

Charmonium Spectral Functions and Transport Coefficients from Quenched Lattice QCD

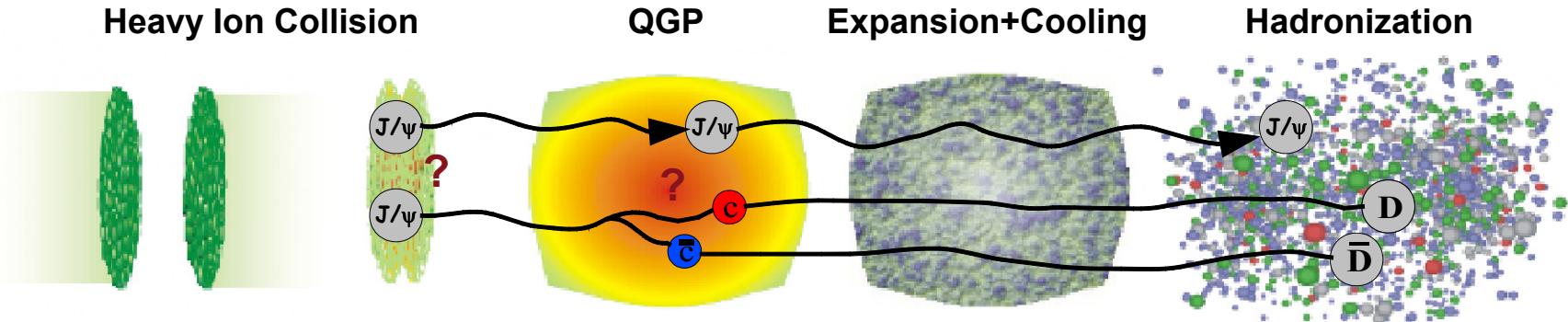
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in collaboration with:

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Frithjof Karsch (BNL+Bielefeld), Edwin Laermann (Bielefeld),
Helmut Satz (Bielefeld), Wolfgang Söldner (Regensburg)



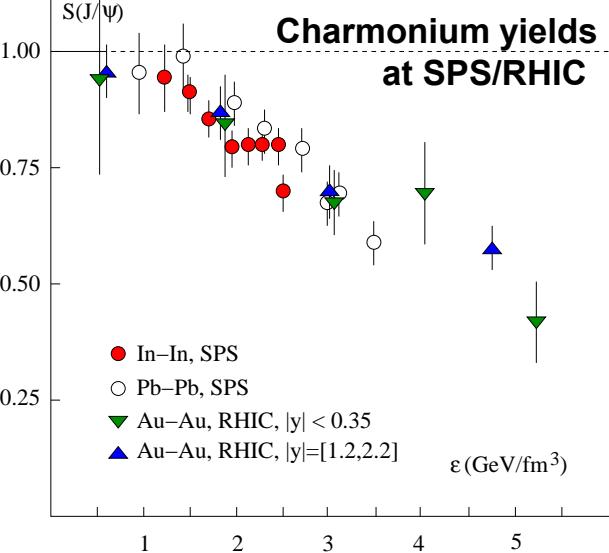
Charmonium in Heavy Ion Collisions – The “simple picture”



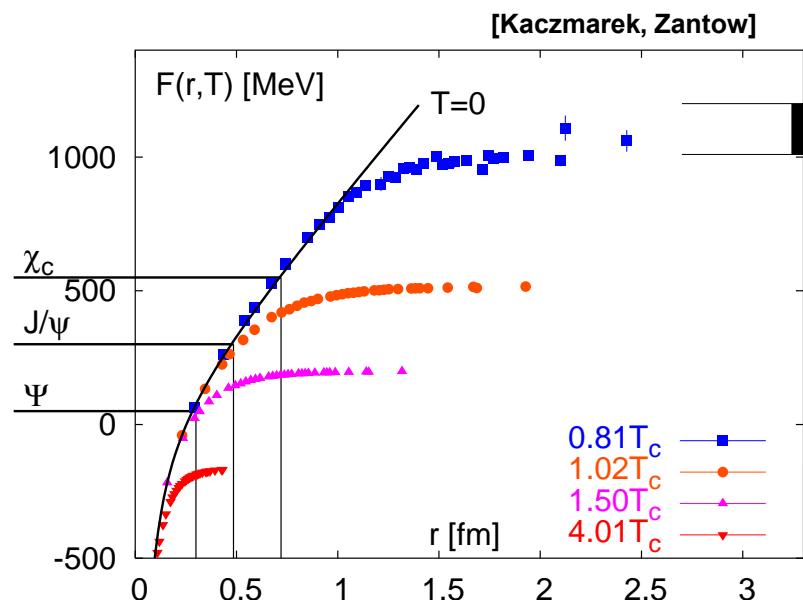
Charmonium+Bottmonium is produced (mainly) in the early stage of the collision

Depending on the Dissociation Temperature

- remain as bound states in the whole evolution
- release their constituents in the plasma



First estimates on Dissociation Temperatures
from detailed knowledge of Heavy Quark Free Energies and Potential Models



Heavy Quark Diffusion constant

Heavy quark diffusion constant \longleftrightarrow slope of spectral function at $\omega=0$ (Kubo formula)

$$D = \frac{\pi}{3\chi_{00}} \lim_{\omega \rightarrow 0} \frac{\rho_{ii}(\omega, \vec{p} = 0, T)}{\omega T}$$

Perturbative estimate ($\alpha_s \sim 0.2$, $g \sim 1.6$):

LO: $2\pi TD \simeq 71.2$

NLO: $2\pi TD \simeq 8.4$

[Moore&Teaney, PRD71(2005)064904,
Caron-Huot&Moore, PRL100(2008)052301]

Strong coupling limit:

$$2\pi TD = 1$$

[Kovtun, Son & Starinets, JHEP 0310(2004)064]

Important input for hydro-models \longrightarrow charm flow v_2 , thermalization rate

relatively small contribution in the low ω -region compared to large ω^2 contribution

\rightarrow effect visible in the correlators around $\tau T \simeq 0.5$

\rightarrow high quality data and large N_t required to resolve this

Vector correlation functions at high temperature

$$G(\tau, \vec{p}, T) = \int_0^\infty \frac{d\omega}{2\pi} \rho(\omega, \vec{p}, T) K(\tau, \omega, T)$$

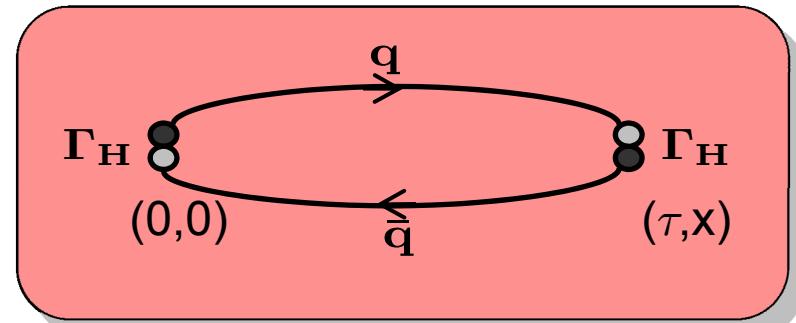
$$K(\tau, \omega, T) = \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh(\frac{\omega}{2T})}$$

Lattice observables:

$$G_{\mu\nu}(\tau, \vec{x}) = \langle J_\mu(\tau, \vec{x}) J_\nu^\dagger(0, \vec{0}) \rangle$$

$$J_\mu(\tau, \vec{x}) = 2\kappa Z_V \bar{\psi}(\tau, \vec{x}) \Gamma_\mu \psi(\tau, \vec{x}) \quad \leftarrow \text{local, non-conserved current, needs to be renormalized}$$

$$G_{\mu\nu}(\tau, \vec{p}) = \sum_{\vec{x}} G_{\mu\nu}(\tau, \vec{x}) e^{i\vec{p}\vec{x}} \quad \leftarrow \text{only } \vec{p} = 0 \text{ used here}$$



