## The Discrete Contribution to $\psi(2 S)$ Decay into J/ $\psi+2 \gamma$

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## Outline

- Background and Motivation
- The Theoretical Framework
- Discrete Contribution to Two Photon Transition
- Comparison with the MC simulation
- Conclusions and Summary


## Motivation and Background

## - Why the two photon transition Process?

## - On theoretical side:

- Two-photon transition among hydrogen system is helpful to study the hydrogen recombination in universe. (Kholupenko, Ivanchik, 2006)
- Similar decay $D^{*} \rightarrow D+2 \gamma$ to extract the couplings $g_{D^{*} D \gamma}$ and $g_{D^{*} D \pi}$. (D.Guetta and P.Singer, 2000)
- Radiative transition may help to test the meson-loop effect in heavy quarkonium states, however its uncertainty is large in one-photon transition. (T.Barnes, 2010 )


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## - On experimental side:

- An $\alpha^{2}$ order subtle QED transition process.
- The two-photon transition has been observed among positronium 1S and 2S states in 80s. (S.Chu, A.P.Mills, 1982)
- CLEO reported $\operatorname{Br}(\Upsilon(3 S) \rightarrow \Upsilon(2 S)+2 \gamma)=(5.0 \pm 0.7) \%$ • ( PDG 2010 )
- In charmonium systems, it is observed by BESIII recently.


## - Preliminary Experimental Result



## box cut:

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\begin{aligned}
& 0.15<\boldsymbol{M}_{r_{r}}<0.51 \mathrm{GeV} \\
& 3.43<\boldsymbol{R} \boldsymbol{M}_{r_{2}}<3.49 \mathrm{GeV}
\end{aligned}
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$$

$\leadsto \operatorname{Br}(\psi(2 S) \rightarrow J / \psi+2 \gamma) \sim 1 \times 10^{-3}$
compatible with CLEO data. (CLEO 2008)

## - Theoretical Picture

## - Discrete part contribution:

- Leading order:
double E-1 transition via discrete nP
( $\mathrm{n}=1,2 \ldots$...) states (virtual and real parts). (including main source of the background)
- Relativistic corrections:
relatively higher order $\mathrm{v}^{2}$ operator corrections



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- Hadron-loop contribution: besides discrete contribution, the DD ${ }^{(*)}$ meson loop effect can also contribute.


Our idea

> Pin down the discrete part contribution, there may left signal of meson loop effect!!

## The Theoretical Frame work

## - Heavy Quarkonium Multiplet I

* For those states have the same $\mathbf{n}$ and L , they can expressed by a single multiplet $J^{\mu_{1} \ldots \mu_{L}}$ :

$$
\begin{aligned}
J^{\mu_{1} \ldots \mu_{L}}= & \frac{1+\psi}{2}\left(H_{L+1}^{\mu_{1} \ldots \mu_{L} \alpha} \gamma_{\alpha}+\frac{1}{\sqrt{L(L+1)}} \sum_{i=1}^{L} \epsilon^{\mu_{i} \alpha \beta \gamma} v_{\alpha} \gamma_{\beta} H_{L \gamma}^{\mu_{1} \ldots \mu_{i-1} \mu_{i+1} \ldots \mu_{L}}\right. \\
& +\frac{1}{L} \sqrt{\frac{2 L-1}{2 L+1}} \sum_{i=1}^{L}\left(\gamma^{\mu_{i}}-v^{\mu_{i}}\right) H_{L-1}^{\mu_{1} \ldots \mu_{i-1} \mu_{i+1} \ldots \mu_{L}}-\frac{2}{L \sqrt{(2 L-1)(2 L+1)}} \\
& \left.\times \sum_{i<j}\left(g^{\mu_{i} \mu_{j}}-v^{\mu_{i}} v^{\mu_{j}}\right) \gamma_{\alpha} H_{L-1}^{\alpha \mu_{1} \ldots \mu_{i-1} \mu_{i+1} \ldots \mu_{j-1} \mu_{j+1} \ldots \mu_{L}}+K_{L}^{\mu_{1} \ldots \mu_{L}} \gamma^{5}\right) \frac{1-\vartheta}{2}
\end{aligned}
$$

