

# Double quarkonium production in hadron colliders from the perspective of NRQCD

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Based on JHEP 1101, 070 (2011);  
PRD83, 054015 (2011)

QWG 2011, GSI, Germany, 4 October 2011

# Motivation

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- Heavy quarkonium: probes perturbative and nonperturbative aspects of QCD.
- NRQCD has achieved great success.
  - resolves **IR divergence** problem.
  - the inclusive production of  $J/\psi$  from KEKB, LEP II, RHIC, HERA, Tevatron and LHC. (See talks by Chao and Butenschoen.)
  - double quarkonium production at B factories.  
He,Fan,Chao(2007);Bodwin,Lee,Yu(2007)
- there are still unresolved puzzles.
  - polarization of prompt  $J/\psi$  at Tevatron.
  - polarized  $J/\psi$  photoproduction. Butenschoen,Kniehl,1109.1476

# Double quarkonium production

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- the double quarkonium production at hadron colliders.
  - test NRQCD.
  - get hints for the puzzles.
- suggested to test the color-octet mechanism at the Tevatron.

$$\sigma(p\bar{p} \rightarrow \psi_{\mu\mu}\psi_{\mu\mu}X) \approx 0.14 \text{ pb.} \quad \text{Barger,Fleming,Phillips('96)}$$

- Color-singlet contribution. [Qiao 2002](#)
- extended to the LHC. [Li,Zhang,Chao\(2009\);Qiao,Sun,Sun\(2009\);](#)  
[Berezhnoy,Likhoded,Luchinsky,Novoselov\(2011\)](#)
  - Polarized  $J/\psi$  pair hadroproduction. [Qiao \(QWG2010\)](#)

# Double quarkonium production

- Double quarkonium production as a probe of double parton scattering

$$d\sigma_{\text{DPS}}^{2 J/\psi} = \frac{d\sigma_{\text{SPS}}^{J/\psi} d\sigma_{\text{SPS}}^{J/\psi}}{2\sigma_{\text{eff}}}$$

Kom, Stirling, Kulesza(2011);  
Novoselov(2011)

 the transverse structures of the proton

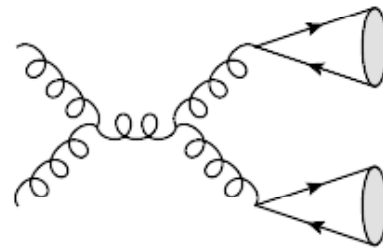
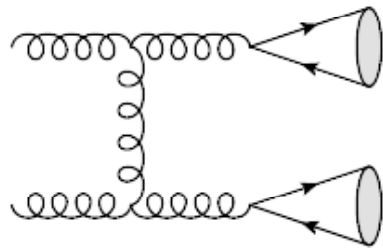
- $k_t$  factorization vs collinear scheme. (See the talk by Baranov.)
- first measurements at the LHCb: LHCb, 1109.0963

$$\sigma^{J/\psi J/\psi} = 5.1 \pm 1.0 \pm 1.1 \text{ nb}, \quad (\text{See the talk by Frosini.})$$

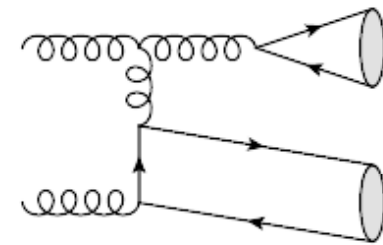
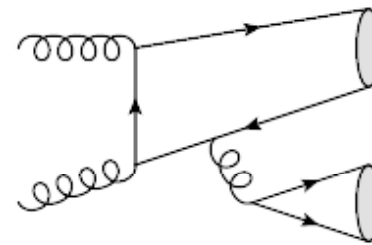
- Previous works based on NRQCD considered only the double quarkonium production of same flavor.
  - gluon fragmentation approximation for the CO contribution.
- In this talk: the double quarkonium production of different flavor.
  - calculate the CO contribution fully instead of gluon frag. approx.

# Typical diagrams

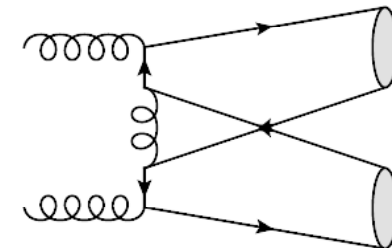
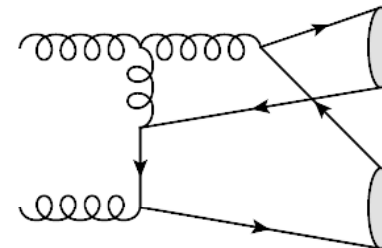
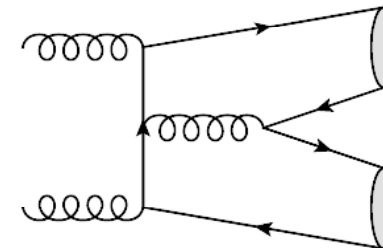
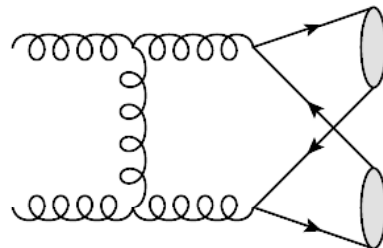
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double fragmentation

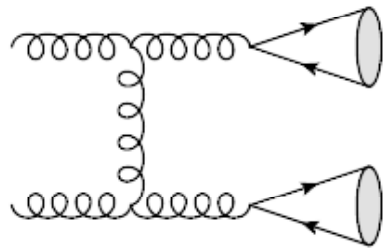


single fragmentation

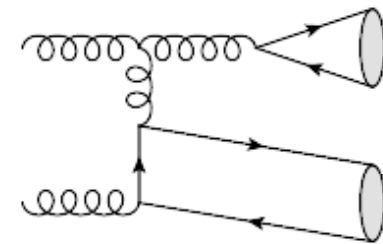
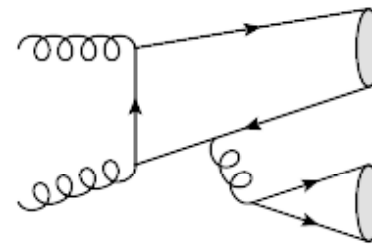
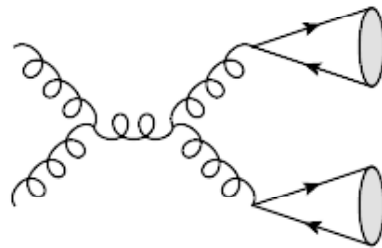


