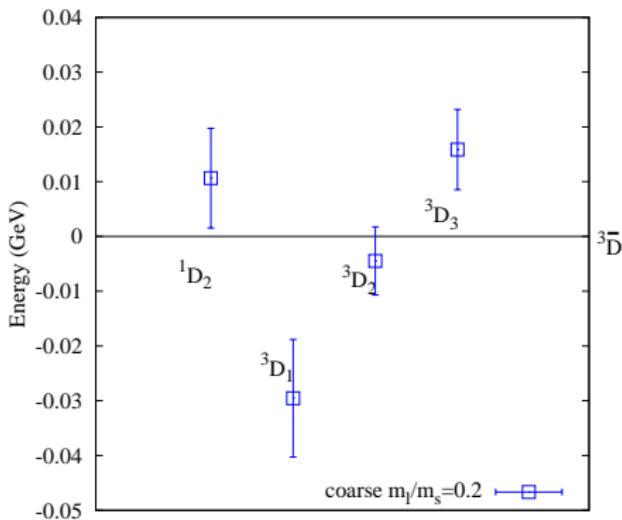
S-wave hyperfine ratio

- ▶ Ratio $\frac{\Upsilon(2S)-\eta_b(2S)}{\Upsilon(1S)-\eta_b(1S)}$ should be independent of c_4
- ▶ Consistent with charmonium hyperfine ratio



D-wave splittings prediction from full QCD

- ▶ Statistical errors dominate
- ▶ Leading systematic error from $O(v^6)$
- ▶ Different lattice irreps in good agreement
- ▶ Other ensembles show little scale or sea quark mass dependence
- ▶ $E(^3D_2) = 10.167(15)_{\text{stat}} \text{ GeV}$
- ▶ Splitting larger than potential model prediction [QWG review (2010)]



B mesons [R. Dowdall et al - in preparation]

Improved B meson spectrum computation in progress

- ▶ NRQCD heavy and HISQ light quarks
- ▶ NRQCD systematics improved as above
- ▶ High statistics calculation: $3 \times 16k$ correlators
- ▶ No free parameters - everything fixed from bottomonium
- ▶ Data from 3 ensembles currently complete

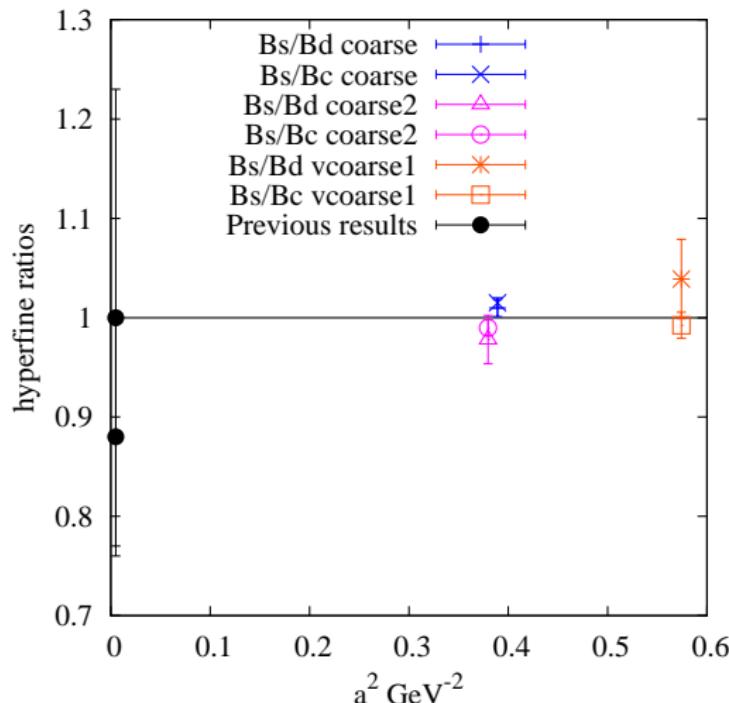
Significant improvement in stat. & syst. errors over previous HPQCD results

[E. Gregory et al (2010)]

B meson hyperfine splittings

Hyperfine splitting ratios consistent with one

$$\frac{M_{B_s^*} - M_{B_s}}{M_{B_d^*} - M_{B_d}}, \quad \frac{M_{B_s^*} - M_{B_s}}{M_{B_c^*} - M_{B_c}}$$



Summary

- ▶ NRQCD systematic errors under much better control
- ▶ Good agreement of pert./non-pert. coefficients
- ▶ First full prediction of bottomonium D-wave states
- ▶ Charm quarks included in the sea for the first time

Future work:

- ▶ B physics
- ▶ $O(v^6)$ terms