



Bound state nature of the exotic Z_b states

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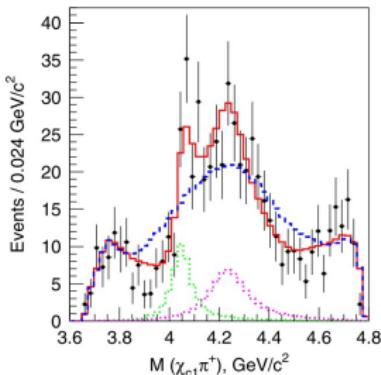
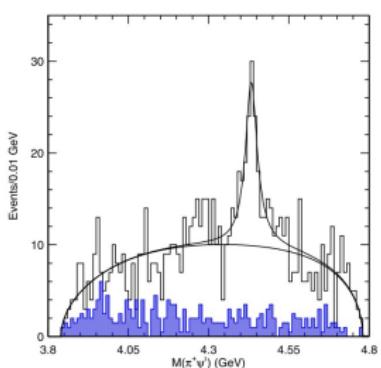
QWG 2011, GSI, 04-07 Oct., 2011

Based on:

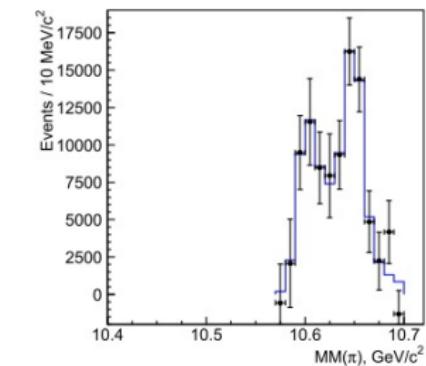
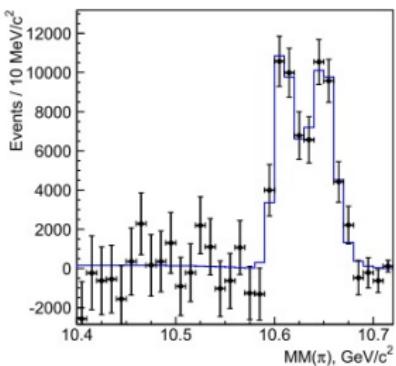
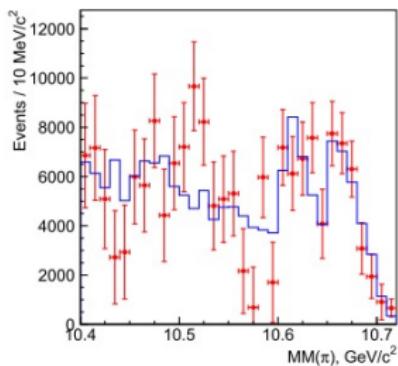
M. Cleven, F.-K.G., C. Hanhart, U.-G. Meißner, arXiv:1107.0254 [hep-ph], to appear in Eur.Phys.J.A

Introduction (I): Smoking guns for exotics once confirmed

Charged charmonia $Z(4430)^{\pm}$, $Z(4050)^{\pm}$ and $Z(4250)^{\pm}$ Belle, PRL100(2008)142001, PRD78(2008)072004



Charged bottomonia $Z_b(10610)^{\pm}$ and $Z_b(10650)^{\pm}$



Introduction (II)

- Analysis favors $I^G(J^P) = 1^+(1^+)$ for both states
- $Z_b(10610)$ and $Z_b(10650)$ are very close to $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds

Measured masses (MeV)	10608.4 ± 2.0	10653.2 ± 1.5
Thresholds (MeV)	$B\bar{B}^*$: 10605	$B^*\bar{B}^*$: 10650

- $\Upsilon(nS)$: $b\bar{b}$ in a spin triplet state; $h_b(nP)$: $b\bar{b}$ in a spin singlet state \Rightarrow transitions $\Upsilon(nS) \rightarrow h_b X$ should be suppressed compared to $\Upsilon(nS) \rightarrow \Upsilon(mS)X$ for breaking spin symmetry

Exp.: $\Upsilon(5S) \rightarrow h_b(1P, 2P)\pi\pi$ have comparative rates as $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi\pi$

$$R_{h_b} = 0.41^{+0.09}_{-0.11}, \quad R_{h_b(2P)} = 0.78^{+0.24}_{-0.13}$$

Belle, arXiv:1103.3419[hep-ex]

Introduction (III): Explanations

- Tetraquark $b\bar{b}q\bar{q}$:

T.Guo et al, arXiv:1105.5935[hep-ph]; Navarra et al., arXiv:1108.1230[hep-ph]; Ali, arXiv:1108.2197[hep-ph]

- Threshold effect:

Bugg, arXiv:1105.5492[hep-ph]; Danilkin et al., arXiv:1106.1552[hep-ph]; Chen, Liu, arXiv:1106.3798[hep-ph]

- Hadronic molecules (due to dynamics of $B^{(*)}\bar{B}^*$ systems):

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Tetraquarks and hadronic molecules

- Both can have $I = 1$ states
- Both can explain the spin symmetry violation in $\Upsilon(5S) \rightarrow h_b^{(\prime)}\pi\pi$
(containing spin triplet and singlet $b\bar{b}$ simultaneously)

$$|Z_b\rangle = \frac{1}{\sqrt{2}} \left(\mathbf{0}_{b\bar{b}}^- \otimes \mathbf{1}_{\bar{q}q}^- + \mathbf{1}_{b\bar{b}}^- \otimes \mathbf{0}_{\bar{q}q}^- \right), \quad |Z'_b\rangle = \frac{1}{\sqrt{2}} \left(\mathbf{0}_{b\bar{b}}^- \otimes \mathbf{1}_{\bar{q}q}^- - \mathbf{1}_{b\bar{b}}^- \otimes \mathbf{0}_{\bar{q}q}^- \right)$$

Bondar, Garmash, Listein, Mizuk, Voloshin, arXiv:1105.4473[hep-ph]

Introduction (IV): Tetraquarks vs hadronic molecules

- ◊ Resonance: pole on the second Riemann sheet, above threshold
- ◊ Virtual state: pole on the second Riemann sheet, below threshold
- ◊ Bound state: pole on the first Riemann sheet, below threshold
- ◊ Attractive one-pion exchange does NOT support a $B\bar{B}^*$ resonance, but a virtual or bound state could exist

Nieves, Pavon Valderrama, arXiv:1106.0600[hep-ph]

Are the data compatible with masses smaller than the corresponding thresholds?

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- Model-independent relation for S-wave loosely bound state (see later)

Weinberg, PR130(1963)776; 131(1963)440; 137(1965)B672; V. Baru et al., PLB586(2004)53;...

$$(z^{\text{eff}})^2 = \frac{8\pi}{\mu^2} \sqrt{-2\mu\mathcal{E}}$$

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Experimental facts:

- ☒ $\Upsilon(5S) \rightarrow \Upsilon(nS)\pi\pi$ (spin symmetry conserving), Z_b as well as other nonresonant contributions
- ☒ $\Upsilon(5S) \rightarrow h_b(1P, 2P)\pi\pi$ (spin symmetry breaking) are dominated by the Z_b states

Check whether the $B^{(*)}\bar{B}^*$ bound-state picture fits $\Upsilon(5S) \rightarrow h_b(1P, 2P)\pi\pi$ data

Lagrangian based on heavy quark spin symmetry

- Bottom meson multiplet $H_a = \frac{1+\gamma}{2}(\vec{B}_a^* - B_a \gamma_5)$ with $a = u, d, s$

In two-component notation, $H_a = \vec{B}_a^* \cdot \vec{\sigma} + B_a$

Hu, Mehen, PRD73(2006)054003

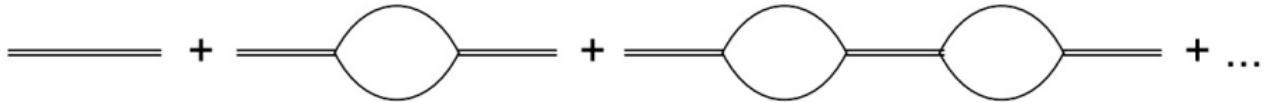
- Isospin triplet of Z_b : $Z_{ba}^i = \begin{pmatrix} \frac{1}{\sqrt{2}}Z^{0i} & Z^{+i} \\ Z^{-i} & -\frac{1}{\sqrt{2}}Z^{0i} \end{pmatrix}_{ba}$

The Z_b states couple to the bottom mesons in an S-wave

$$\mathcal{L}_Z = i \frac{z^{\text{bare}}}{2} \text{Tr} [Z_{ba}^{\dagger i} H_a \sigma^i \bar{H}_b] + \text{H.c.}$$

The bare coupling z^{bare} will be renormalized by the bottom meson loops.