nonfragmentation

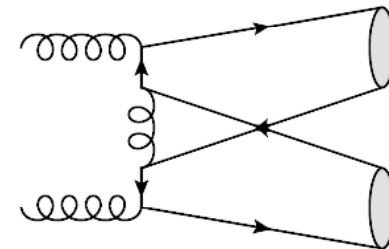
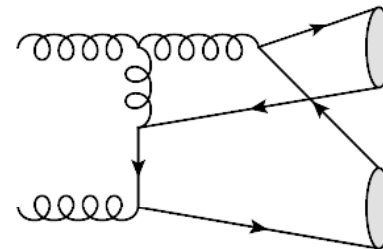
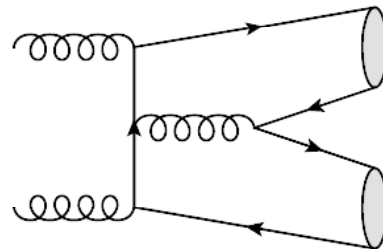
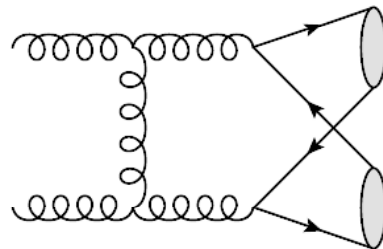
# Typical diagrams



double fragmentation



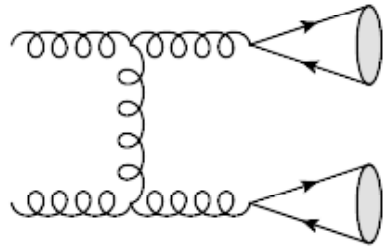
single fragmentation



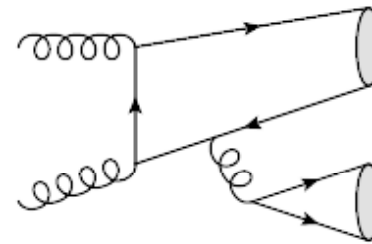
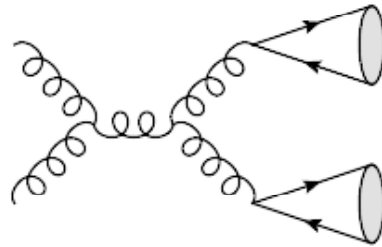
nonfragmentation

Color singlet

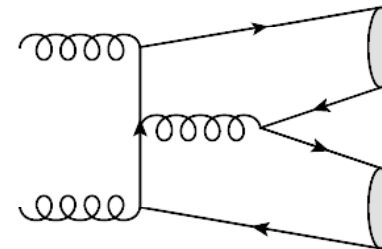
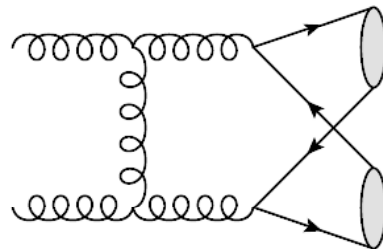
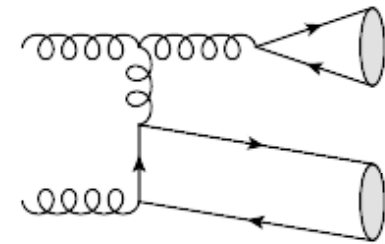
# Typical diagrams



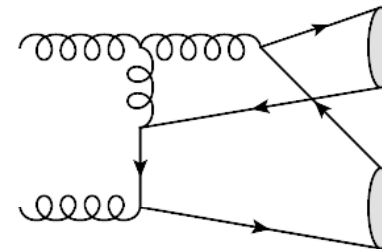
double fragmentation



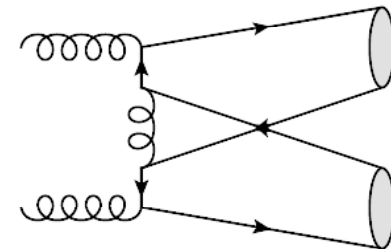
single fragmentation



nonfragmentation



Color singlet



Color octet

# Double quarkonium production

- The schematic form of the cross section is

$$d\sigma[pp \rightarrow H_1(P_1) + H_2(P_2)] = \sum_{i,j,n_1,n_2} \underbrace{f_{i/p} \otimes f_{j/p} \otimes d\hat{\sigma}[ij \rightarrow Q_1^{n_1} + Q_2^{n_2}]}_{\text{at least } \alpha_s^4(\mu)} \underbrace{\langle O_{n_1} \rangle_{H_1} \langle O_{n_2} \rangle_{H_2}}_{\text{NRQCD MEs}}$$

- potential large uncertainties come from the scale dependence of the strong coupling constant and NRQCD matrix elements.

- $\mu_r = \mu_f = m_T = \sqrt{m_Q^2 + p_T^2}.$

- NRQCD matrix elements

- CS: could be determined from EM decays of the quarkonium.  
(See the talks by Chung and Ma.)
- CO: lattice simulation, global fit from single quarkonium productions.  
(See the talk by Butenschoen.)

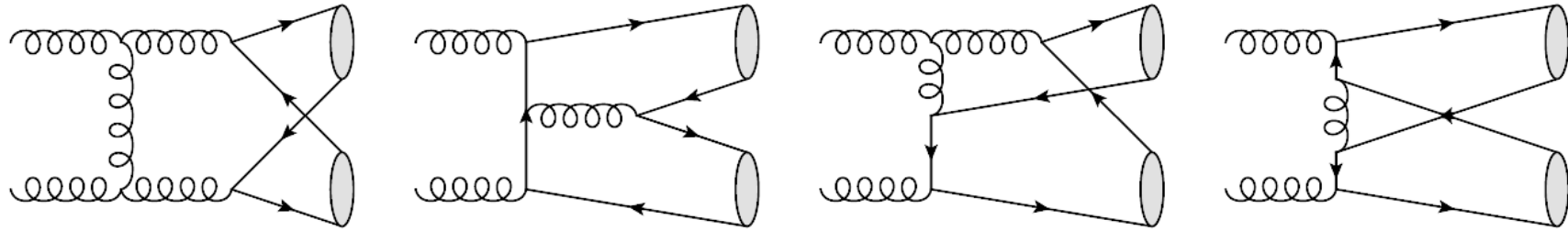


## Double quarkonium production of same flavor

$$pp \rightarrow J/\psi + J/\psi + X,$$

$$pp \rightarrow \Upsilon + \Upsilon + X$$

# Color-singlet channel



- The leading processes are of order  $\alpha_s^4$ .

