Classicalization and unitarization of wee partons in QCD and gravity: The CGC-Black Hole correspondence

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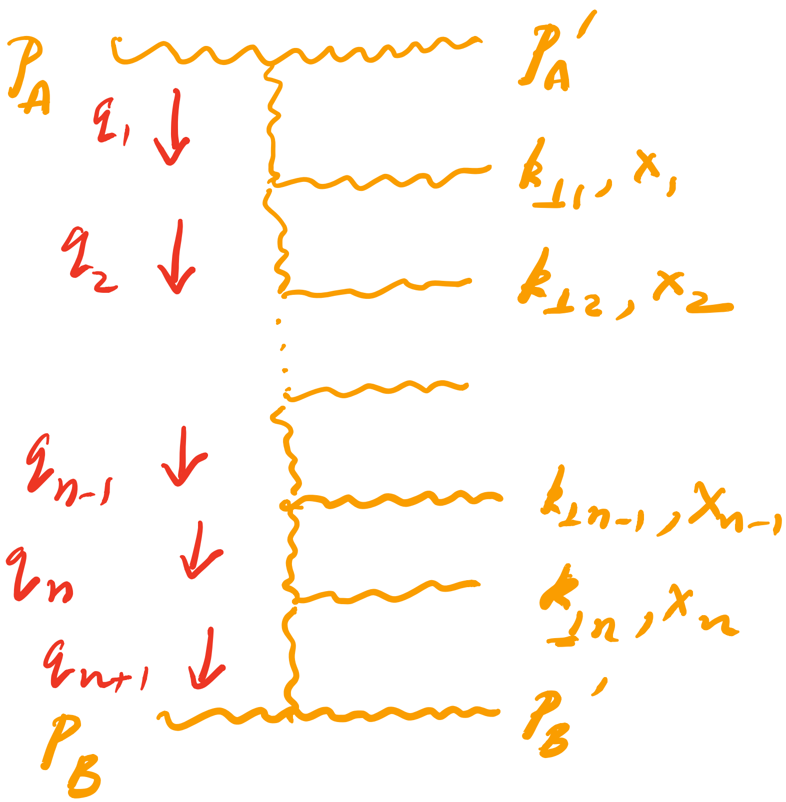
EMMI RRTF Seminar, March 4, 2021

Talk Outline

- $2 \rightarrow \mathrm{N}$ amplitudes in the Regge limit of QCD: the BFKL equation
- Classicalization and perturbative unitarization in QCD: the Color Glass Condensate
- $2 \rightarrow \mathrm{N}$ amplitudes in the Regge limit of Gravity: the Lipatov-ACV approach and the double copy
- Classicalization and unitarization in Gravity: The Black Hole N Portrait
- The Bekenstein-Hawking bound and unitarization in the BHNP
- The CGC/BHNP correspondence
- Coda: Gravitation memory and color memory

$2 \to N + 2$ amplitude in the Regge limit of QCD: the BFKL equation

How does one compute multiparticle production in pQCD at high energies ?



Kuraev,Lipatov,Fadin (1977)
Balitsky, Lipatov (1978)

$$s = (P_A + P_B)^2 = 2 P_A \cdot P_B$$

$$s_i = (k_i + k_{i+1})^2 = 2 k_i \cdot k_{i+1}$$

$$k_i = (q_i - q_{i+1}) \; ; \; k_0 = P_A' \, , \; k_{n+1} \equiv P_B'$$

Multi - Regge limit:

$$s \gg s_1 \sim s_2 \sim \dots \sim s_{n+1} \gg q_1^2 \sim q_2^2 \sim \dots q_{n+1}^2$$

$$q_i^2 = - q_{i\perp}^2 \; ; \; k_i = \alpha_i P_B + \beta_i P_A + k_{\perp i}$$

$$s \prod_{i=1}^{n} (k_{\perp i})^2 = s_1 s_2 \dots s_{n+1} \; ; \; k_{\perp i} \cdot P_A = k_{\perp i} \cdot P_B = 0$$

Alternately express in terms of rapidities

$$q_i \simeq (k_{\perp i} e^{y_i}, - k_{\perp i-1} e^{-y_{i-1}}, - k_{\perp i-1})$$

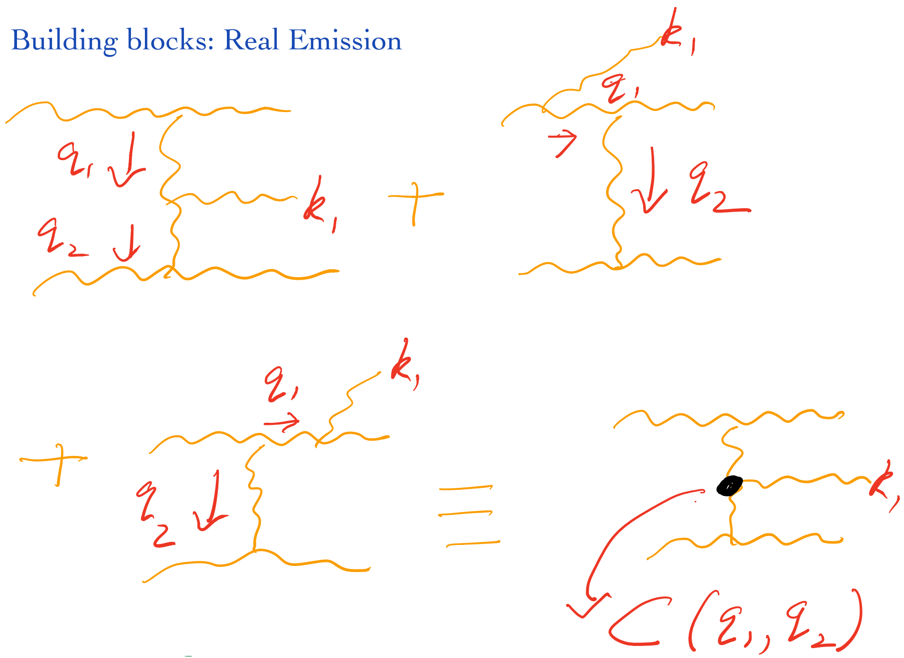
$$q_i^2 \simeq - k_{\perp i-1}^2 = t_i$$