Renormalization (I)



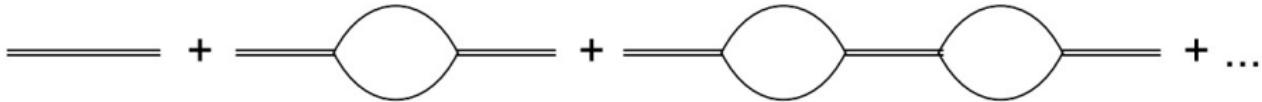
Bare propagator is **renormalized by the bottom meson loops**

$$G_Z(E) = \frac{1}{2} \frac{i}{E - \mathcal{E}_0 - \Sigma(E)}$$

Self-energy:

$$\begin{aligned}\Sigma(E) &= i \frac{(z^{\text{bare}})^2}{4} \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^0 - \vec{l}^2/(2m_1) + i\epsilon} \frac{1}{E - l^0 - \vec{l}^2/(2m_2) + i\epsilon} \\ &= (z^{\text{bare}})^2 \frac{\mu}{8\pi} \left[\underbrace{\sqrt{-2\mu E} \theta(-E)}_{\text{below th.}} - i \underbrace{\sqrt{2\mu E} \theta(E)}_{\text{above th.}} \right] \text{ (in } \overline{\text{MS}} \text{ scheme)}\end{aligned}$$

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Renormalization condition:

$$\mathcal{E} \equiv M_Z - m_1 - M_2 = \mathcal{E}_0 + \text{Re}\Sigma(\mathcal{E})$$

Renormalization (II)

Expanding the real part of $\Sigma(E)$ around $E = \mathcal{E} \implies$

$$G_Z(E) = \frac{1}{2} \frac{iZ}{E - \mathcal{E} - Z\tilde{\Sigma}(E)}$$

where $\tilde{\Sigma}(E) = \Sigma(E) - \text{Re}(\Sigma(\mathcal{E})) - (E - \mathcal{E})\text{Re}(\Sigma'(\mathcal{E}))$

Wave function renormalization constant

$$Z = \frac{1}{1 - \text{Re}(\Sigma'(\mathcal{E}))} = \underbrace{\left[1 + \frac{\mu^2(z^{\text{bare}})^2}{8\pi\sqrt{-2\mu\mathcal{E}}} \right]^{-1}}_{\text{below threshold}} \theta(-\mathcal{E}) + 1 \times \theta(\mathcal{E})$$

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For a pure bound state, $z^{\text{bare}} = \infty$ or $Z = 0$ [see e.g. Weinberg, PR130(1963)776]

Physical coupling constant $(z^{\text{eff}})^2 = \lim_{|z^{\text{bare}}| \rightarrow \infty} Z(z^{\text{bare}})^2 = \frac{8\pi}{\mu^2} \sqrt{-2\mu\mathcal{E}}$

Taking into account decays into Υ, h_b

$$G_Z(E) = \frac{1}{2} \frac{iZ}{E - \mathcal{E} - Z\tilde{\Sigma}(E) + i\Gamma^{\text{phys}}(E)/2}$$

For hadronic molecular Z_b , Γ^{phys} is dominated by $Z_b \rightarrow h_b^{(\prime)}\pi$ (see below)

Power counting of one-loop diagrams

For details of power counting, see F.-K.G., Hanhart, Li, Meißner, Zhao, PRD83(2011)034013

Ingredients of the nonrelativistic power counting:

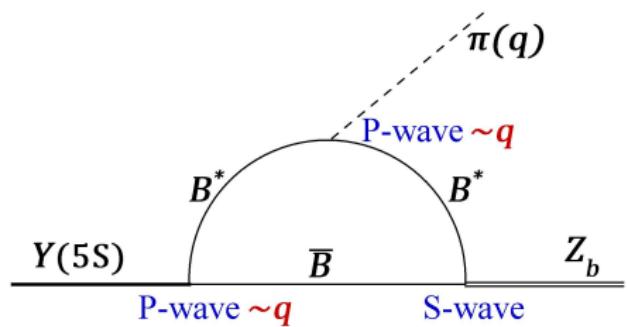
- Nonrelativistic: $M_{\Gamma(5S)(Z_b, h_b)} - 2M_B \ll M_B$
- Energy $\sim v_B^2$, loop momentum $\sim v_B$, external momentum: q
- Loop measure $\sim v_B^5$, propagator $\sim v_B^{-2}$

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$$\frac{v_B^5}{(v_B^2)^3} \frac{q^2}{M_B^2} = \frac{q^2}{v_B M_B^2} \ll 1$$

