Charmonium Spectroscopy from Lattice QCD

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Outline

- Motivation
- Techniques
- Results: charmonium spectroscopy
- Comment on some systematics: HFS, disconnected diagrams, finite volume, light quark mass.
- Summary

Introduction: the charmonium renaissance

New experimental results have motivated a reexamination of charm spectroscopy.

Anti-Proton
ANnihilation at
DArmstadt

At full luminosity PANDA will collect several thousand cc states per day. By means of fine scans it will be possible to measure masses with an accuracy of the order of 100 keV and widths to 10% or better. PANDA will explore the entire energy region below and above the open charm threshold, to find the missing D- and F- wave states and unravel the nature of the newly discovered X, Y, Z states.

http://www-panda.gsi.de/

The Hadron Spectrum Collaboration lattices

The TCD branch

Liuming Liu, Graham Moir, Mike Peardon, S.R, Pol Vilaseca

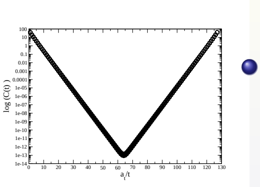
Lattices for charm physics

- $2+1$ dynamical flavours
- Anisotropy: $\xi = a_s/a_t = 3.5$
- Distillation [arXiv.0905.2160] with $N_{\mathrm{ev}} = 64$ and inversions on all timeslices
- Tree-level Synamizik-improved gauge action
- Sheikholeslami-Wohlert fermion action with spatial stout-link smearing [PRD69:054501,2004]

Volume	m_π	Configs	a_s	$a_t^{-1}(m_\Omega)$
$16^3 \times 128$	396MeV	96	0.12 fm	5.667 GeV

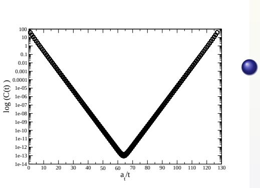
Spectroscopy

Spectroscopy - making measurements



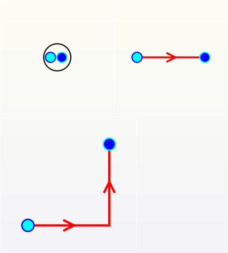
- Energy of a (colorless) QCD state extracted from a two-point function in Euclidean time, $C(t) = \langle \phi(t) | \phi^{\dagger}(0) \rangle$.
- Inserting a complete set of states, $\lim_{t \to \infty} C(t) = Ze^{-E_0 t}$.
- Observing the exponential fall of $C(t)$ at large t, the energy can be measured.

Spectroscopy - making measurements



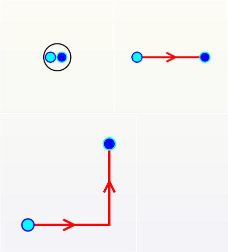
- Energy of a (colorless) QCD state extracted from a two-point function in Euclidean time, $C(t) = \langle \phi(t) | \phi^\dagger(0) \rangle$.
- Inserting a complete set of states, $\lim_{t\to\infty} C(t) = Z e^{-E_0 t}$.
- Observing the exponential fall of $C(t)$ at large t, the energy can be measured.
- Excited state energies from a matrix of correlators: $C_{ij}(t) = \langle \phi_i(t) | \phi_j^\dagger(0) \rangle$.
- Solving a generalised eigenvalue problem $C(t_1)\mathbf{v} = \lambda C(t_0)\mathbf{v}$ gives
$$\lim_{(t_1-t_0)\to\infty} \lambda_n = e^{-E_n(t_1-t_0)} \; .$$

Spectroscopy - making measurements



- Lattice operators: bilinears with path-ordered products between q and $\bar{q}$ fields; different offsets, connecting paths and spin contractions give different projections into lattice symmetry channels.
- Want ops with overlap onto low-lying spectrum
- Smooth fields spatially before measuring: smearing

Spectroscopy - making measurements



- Lattice operators: bilinears with path-ordered products between q and $\bar{q}$ fields; different offsets, connecting paths and spin contractions give different projections into lattice symmetry channels.
- Want ops with overlap onto low-lying spectrum
- Smooth fields spatially before measuring: smearing
    - Distillation [Hadron Spectrum Collab.]
    - Reduce the size of space of fields (on a time-slice) preserving important features.
    - all elements of the (reduced) quark propagator can be computed: allows for many operators, disconnected diagrams and multi-hadron operators.
    - combined with stochastic methods to improve volume scaling.