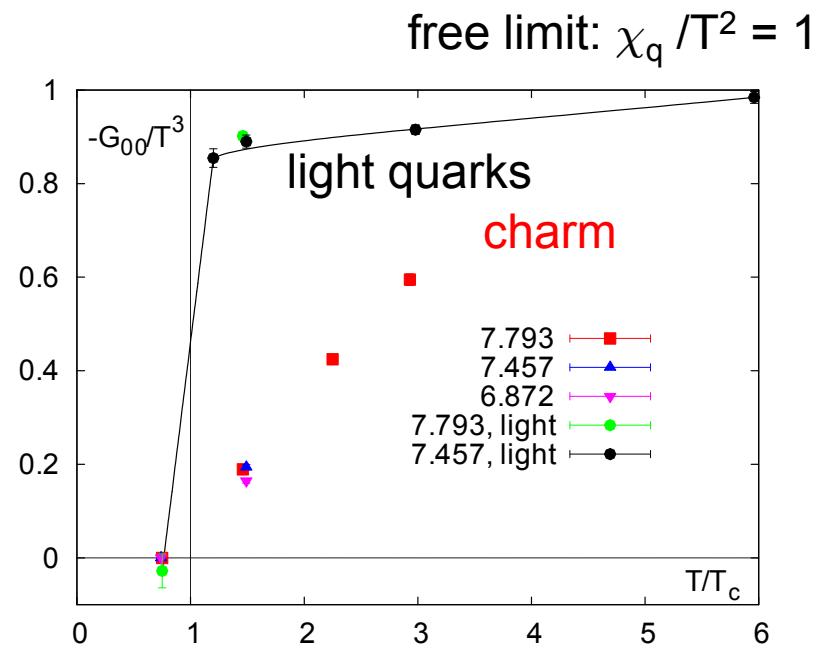
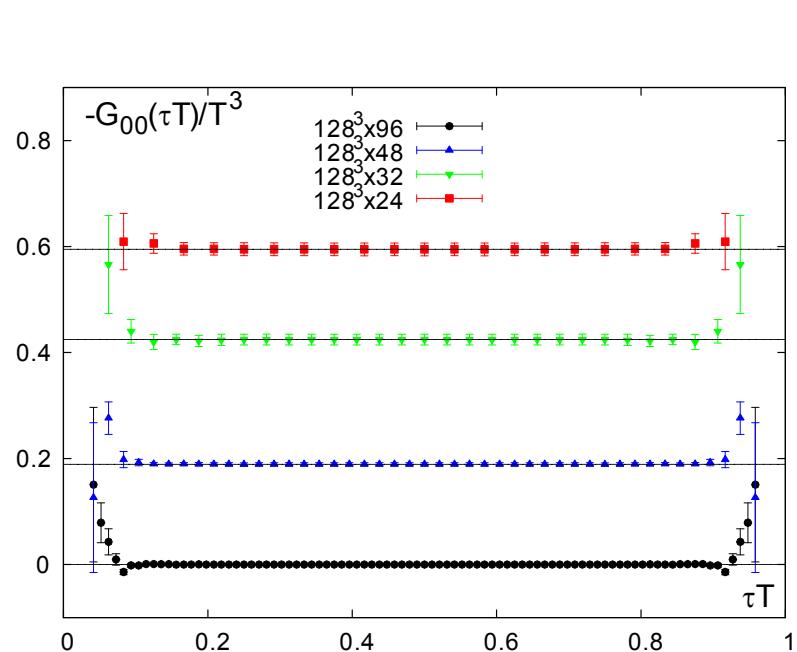
Vector correlation functions at high temperature

$$G_V(\tau, \vec{p}) = -G_{00}(\tau, \vec{p}) + G_{ii}(\tau, \vec{p})$$

J_0 is a conserved current

→ G_{00} is τ independent at $p=0 \sim$ quark number susceptibility χ_q

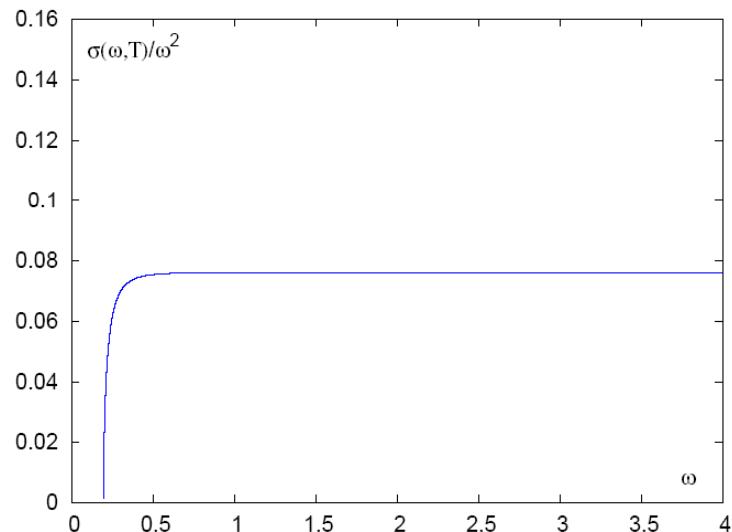
$$G_{00}(\tau, \vec{p}=0) \equiv \chi_q T + \mathcal{O}(a^2)$$



Free spectral functions – lattice vs. continuum

Free (non-interacting) spectral function [Karsch et al. 03, Aarts et al. 05]

$$\begin{aligned}\sigma_H = & \frac{N_c}{8\pi^2} \Theta(\omega^2 - 4m^2) \omega^2 \tanh\left(\frac{\omega}{4T}\right) \\ & \times \sqrt{1 - \left(\frac{2m}{\omega}\right)^2} \left[a_H + \left(\frac{2m}{\omega}\right)^2 b_H \right] \\ & + \frac{N_c}{3} \frac{T^2}{2} f_H \omega \delta(\omega)\end{aligned}$$



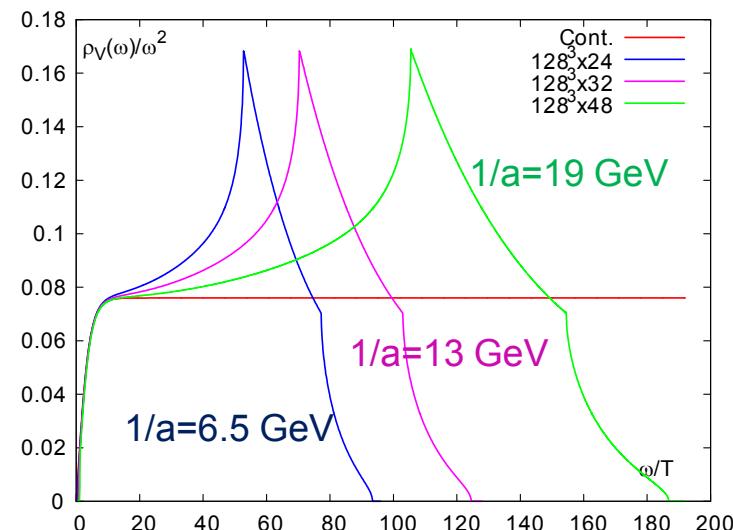
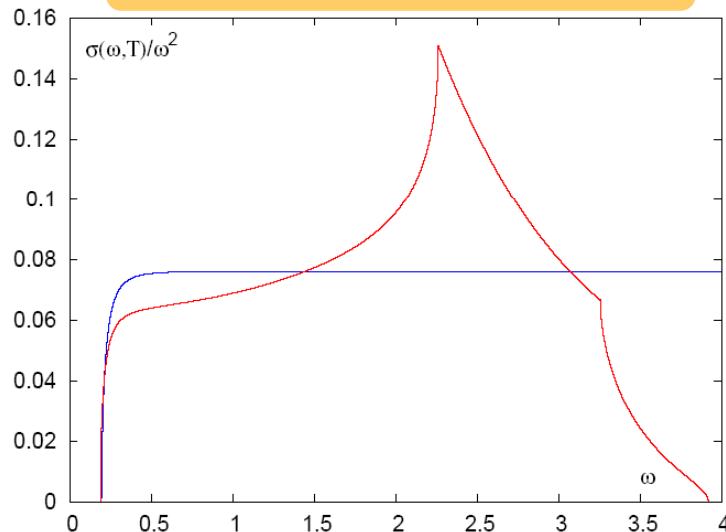
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Lattice cut-off effects:

$$\omega_{max} = 2 \log(7 + ma)$$



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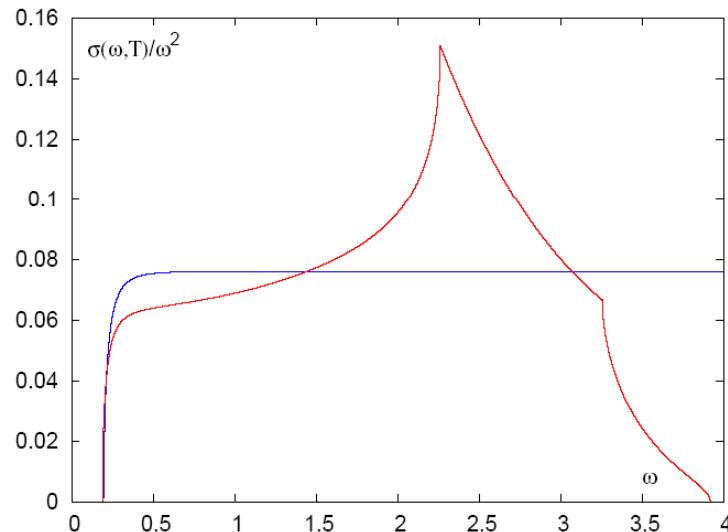
$$+ \frac{N_c}{3} \frac{T^2}{2} f_H \omega \delta(\omega)$$

zero mode contribution at $\omega \simeq 0$ [Umeda 07]
 (not present in pseudo-scalar channel)

with interactions:

$$\delta(\omega) \rightarrow \frac{1}{\pi} \frac{\eta}{\omega^2 + \eta^2}$$

[Petreczky+Teaney 06
 Aarts et al. 05]



Lattice set-up

Quenched SU(3) gauge configurations (separated by 500 updates) at 4 temperatures

Lattice size $N_\sigma^3 N_\tau$ with $N_\sigma = 128$
 $N_\tau = 16, 24, 32, 48, 96$

Non-perturbatively O(a) clover improved Wilson fermions

Non-perturbative renormalization constants

Quark masses close to charm quark mass

β	Mass in GeV				
	J/ψ	η_c	χ_{c1}	χ_{c0}	
6.872	3.1127(6)	3.048(2)	3.624(38)	3.540(25)	
7.457	3.147(1)(25)	3.082(2)(21)	3.574(8)	3.488(4)	
7.793	3.472(2)(114)	3.341(2)(104)	4.02(2)(23)	4.52(2)(37)	

cut-off dependence

volume dependence

β	$a[\text{fm}]$	$a^{-1}[\text{GeV}]$	$L_\sigma [\text{fm}]$	c_{SW}	κ	$N_\sigma^3 \times N_\tau$	T/T_c	N_{config}
6.872	0.031	6.432	3.93	1.412488	0.13035	$128^3 \times 32$	0.74	128
						$128^3 \times 16$	1.49	198
7.457	0.015	12.864	1.96	1.338927	0.13179	$128^3 \times 64$	0.74	179
						$128^3 \times 32$	1.49	250
7.793	0.010	18.974	1.33	1.310381	0.13200	$128^3 \times 96$	0.73	234
						$128^3 \times 48$	1.46	461
						$128^3 \times 32$	2.20	105
						$128^3 \times 24$	2.93	81

close to continuum
 $(m_c a \ll 1)$

Temperature dependence

Spectral Functions – Maximum Entropy Method

How to obtain continuous spectral function $\sigma(\omega, T)$
from discrete (and rather small) number of distances in the correlator?