$$y_0 \gg y_1 \gg y_2 \dots \gg y_{n+1} \; ; \; y_i \sim \ln\left(\frac{x_{i-1}}{x_i}\right)$$

$2 \rightarrow N + 2$ amplitude in the Regge limit of QCD: the BFKL equation

To build in real and virtual corrections to all orders in α_S, first focus on one rung of $2 \rightarrow$ N+2 ladder

Building blocks: Real Emission



Lipatov vertex

$$C(q_{i+1}, q_i) = -q_{\perp i+1} - q_{\perp i} + p_A \left(\frac{2 q_i^2}{\alpha_i s} + \beta_i \right) - p_B \left(\frac{2 q_{i+1}^2}{\beta_i s} + \alpha_i \right)$$

Non-local - gauge invariant!

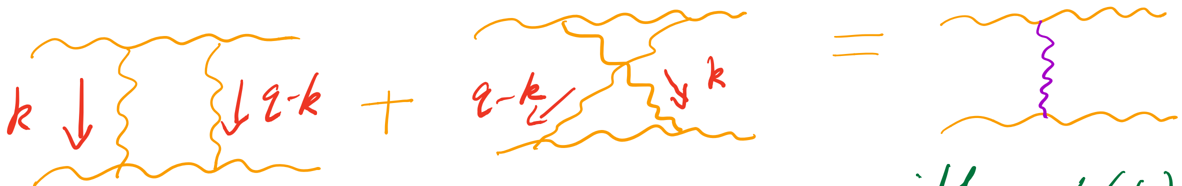
Ward identity: $k_i \cdot C = 0$

$$A^{2\rightarrow 3}(s,t) \sim 2s \frac{1}{t_1} C(q_2, q_1) \frac{1}{t_2}$$

BFKL pedagogical review:
DelDuca, hep-ph/9503226

$2 \rightarrow N + 2$ amplitude in the Regge limit of QCD: the BFKL equation

Building Blocks: virtual corrections

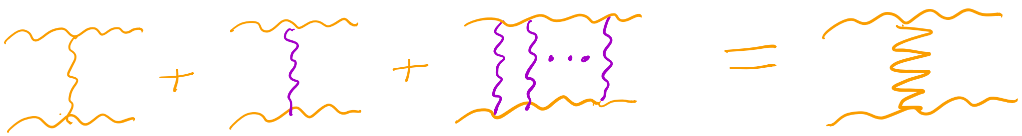


$$\frac{1}{t} \to \frac{1}{t} \ln\frac{s}{-t}\,\alpha(t)$$

with $\alpha(t) \propto \ln\frac{-t}{\mu^2}$

Infrared cut off

Reggeization ansatz:



$$\equiv \frac{1}{t_i} \to \frac{1}{t_i}\, e^{\alpha(t_i)(y_{i-1}-y_i)}$$

Double log structure:
Sudakov form factor

→ infrared sensitive

→ $2 \to 2$ amp. vanishes for $\mu \to 0$

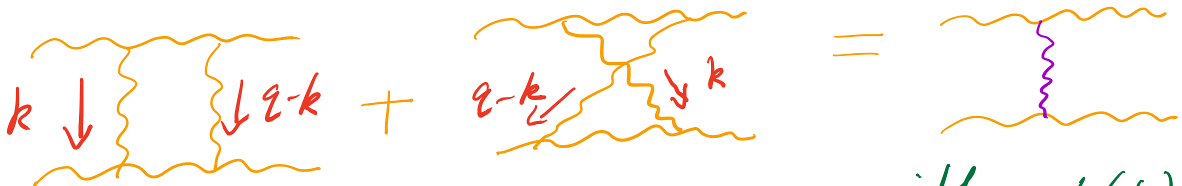
In general

$$\alpha(t) = \tilde{g}_S^2 \alpha^{(1)}(t) + \tilde{g}_S^4 \alpha^{(2)}(t) + \mathcal{O}(\tilde{g}_S^6)$$

V. Fadin, hep-ph/9807528

$2 \to N + 2$ amplitude in the Regge limit of QCD: the BFKL equation

Building Blocks: virtual corrections



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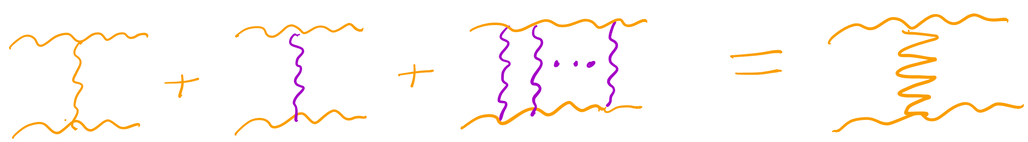

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Double log structure:
Sudakov form factor