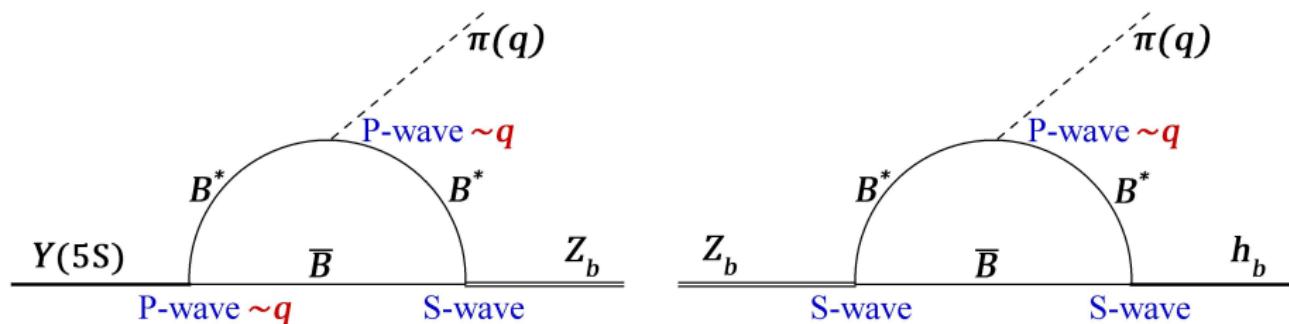
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$$\frac{v_B^5}{(v_B^2)^3} q = \frac{q}{v_B}$$

Can be confirmed by explicit calculation.

Decay mechanism

- Bottom meson loops are highly suppressed in $\Upsilon(5S) \rightarrow Z_b\pi \implies$
 $\Upsilon(5S) \rightarrow Z_b\pi$ could be dominated by a subleading component of Z_b , such as a tetraquark, which does not require a meson loop
- $Z_b \rightarrow h_b\pi$ is given by the bottom meson loops

$$\Gamma(Z_b \rightarrow h_b\pi) = 140 \left(\frac{g_1 z_1}{\text{GeV}^{-1}} \right)^2 \text{ MeV}, \quad \Gamma(Z'_b \rightarrow h_b\pi) = 169 \left(\frac{g_1 z_2}{\text{GeV}^{-1}} \right)^2 \text{ MeV}$$

Coupling constants: g_1 for $h_b B^{(*)} \bar{B}^*$, $z_{1[2]}$ for $Z_b^{[i]} B^{(*)} \bar{B}^*$

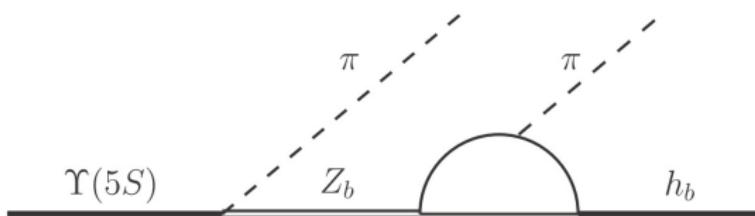
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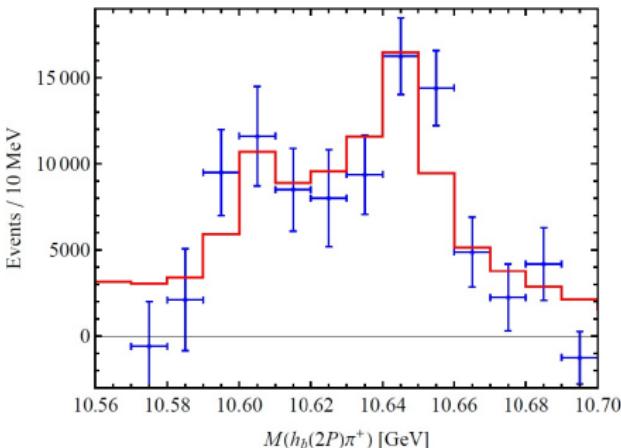
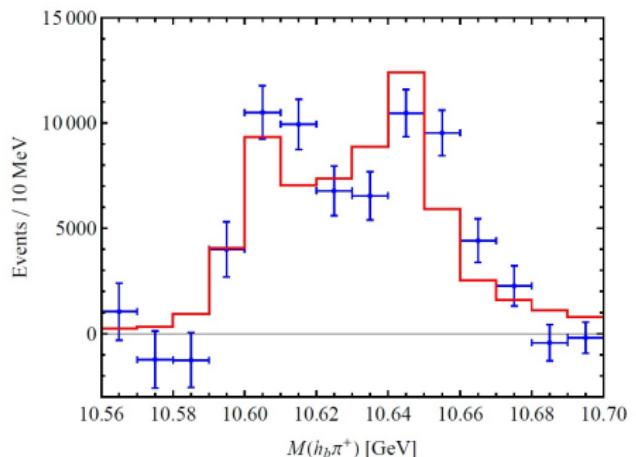
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Decay mechanism of $\Upsilon(5S) \rightarrow Z_b \pi \rightarrow h_b \pi \pi$



