Precision determination of $r_0\Lambda_{\overline{\rm MS}}$ from the QCD static energy

Xavier Garcia i Tormo
Universität Bern
(work done with Nora Brambilla, Joan Soto and Antonio Vairo)

Phys. Rev. Lett. **105** (2010) 212001. [arXiv:1006.2066 [hep-ph]]

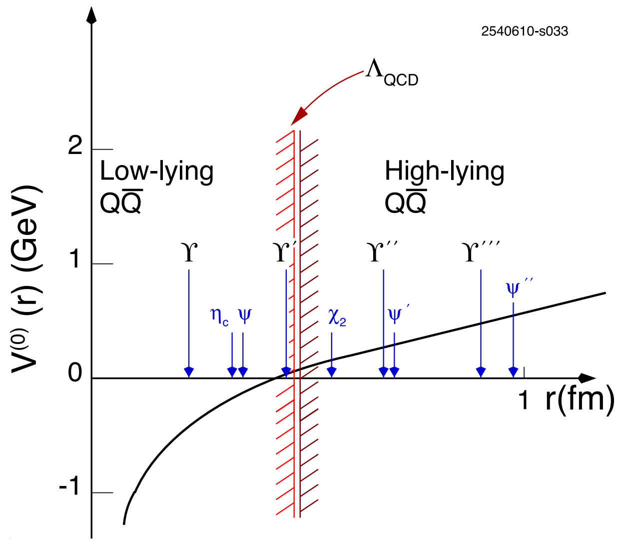
UNIVERSITÄT BERN

Introduction

Energy between a static quark and a static antiquark separated a distance r, *QCD static energy* $E_0(r)$. Basic object to understand the behavior of QCD

Introduction

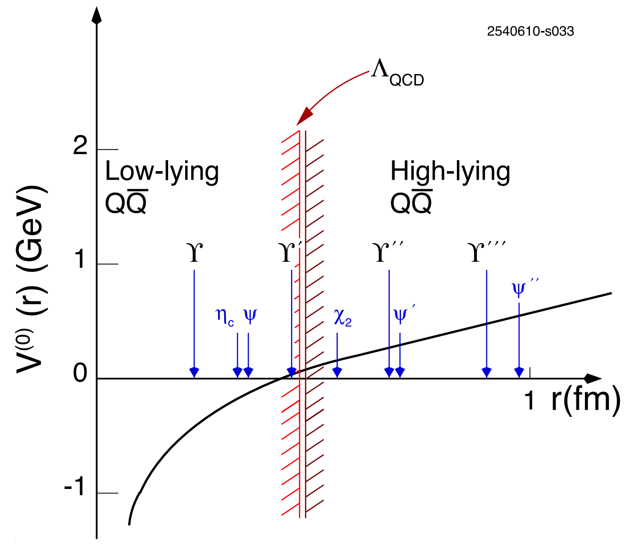
Energy between a static quark and a static antiquark separated a distance r, *QCD static energy* $E_0(r)$. Basic object to understand the behavior of QCD



From N. Brambilla *et al.*, Eur. Phys. J. **C71** (2011) 1534

Introduction

Energy between a static quark and a static antiquark separated a distance r, *QCD static energy* $E_0(r)$. Basic object to understand the behavior of QCD

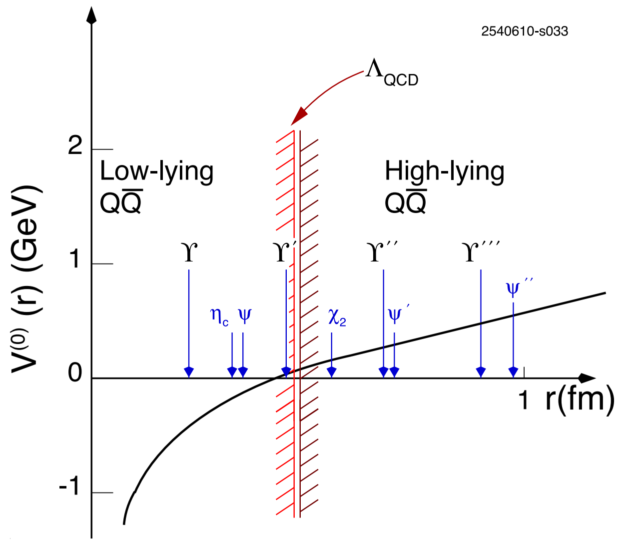


From N. Brambilla *et al.*, Eur. Phys. J. **C71** (2011) 1534

Short-distance part $\longleftrightarrow$ Long-distance part

Introduction

Energy between a static quark and a static antiquark separated a distance r, *QCD static energy* $E_0(r)$. Basic object to understand the behavior of QCD



From N. Brambilla *et al.*, Eur. Phys. J. **C71** (2011) 1534

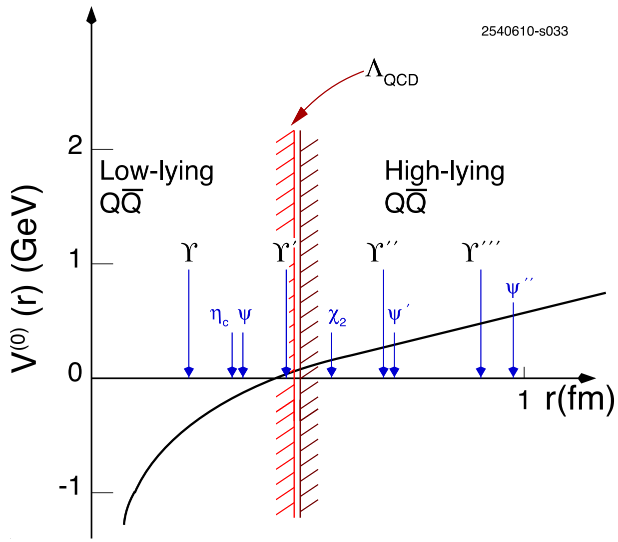
Short-distance part $\longleftrightarrow$ Long-distance part