→ infrared sensitive

→ $2 \to 2$ amp. vanishes for $\mu \to 0$

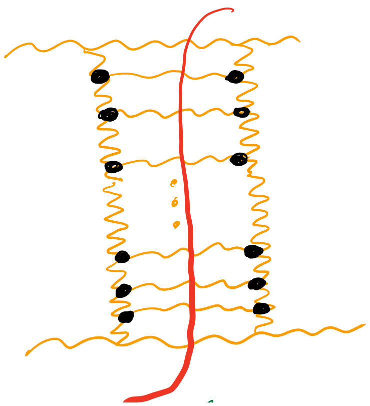
Parenthetical remark:
To this 2-loop order, this Regge trajectory can be obtained from the "cusp" anomalous dimension of the product of two Wilson lines: "Infrared factorization"

Korchemsky,Korchemskaya, hep-ph/9607229

$2 \rightarrow N + 2$ amplitude in the Regge limit of QCD: the BFKL equation

Putting together real and virtual contributions in this *leading logs in x* approximation: $(\alpha_S Y)^n$

BFKL Pomeron



$$\mathrm{Im}\, A(s,t) \propto \sum_{n=0}^{\infty} (\alpha_S C_T)^{n+2}$$

C_T is color factor

$$\times \int \prod_{i=1}^{n} \frac{dy_i}{4\pi} \prod_{j=1}^{n+1} \frac{d^2 q_{j\perp}}{(2\pi)^2}$$

Phase space factors

$$\times\ 2is \prod_{\ell=1}^{n+1} \frac{1}{t_\ell t_{\ell'}} e^{(y_{\ell-1}-y_\ell)[\alpha(t_\ell)+\alpha(t_\ell')]}$$

Reggeized propagators on both sides of cut

$$\times \prod_{m=1}^{n} (C_\mu C^\mu)[q_m, q_{m+1}]$$

Product of Lipatov vertices

Laplace Transform:

$$F_\ell(t) = -2it(4\pi\alpha_S)^2 C_T^2 \times \text{phase space}$$

$$\times \boxed{\frac{1}{t_1 t_1'} \frac{(-2\alpha_S C_T)\, C_\mu C^\mu}{\ell - 1 - \alpha(t_1) - \alpha(t_2)}}$$

$$\times \ \ldots$$

Defining $F_\ell(t) \propto \int \frac{d^2 q_{1\perp}}{(2\pi)^2} \frac{f_\ell(q_\perp, t)}{q_{1\perp}^2 (q-q_1)_\perp^2}$

Obtain BFKL integral eqn.

$$[\ell - 1 - \alpha(t) - \alpha(t')]\, f_\ell(t, q_1)$$

$$= 1 - 2\alpha_S C_T \int \frac{d^2 q_{2\perp}}{(2\pi)^2} \frac{C_\mu C^\mu(q_1, q_2)}{q_{2\perp}^2 (q-q_2)_\perp^2} f_\ell(q_2, t)$$

$2 \rightarrow N + 2$ amplitude in the Regge limit of QCD: the BFKL equation

Putting together real and virtual contributions in this *leading logs in x* approximation: $(\alpha_S Y)^n$

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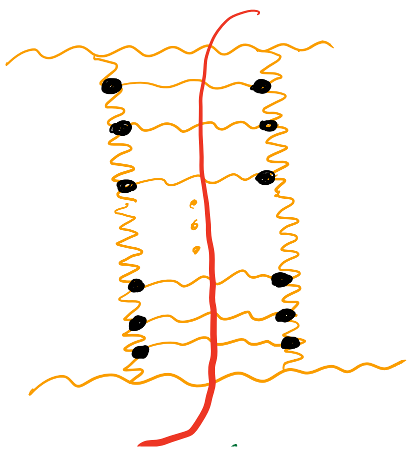

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Reggeized propagators
on both sides of cut

$$\times \prod_{m=1}^{n} (C_\mu C^\mu)[q_m, q_{m+1}]$$

Product of Lipatov vertices

Inverse Laplace Transform:

$$\mathrm{Im}\, A(s,t) = \oint_C \frac{d\ell}{2\pi i} e^{\ell y} F_\ell(t)$$

$$\sigma_{tot} = 2\, \mathrm{Im}\, A(s, t=0)$$

$$= s^\lambda \text{ with } \lambda = \frac{4\alpha_S N_c \ln 2}{\pi}$$

$$\simeq 0.5 \text{ for } \alpha_S = 0.2$$

Note:
Real and virtual corrections combine to cancel Infrared divergence !

Strong violation of Froissart bound

NLO BFKL : $\lambda \approx 0.3$

Classicalization and perturbative unitarization in QCD: the Color Glass Condensate

When N becomes large, partons start to recombine and screen – “shadowing”