The “naive” spectroscopy of single particles (no multi-hadrons):
As well as control of usual lattice systematics ($a \to 0$,$L \to \infty$, m_π realistic) need

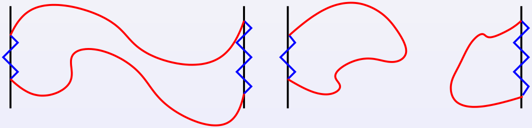
- statistical precision at % percent level
- reliable spin identification
- heavy quark methods

The “naive” spectroscopy of single particles (no multi-hadrons):
As well as control of usual lattice systematics ($a \to 0$,$L \to \infty$, $m_q \sim m_\pi$) need

- statistical precision at percent level
  - to include multi-hadrons and study resonances
- reliable spin identification
- heavy quark methods

statistical precision at percent level

- "distillation" - a new approach to simulating correlators. Particularly good for spectroscopy.
- enables precision determination of disconnected diagrams, crucial for isoscalar spectroscopy. [see Dudek et al (Hadron Spectrum Collab.) arXiv.1102.4299].



- large bases of interpolating operators now feasible, for better determination of excited states via variational method.

Spectroscopy

The "naive" spectroscopy of single particles (no multi-hadrons):
As well as control of usual lattice systematics ($a \rightarrow 0$,$L \rightarrow \infty$, m_π realistic) need

- statistical precision at % percent level
- reliable spin identification
  - understanding symmetries and connection between lattice and continuum
  - designing operators with overlap onto J^{PC} of interest.
- heavy quark methods

Reliable spin identification

- Continuum: states classified by irreps (J^P) of $O(3)$. On the lattice: $O(3) \to O_h$.
- O_h has 10 irreps: $\{A_1^{(g,u)}, A_2^{(g,u)}, E^{(g,u)}, T_1^{(g,u)}, T_2^{(g,u)}\}$
- Continuum spin assignment then by subduction

J	0	1	2	3	4	$\ldots$
A_1	1				1	$\ldots$
A_2				1		$\ldots$
E			1		1	$\ldots$
T_1		1		1	1	$\ldots$
T_2			1	1	1	$\ldots$

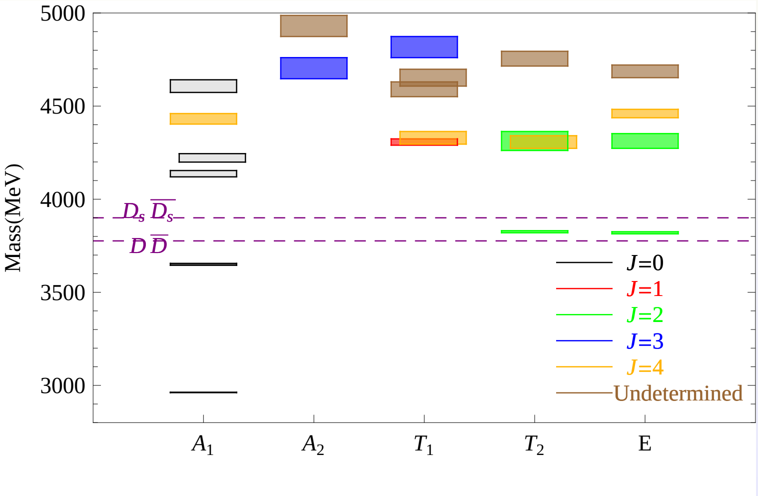
- Design good operators: start from continuum, "latticize" (D_{latt} for D) continuum operators.
- These lattice operators subduced from J should have good overlap with states of continuum spin J. Study overlaps (Z) to identify spin across irreps.

The “naive” spectroscopy of single particles (no multi-hadrons):
As well as control of usual lattice systematics ($a \to 0$,$L \to \infty$, m_π realistic) need

- statistical precision at % percent level
- reliable spin identification
- heavy quark methods
  - relativistic action for charm ($a_t m_c < 1$).