$\downarrow$

Perturbation theory

Introduction

Energy between a static quark and a static antiquark separated a distance r, *QCD static energy* $E_0(r)$. Basic object to understand the behavior of QCD



From N. Brambilla *et al.*, Eur. Phys. J. **C71** (2011) 1534

Short-distance part $\longleftrightarrow$ Long-distance part

$\downarrow$

Perturbation theory

$$E_0(r) \sim -C_F \frac{\alpha_s}{r} \left(1 + O(\alpha_s) + O(\alpha_s^2) + O(\alpha_s^3, \alpha_s^3 \ln \alpha_s) + \cdots\right)$$

$$E_0(r) \sim -C_F \frac{\alpha_s}{r} \left(1 + O(\alpha_s) + O(\alpha_s^2) + O(\alpha_s^3, \alpha_s^3 \ln \alpha_s) + \cdots\right)$$

Completely known

$$E_0(r) \sim -C_F \frac{\alpha_s}{r} \left(1 + O(\alpha_s) + O(\alpha_s^2) + O(\alpha_s^3, \alpha_s^3 \ln \alpha_s) + \cdots\right)$$

Ultrasoft gluons. Virtual emissions that change the color state of the pair

$$E_0(r) \sim -C_F \frac{\alpha_s}{r} \left(1 + O(\alpha_s) + O(\alpha_s^2) + O(\alpha_s^3, \alpha_s^3 \ln \alpha_s) + \cdots\right)$$

Ultrasoft gluons. Virtual emissions that change the color state of the pair

Nomenclature used:

Energy $\sim$ Potential $+$ Ultrasoft contribution

$$E_0(r) \sim -C_F \frac{\alpha_s}{r} \left(1 + O(\alpha_s) + O(\alpha_s^2) + O(\alpha_s^3, \alpha_s^3 \ln \alpha_s) + \cdots\right)$$

Ultrasoft gluons. Virtual emissions that change the color state of the pair

Nomenclature used:

$$\text{Energy} \sim \text{Potential} + \text{Ultrasoft contribution}$$

physical observable

$$E_0(r) \sim -C_F \frac{\alpha_s}{r} \left(1 + O(\alpha_s) + O(\alpha_s^2) + O(\alpha_s^3, \alpha_s^3 \ln \alpha_s) + \cdots\right)$$

Ultrasoft gluons. Virtual emissions that change the color state of the pair

Nomenclature used:

$$\text{Energy} \sim \text{Potential} + \text{Ultrasoft contribution}$$

physical observable

IR divergent

$$E_0(r) \sim -C_F \frac{\alpha_s}{r}\left(1 + O(\alpha_s) + O(\alpha_s^2) + O(\alpha_s^3, \alpha_s^3 \ln \alpha_s) + \cdots\right)$$

Ultrasoft gluons. Virtual emissions that change the color state of the pair

Nomenclature used:

$$\text{Energy} \sim \text{Potential} + \text{Ultrasoft contribution}$$

physical observable

IR divergent

UV divergent

$$E_0(r) \sim -C_F \frac{\alpha_s}{r} \left(1 + O(\alpha_s) + O(\alpha_s^2) + O(\alpha_s^3, \alpha_s^3 \ln \alpha_s) + \cdots\right)$$

Ultrasoft gluons. Virtual emissions that change the color state of the pair

Nomenclature used:

$$\text{Energy} \sim \text{Potential} + \text{Ultrasoft contribution}$$

physical observable

$$\left.\begin{array}{l}\text{IR divergent}\\ \text{UV divergent}\end{array}\right\} \text{Require regularization. Scheme dependent}$$

$$E_0(r) \sim -C_F \frac{\alpha_s}{r} \left(1 + O(\alpha_s) + O(\alpha_s^2) + O(\alpha_s^3, \alpha_s^3 \ln \alpha_s) + \cdots\right)$$

Ultrasoft gluons. Virtual emissions that change the color state of the pair

Nomenclature used:

$$\text{Energy} \sim \text{Potential} + \text{Ultrasoft contribution}$$

physical observable

IR divergent

UV divergent. Conveniently calculated with effective theory *potential Non-Relativistic QCD*

$$E_0(r) \sim -C_F \frac{\alpha_s}{r} \left(1 + O(\alpha_s) + O(\alpha_s^2) + O(\alpha_s^3, \alpha_s^3 \ln \alpha_s) + \cdots\right)$$