$$= -\frac{[\alpha_s G(x,Q^2)]^2}{Q^2 R^2}$$

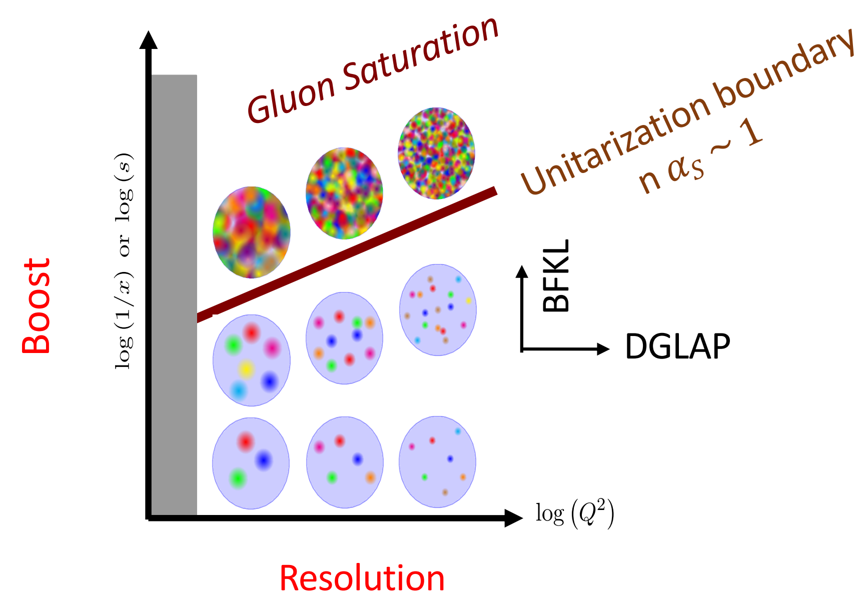
Recombination

Screening

Rapid growth in gluon occupancy with BFKL countered by ”higher twist” recombination and screening, creating state of maximal occupancy

$N = \frac{1}{\alpha_S}$ on **saturation line Q = Q_S(x)**

Classicalization ! Yang-Mills non-linear dynamics of glue



Classicalization and perturbative unitarization in QCD: the Color Glass Condensate

Semi-classical picture: Given a non-perturbative distribution of classical color sources coupled to overoccupied background glue, compute small quantum fluctuations in a small rapidity window, and absorb these into the classical source distribution for the next window in rapidity – *interate* (RG)

$$\langle \mathrm{d}\sigma_{\mathrm{LO}} \rangle = \int [\mathcal{D}\rho_A]\, W_{\Lambda_0^-}[\rho_A]\, \mathrm{d}\hat{\sigma}_{\mathrm{LO}}[\rho_A]\,. \Longrightarrow \langle \mathrm{d}\sigma_{\mathrm{LO}} + \delta\sigma_{\mathrm{NLO:1}} \rangle = \int [\mathcal{D}\rho_A] \underbrace{\left\{ \Big(1 + \ln(\Lambda_1^-/\Lambda_0^-)\mathcal{H}_{\mathrm{LO}}\Big) W_{\Lambda_0^-}[\rho_A] \right\}}_{\Big(1 + \ln(\Lambda_1^-/\Lambda_0^-)\mathcal{H}_{\mathrm{LO}}\Big) W_{\Lambda_0^-}[\rho_A] = W_{\Lambda_1^-}[\rho_A]} \mathrm{d}\hat{\sigma}_{\mathrm{LO}}[\rho_A]$$

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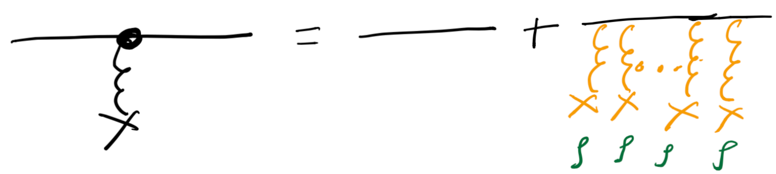
$$\Big(1+\ln(\Lambda_1^-/\Lambda_0^-)\mathcal{H}_{LO}\Big)W_{\Lambda_0^-}[\rho_A] = W_{\Lambda_1^-}[\rho_A]$$



$$= S_0(p)\, T_q(p,q)\, S_0(q)$$

$$T_q(p,q) = (2\pi)\delta(p^- - q^-)\gamma^- \mathrm{sign}(p^-) * \int d^2 z_\perp\, e^{-i(p_\perp - q_\perp)z_\perp}\, V^{\mathrm{sign}(p^-)}(z_\perp)$$

$$= G_0(p)\, T_g\, G_0(q)$$

$$T_g^{\mu\nu}(p,q) = (2\pi)\delta(p^- - q^-)(2p^-)\,\mathrm{sign}(p^-) * \int d^2 z_\perp\, e^{-i(p_\perp - q_\perp)z_\perp}\, U^{\mathrm{sign}(p^-)}(z_\perp)$$

U = P exp ($i\int_y^\infty dy' \frac{\rho(x_t, y')}{\nabla_t^2}$) path ordered exponential in adjoint rep. sources (likewise V in fundamental rep.) capturing "coherent" multiple scattering to all orders in ρ/∇_t^2

Classicalization and perturbative unitarization in QCD: the Color Glass Condensate

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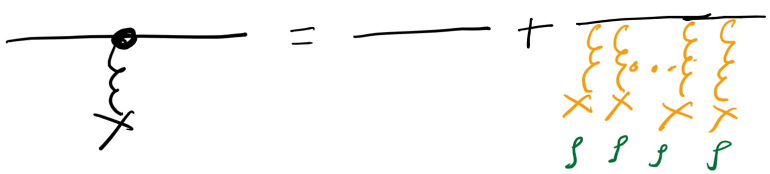

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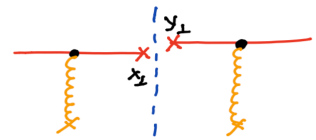
Parenthetical remark: In Lipatov's Reggeon EFT, these are quark-quark-reggeon and gluon-gluon-reggeon vertices respectively

Bondarenko,Lipatov,Pozdynyakov, Prygarin, arXiv:1708.05183
Hentschinski, arXiv:1802.06755

Classicalization and perturbative unitarization in QCD: the Color Glass Condensate

Consider quark scattering on dense target (forward physics)