- Two subprocesses at this order contribute

$$g + g \rightarrow Q_1 + Q_2, \quad \text{dominant}$$

$$q + \bar{q} \rightarrow Q_1 + Q_2. \quad \text{subdominant}$$

- The leading contribution is the color-singlet channel

$$(Q_1^{n_1}, Q_2^{n_2}) = [Q\bar{Q}_1(^3S_1), Q\bar{Q}_1(^3S_1)]$$

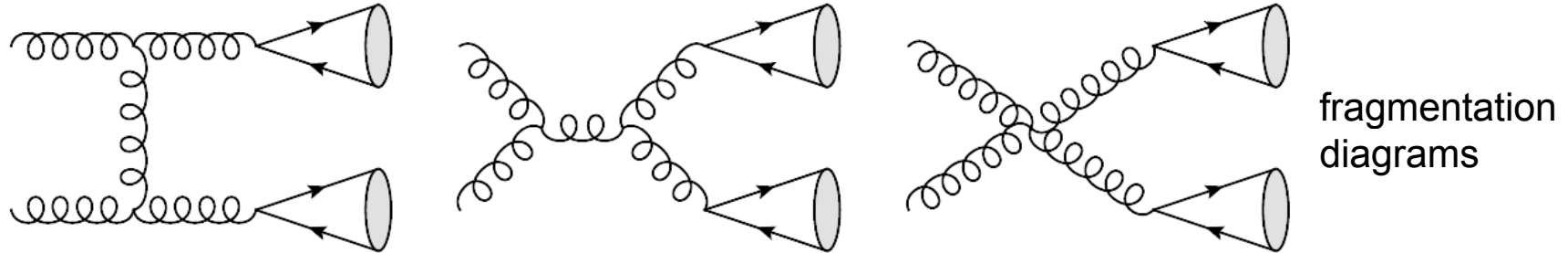
- Color-octet channels are suppressed.

$$(Q_1^{n_1}, Q_2^{n_2}) = [Q\bar{Q}_8(^3S_1), Q\bar{Q}_8(^3S_1)] \quad \text{by } v^8$$

$$(Q_1^{n_1}, Q_2^{n_2}) = [Q\bar{Q}_1(^3S_1), Q\bar{Q}_8(^3S_1)] \quad \text{by } v^4$$

- 31 Feynman diagrams in the color-singlet channel.

## Color-octet channel (gluon fragmentation approx.)



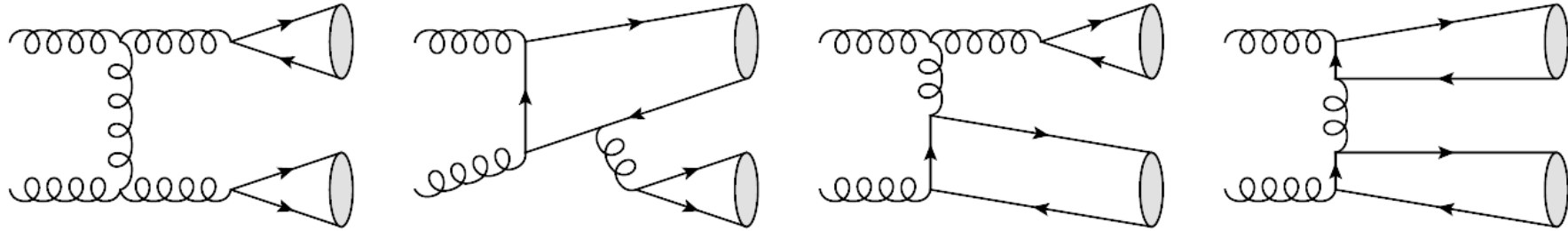
- The leading processes are of order  $\alpha_s^4$ .
- gluon fragmentation approximation.
  - two real gluon production, followed by the fragmentation of each gluon into a quarkonium in the  $^3S_1$  color-octet state.
- 4 Feynman diagrams.
- The schematic form of the cross section is

$$d\sigma^{H_1(P_1)+H_2(P_2)} = f_{g/p} \otimes f_{g/p} \otimes d\hat{\sigma}_{gg \rightarrow gg} \otimes D_{g \rightarrow H_1} \otimes D_{g \rightarrow H_2},$$

$$D_{g \rightarrow H_i}(z_i, m_{H_i}) = \frac{\pi \alpha_s}{24 m_{H_i}^3} \delta(1 - z_i) \langle O_8(^3S_1) \rangle_{H_i}.$$

- It is necessary to evaluate the frag. func. at the fac. scale  $\mu_f \sim p_T \gg m_Q$ . by making use of Altarelli-Parisi evolution equation.