$v^{\mu}$ is the four-velocity of the multiplet state and

$$
v_{\mu_{i}} K_{L}^{{ }^{\mu_{1} \ldots \mu_{i} \ldots \mu_{L}}=0, v_{\mu_{i}} H_{L}{ }^{\mu_{1} \ldots \mu_{i} \ldots \mu_{L}}=0 ~}
$$

## - Heavy Quarkonium Multiplet II

- Explicit Expression for $\mathrm{L}=\mathrm{S}, \mathrm{P}$ case:
- For L=S: $\quad J=\frac{1+\not \subset}{2}\left(H_{1}^{\mu} \gamma_{\mu}-K_{0} \gamma^{5}\right) \frac{1-\not \subset}{2}$
- For L=P:

$$
J^{\mu}=\frac{1+\not \not)}{2}\left(H_{2}^{\mu \alpha} \gamma_{\alpha}+\frac{1}{\sqrt{2}} \varepsilon^{\mu \alpha \beta \beta \gamma} v_{\alpha} \gamma_{\beta} H_{1 \gamma}+\frac{1}{\sqrt{3}}\left(\gamma^{\mu}-v^{\mu}\right)+K_{0} \gamma^{5}\right) \frac{1-\not x}{2}
$$

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$$

- Effective Lagrangian for radiative transition among Sand $P$-wave states:

$$
L=\sum_{m, n} \delta^{n P, m S} \operatorname{lr}\left[\bar{J}(m S) J_{\mu}(n P)\right] v{ }_{v} F^{\mu v}+\text { H.c. }
$$

Preserve CPT, gauge invariance, and heavy quark spin symmetry
coupling constant $\quad$ Electromagnetic tensor

## - E1 Transition Formula

- Decay widths for the one-photon emission:

$$
\begin{aligned}
& \text { mS--->nP+ }: \\
& \Gamma\left(m^{3} S_{1} \rightarrow n^{3} P_{J}+\gamma\right)=(2 J+1) \frac{\left(\delta_{J}^{n P, m S}\right)^{2}}{144} k_{\gamma}^{3} \frac{\left(M_{m S}+M_{n P}\right)^{4}}{M_{m S}^{3} M_{n P}} \\
& \mathrm{nP}^{--->\mathrm{mS}+\gamma:} \\
& \Gamma\left(n^{3} P_{J} \rightarrow m^{3} S_{1}+\gamma\right)=\frac{\left(\delta_{J}^{n P, m S}\right)^{2}}{48} k_{\gamma}^{3} \frac{\left(M_{m S}+M_{n P}\right)^{4}}{M_{m S} M_{n P}^{3}} \quad \text { J-dependent }
\end{aligned}
$$

## - E1 Transition Formula

## Decay widths for the one-photon emission:

- mS--->nP+ $\gamma$ :
- nP--->mS + :
$\Gamma\left(n^{3} P_{J} \rightarrow m^{3} S_{1}+\gamma\right)=\frac{\left(\delta_{J}^{r P, m S}\right)^{2}}{48} k_{\gamma}^{3} \frac{\left(M_{m S}+M_{n P}\right)^{4}}{M_{m S} M_{n P}^{3}}$
- Numerical result of coupling constant:

TABLE I. The numerical values of the coupling constants $\delta_{J}^{n P, m S}\left(\mathrm{GeV}^{1}\right)$ are shown. For the $n=1$ case, the results are obtained by fitting the experimental data, and for $n=2$, the results are determined by comparing with the potential model predictions [26].