LO



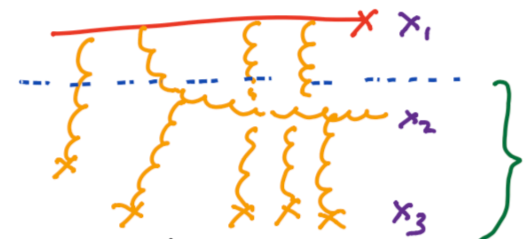
$$\text{Amp.} \times \text{c.c.} \to 1 - \frac{1}{N_c}\left\langle \mathrm{Tr}\left(V(x_1)\,V^{\dagger}(y_1)\right)\right\rangle_{x_2}$$

DIPOLE CORRELATOR

BUILDING BLOCK OF HIGH ENERGY QCD

NLO

if $x_1 >> x_2 >> x_3$



These soft bremsstrahlung contributions absorbed in RG evolution of DIPOLE CORRELATOR

$$S_{y_1} \equiv \frac{1}{N_c}\left\langle \mathrm{Tr}\left(V_{x_1} V^{\dagger}_{y_1}\right)\right\rangle_{y_1}$$

$$\to \frac{1}{N_c}\left\langle \mathrm{Tr}\left(V_{x_1} V^{\dagger}_{y_1}\right)\right\rangle_{y_2}$$

Balitsky-Kovchegov equation: RG evolution of dipole correlator

$$\frac{ds}{dy} = K_{BFKL} \otimes (s - ss)$$

SOFT BREMSSTRAHLUNG

+ MULTIPLE SCATTERING $\equiv$ SHADOWING

Classicalization and perturbative unitarization in QCD: the Color Glass Condensate

Consider quark scattering
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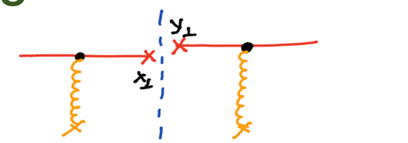
LO

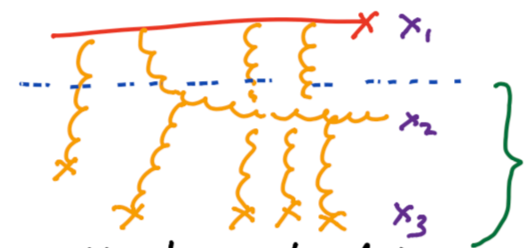


Amp. × c.c. $\rightarrow 1 - \frac{1}{N_c}\langle \mathrm{Tr}\,(V(x_\perp)\,V^\dagger(y_\perp))\rangle_{x_2}$

DIPOLE CORRELATOR

BUILDING BLOCK OF HIGH ENERGY QCD

NLO if $x_1 \gg x_2 \gg x_3$



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$$\rightarrow \frac{1}{N_c}\langle \mathrm{Tr}\,(V_{x_\perp} V^\dagger_{y_\perp})\rangle_{y_2}$$

BK unitarizes cross-section when $S \rightarrow 0$: "Black disc"

This occurs when $N \sim 1/\alpha_S$

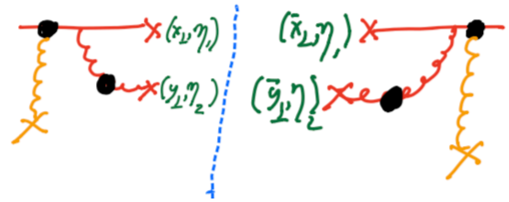
when dipole size $r_\perp = x_\perp - y_\perp \equiv \frac{1}{Q_S}$

In "dilute" regime, expanding around $S \sim 1$, to lowest order in $S/\nabla_\perp^2$, obtain BFKL eqn!

Classicalization and perturbative unitarization in QCD: the Color Glass Condensate

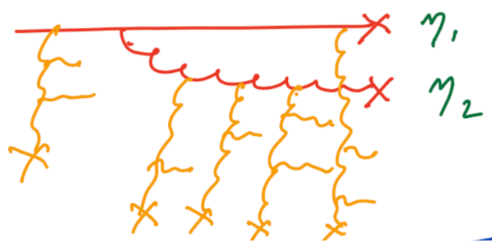
Consider double inclusive now
$pA \to qg + X$

CROSS-SECTION



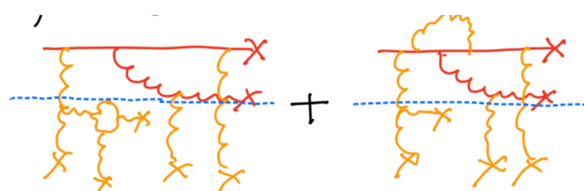
SENSITIVE TO BOTH 2-POINT DIPOLE WILSON LINE CORRELATOR & NOVEL 4-POINT QUADRUPOLE CORRELATOR

SOFT BREMSSTRAHLUNG ALSO SHADOWS SUCH CORRELATORS



JIMWLK RG equations describe bremsstrahlung of DIPOLES, QUADRUPOLES, SEXTUPOLES, ... in an infinite hierarchy

Sadly, LLx resummation not good enough for precision studies



NLLx corrections to RG equations
UNIVERSAL

NLO corrections to observable
PROCESS DEPENDENT

* PARTS OF THIS ARE COMPUTED
  → NLO JIMWLK "available"
  NLO BK more accessible
* PARTS IMPOSED: Eg, RUNNING COUPLING

$2 \to N + 2$ amplitudes in the Regge limit of gravity: the Lipatov-ACV approach and the double copy

HIGH-ENERGY SCATTERING IN QCD AND IN QUANTUM GRAVITY AND TWO-DIMENSIONAL FIELD THEORIES

L.N. LIPATOV*

We construct effective actions describing high-energy processes in QCD and in quantum gravity with intermediate particles (gluons and gravitons) having the multi-Regge kinematics. The *S*-matrix for these effective scalar field models contains the results of the leading logarithmic approximation and is unitary. It can be expressed in terms of correlation functions for two field theories acting in longitudinal and transverse two-dimensional subspaces.