$$G(\tau, T) = \int_0^\infty d\omega K(\tau, \omega, T) \sigma(\omega, T)$$

$$K(\tau, \omega, T) = \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh(\frac{\omega}{2T})}$$

Best method on the market: Maximum Entropy Method (MEM)
based on Bayesian theorem [Asakawa et al. 01] → most probable spectral function
properly renormalized correlators as input

non-perturbative renormalization constants for vector [Lüscher et al. 1997]

TI perturbative renormalization constants for pseudo-scalar
prior knowledge needed as input → default model $m(\omega)$
result should be independent of default model ← usually not the case

Maximum Entropy Method

Maximize the conditional probability $P[\sigma | GH] \sim \exp(\alpha S - L)$

- G is the input correlator and H the additional prior information
- L is the standard chi²
- S is the Shannon-Jaynes entropy:

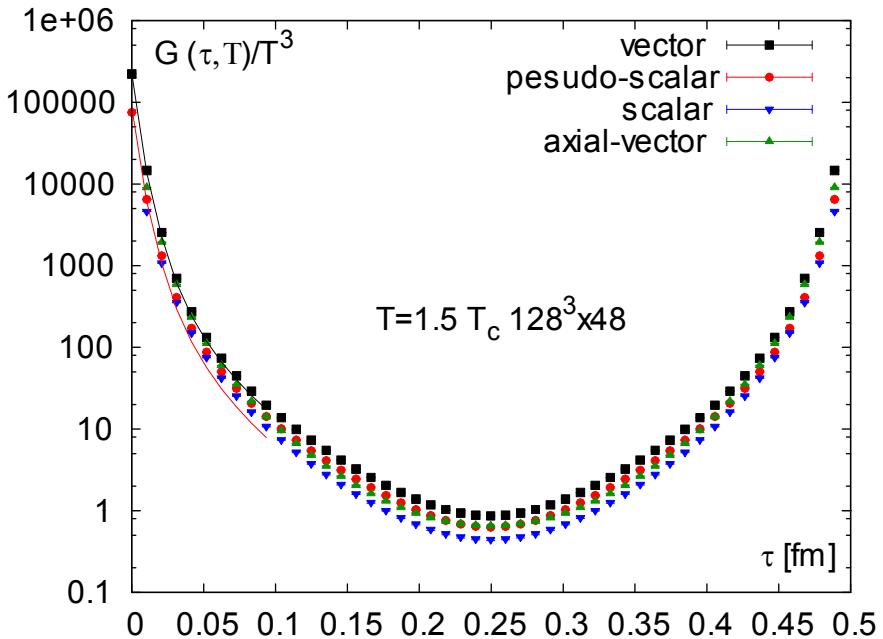
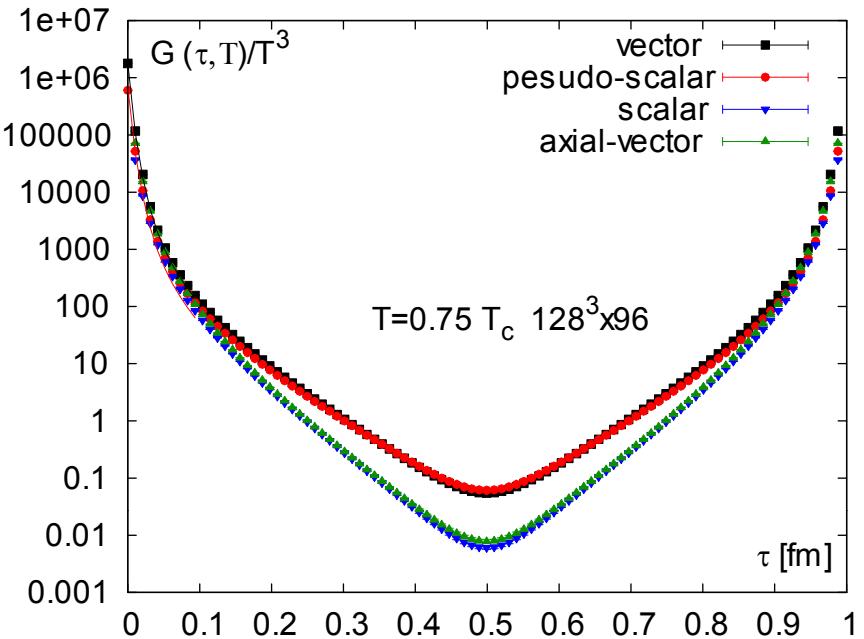
$$S = \int_0^\infty \left[\sigma(\omega) - m(\omega) - \sigma(\omega) \log\left(\frac{\sigma(\omega)}{m(\omega)}\right) \right]$$

- prior knowledge through default model $m(\omega)$
- α is the relative weight of S and L, integrated out at the end

Prior information necessary and important

Should have the correct large ω behavior and functional form

Charmonium Correlators – Temporal Correlators



non-degenerate states still at $1.50 T_c$

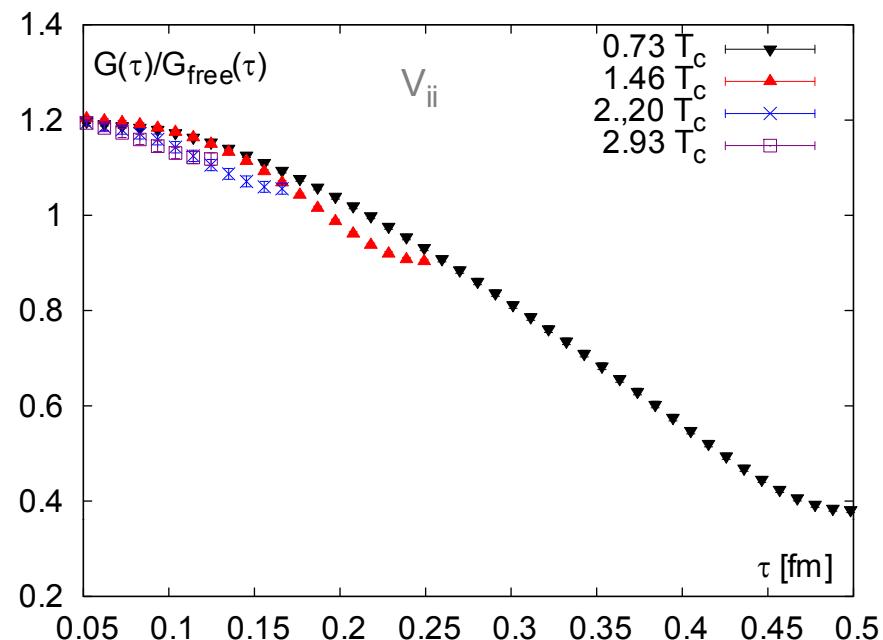
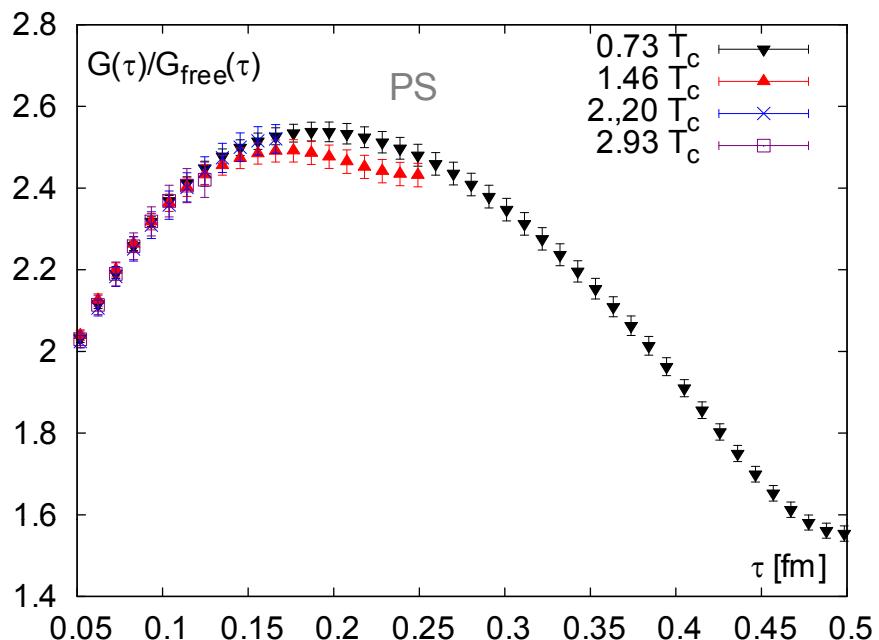
(almost) close to free correlators at (very) small separations