Ultrasoft gluons. Virtual emissions that change the color state of the pair

Nomenclature used:

$$\text{Energy} \sim \text{Potential} + \text{Ultrasoft contribution}$$

physical observable

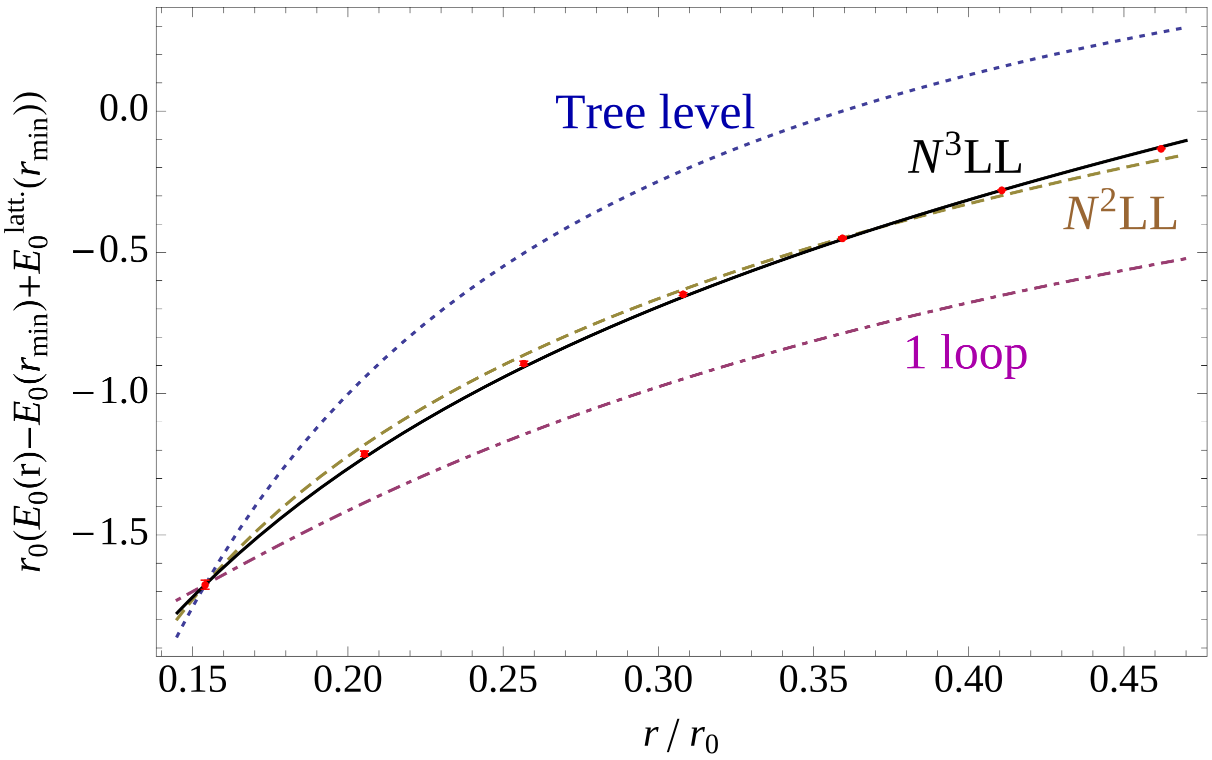
IR divergent

UV divergent. Conveniently calculated with effective theory *potential Non-Relativistic QCD*

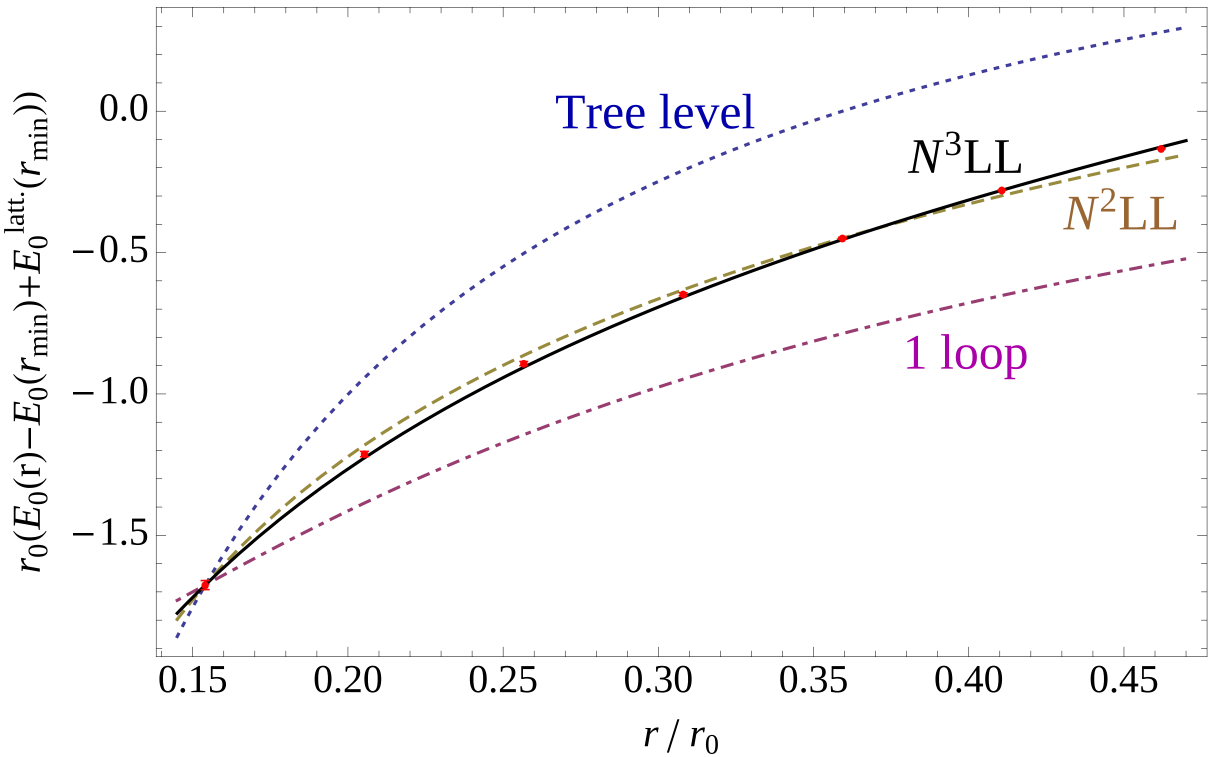
Also use pNRQCD to perform resummation of logarithms