NPB 364 (1991) 614; 157 cites in INSPIRE

Effective action and all-order gravitational eikonal at planckian energies

AMATI,CIAFALONI,VENEZIANO **NPB403 (1993)707**

Building on previous work by us and by Lipatov, we present an effective action approach to the resummation of all semiclassical (i.e. $O(\hbar^{-1})$) contributions to the scattering phase arising in high-energy gravitational collisions. By using an infrared-safe expression for Lipatov's effective action, we derive an eikonal form of the scattering matrix and check that the superstring amplitude result is reproduced at first order in the expansion parameter R^2/b^2, where R, b are the gravitational radius and the impact parameter, respectively. If rescattering of produced gravitons is neglected, the longitudinal coordinate dependence can be explicitly factored out and exhibits the characteristics of a shock-wave metric while the transverse dynamics is described by a reduced two-dimensional effective action. Singular behaviours in the latter, signalling black hole formation, can be looked for.

The World as a Hologram

LEONARD SUSSKIND

Wee partons, by contrast, are not subject to Lorentz contraction. This implies that in the Feynman Bjorken model, the halo of wee partons eternally "floats" above the horizon at a distance of order $10^{-13}cm$ as it transversley spreads. The remaining valence partons carry the various currents which contract onto the horizon as in the Einstein Lorentz case.

By contrast, both the holographic theory and string theory require all partons to be wee. No Lorentz contraction takes place and the entire structure of the string floats on the stretched horizon. I have explained in previous articles how this behavior prevents the accumulation of arbitrarily large quantities of information near the horizon of a black hole. Thus we are led full circle back to Bekenstein's principle that black holes bound the entropy of a region of space to be proportional to its area.

J.Math.Phys. **36 (1995) 6377; 3048 cites**

In Acknowledgements:

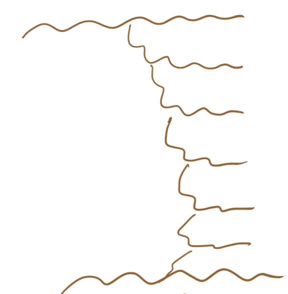
Finally I benefitted from discussions with Kenneth Wilson and Robert Perry, about boosts and renormalization fixed points in light front quantum mechanics and Lev Lipatov about high energy scattering.

These works do not explicitly discuss parton saturation which leads to a perspective I will discuss shortly

$2 \to N + 2$ amplitudes in the Regge limit of gravity: the Lipatov-ACV approach and the double copy

Double copy between QCD and Gravity amplitudes

High energy scattering of Gravitons



Same multi-Regge kinematics as previously

Old idea (Kawai-Lewellyn-Tye) based on relations between closed and open string amplitudes – in "low energy" limit between Einstein & Yang-Mills amplitudes

$$M_4^{\rm tree}(1,2,3,4) = \left(\frac{\kappa}{2}\right)^2 s\, A_4^{\rm tree}(1,2,3,4)\, A_4^{\rm tree}(1,2,4,3) \qquad \kappa = 32\,\pi^2\, G_N$$

A remarkable "BCJ" color-kinematics duality

Bern, Carrasco,Johansson,arXiv:0805.3993

Tree level $gg \to gg$ amplitudes (with on shell legs) can be written as

$$i\mathcal{A}_4^{\rm tree} = g^2 \left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right)$$

with the s channel color factor $c_s = -2 f^{a_1 a_2 b} f^{b a_3 a_4}$

kinematic factor
$$n_s = -\frac{1}{2}\Big\{ \Big[(\epsilon_1.\epsilon_2)p_1^\mu + 2(\epsilon_1.p_2)\epsilon_2^\mu - (1 \leftrightarrow 2)\Big]\Big[(\epsilon_3.\epsilon_4)p_3^\mu + 2(\epsilon_3.p_4)\epsilon_4^\mu - (3 \leftrightarrow 4)\Big] + s\Big[(\epsilon_1.\epsilon_3)(\epsilon_2.\epsilon_4) - (\epsilon_1.\epsilon_4)(\epsilon_2.\epsilon_3)\Big]\Big\}$$

Tree level gravity amplitude obtained fron replacing color factors by kinematic factors

$$i\mathcal{A}_4^{\rm tree}|_{c_i \to n_i, g \to \kappa/2} = i\mathcal{M}_4^{\rm tree} = \left(\frac{\kappa}{2}\right)^2 \left(\frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}\right)$$

Significant on-going work on extension to loop amplitudes

Review: Bern et al., arXiv: 1909.01358

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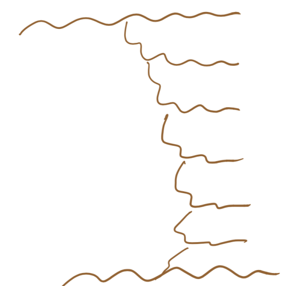
Double copy between QCD and Gravity amplitudes

High energy scattering of Gravitons



Same multi-Regge kinematics as previously

Old idea (Kawai-Lewellyn-Tye) based on relations between closed and open string amplitudes – in "low energy" limit between Einstein & Yang-Mills amplitudes

$$M_4^{\rm tree}(1,2,3,4) = \left(\frac{\kappa}{2}\right)^2 s\, A_4^{\rm tree}(1,2,3,4)\, A_4^{\rm tree}(1,2,4,3) \qquad \kappa = 32\,\pi^2\, G_N$$