## Color-octet channel (full calculation)



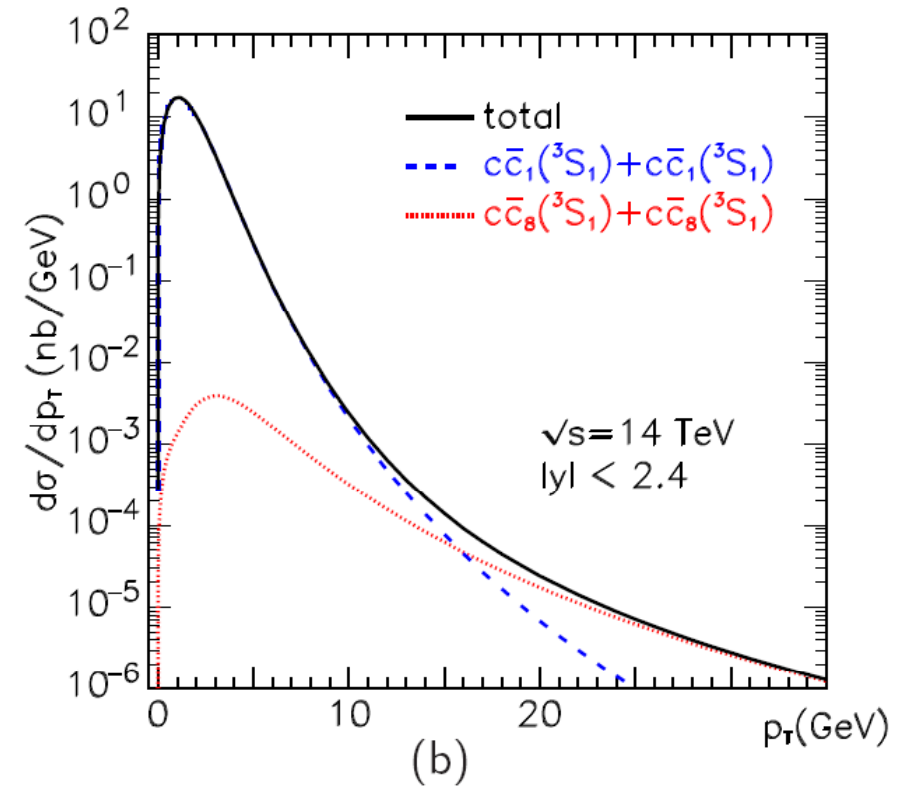
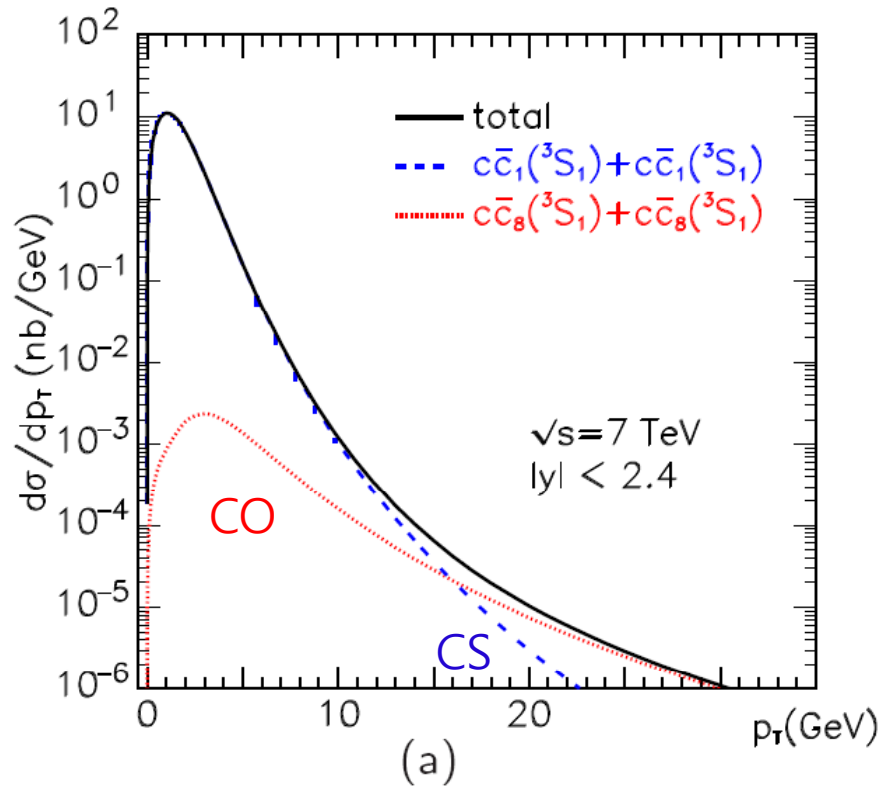
- various combinations of intermediate states are allowed.

$$^3S_1^{(8)} + ^3S_1^{(8)}, ^3S_1^{(8)} + ^1S_0^{(8)}, ^3S_1^{(8)} + ^3P_J^{(8)},$$

$$^1S_0^{(8)} + ^3P_0^{(8)}, ^3S_1^{(1)} + ^3S_1^{(8)}, ^3S_1^{(1)} + ^1S_0^{(8)}, \dots$$

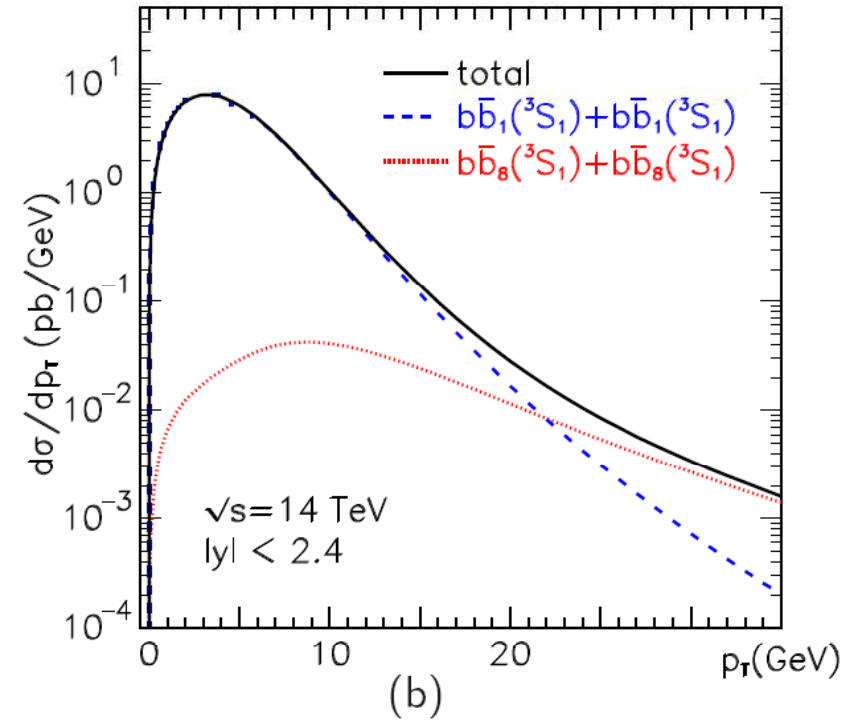
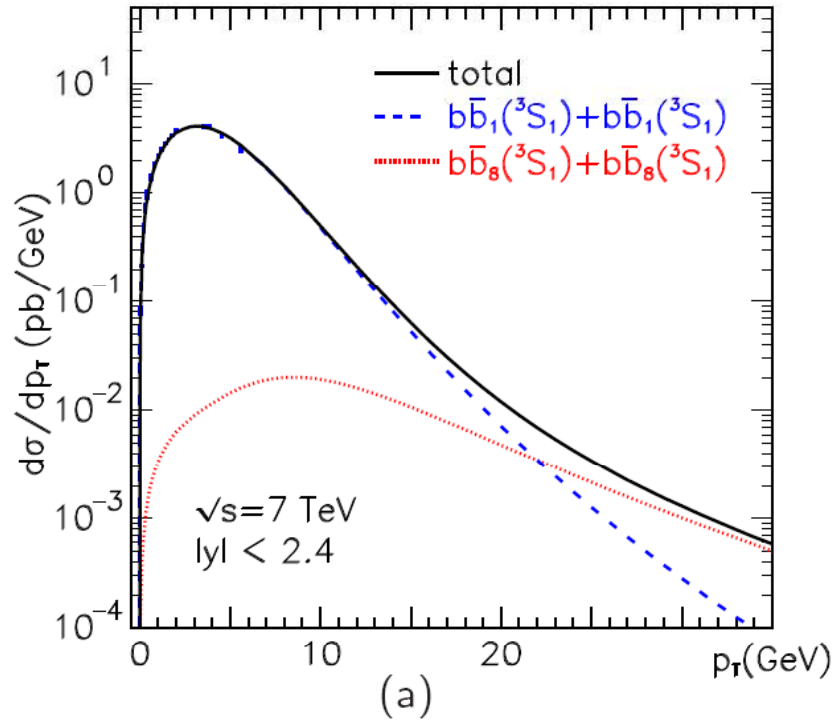
- We consider only  $^3S_1^{(8)} + ^3S_1^{(8)}$  combination because
  - $^1S_0$  and  $^3P_0$  color-octet matrix elements may be much suppressed.
  - $^3S_1^{(8)} + ^3S_1^{(8)}$  combination could be dominant at large  $p_T$ .
- 72 Feynman diagrams.