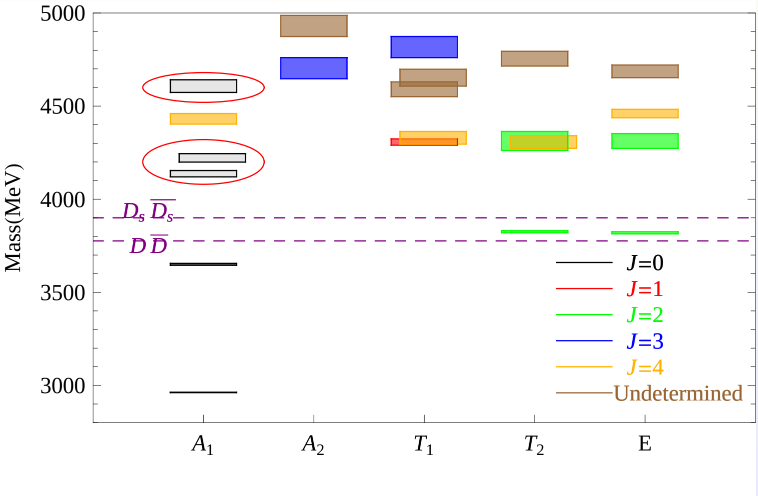
Results

J^{-+}



5000
4500
4000
3500
3000
Mass(MeV)
$D_s \bar{D}_s$
$D \bar{D}$
J=0
J=1
J=2
J=3
J=4
Undetermined
A_1
A_2
T_1
T_2
E

J^{-+}



5000
4500
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Mass(MeV)
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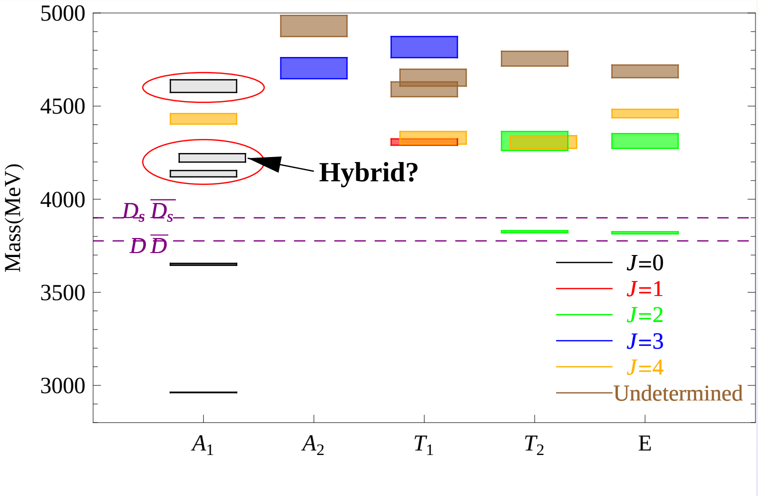
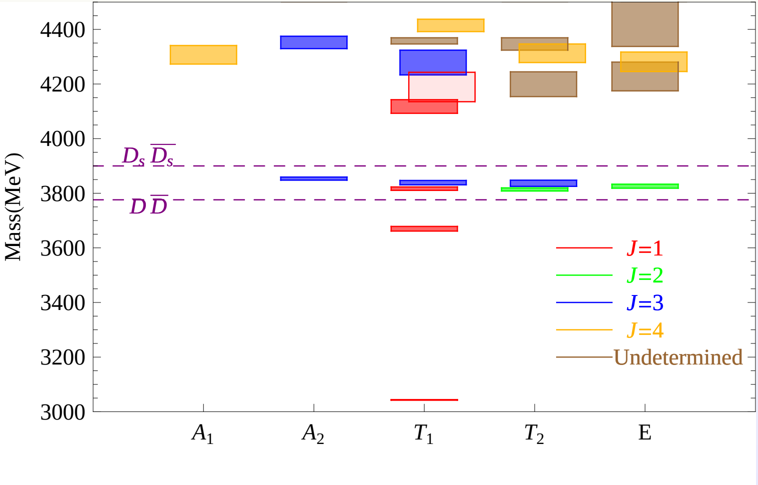


