A non-perturbative look at heavy quark diffusion

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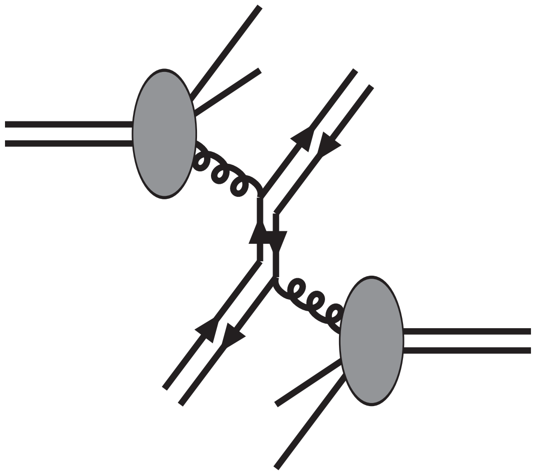
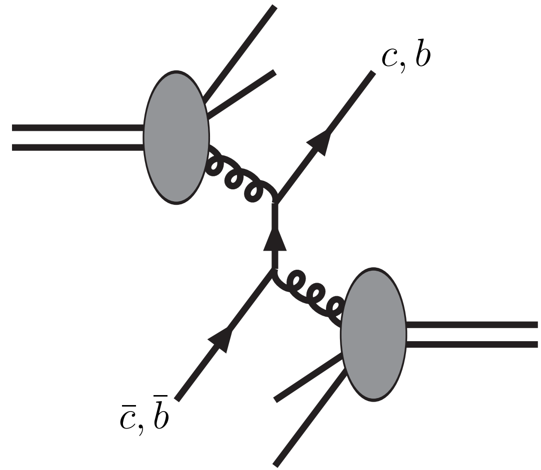
Based on: A. Francis, O. Kaczmarek, ML, J. Langelage, *Towards a non-perturbative measurement of the heavy quark momentum diffusion coefficient,* 1109.3941.

Other recent work: H. Meyer, *The errant life of a heavy quark in the quark-gluon plasma,* 1012.0234; D. Banerjee, S. Datta, R. Gavai, P. Majumdar, *Heavy quark momentum diffusion coefficient from lattice QCD,* 1109.5738.

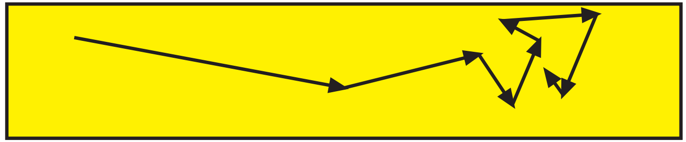
Quarkonium: Open Charm/Bottom:



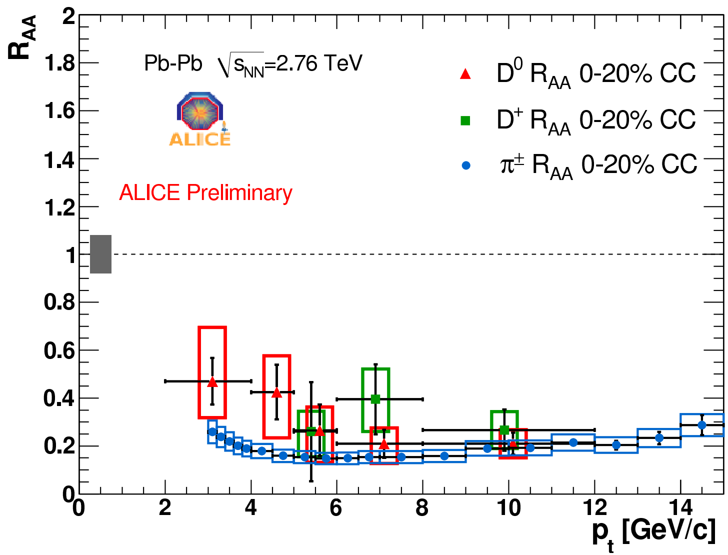


Interesting thermal physics takes place in the latter case as well!

The heavy quark jets get slowed down and eventually stopped, by bremsstrahlung and by elastic scatterings.