Gravity-QCD double copy in the Regge limit

Lipatov, PLB 116B (1982) 411
Spinor helicity derivation, Liu, arXiv:1811.11710

$$\mathcal{M}_n \simeq -s^2\, \mathcal{C}(2;3) \frac{-1}{|q_4^\perp|^2} \mathcal{V}(q_4;4;q_5) \cdots \frac{-1}{|q_{n-1}^\perp|^2} \mathcal{V}(q_{n-1};n-1;q_n) \frac{-1}{|q_n^\perp|^2}\, \mathcal{C}(1;n)$$

Gravitational effective vertex: $\mathcal{V}(q_i, i, q_{i+1}) = \Gamma_{i,\mu\nu}(q_i, q_{i+1})\, \epsilon_i^{\mu\nu}(k_i)$ is double copy $\Gamma_i^{\mu\nu}(q_i, q_{i+1}) \equiv 2\left(C_i^\mu C_i^\nu - N_i^\mu N_i^\nu\right)$

Lipatov Vertex !

QED Bremsstrahlung vertex

The C(2;3) and C(1;n) are gravitational "impact factors" that can also be expressed as a double copy of Lipatov's gluon-gluon-reggeized gluon vertex

Writing it as a color kinematics duality is more tricky but can be done

Johansson,Sabio Vera,, Campillo, Vasquez-Mozo, arXiv:1310.1680

Classicalization and unitarization in Gravity: The Black Hole N Portrait

The double copy between QCD and gravity amplitudes in the Regge limit suggests that $2 \rightarrow \mathrm{N}+2$ grow much faster than QCD amplitudes

When $\mathrm{N} = \frac{1}{\alpha_{gr}}$, the dense graviton state unitarized by forming a Black Hole

The Black Hole N Portrait suggests gravity is UV self-complete due classicalization at a weak coupling scale

In the Black Hole N Portrait,

Dvali, Gomez, arXiv:1112.3359
Dvali,Guidice,Gomez,Kehagis, arXiv:1010.1415

$R_S = 2GM$ Schwarzchild radius
$L_P = \sqrt{\hbar G}$ Planck Length
$M_P = \sqrt{\hbar / G}$ Planck Mass

Gravitons have wavelengths $\lambda = R_S = \sqrt{N}\, L_P$

$$\alpha_{gr} = \frac{L_P^2}{\lambda^2} = \frac{1}{N} \xrightarrow{N\to\infty} 0$$

Black Hole evaporation Rate

$$\Gamma = 6\, n_h (1+N) \sim \alpha_{gr} R_S^2 \frac{N^2}{R_S^3} = \frac{1}{R_S}$$

For $T_H = \frac{\hbar}{R_S} = \frac{\hbar}{\sqrt{N}\, L_P}$

obtain familiar expressions

$$\frac{dM}{dt} = -\frac{T^2}{\hbar}$$

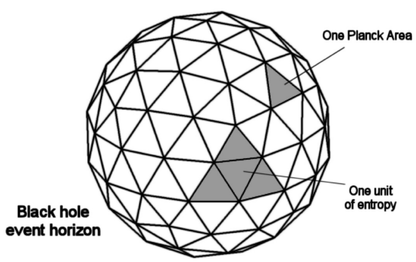
and half-life

$$\tau_{BH} = \frac{\hbar^2}{T^3 G} \equiv N^{3/2} L_P$$

Classicalization and unitarization in Gravity: The Black Hole N Portrait

Bekenstein-Hawking bound

(for a nice discussion, see Bousso, arxlv:1810.01880)



The Bekenstein entropy bound is given by $S \leq 2\pi E R/\hbar$

In the Black Hole N portrait, this is given by $S \leq 2\pi\, N\, Q_S\, R_S$

which is saturated when $N = \frac{1}{\alpha_{gr}} \rightarrow S_{Bek} = \frac{1}{\alpha_{gr}}$

Here $E = N\, Q_S$ is the energy in a critically packed volume = R_S^3 of quanta, and $Q_S = 1/R_S$

From our previous discussion, $S_{Bek} = \frac{1}{\alpha_{gr}} = \frac{R_S^2}{L_P^2} = \frac{Area}{4G} = S_{BH}$

The entropy can also be expressed in terms of a Goldstone decay constant corresponding to spontaneous breaking of Poincare invariance by the graviton condensate: $f^2 = N\, Q_S^2$

The entropy in these units is $S_{BH} = \text{Area} \times f^2$

Dvali, arXiv:0907.07332

At the critical point $N = \frac{1}{\alpha_{gr}}$, the saturation of the BH bound also leads to unitarization !

$$\sigma^{BHNP}_{2\to N} = N!\, \alpha_{gr}^{N}\, e^{S_{BH}} \simeq \cancel{N^N} e^{-N} \cancel{\left(\frac{1}{N}\right)^N} e^{\frac{1}{\alpha_{gr}}} = e^{-\frac{1}{\alpha_{gr}} + \frac{1}{\alpha_{gr}}} \sim O(1)$$

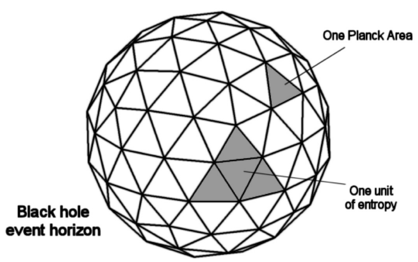
Dvali,Gomez,Isermann,Lust,Stieberger arXiv:1409.7405

Away from the critical point, the cross-section for BH formation drops exponentially !