largest distance 0.25 fm due to compact temporal direction

only small distance regime (0.1-0.25 fm) relevant

for thermal effects and bound state effects

Charmonium Correlators vs Free Correlators



small temperature effects visible in both channels

relevant distance regime compressed to smaller distances

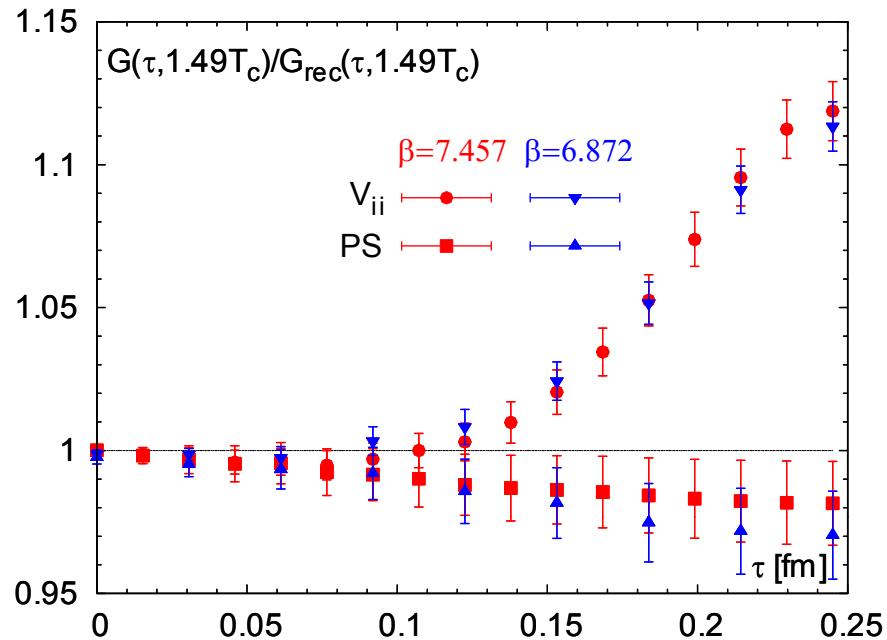
largest distance $1/(2T)$ due to compact temporal direction

only small distance regime relevant

for thermal effects and bound state effects

Problem: quark mass dependence of free correlation function!

Charmonium Correlators – cut-off dependence

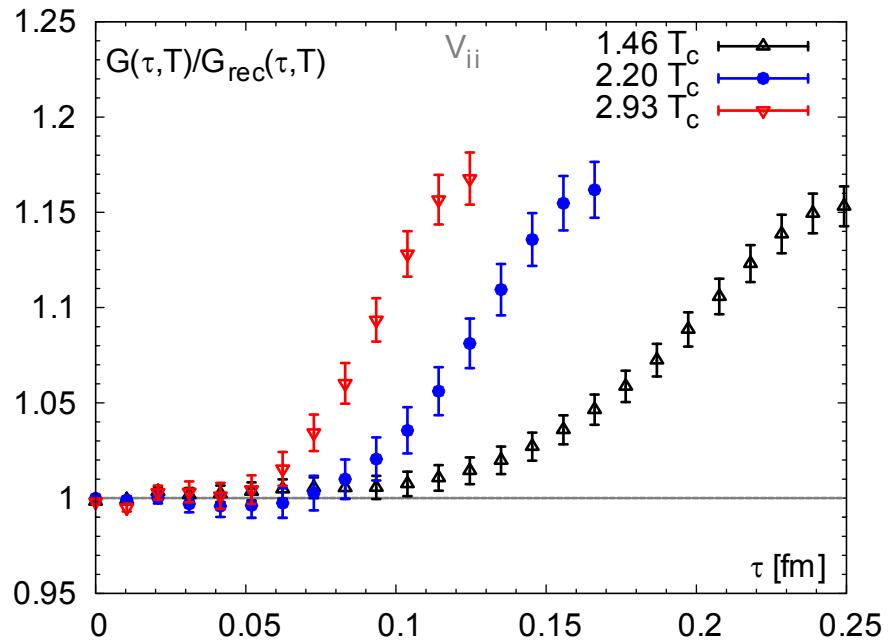
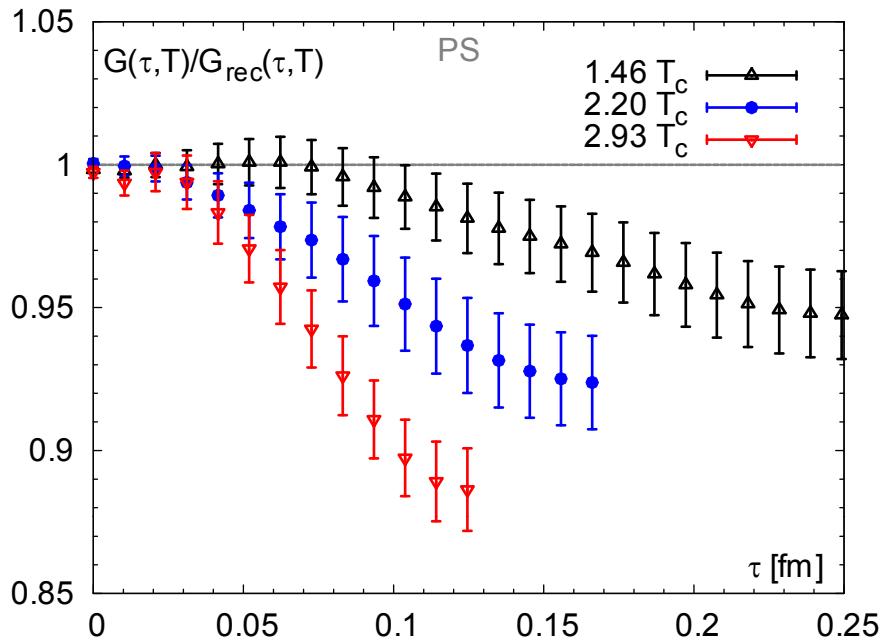


$$G_{rec}(\tau, T) = \int \sigma_0(\omega, 0.75T_c) K(\omega, \tau, T)$$

$$G_{\mu\mu}(\tau, T) = G_{ii}(\tau, T) + G_{00}(\tau, T)$$

only small cut-off/volume dependence
in vector and pseudo-scalar channel
at $1.5 T_c$

Charmonium Correlators vs Free Correlators

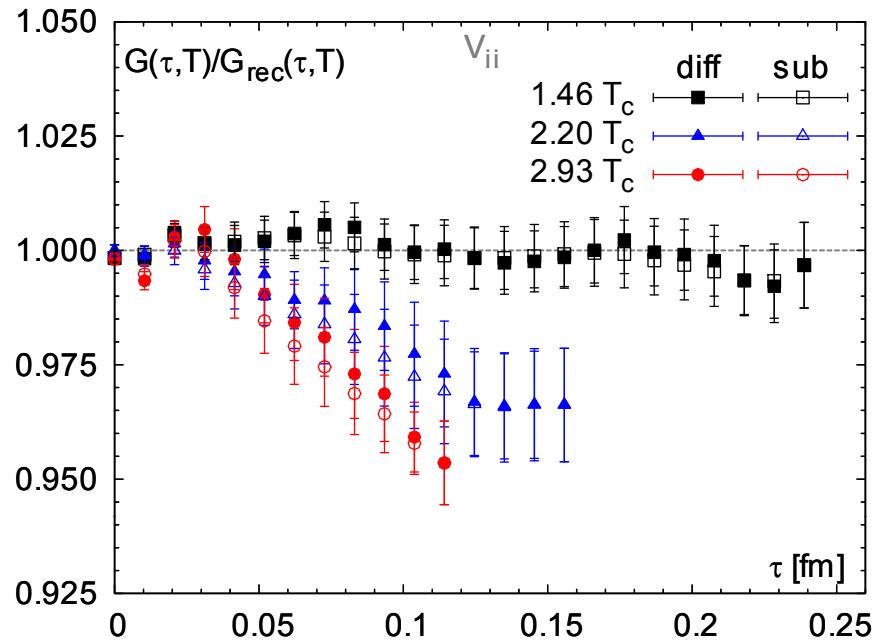
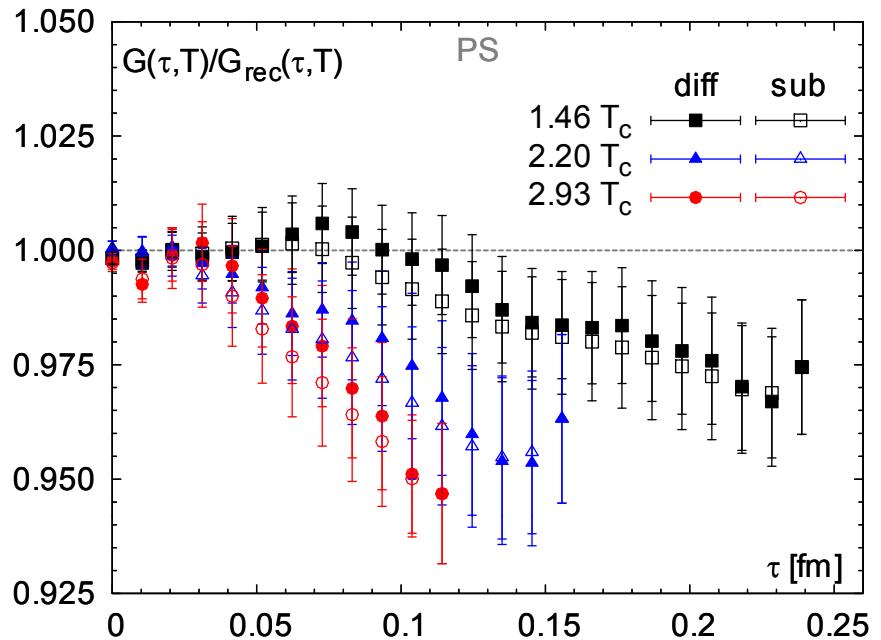


$$G_{\text{rec}}(\tau, T) = \int \sigma_0(\omega, 0.75T_c) K(\omega, \tau, T)$$

$$G_{\mu\mu}(\tau, T) = G_{ii}(\tau, T) + G_{00}(\tau, T)$$

- main T-effect due to zero-mode contribution
- well described by small ω -part of $\sigma_T(\omega, T)$
- explains the rise in the vector channel
- no zero-mode contribution in PS-channel
(similar to discussions by Umeda, Petreczky)