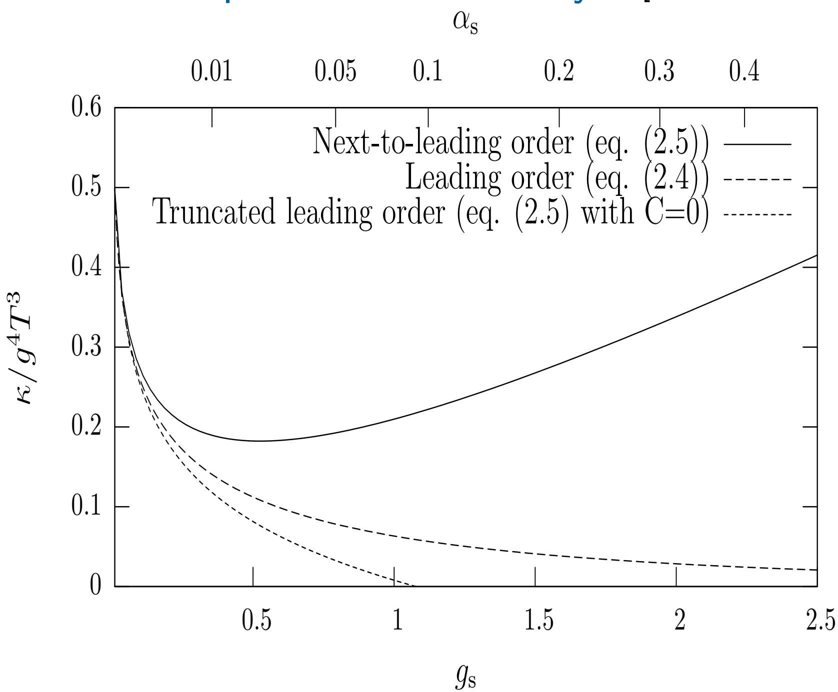


Very strong interactions are seen in experiment: [ALICE 1106.4042]



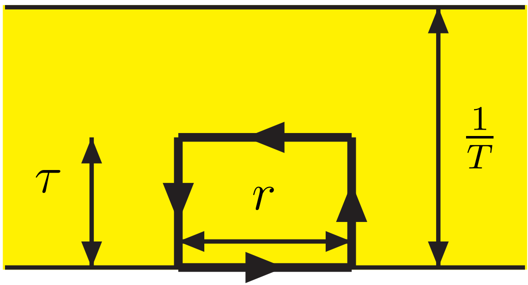
This physics can be captured by a "transport coefficient", called the "momentum diffusion coefficient" and often denoted by κ (the "usual diffusion coefficient" is given by $D = 2T^2/\kappa$).

At NLO in perturbation theory: [Caron-Huot Moore 0801.2173]



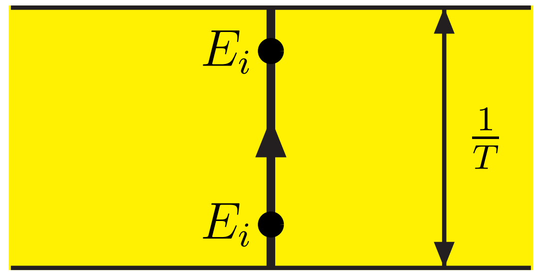
$\Rightarrow$ Huge effects in a good direction — but very poor convergence so a **lattice study is needed**.

How does it look in imaginary time? (Assuming $M \gg \pi T$.)



A quark-antiquark pair, separated by a distance r.

[e.g. Rothkopf et al 1108.1579]



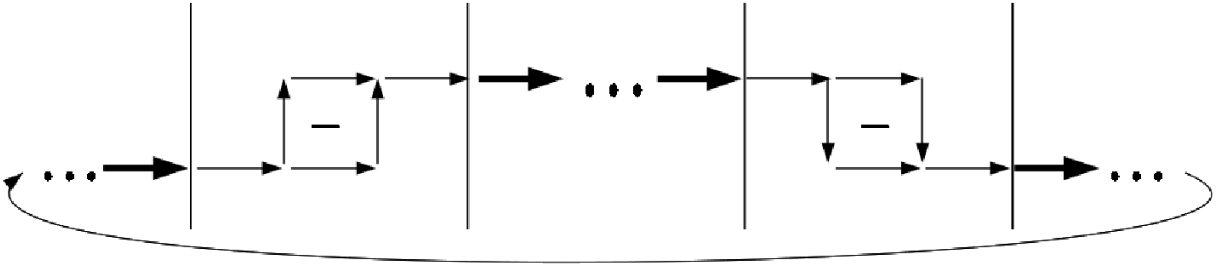
A single quark, kicked by the Lorentz force $F_i = gE_i$.