|  | $\chi_{c 0}(1 P)$ | $\chi_{c 1}(1 P)$ | $\chi_{c 2}(1 P)$ | $\chi_{c 0}(2 P)$ | $\chi_{c 1}(2 P)$ | $\chi_{c 2}(2 P)$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $J / \psi$ | 0.211 | 0.230 | 0.228 | $5.27 \times 10^{2}$ | $5.30 \times 10^{2}$ | $5.34 \times 10^{2}$ |
| $\psi(2 S)$ | 0.224 | 0.235 | 0.273 | 0.410 | 0.413 | 0.416 |

## Discrete Contribution to Two Photon Transition

## - Feynman Diagrams and Amplitude

## - Two Feynman diagrams



FIG. 1. The Feynman diagrams for $\psi(2 S)$ decay into $J / \psi+2 \gamma$ via intermediate states $\chi_{c J}(n P)$.

- Feynman amplitude:

$$
M^{\mathrm{Tot}}=M^{x_{0}(1 P)}+M^{\chi_{c_{1}}(1 P)}+M^{\chi_{2}(1 P)}+M^{\chi_{c}(2 P)}
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- Decay width is divided into four part:

$$
\Gamma_{\mathrm{dis}}(\psi(2 S) \rightarrow J / \psi+\psi)=\Gamma_{\mathrm{ind}}^{P}+\Gamma_{\mathrm{int}}^{1 P}+\Gamma^{2 P} \pm \Gamma_{\mathrm{int}}^{1,2 P}
$$

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$\Gamma_{\text {dis }}(\psi(2 S) \rightarrow J / \psi+\gamma)=\Gamma_{\text {ind }}^{I P}+\Gamma_{\text {init }}^{P P}+\Gamma^{\Gamma P}+\Gamma_{\text {int }}^{1,2 P}$

Account for the relative phase

Sum of three individual 1P
1P interferences

Only 2P
1P,2P Interference

## - Numerical Result I

- Whole phase space region result:

$$
\begin{gathered}
\Gamma_{\text {ind }}^{1 P}=15.14 \mathrm{keV} \simeq \sum_{J} \Gamma\left(\psi(2 S) \rightarrow \chi_{c J}+\gamma\right) \times \operatorname{Br}\left(\chi_{c J} \rightarrow J / \psi+\gamma\right) \\
\Gamma_{\text {int }}^{P}=5.95 \times 10^{-2} \mathrm{keV}, \Gamma_{\text {int }}^{1,2 P}=4.13 \times 10^{-2} \mathrm{keV}, \Gamma^{2 P}=2.80 \times 10^{-3} \mathrm{keV} \\
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- Cut region:

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\begin{gathered}
0.15 \mathrm{GeV}<M_{\gamma r}<0.51 \mathrm{GeV}, 3.43 \mathrm{GeV}<M_{J / \psi V}<3.49 \mathrm{GeV}, \\
\Gamma_{\text {ind }}^{1 P}=4.68 \times 10^{-2} \mathrm{keV}, \Gamma_{\text {int }}^{1 P}=6.5 \times 10^{-3} \mathrm{keV} \\
\Gamma^{2 P}=1.82 \times 10^{-4} \mathrm{keV}, \Gamma_{\text {int }}^{1,2}=4.78 \times 10^{-3} \mathrm{keV} \\
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In cut region the interference effect can reach 20\%!!

## - Numerical Result II

- Branching ratio in cut region:

$$
B r_{\text {dis }}^{\text {cut }}(\psi(2 S) \rightarrow J / \psi+2 \gamma)=\left\{\begin{array}{l}
1.92 \times 10^{-4} \text { for } \theta=0, \\
1.60 \times 10^{-4} \text { for } \theta=\pi .
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$$

The polarization of $J / \psi$ in final state:

$$
\alpha=\frac{\Gamma_{T}-2 \Gamma_{L}}{\Gamma_{T}+2 \Gamma_{L}}, \alpha=\left\{\begin{array}{c}
-1, \text { Longitudinal } \\
0, \text { Unpolarized } \\
1, \text { Transverse }
\end{array}\right\}
$$

- If we use the same $v^{\mu}, \alpha=0$.
- In the whole region: $\alpha=-0.16$.
- In the cut region, $\alpha=-0.122$ and $\alpha=-0.107$ for $\theta=0$ and $\theta=\pi$, respectively. If only include three individual 1P contribution, $\alpha=-0.078$.