Charmonium Correlators vs Free Correlators



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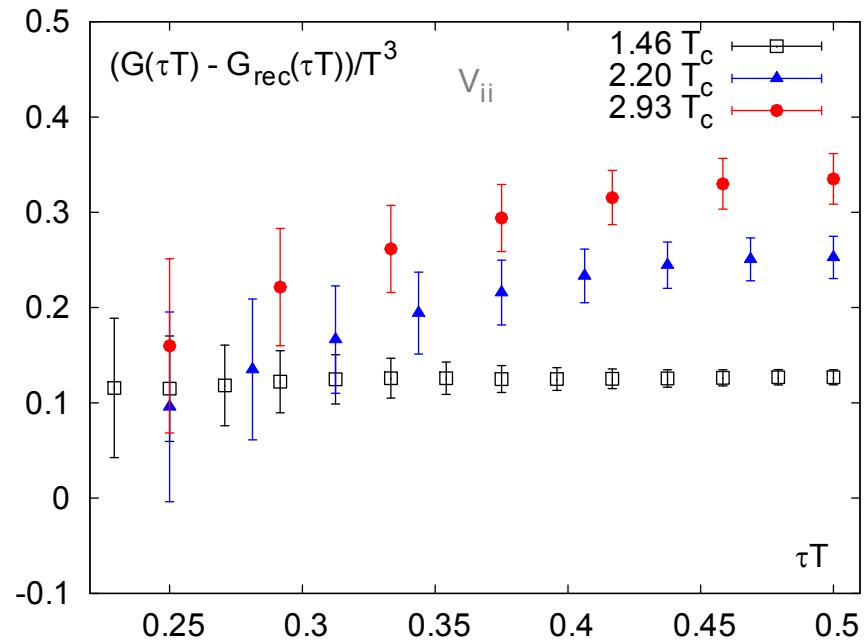
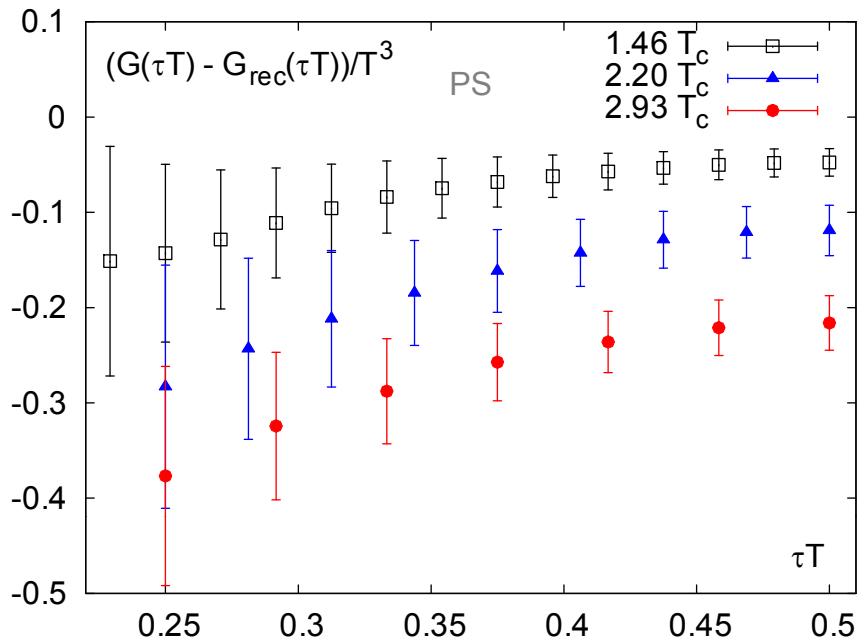
$$G_{\mu\mu}(\tau, T) = G_{ii}(\tau, T) + G_{00}(\tau, T)$$

$$G_{\text{diff}}(\tau, T) = G(\tau, T) - G(\tau + 1, T)$$

$$G_{\text{sub}}(\tau, T) = G(\tau, T) - G(\tau = N_t/2, T)$$

- main T-effect due to zero-mode contribution
- effectively removed by diff/sub correlators
- almost constant small- ω contribution
- similar T-dependence in PS and V channel in the large frequency part of spectral fct.

Charmonium Correlators vs Free Correlators

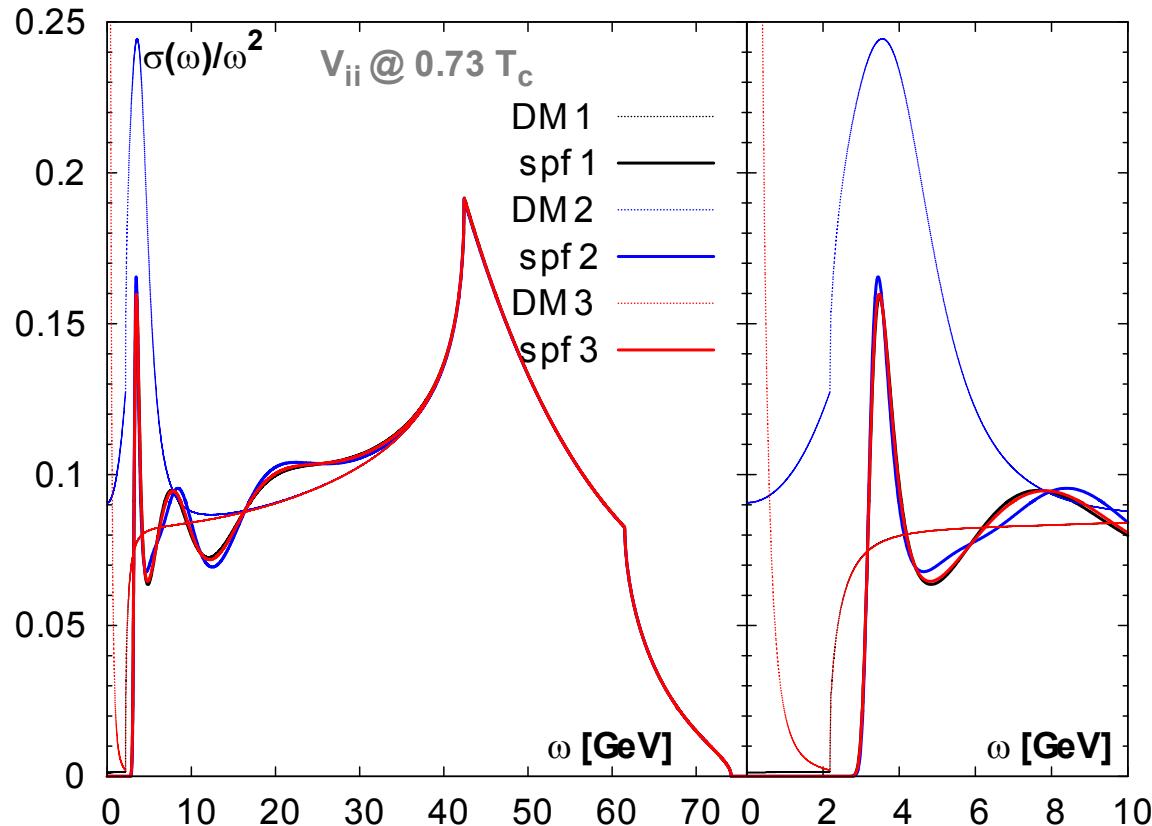


- negative difference for all T
- indications for thermal modifications in the bound state frequency region
- remember: no transport contribution in this channel

- positive diff. due to small- ω contr.
- positive slope indicates modifications in the bound state frequency region
- remember: small- ω contribution determines transport coefficient