[Caron-Huot et al 0901.1195]

The latter as an equation:

$$G_E(\tau) = -\frac{1}{3}\sum_{i=1}^{3} \frac{\langle \mathrm{Re}\,\mathrm{Tr}[U_{\beta;\tau}\, gE_i(\tau,\mathbf{0})\, U_{\tau;0}\, gE_i(0,\mathbf{0})]\rangle}{\langle \mathrm{Re}\,\mathrm{Tr}[U_{\beta;0}]\rangle} \,.$$

Discretized version:



Special techniques:

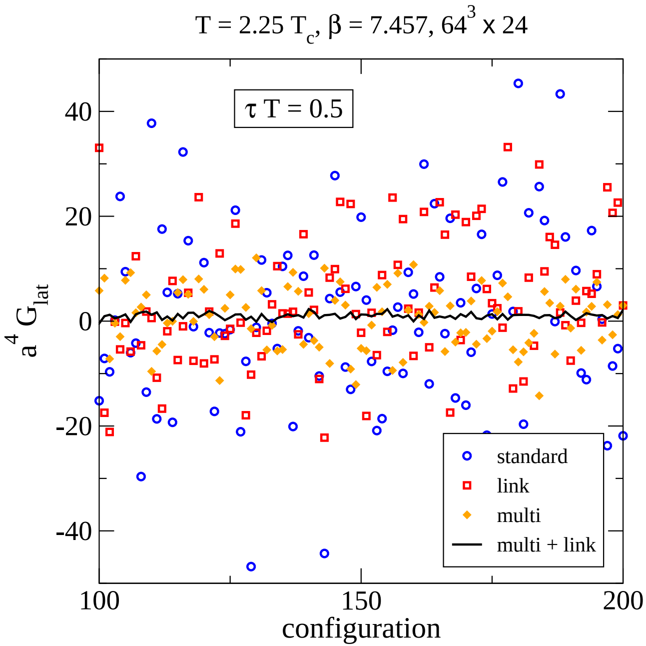
- Links between electric fields handled through “link integration”

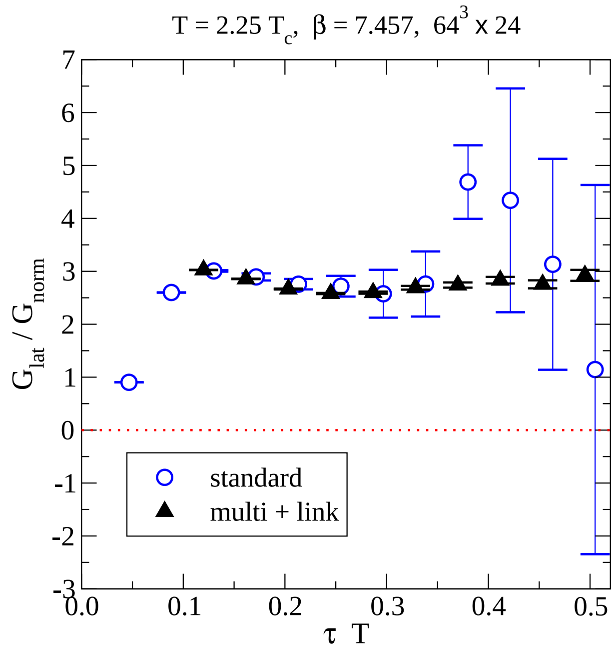
  [Parisi Petronzio Rapuano PLB 128 (1983) 418; de Forcrand Roiesnel PLB 151 (1985) 77]

- Time intervals enclosing electric fields subjected to ~ 10 extra updates with fixed boundary conditions

  [Lüscher Weisz hep-lat/0108014; Meyer 0704.1801]