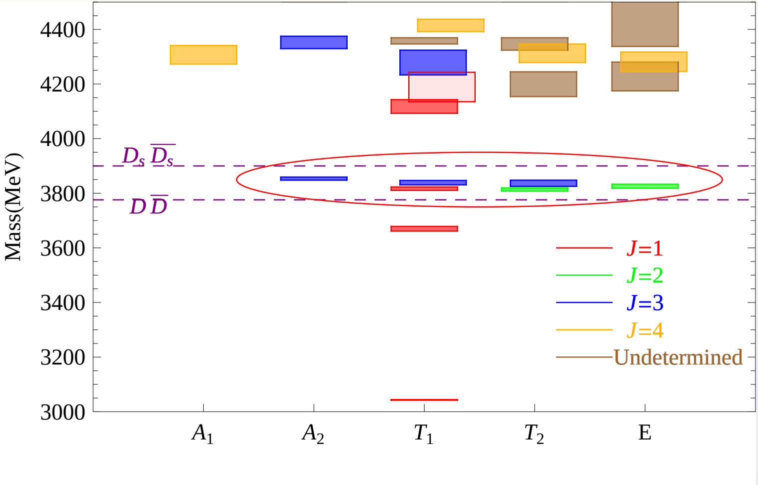
Mass(MeV)
5000
4500
4000
3500
3000
Hybrid?
$D_s\,\overline{D_s}$
$D\,\overline{D}$
J=0
J=1
J=2
J=3
J=4
Undetermined
A_1
A_2
T_1
T_2
E

J^{--}



Mass(MeV)
4400
4200
4000
3800
3600
3400
3200
3000
$D_s\,\overline{D_s}$
$D\,\overline{D}$
J=1
J=2
J=3
J=4
Undetermined
A_1
A_2
T_1
T_2
E

J^{--}



Mass(MeV)
4400
4200
4000
3800
3600
3400
3200
3000
$D_s\,\overline{D}_s$
$D\,\overline{D}$
J=1
J=2
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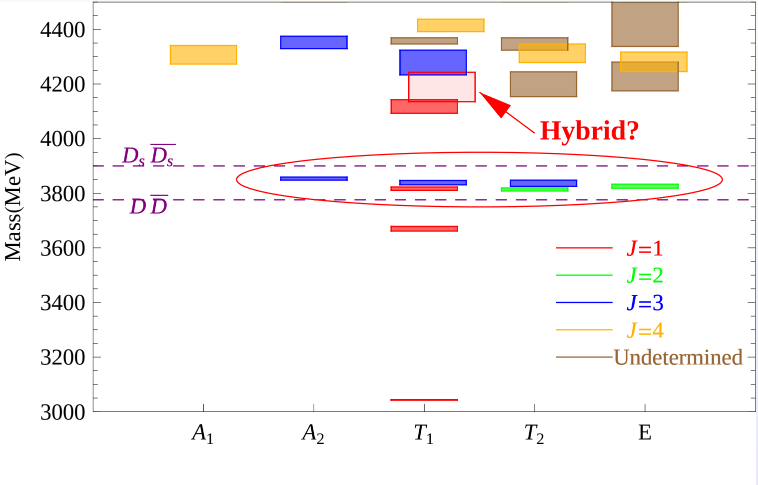
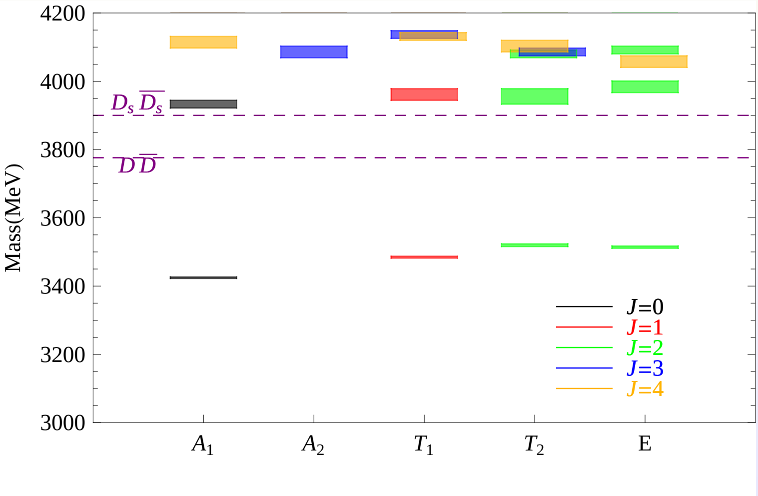
J=4
Undetermined
A_1
A_2
T_1
T_2
E