## $J/\psi J/\psi$ production (full calculation)



- The color-singlet contribution dominates at small and moderate  $p_T$ .
- The CO contribution dominates over the CS contribution at  $p_T > 16$  GeV.
- the spectrum vanishes at  $p_T = 0$ , because both channels are free of infrared divergence.

## $\Upsilon\Upsilon$ production (full calculation)



- essentially the same as for the double  $J/\psi$  production.
- The CO contribution dominates over the CS contribution at  $p_T > 24$  GeV.

## Double quarkonium production (same flavor)

$$\sigma(pp \rightarrow 2J/\psi + X)$$

$\sqrt{s} \setminus \sigma$ (nb)	$c\bar{c}_1(^3S_1) + c\bar{c}_1(^3S_1)$	$c\bar{c}_8(^3S_1) + c\bar{c}_8(^3S_1)$	total
7 TeV	22.3	0.011	22.3
14 TeV	34.8	0.019	34.8

$$\sigma(pp \rightarrow 2\Upsilon + X)$$

$\sqrt{s} \setminus \sigma$ (pb)	$b\bar{b}_1(^3S_1) + b\bar{b}_1(^3S_1)$	$b\bar{b}_8(^3S_1) + b\bar{b}_8(^3S_1)$	total
7 TeV	24.1	0.27	24.4
14 TeV	47.9	0.60	48.5

- double quarkonium production of same flavor can be tested at the LHC.
- If we consider the contributions from feeddown of  $\psi(2S)$  and  $\chi_{cJ}$ , it seems that the color-octet mechanism may be testable at the LHC
- The CS contribution might contaminate the CO contribution.
- We suggest the  $J/\psi + \Upsilon$  production at the LHC **as a clean probe of the color-octet mechanism.**

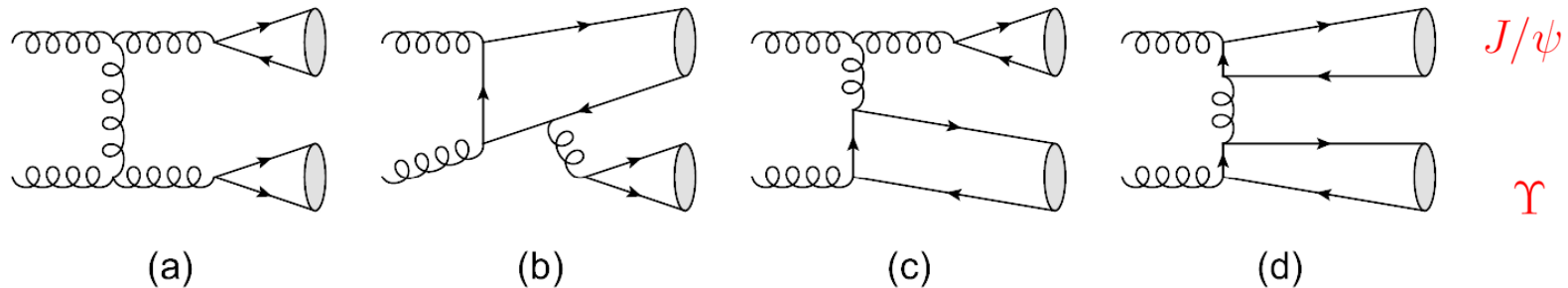
## Double quarkonium production of different flavor

$$pp \rightarrow J/\psi + \Upsilon + X$$



# Double quarkonium production (different flavor)

**Color octet channel**  $pp \rightarrow J/\psi + \Upsilon + X$



- The leading processes are of order  $\alpha_s^4$ .

$c\bar{c}_8(^3S_1) + b\bar{b}_8(^3S_1) : (a),(b),(c),(d), \quad v_c^4 v_b^4$  , double fragmentation

$c\bar{c}_1(^3S_1) + b\bar{b}_8(^3S_1) : (b), \quad v_b^4$  , single fragmentation

$c\bar{c}_8(^3S_1) + b\bar{b}_1(^3S_1) : (b), (c), (d), \quad v_c^4$  , single fragmentation

$c\bar{c}_8(^3S_1) + b\bar{b}_8(^1S_0) : (b),(c),(d), \quad v_c^4 v_b^3$  , single fragmentation

$c\bar{c}_8(^3S_1) + b\bar{b}_8(^3P_J) : (b),(c),(d), \quad v_c^4 v_b^4$  , single fragmentation

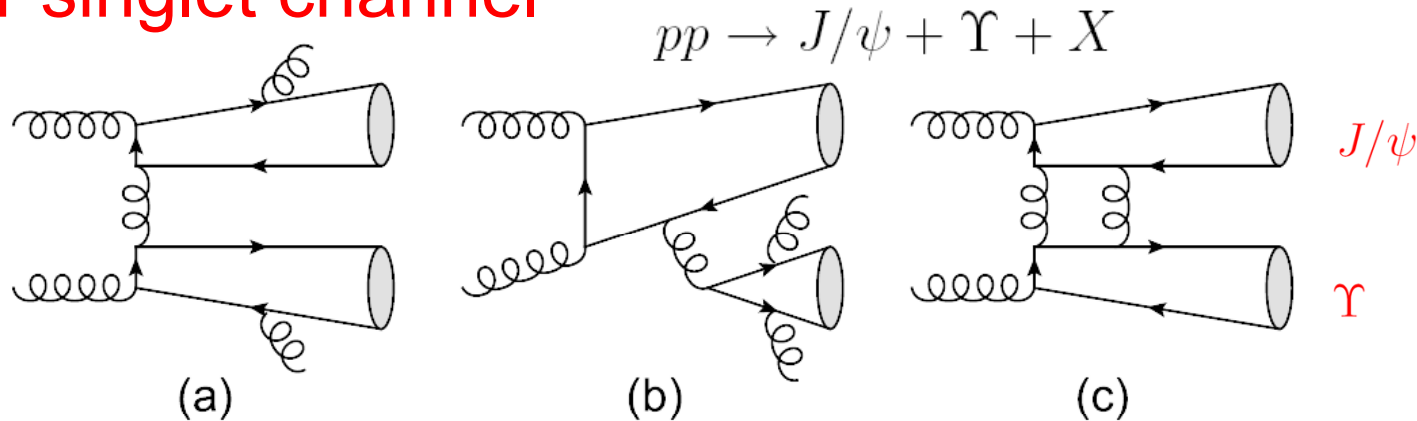
etc.