J/ $\Psi$ tends to be in longitudinal polarization state!!

## - Photon Spectrum I

## - 1P contribution in whole region:



FlG. 2. The partial decay width as a function of the photon energy fraction $x_{2}$ : (a) the individual contribution of the three $\chi_{c J}(J=0,1,2)$ states, corresponding to $\Gamma_{\operatorname{Ind}}^{1 P}$ in (15), (b) the contribution of the interference terms between the three $\chi_{c J}(J=0,1,2)$ states, corresponding to $\Gamma_{\text {Int }}^{1 P}$ in (15).

## - Photon Spectrum II

## - 2P effect in whole region:



FIG. 3. The partial decay width as a function of the photon energy fraction $x_{2}$ : (a) the contribution of the $2 P$ states, corresponding to $\Gamma^{2 P}$ in (15), (b) the contribution of the interference terms between the $2 P$ and the three $1 P$ states, corresponding to $\Gamma_{\mathrm{Int}}^{1,2 P}$ in (15).

## - Photon Spectrum III

## - Summary in cut region:



FIG. 4. The discrete contributions to the photon energy spectrum of the $\psi(2 S) \rightarrow J / \psi+\gamma \gamma$ process in the cut region: (a) the contribution of the $1 P$ states, corresponding to $\Gamma_{\text {Ind }}^{1 P}$ and $\Gamma_{\text {Int }}^{P P}$ in (15), (b) the contribution of the $2 P$ states and of the interference terms between $1 P$ and $2 P$ states, corresponding to $\Gamma^{2 P}$ and to $\Gamma_{\mathrm{Int}}^{1,2 P}$ in (15), (c) the total contribution for a different relative phase angle $\theta$, corresponding to the $\pm$ sign in (15).

## Comparison with MC simulation

## - Description about the MC

- In MC simulation, normally, only the individual part $\Gamma_{\text {ind }}^{1 P}$ is taken into account.
- The non-relativistic Breit-Wigner is used to describe the line shape of $\chi_{\mathrm{cJ}}(1 \mathrm{P})$ state.
- Double E1 transition factor $k_{y 1}^{3} k_{y 2}^{3}$ should be included.
- The decay widths and masses are from PDG 2010.


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Double E1 transition factor $k_{\gamma 1}^{3} k_{\gamma 2}^{3}$ should be included.
The decay widths and masses are from PDG 2010.
- Two questions left:

0

- Effect of the interference and higher excited states.
- Is the non-relativistic Breit-Wigner a good approximation in cut region?


## - Comparison in Cut Region

## - Photon spectrum in cut region:



FIG. 5. The MC simulation of the cascade decay of $\psi(2 S) \rightarrow$ $\left(J \psi \gamma_{1}\right)_{\chi_{c J}} \gamma_{2}$ in the cut region, where the branching fractions are from PDG2010 [45]. The dotted line denotes the naive nonrelativistic Breit-Wigner simulation, the solid line is the simulation including the $k_{\gamma_{1}}^{3} k_{\gamma_{2}}^{3}$ factor, and the dashed line is the contribution of the three individual $\chi_{c J}(1 P)$ states, corresponding to $\Gamma_{\text {Ind }}^{1 P}$ in (15), calculated in this paper.

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In cut region, the non-relativistic together with energy factor is not enough to study the individual contribution.

## Conclusions and Summary

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- In the two photon transition process, the effect of interference and higher excited states (2P) are very tiny.
- In the experimental cut region, the contribution of the interference and higher excited states is sizeable.
- The $\mathrm{J} / \psi$ tends to be in longitudinal polarization state.
- The experimental MC simulation still gets potential to be improved in the cut region.
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## Thank You!