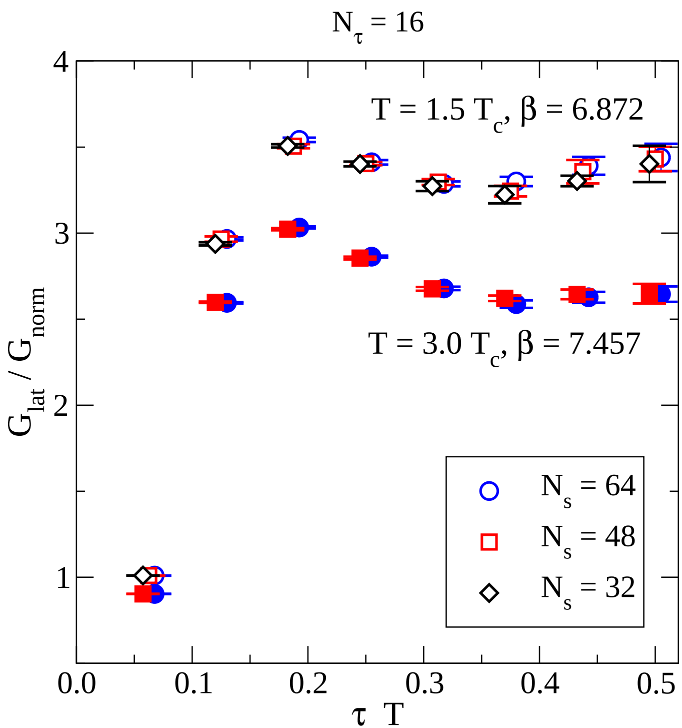
Combining both techniques we get a signal:





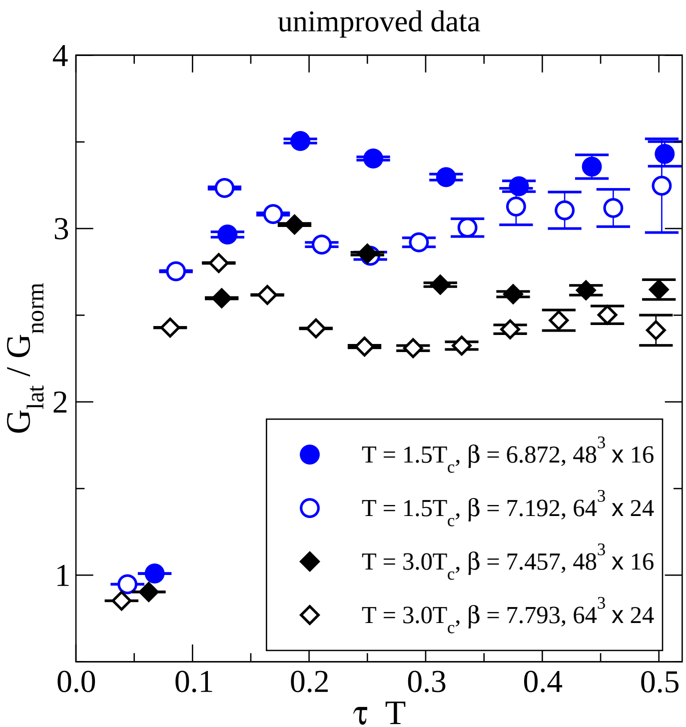
$$G_{\mathrm{norm}}(\tau T) \equiv \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)} \right] .$$

Finite-volume effects ($V = N_\tau \times N_s^3$):



These seem to be harmless at the current resolution.