- Among various combinations of intermediate states, we consider

$$^3S_1^{(8)} + ^3S_1^{(8)}, ^3S_1^{(1)} + ^3S_1^{(8)}, ^3S_1^{(8)} + ^3S_1^{(1)}.$$

# Double quarkonium production (different flavor)

## Color singlet channel



- Tree-level color-singlet contribution accompanies at least two hard gluons.
  - extra hard jets in the final state.
- The color-singlet contribution at one-loop level can appear via two-gluon exchange.

## Double quarkonium production (different flavor)

- The relative size of the CS contribution to the CO contribution  $c\bar{c}_8(^3S_1) + b\bar{b}_8(^3S_1)$  at large  $p_T$  is

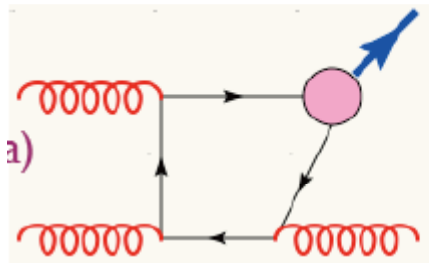
$$\frac{1}{(4\pi)^2} \frac{\alpha_s^2}{v_c^4 v_b^4} \left( \frac{m_\psi}{p_T} \frac{m_\Upsilon}{p_T} \right)^4 \sim 0.005 \text{ at } p_T = 5 \text{ GeV.}$$

- At small  $p_T$ , the factor  $p_T$  should be of order  $m_H$ .
  - the suppression factors of the CS contribution to the mixed contributions  $c\bar{c}_1(^3S_1) + b\bar{b}_8(^3S_1)$  and  $c\bar{c}_8(^3S_1) + b\bar{b}_1(^3S_1)$  are

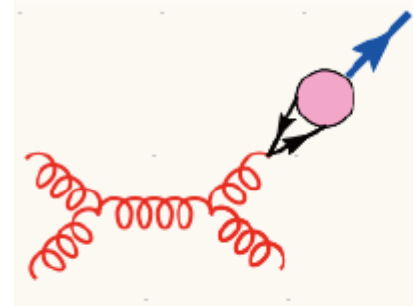
$$\frac{\alpha_s^2}{(4\pi)^2 v_b^4} \text{ or } \frac{\alpha_s^2}{(4\pi)^2 v_c^4} \sim 0.3 \text{ or } 0.003.$$

# Single vs. double quarkonium production (diff. flavor)

## Singlet quarkonium production



$$\alpha_S^3 \frac{(2m_c)^4}{p_T^8}$$



$$\alpha_S^3 \frac{(2m_c)^2 v^4}{p_T^4}$$

- The ratio of the CS and CO contributions for the single quarkonium production at large  $p_T$  is approximately

$$\frac{m_c^4}{v^4 p_T^4} \sim 0.07 \text{ at } p_T = 5 \text{ GeV.}$$

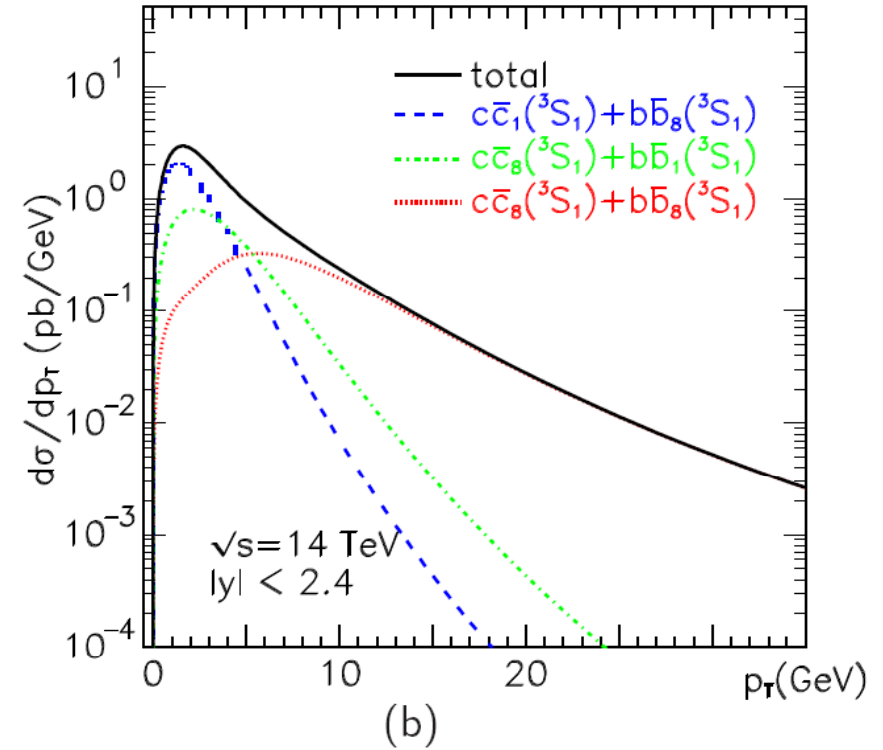
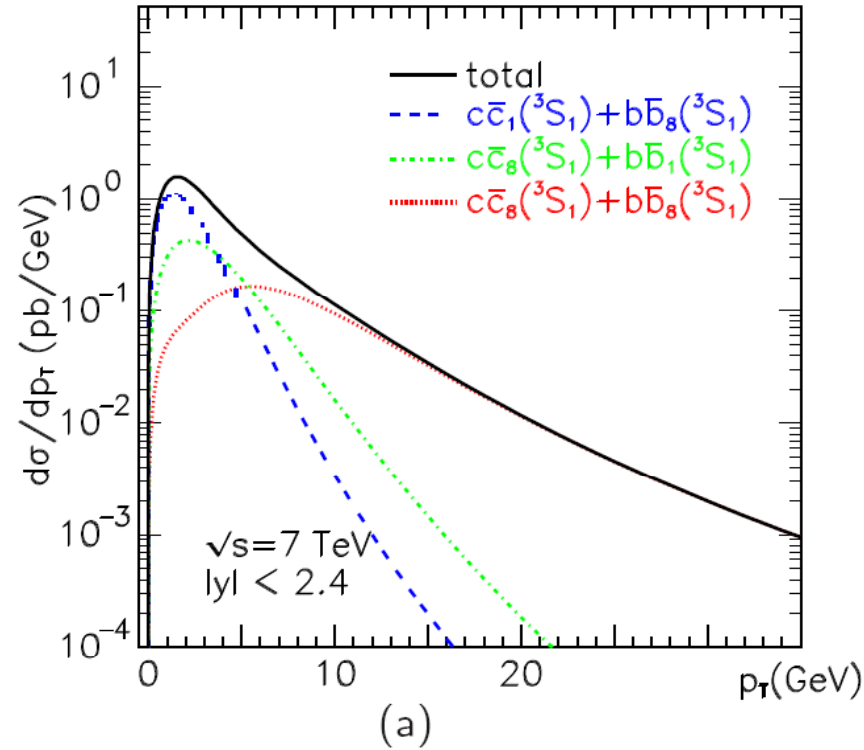
cf.  $\sim 0.005$  at  $p_T = 5$  GeV  
for double quarkonium  
production.