Compare the static energy with lattice data (of Necco, Sommer'02, $n_f = 0$)

Compare the static energy with lattice data (of Necco, Sommer'02, $n_f = 0$)

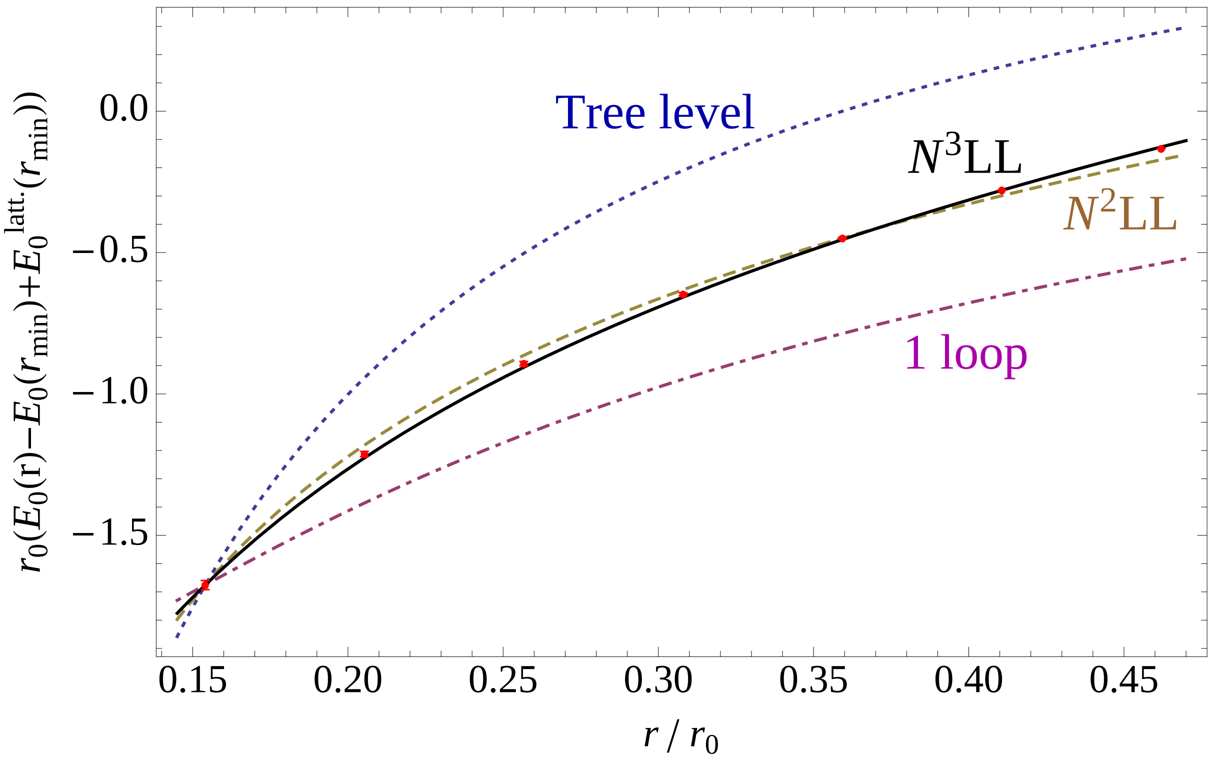


Compare the static energy with lattice data (of Necco, Sommer'02, $n_f = 0$)



All curves coincide at the first point by construction

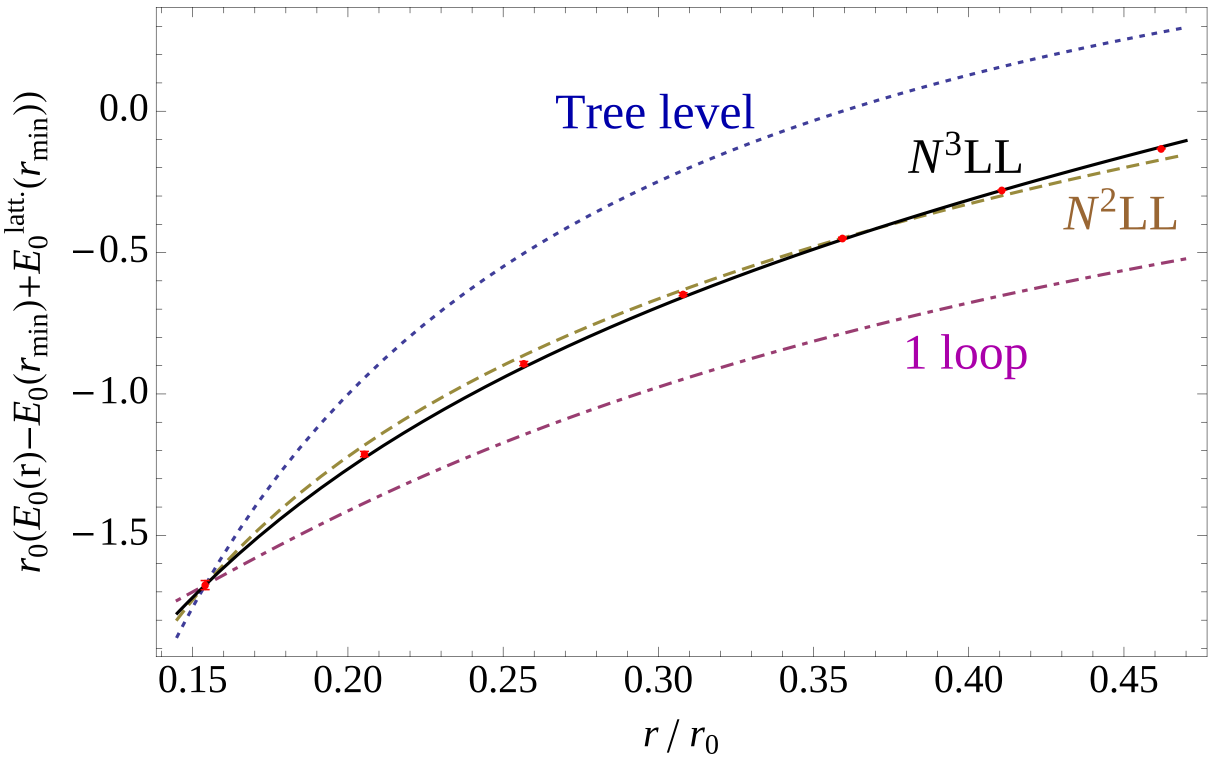
Compare the static energy with lattice data (of Necco, Sommer'02, $n_f = 0$)