First estimate from fit to vector channel: $2\pi T D \approx 0.6 - 3.4$

Charmonium Spectral function below T_c



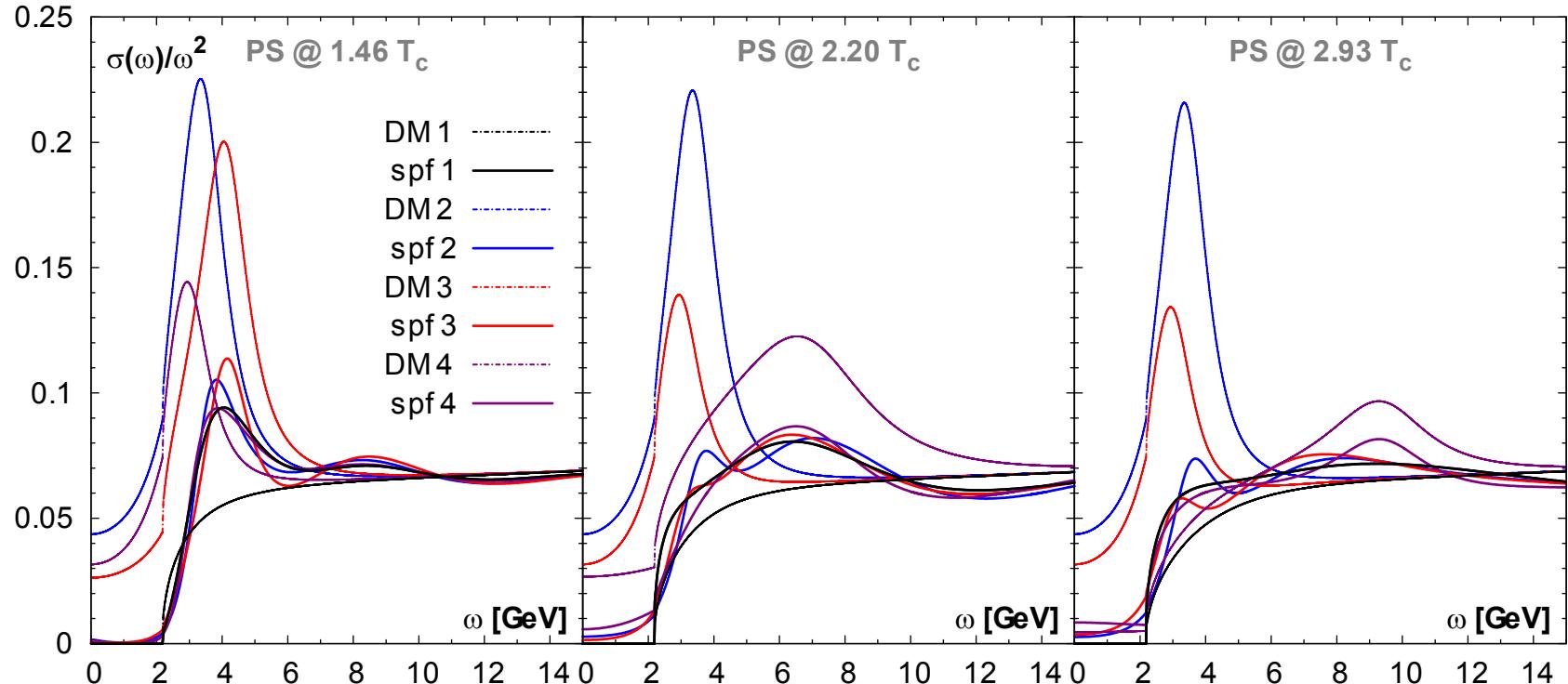
$N_\sigma=128$ and $N_\tau=96$ ($a^{-1} \approx 19$ GeV) \rightarrow cut-off effects well separated

Pronounced ground state peak close to J/ψ mass

no zero-mode/transport contribution observed below T_c in any channel

structure at small and intermediate ω independent of default model

Charmonium Spectral function above T_c



various default models used to understand systematic uncertainties

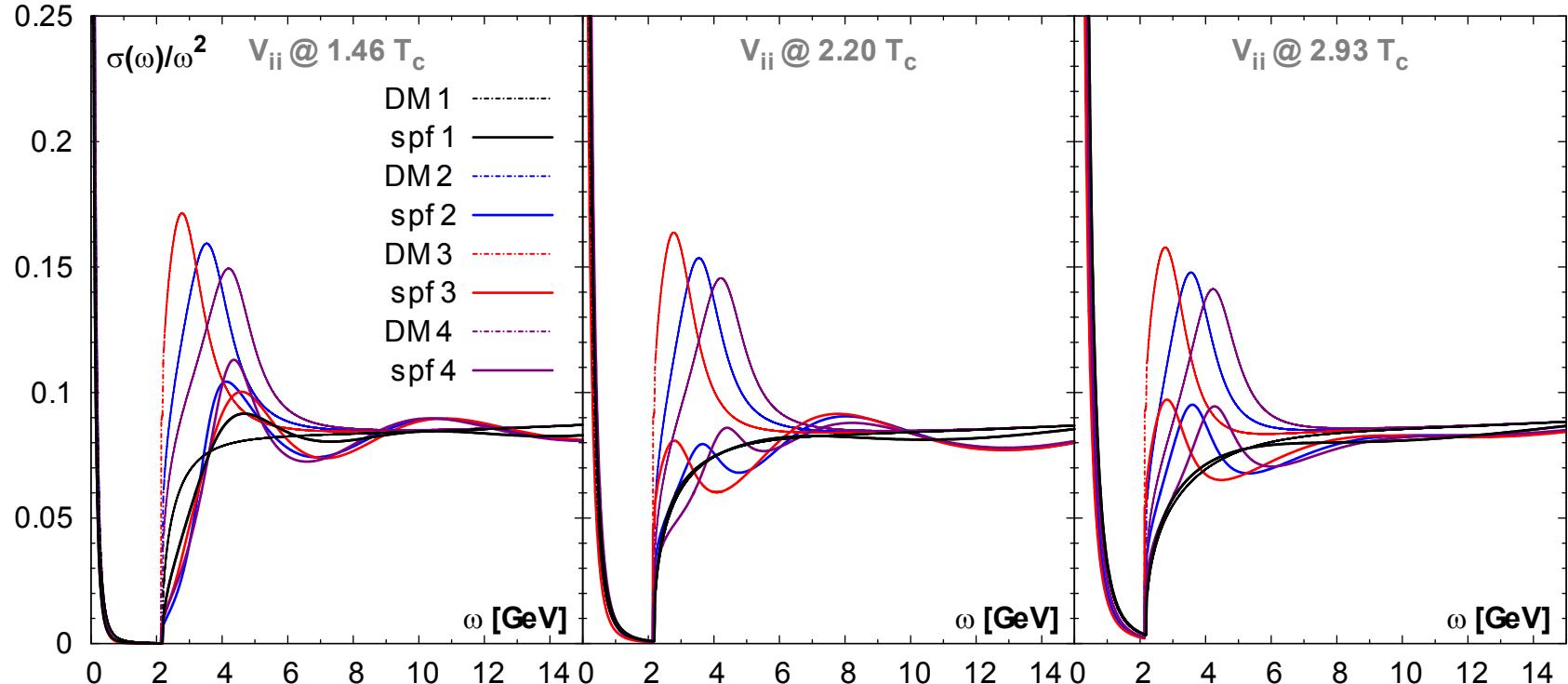
small default model dependence only in small- ω region

no strong default model dependence in the intermediate ω region

no zero-mode contribution in pseudo-scalar channel observed

no pronounced peak \rightarrow bound states melted? / threshold enhancement?

Charmonium Spectral function above T_c



small- ω region accessible → hope to extract transport properties

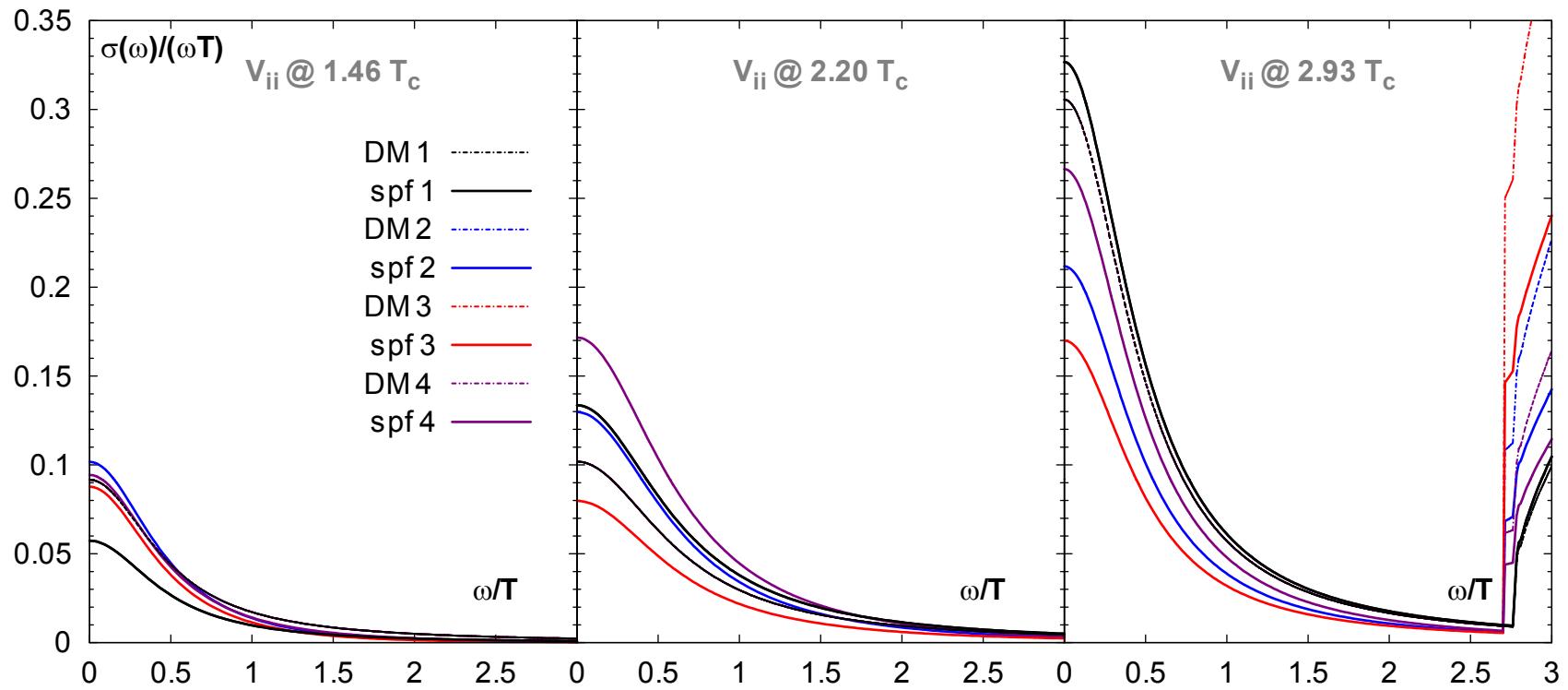
small default model dependence only in small- ω region

no strong default model dependence in the intermediate ω region

unique transport contribution in vector channel observed

no pronounced peak → bound states melted? / threshold enhancement?