- At small  $p_T$ , there is no kinematic enhancement factor for the color-octet contribution.

## Double quarkonium production (different flavor)

- Thus we conclude that the color-singlet contribution is fully suppressed and also easily distinguishable.
- The  $J/\psi + \Upsilon$  production at the LHC will provide good tests for the color-octet mechanism **with less backgrounds and without color-singlet contamination**.
- If we cannot observe the events at the expected level, it would imply that the current values of the color-octet matrix elements are overestimated.

# $J/\psi\Upsilon$ production



- $c\bar{c}_8(^3S_1) + b\bar{b}_8(^3S_1)$  dominates at  $p_T > 6$  GeV.
- $c\bar{c}_1(^3S_1) + b\bar{b}_8(^3S_1)$  dominates at  $p_T < 4$  GeV.
- Three contributions compete among one another at  $4 \text{ GeV} < p_T < 6 \text{ GeV}$ .

## Double quarkonium production (different flavor)

$\sqrt{s} \setminus \sigma \text{ (pb)}$	$c\bar{c}_1(^3S_1) + b\bar{b}_8(^3S_1)$	$c\bar{c}_8(^3S_1) + b\bar{b}_1(^3S_1)$	$c\bar{c}_8(^3S_1) + b\bar{b}_8(^3S_1)$	total
7 TeV	3.18	1.95	1.63	6.76
14 TeV	6.00	3.72	3.36	13.08

- expects 7 pb  $\sim$  13 pb at the LHC.
- about 1900 events assuming the integrated luminosity  $\sim 100^{-1}\text{fb}$  and considering branching fractions of J/ $\psi$  and Y into a muon pair.
- Double quarkonium production of different flavor can be observed at the LHC.

## Color octet mechanism

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- The total cross section for  $J/\psi + \Upsilon$  production is much less than that for double quarkonium production of same flavor.
- In order to probe the color-octet mechanism more accurately, we impose a lower  $p_T$  cut to remove most of the color-singlet contribution.
- At the c.m. energy 14 TeV,

$$\sigma[pp \rightarrow 2J/\psi + X]_{p_T \gtrsim 16 \text{ GeV}} = 0.2 \text{ pb}$$

$$\sigma[pp \rightarrow 2\Upsilon + X]_{p_T \gtrsim 24 \text{ GeV}} = 0.05 \text{ pb}$$

$$\sigma[pp \rightarrow J/\psi + \Upsilon + X] = 13 \text{ pb}$$

- conclude that the  $pp \rightarrow J/\psi + \Upsilon + X$  channel is the most sensitive to the color-octet matrix elements among the three double-quarkonium final states.



# Double quarkonium production

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- CO matrix element

$$\begin{aligned}\langle O_8^{J/\psi}(^3S_1) \rangle &= 3.9 \times 10^{-3} \text{ GeV}^3 \\ \langle O_8^\Upsilon(^3S_1) \rangle &= 1.5 \times 10^{-1} \text{ GeV}^3\end{aligned}$$



$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle$	$(4.97 \pm 0.44) \times 10^{-2} \text{ GeV}^3$
$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle$	$(2.24 \pm 0.59) \times 10^{-3} \text{ GeV}^3$
$\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle$	$(-1.61 \pm 0.20) \times 10^{-2} \text{ GeV}^5$

In this talk

Global fit

[Butenschoen, Kniehl, 1109.1476](#)

- may decrease the cross section for the  $2J/\psi$  and  $J/\psi + \Upsilon$  productions.
- Our calculation: considers the leading contribution of the expansion.
- ignored the  $^1S_0^{(8)}$  and  $^3P_J^{(8)}$  contribution but they could contribute to the double quarkonium production if the global fit values are correct.
- Feeddown, NLO corrections and relativistic corrections may be important.

# Conclusions

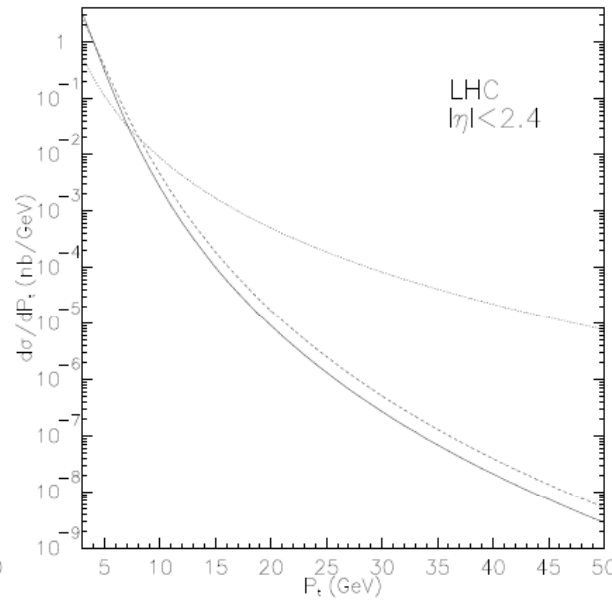
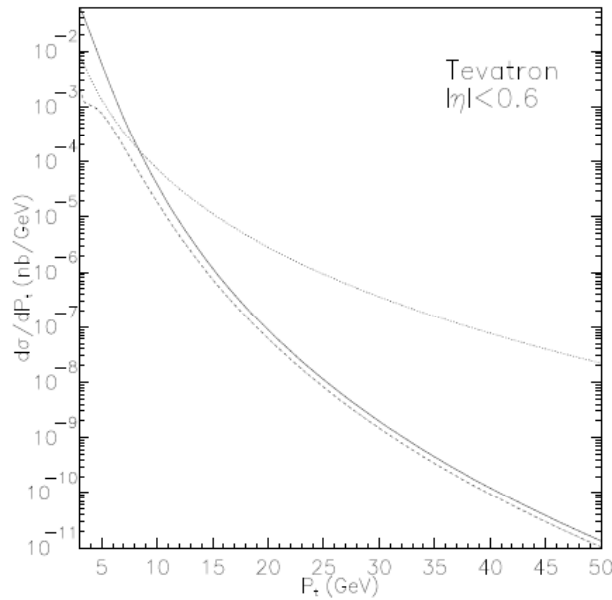
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- Double quarkonium production at hadron colliders provides another test ground of NRQCD.
- $J/\psi + \Upsilon$  production may be used to test the color-octet mechanism with less backgrounds and without color-singlet contamination.
- If one cannot see the  $J/\psi + \Upsilon$  events at the expected level, it would imply that the current color-octet matrix elements are overestimated.
- requires the higher-order contributions to predict the cross sections more precisely.