$$\mathcal{L}_{\Upsilon Z_b \pi} = c \Upsilon^i Z_{ba}^{\dagger i} \partial^0 \pi_{ab} + \text{H.c.}$$

Fit to the Belle data

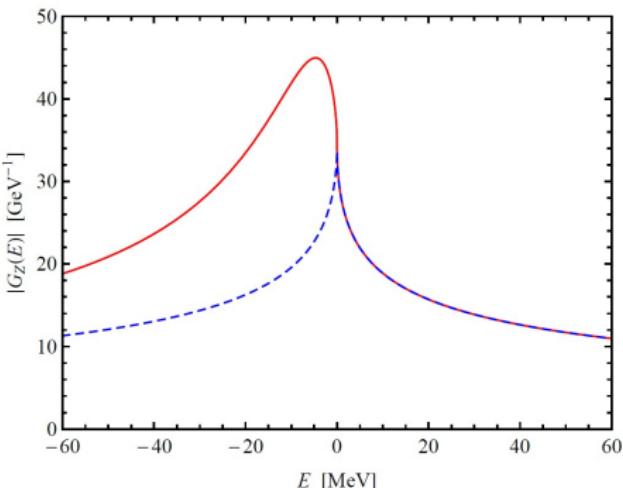
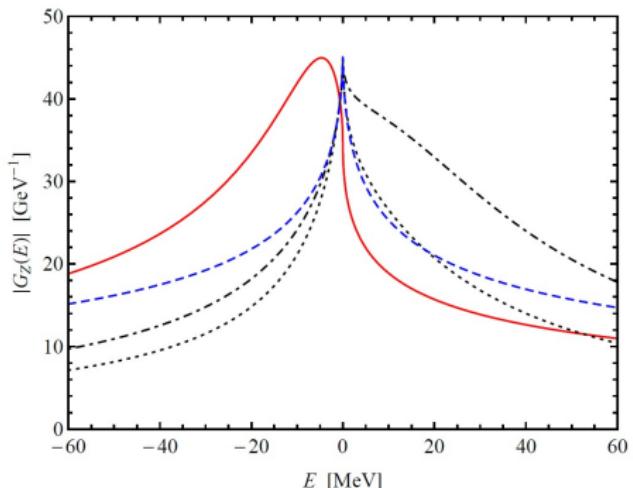


The best fit with $\chi^2/\text{d.o.f.} = 54.1/22 = 2.45$:

$$z_1 = 0.75_{-0.11}^{+0.08} \text{ GeV}^{-1/2}, \quad g_1 z_1 = 0.40 \pm 0.06 \text{ GeV}^{-1}, \quad r_z = \frac{z_2}{z_1} = -0.39_{-0.07}^{+0.06}$$

Small binding energies: $\mathcal{E}_{Z_b} = -4.7_{-2.3}^{+2.2} \text{ MeV}$, $\mathcal{E}_{Z'_b} = -0.11_{-0.14}^{+0.06} \text{ MeV}$

Breit-Wigner is NOT proper here



Solid (red): Bound state with $\mathcal{E} = -4.7 \text{ MeV}$

Dashed (blue): Virtual state with $\mathcal{E} = -4.7 \text{ MeV}$

Dotted and Dot-dashed (black): Resonances with $\mathcal{E} = 8 \text{ and } 20 \text{ MeV}$

Summary

- Explored the consequences of the **bound-state assumption** for the Z_b states
- The assumption is compatible with the data in the $h_b^{(\prime)}\pi^+\pi^-$ channels. The $Z_b^{(\prime)}$ states could be slightly below the $B^{(*)}\bar{B}^*$ thresholds
- Breit-Wigner parametrization is not proper for a near-threshold state coupled strongly to the corresponding hadronic channel
Analogue: $f_0(980)$ near the $K\bar{K}$ threshold, better to use **Flatte parameterization**.