All curves coincide at the first point by construction

r_0 is the lattice reference scale

Compare the static energy with lattice data (of Necco, Sommer'02, $n_f = 0$)



All curves coincide at the first point by construction

r_0 is the lattice reference scale

N$^{3(2)}$LL accuracy: $\alpha_s^{1+[3(2)+n]} \ln^n \alpha_s$ with $n \geq 0$

$r_0\Lambda_{\overline{\mathrm{MS}}}$ determination

To do the previous lattice comparison we need $r_0\Lambda_{\overline{\mathrm{MS}}}$ as input

$$r_0\Lambda_{\overline{\mathrm{MS}}} = 0.602 \pm 0.048$$

Capitani *et al*. [ALPHA Collaboration]'99

$r_0\Lambda_{\overline{\rm MS}}$ determination

To do the previous lattice comparison we need $r_0\Lambda_{\overline{\rm MS}}$ as input

$$r_0\Lambda_{\overline{\rm MS}} = 0.602 \pm 0.048$$

Capitani *et al.* [ALPHA Collaboration]'99

But uncertainty, induced by it, is larger than unknown higher order terms in the static energy.

$r_0\Lambda_{\overline{\mathrm{MS}}}$ determination

To do the previous lattice comparison we need $r_0\Lambda_{\overline{\mathrm{MS}}}$ as input

$$r_0\Lambda_{\overline{\mathrm{MS}}} = 0.602 \pm 0.048$$

Capitani *et al.* [ALPHA Collaboration]'99

But uncertainty, induced by it, is larger than unknown higher order terms in the static energy. Use lattice comparison to extract $r_0\Lambda_{\overline{\mathrm{MS}}}$

$r_0\Lambda_{\overline{\rm MS}}$ determination

To do the previous lattice comparison we need $r_0\Lambda_{\overline{\rm MS}}$ as input

$$r_0\Lambda_{\overline{\rm MS}} = 0.602 \pm 0.048$$

Capitani *et al.* [ALPHA Collaboration]'99

But uncertainty, induced by it, is larger than unknown higher order terms in the static energy. Use lattice comparison to extract $r_0\Lambda_{\overline{\rm MS}}$

- Lattice comparison requires scheme that cancels leading renormalon

$r_0\Lambda_{\overline{\text{MS}}}$ determination

To do the previous lattice comparison we need $r_0\Lambda_{\overline{\text{MS}}}$ as input

$$r_0\Lambda_{\overline{\text{MS}}} = 0.602 \pm 0.048$$

Capitani *et al.* [ALPHA Collaboration]'99

But uncertainty, induced by it, is larger than unknown higher order terms in the static energy. Use lattice comparison to extract $r_0\Lambda_{\overline{\text{MS}}}$

- Lattice comparison requires scheme that cancels leading renormalon $\longrightarrow$ introduces dimensional scale, ϱ

$r_0\Lambda_{\overline{\rm MS}}$ determination

To do the previous lattice comparison we need $r_0\Lambda_{\overline{\rm MS}}$ as input

$$r_0\Lambda_{\overline{\rm MS}} = 0.602 \pm 0.048$$

Capitani *et al.* [ALPHA Collaboration]'99

But uncertainty, induced by it, is larger than unknown higher order terms in the static energy. Use lattice comparison to extract $r_0\Lambda_{\overline{\rm MS}}$

- Lattice comparison requires scheme that cancels leading renormalon $\longrightarrow$ introduces dimensional scale, ϱ
  - Natural value around the inverse of the center of the r-range, $\varrho \sim 3.25 r_0^{-1}$

$r_0\Lambda_{\overline{\text{MS}}}$ determination

To do the previous lattice comparison we need $r_0\Lambda_{\overline{\text{MS}}}$ as input

$$r_0\Lambda_{\overline{\text{MS}}} = 0.602 \pm 0.048$$

Capitani *et al.* [ALPHA Collaboration]'99

But uncertainty, induced by it, is larger than unknown higher order terms in the static energy. Use lattice comparison to extract $r_0\Lambda_{\overline{\text{MS}}}$