Backup slides

# Comparison

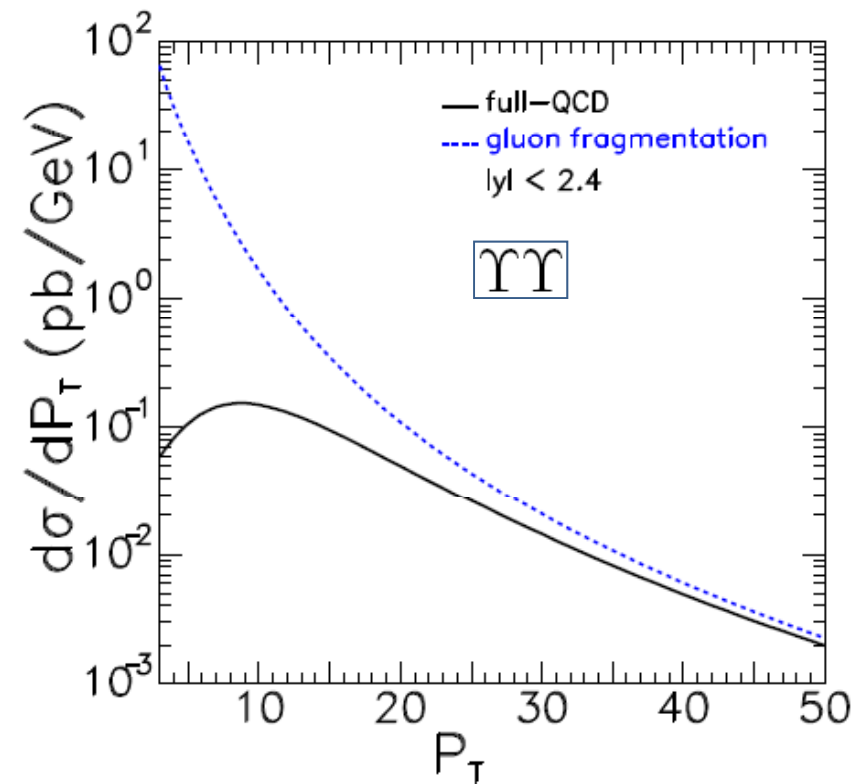
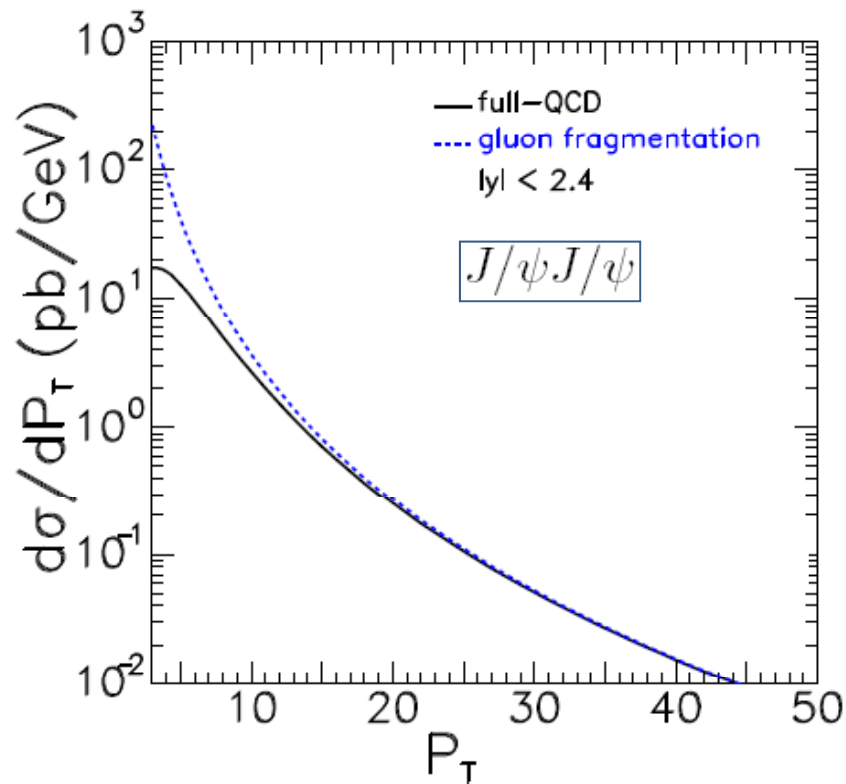
- 2 J/ψ production. [Ma,Zhang,Chao, PRL102, 162002 \(2009\)](#)



<i>Final States</i>	$\sigma_{Tevatron}[nb]$	$\sigma_{LHC}[nb]$
$\eta_c \eta_c$	$3.32 \times 10^{-3}$	2.73
$J/\psi J/\psi$	$5.63 \times 10^{-2}$	2.83
$\eta_b \eta_b$	$1.87 \times 10^{-5}$	$7.36 \times 10^{-3}$
$\Upsilon \Upsilon$	$1.23 \times 10^{-4}$	$1.51 \times 10^{-2}$
$B_c \bar{B}_c$	$3.86 \times 10^{-3}$	$2.72 \times 10^{-1}$
$B_c \bar{B}_c^*$	$1.00 \times 10^{-3}$	$8.37 \times 10^{-2}$
$B_c^* \bar{B}_c^*$	$8.23 \times 10^{-3}$	$7.08 \times 10^{-1}$

- CO: based on the gluon fragmentation approximation.

## gluon frag. approx. vs. full calculation



- $$\alpha_s = \begin{cases} \alpha_s(m_{J/\psi}), \alpha_s(m_\Upsilon), & \text{for fragmentation processes,} \\ \alpha_s(m_T), & \text{for nonfragmentation processes.} \end{cases}$$
- This choice violates gauge invariance with an error of  $O(m_c^2/\hat{s})$ .
  - overestimates the cross section by about a factor of 6 or 40.