Charmonium Spectral function – Transport Peak



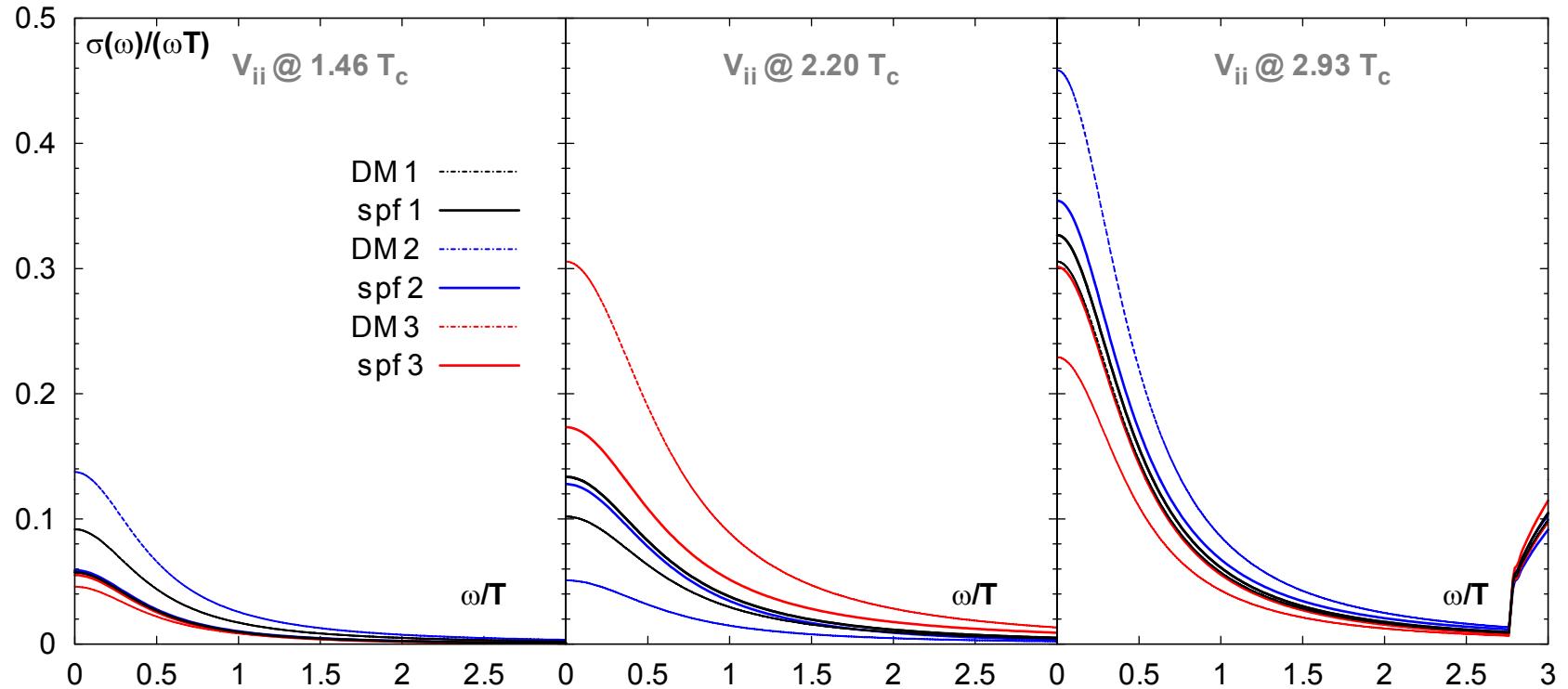
vary the large- ω behavior of the default models

transport peak is a unique feature for all DMs

increases with increasing temperature

at least lower and upper limits for transport coefficients should be possible

Charmonium Spectral function – Transport Peak



vary the low- ω behavior of the default models

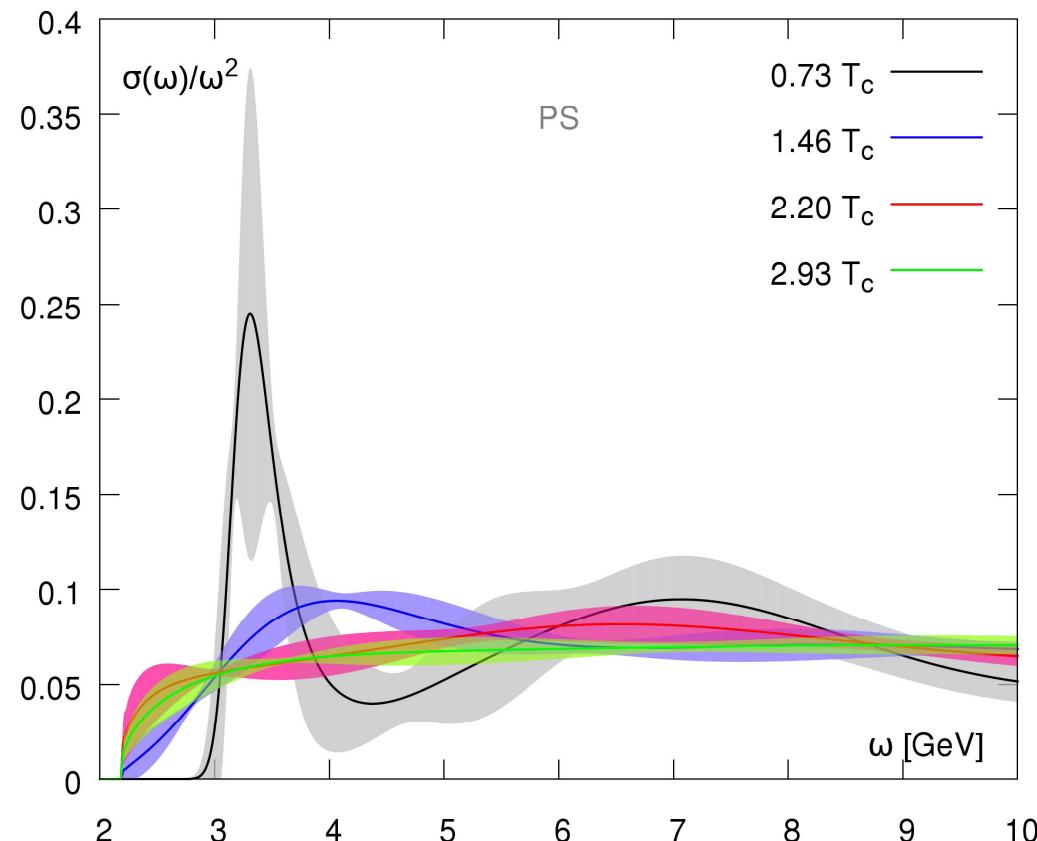
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Charmonium Spectral function

our current best guess:



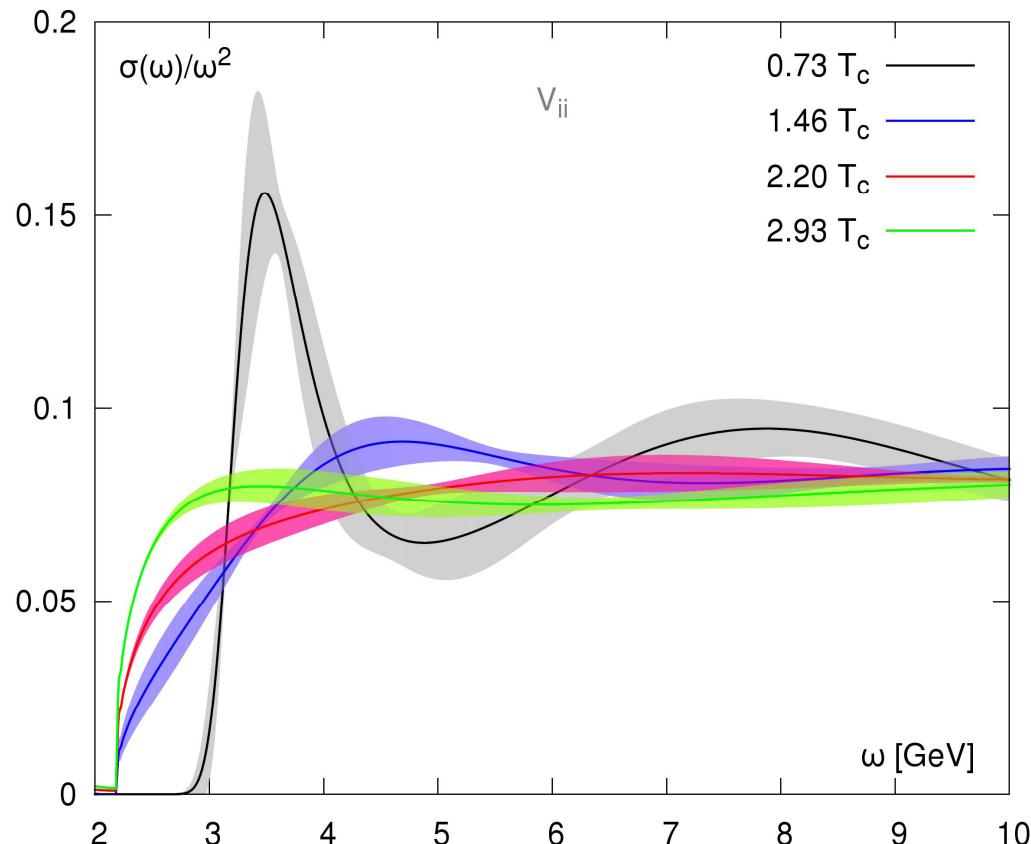
statistical error band from Jackknife analysis

no clear signal for bound states above $1.46 T_c$

systematic uncertainties still need to be analyzed in more detail!