- Lattice comparison requires scheme that cancels leading renormalon $\longrightarrow$ introduces dimensional scale, ϱ
  - Natural value around the inverse of the center of the r-range, $\varrho \sim 3.25 r_0^{-1}$
- Structure of renormalization group equations (singlet-octet mixing) introduces dependence on a constant, K_2

$r_0\Lambda_{\overline{\mathrm{MS}}}$ determination

To do the previous lattice comparison we need $r_0\Lambda_{\overline{\mathrm{MS}}}$ as input

$$r_0\Lambda_{\overline{\mathrm{MS}}} = 0.602 \pm 0.048$$

Capitani *et al.* [ALPHA Collaboration]'99

But uncertainty, induced by it, is larger than unknown higher order terms in the static energy. Use lattice comparison to extract $r_0\Lambda_{\overline{\mathrm{MS}}}$

- Lattice comparison requires scheme that cancels leading renormalon $\longrightarrow$ introduces dimensional scale, ϱ
  - Natural value around the inverse of the center of the r-range, $\varrho \sim 3.25 r_0^{-1}$
- Structure of renormalization group equations (singlet-octet mixing) introduces dependence on a constant, K_2
  - Power counting: $K_2 \sim \Lambda_{\overline{\mathrm{MS}}}$

Find values of $r_0\Lambda_{\overline{\mathrm{MS}}}$ that are allowed by lattice data

Find values of $r_0\Lambda_{\overline{\rm MS}}$ that are allowed by lattice data

Convergent perturbative series and agreement with lattice improves when perturbative order is increased

Find values of $r_0\Lambda_{\overline{\mathrm{MS}}}$ that are allowed by lattice data

Convergent perturbative series and agreement with lattice improves when perturbative order is increased

Procedure in detail:

Find values of $r_0\Lambda_{\overline{\mathrm{MS}}}$ that are allowed by lattice data

Convergent perturbative series and agreement with lattice improves when perturbative order is increased

Procedure in detail:

1. Vary ϱ (by $\pm 25\%$) around natural value

Find values of $r_0\Lambda_{\overline{\rm MS}}$ that are allowed by lattice data

Convergent perturbative series and agreement with lattice improves when perturbative order is increased

Procedure in detail:

1. Vary ϱ (by $\pm 25\%$) around natural value
2. Fit $r_0\Lambda_{\overline{\rm MS}}$ for each value of ϱ and at each order in pert. th.

Find values of $r_0\Lambda_{\overline{\mathrm{MS}}}$ that are allowed by lattice data

Convergent perturbative series and agreement with lattice improves when perturbative order is increased

Procedure in detail:

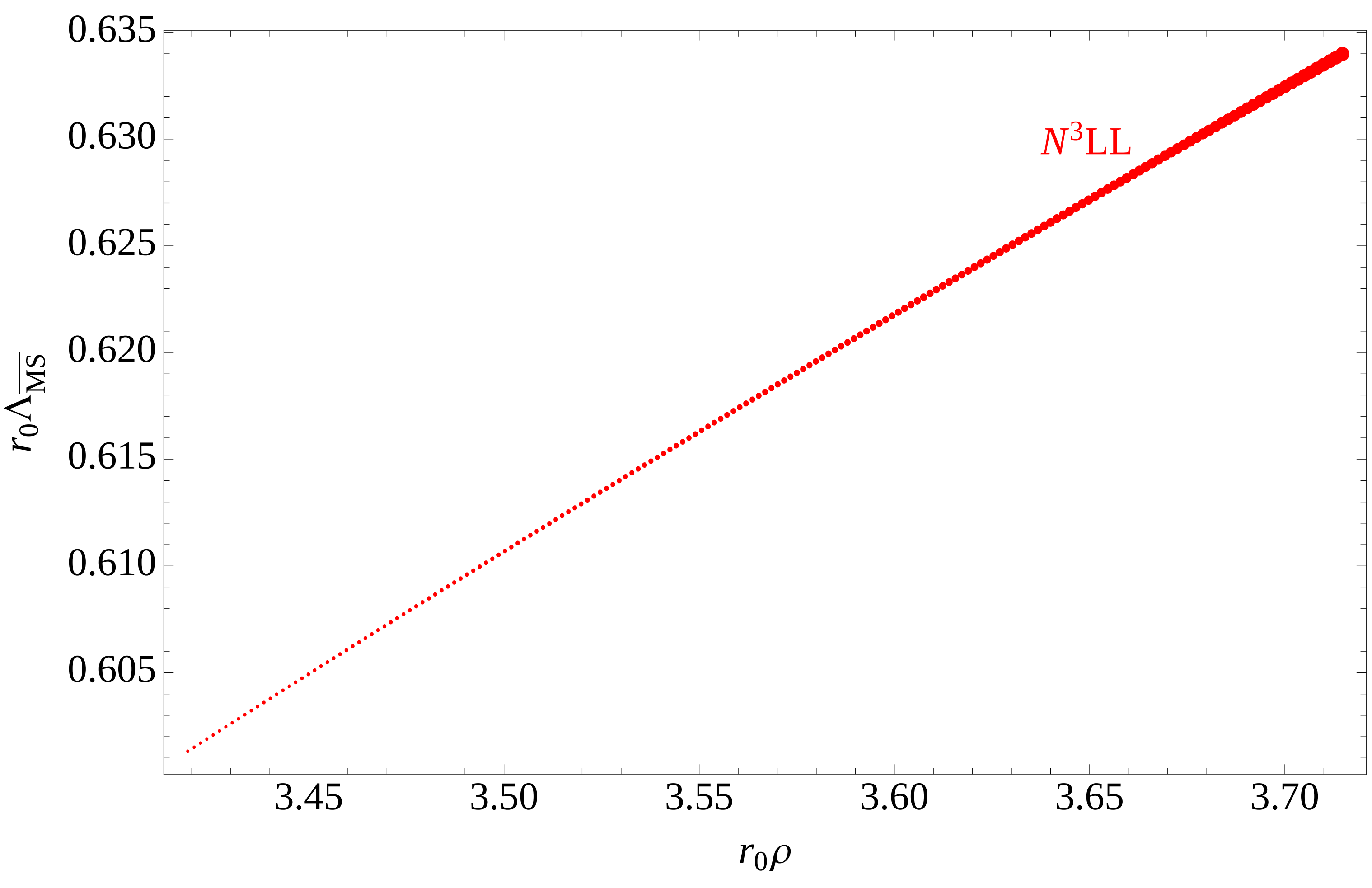
1. Vary ϱ (by $\pm 25\%$) around natural value
2. Fit $r_0\Lambda_{\overline{\mathrm{MS}}}$ for each value of ϱ and at each order in pert. th.
3. Select ϱ's for which χ^2 decreases when increasing pert. order