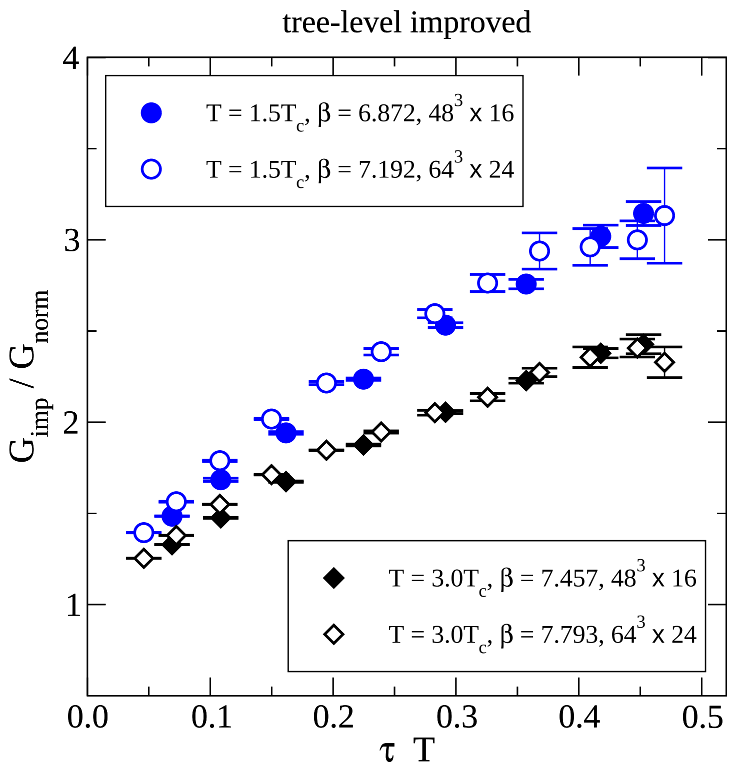
A bigger problem concerns the dependence on the lattice spacing:



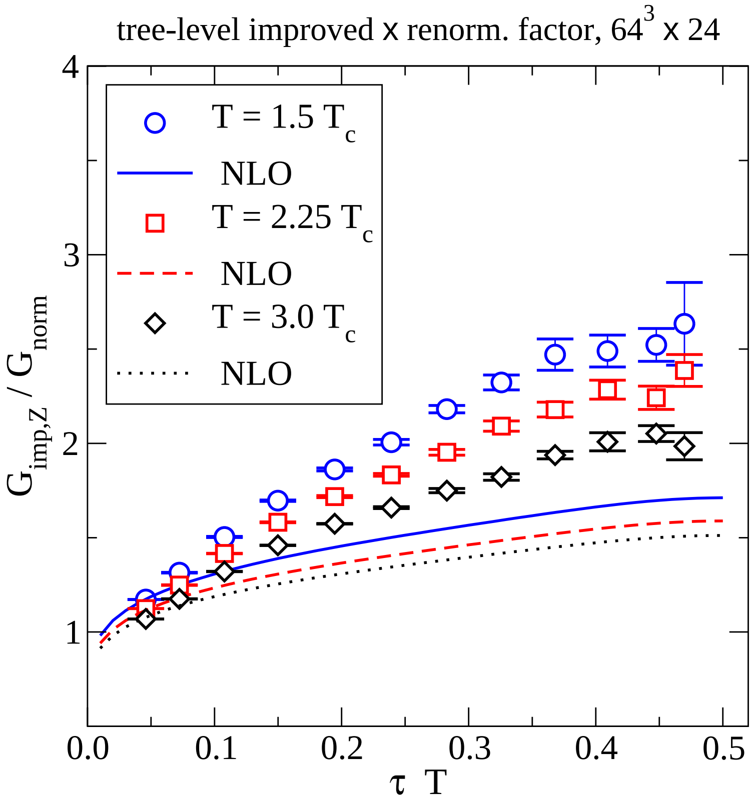
Either need several more lattice spacings, or do something clever...

With "tree-level improvement" [Sommer hep-lat/9310022; Meyer 0904.1806]:

(i) $G^{\rm LO}_{\rm cont}(\overline{\tau T}) = G^{\rm LO}_{\rm lat}(\tau T)$; (ii) $G_{\rm imp}(\overline{\tau T}) \equiv G_{\rm lat}(\tau T)$.