J^{--}



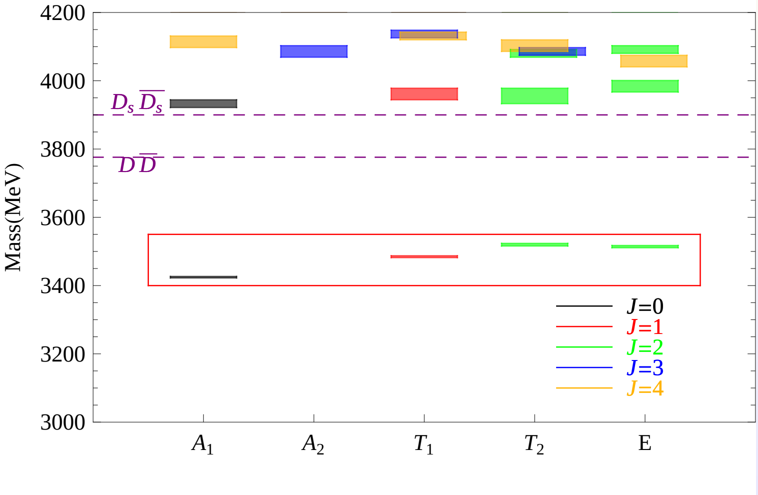
Mass(MeV)
4400
4200
4000
3800
3600
3400
3200
3000
$D_s \overline{D_s}$
$D \overline{D}$
Hybrid?
J=1
J=2
J=3
J=4
Undetermined
A_1
A_2
T_1
T_2
E

J^{++}



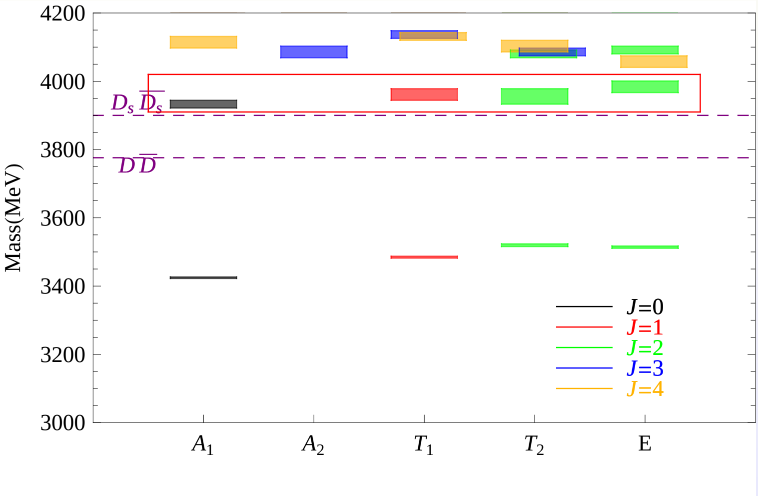
4200
4000
3800
3600
3400
3200
3000
Mass(MeV)
$D_s\,\overline{D_s}$
$D\,\overline{D}$
A_1
A_2
T_1
T_2
E
J=0
J=1
J=2
J=3
J=4

J^{++}



4200
4000
3800
3600
3400
3200
3000
Mass(MeV)
$D_s\overline{D_s}$
$D\overline{D}$
J=0
J=1
J=2
J=3
J=4
A_1
A_2
T_1
T_2
E

J^{++}



4200
4000
3800
3600
3400
3200
3000
Mass(MeV)
$D_s\,\overline{D_s}$
$D\,\overline{D}$
A_1
A_2
T_1
T_2
E
J=0
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J=2
J=3
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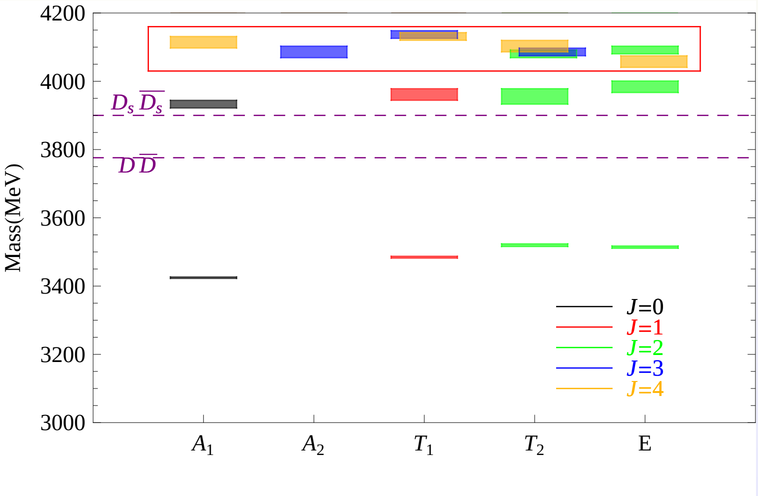
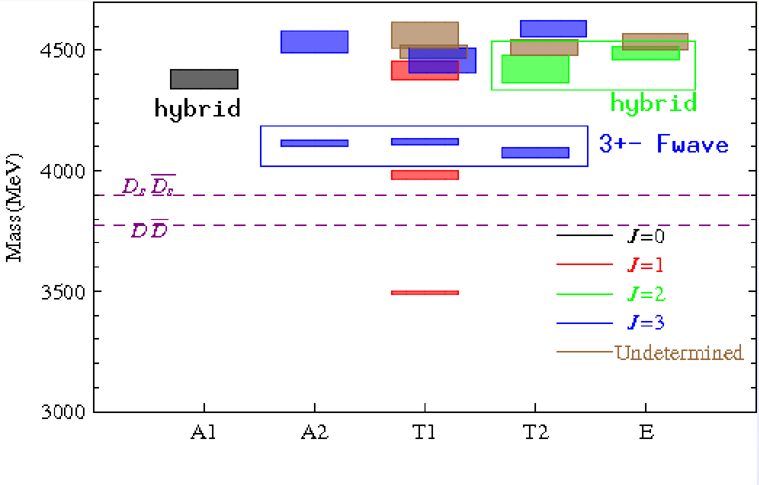
J^{++}