Find values of $r_0\Lambda_{\overline{\rm MS}}$ that are allowed by lattice data

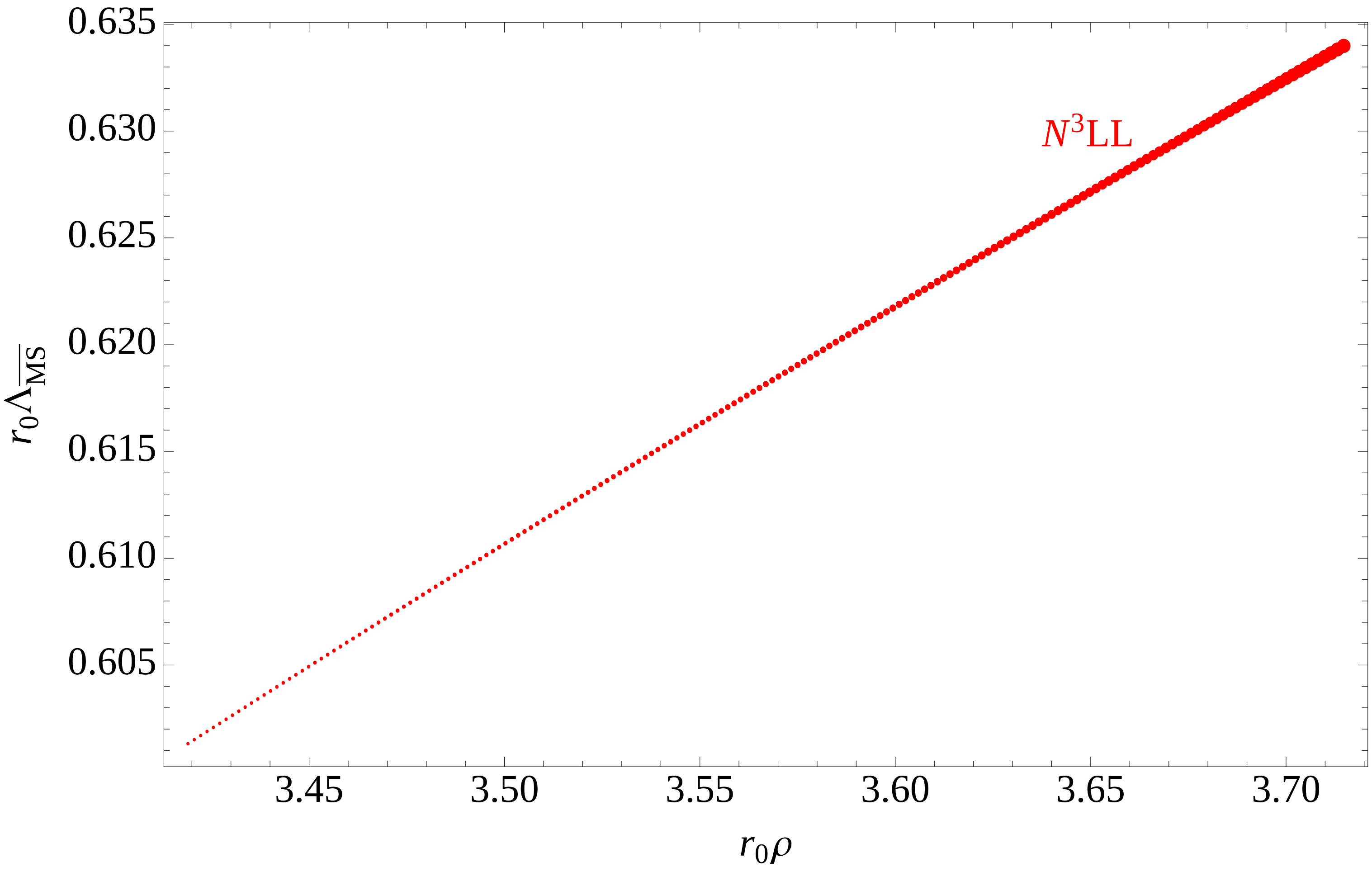
Convergent perturbative series and agreement with lattice improves when perturbative order is increased

Procedure in detail:

1. Vary ϱ (by $\pm 25\%$) around natural value
2. Fit $r_0\Lambda_{\overline{\rm MS}}$ for each value of ϱ and at each order in pert. th.
3. Select ϱ's for which χ^2 decreases when increasing pert. order
4. Select values that respect power counting for K_2



0.635
0.630
0.625
0.620
0.615
0.610
0.605
$r_0\Lambda_{\overline{\text{MS}}}$
3.45
3.50
3.55
3.60
3.65
3.70
$r_0\rho$
N^3LL



Use weighted (inverse χ^2) average for the central value

To assign the error:

To assign the error:

- Weighted standard deviation

To assign the error:

- Weighted standard deviation
- Difference with weighted average at previous order

To assign the error:

- Weighted standard deviation
- Difference with weighted average at previous order
- Additionally, redo analysis with alternative weight assignments (p-value, constant)

To assign the error:

- Weighted standard deviation
- Difference with weighted average at previous order
- Additionally, redo analysis with alternative weight assignments (p-value, constant). Results are compatible

To assign the error:

- Weighted standard deviation
- Difference with weighted average at previous order
- Additionally, redo analysis with alternative weight assignments (p-value, constant). Results are compatible. Quote error to cover whole range

UNIVERSITÄT BERN

To assign the error:

- Weighted standard deviation
- Difference with weighted average at previous order
- Additionally, redo analysis with alternative weight assignments (p-value, constant). Results are compatible. Quote error to cover whole range

Error assigned must account for uncertainties due to neglected higher order terms

UNIVERSITÄT BERN

Final result:

$$r_0\Lambda_{\overline{\rm MS}} = 0.622^{+0.019}_{-0.015}$$

Final result:

$$r_0\Lambda_{\overline{\mathrm{MS}}} = 0.622^{+0.019}_{-0.015}$$

Compatible but more precise than number used previously ($r_0\Lambda_{\overline{\mathrm{MS}}} = 0.602 \pm 0.048$)

Remarks

Remarks

Such a higher order calculation of the static energy is useful in practice and is needed

Remarks

Such a higher order calculation of the static energy is useful in practice and is needed

Accuracy	$r_0\Lambda_{\overline{\rm MS}}$
$\mathrm{N}^2\mathrm{LL}$	0.619 ± 0.13
$\mathrm{N}^3\mathrm{LL}$	0.622 ± 0.012

Remarks

Such a higher order calculation of the static energy is useful in practice and is needed

Accuracy	$r_0\Lambda_{\overline{\rm MS}}$
N^{2}LL	0.619 ± 0.13
N^{3}LL	0.622 ± 0.012

N^{3}LL result improves the precision of the N^{2}LL determination by an order of magnitude

Exactly the same procedure can be done in the unquenched case

Exactly the same procedure can be done in the unquenched case

$$r_0 \Lambda^{unq.}_{\overline{\rm MS}}$$

Exactly the same procedure can be done in the unquenched case

$$r_0 \Lambda_{\overline{\mathrm{MS}}}^{unq.} \longrightarrow r_0$$

Using value of α_s as input, determination of r_0

Exactly the same procedure can be done in the unquenched case

$$r_0\Lambda^{unq.}_{\overline{\mathrm{MS}}} \longrightarrow r_0$$

Using value of α_s as input, determination of r_0

$$r_0\Lambda^{unq.}_{\overline{\mathrm{MS}}} \longrightarrow \Lambda_{\overline{\mathrm{MS}}}$$

Exactly the same procedure can be done in the unquenched case

$$r_0 \Lambda_{\overline{\mathrm{MS}}}^{unq.} \longrightarrow r_0$$

Using value of α_s as input, determination of r_0

$$r_0 \Lambda_{\overline{\mathrm{MS}}}^{unq.} \longrightarrow \Lambda_{\overline{\mathrm{MS}}}$$

Using value of r_0 as an input allows for a novel independent determination of α_s

Conclusions

- Static energy is now known at high orders in perturbation theory

Conclusions

- Static energy is now known at high orders in perturbation theory
- Such high order calculation is needed

Conclusions

- Static energy is now known at high orders in perturbation theory
- Such high order calculation is needed
  - Agreement with lattice

Conclusions

- Static energy is now known at high orders in perturbation theory
- Such high order calculation is needed
  - Agreement with lattice
  - Extraction of $r_0\Lambda_{\overline{\mathrm{MS}}}$

UNIVERSITÄT BERN

Conclusions

- Static energy is now known at high orders in perturbation theory
- Such high order calculation is needed
  - Agreement with lattice
  - Extraction of $r_0\Lambda_{\overline{\rm MS}}$
- New determination: $r_0\Lambda_{\overline{\rm MS}} = 0.622^{+0.019}_{-0.015}$

Conclusions

- Static energy is now known at high orders in perturbation theory
- Such high order calculation is needed
  - Agreement with lattice
  - Extraction of $r_0\Lambda_{\overline{\mathrm{MS}}}$
- New determination: $r_0\Lambda_{\overline{\mathrm{MS}}} = 0.622^{+0.019}_{-0.015}$
- Same procedure could be performed with unquenched data and would provide a determination of α_s

UNIVERSITÄT BERN

Conclusions

- Static energy is now known at high orders in perturbation theory
- Such high order calculation is needed
  - Agreement with lattice
  - Extraction of $r_0\Lambda_{\overline{\rm MS}}$
- New determination: $r_0\Lambda_{\overline{\rm MS}} = 0.622^{+0.019}_{-0.015}$
- Same procedure could be performed with unquenched data and would provide a determination of α_s

Thank you