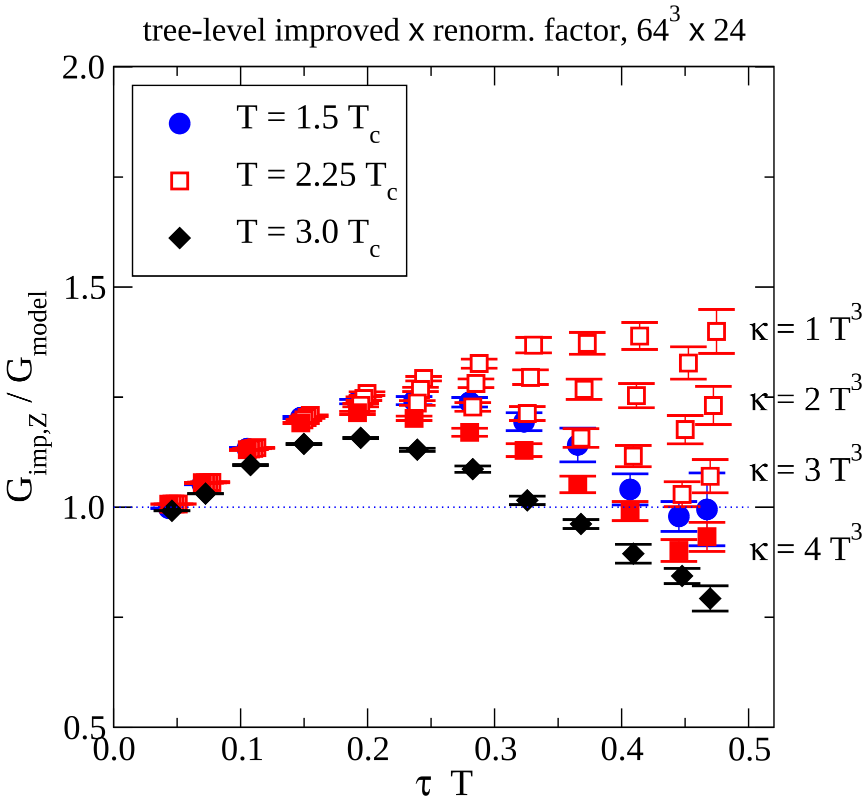


After perturbative renormalization [Eichten Hill PLB 240 (1990) 193] , with $c_2^2 \approx (1 - \frac{0.59777}{\beta})^2$, can compare with NLO [Burnier et al 1006.0867]:



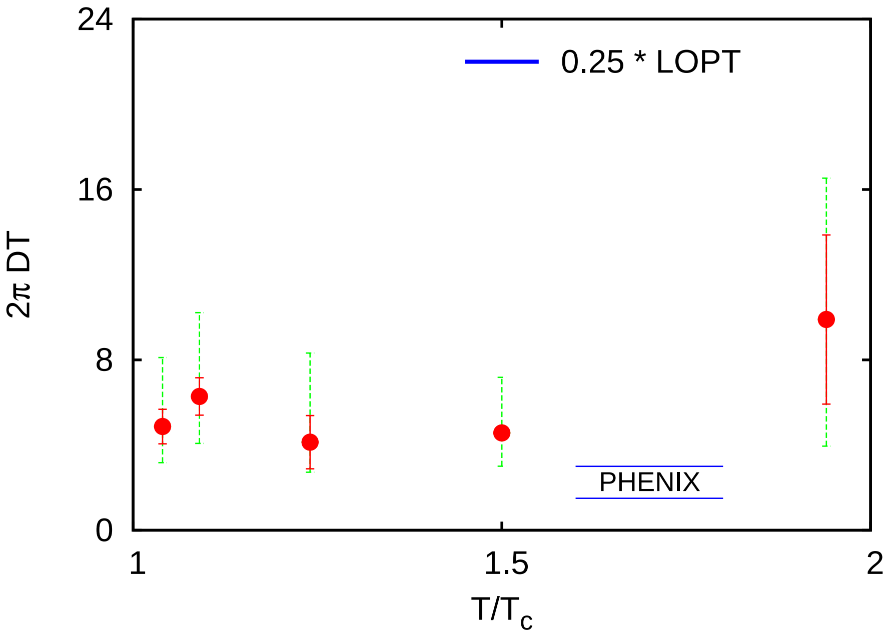
A clear enhancement at large time separations!

A rough model: enhance IR as $\rho_{\text{model}}(\omega) \equiv \max\{\frac{\omega\kappa}{2T}, \rho_{\text{NLO}}(\omega)\}$.



$\kappa \overset{?}{>} \kappa^{\text{max}}_{\text{NLO}} \sim 2T^3$;
$D \sim (0.5 \ldots 0.8)/T$;
$2\pi DT \sim 3 \ldots 5$.

Compares reasonably with the other paper [Banerjee et al 1109.5738]:



However **renormalization of both results** and particularly their **analytic continuation** require further work.

Conclusions

Single heavy quarks experience modifications at finite temperature which are conceptually somewhat simpler than with quarkonium, although still of non-perturbative nature.

There is hope for a comparison with experiment in the future.

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