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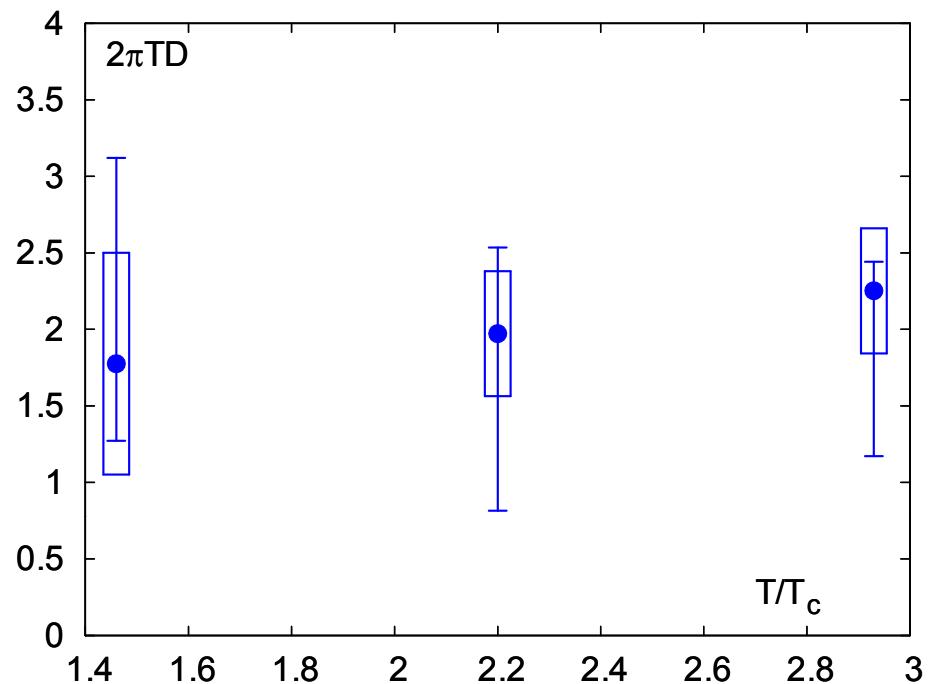
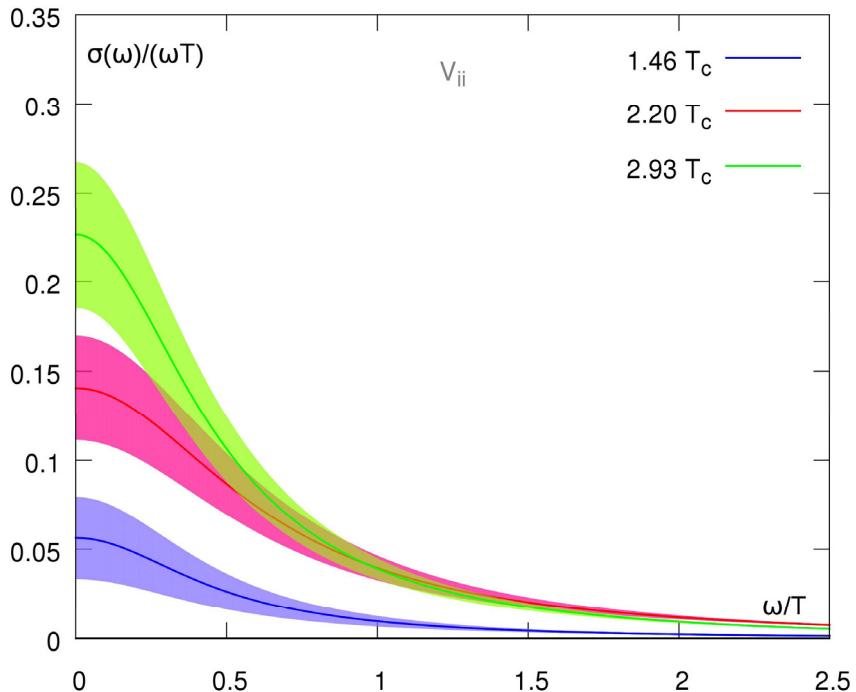


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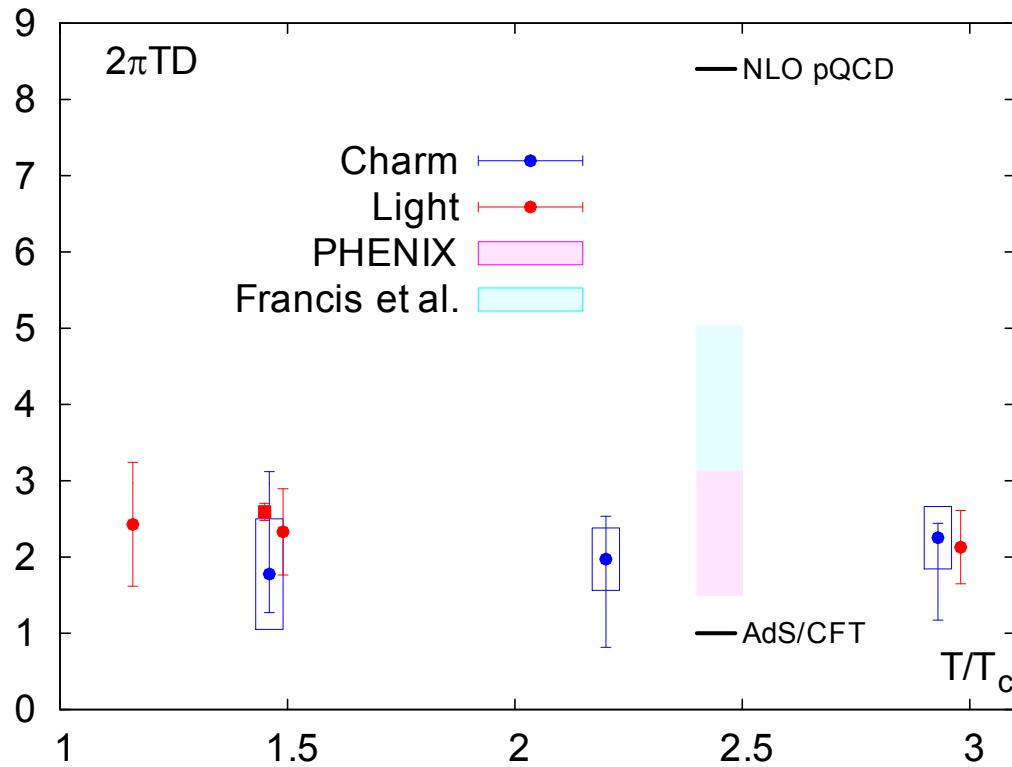
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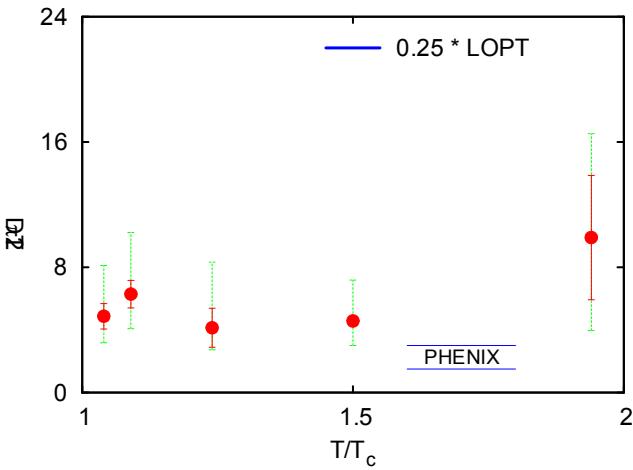
Heavy Quark Diffusion constant



estimate from HQ momentum diffusion
Francis, OK, Laine, Langelage, arXiv:1109.3941
(see talk by Mikko Laine)

estimate from charmonium correlation fct.
Ding, Francis, OK, Karsch, Satz, Soeldner,
arXiv:1107.0311

estimate from light quark correlation fct.
Ding, Francis, OK, Karsch, Laermann, Soeldner,
PRD83 (2011) 014504
OK, Francis, arXiv:1109.4054
(see also Aarts et al. PRL 99 (2007) 022002)



similar calculation on HQ momentum diffusion by
Banerjee, Datta, Gavai, Majumdar, arXiv:1109.5738
(see also Meyer, New J.Phys. 13 (2011) 035008)

There is improvement in extracting transport coefficients from LQCD!
still more improvement needed to reduce systematic uncertainties
and to move from estimates to real numbers!

Conclusions and Outlook – Charm Quarks

Conclusions:

Detailed knowledge of the **vector correlation function** at various T in quenched QCD

→ **continuum extrapolation** of correlation function still needed!

Results so far depend on MEM analysis → Ansätze more difficult due to m_q dependence

→ **Heavy quark diffusion constant:** $2\pi DT \approx 2$

→ **No signs for bound states at and above $1.46 T_c$**

Outlook:

understand systematic uncertainties in more detail

need more sophisticated Ansätze for the spectral function

vector correlation function at **non-zero momentum**