4200
4000
3800
3600
3400
3200
3000
Mass(MeV)
$D_s\,\overline{D_s}$
$D\,\overline{D}$
A_1
A_2
T_1
T_2
E
J=0
J=1
J=2
J=3
J=4

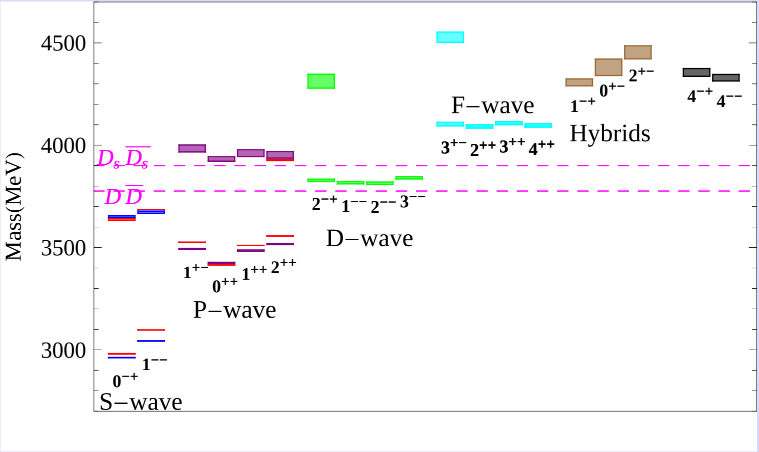
J^{+-}



4500
4000
3500
3000
Mass (MeV)
hybrid
hybrid
3+- Fwave
$D_s\overline{D_s}$
$D\overline{D}$
J=0
J=1
J=2
J=3
Undetermined
A1
A2
T1
T2
E

Summary: The charmonium spectrum

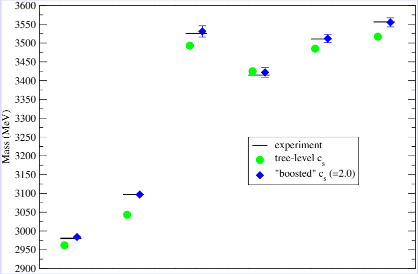
from Liuming Liu



Some (preliminary) extras

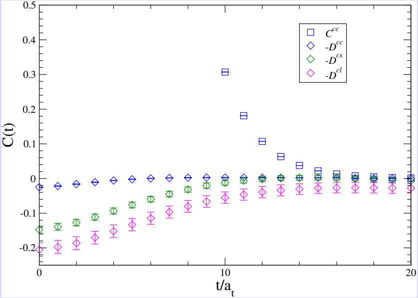
hyperfine splitting

a finite a effect. Boosting c_s fixes the splitting in S and P waves.



disconnected diagrams

flavour mixing analysis for disconnected contributions to η_c almost done.



Conclusions

- Distillation working very well for charmonium
- All S,P,D waves with excited states as well as hybrids (exotic and non-exotic), F and G waves.
- Precision on S and P waves < 1 MeV. On $1^{-+} \sim 15$ MeV.
- Anisotropic lattice very useful for good resolution in temporal direction
- Different volume: starting
- D meson spectroscopy underway
- Multihadron operators and lighter quark masses coming