Classicalization and unitarization in Gravity: The Black Hole N Portrait

Bekenstein-Hawking bound

(for a nice discussion, see Bousso, arxIv:1810.01880)



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How to connect to Lipatov-ACV amplitude language? This entropy factor arises from cancellation of infrared divergences In the Sudakov form factor with real soft contributions – to give a finite result a la BFKL

Addazi,Bianchi,Veneziano, arXiv:1611.03643

The CGC/BHNP correspondence

Dvali, RV, in preparation

From the previous discussion, we begin to glimpse the elements of the CGC/BHNP correspondence with a region of space $R_S = Q_S^{-1}$ of the CGC mapped on to a Black Hole on analogous radius determined by the saturation of the graviton condensate:

The scale at which this occurs is a weak coupling scale in both theories:

$$\text{for BHNP}: \quad Q_S^{-1} = \sqrt{N} M_P^{-1};$$
$$\text{for CGC}: \quad Q_S = \sqrt{N} \Lambda_{\rm QCD}$$

This scale is determined by a maximal occupancy of $\mathrm{N} = \frac{1}{\alpha}$ which is where (perturbative) unitarization occurs
Its energy-dependent: the "wee-er" the parton, the quicker it saturates…

The direct correspondence between the two theories only occurs at $Q=Q_S$ where the physics is universal
- It is independent of the details of the dynamics of the wee partons

The existence of a double copy between the two theories is however suggestive that it can be extended to a "classical" double copy between the gluon and graviton classical shockwaves

Review on classical double copy: C. White, arXiv:1708.07056

The CGC/BHNP correspondence: Bekenstein entropy

Dvali, RV, in preparation

We can use this direct correspondence to conjecture the maximal Bekenstein entropy of the CGC

Since the typical momenta of saturated wee gluons is Q_S , the CGC energy is N Q_S in a "color domain" of radius Q_S^{-1} - and the entropy argument goes through as previously:

$$S_{CGC} = \frac{1}{\alpha_S}$$

Inside a confining radius of Λ_{QCD}^{-1}, of the Lorentz contracted proton at high energies, there can be $\frac{Q_S^2}{\Lambda_{QCD}^2}$ = N domains

Hence,

$$S_{proton} = \mathrm{N}\, S_{CGC} = \frac{1}{\alpha_S^2}$$

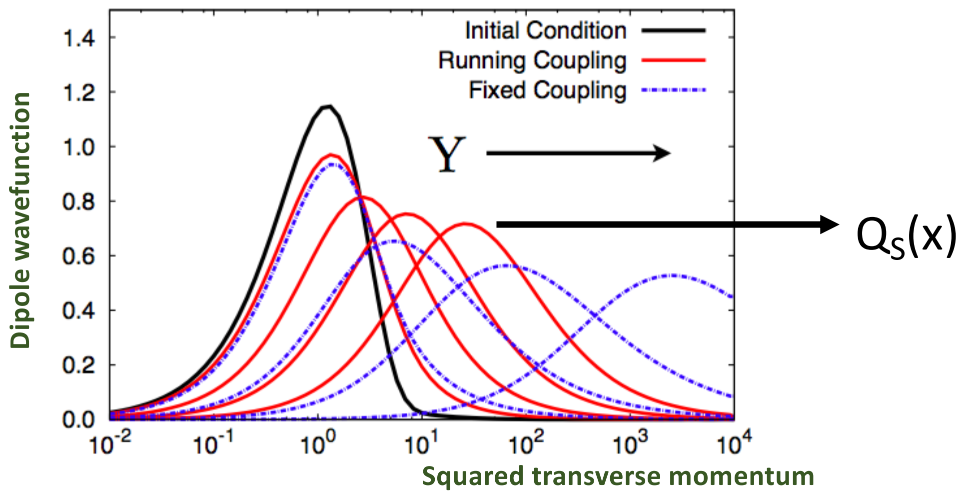
Further, since one can have 1/ α_S wee partons in a rapidity interval Y, this gives $\frac{dS_{proton}}{dY} = \frac{1}{\alpha_S}$

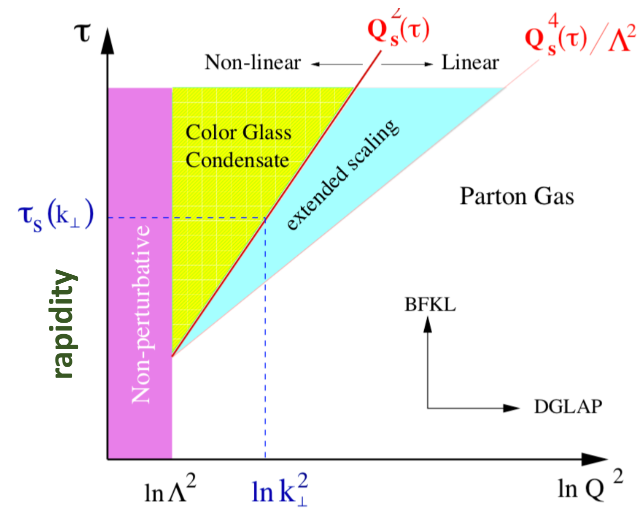
Other discussions in the literature:
Kovner, Lublinsky, Serino, arXiv:1806.01089
Armesto,Dominguez,Kovner,Lublinsky,Skokov, arXiv:1901.08080

The CGC/BHNP correspondence: Bekenstein entropy

Dvali, RV, in preparation

One can also understand the CGC entropy in the language of a Goldstone decay constant characterizing the breaking of the Poincare symmetry of partons due to the formation of the “soliton-like” CGC state





This ”UV” scale characterizes the onset of screening (shadowing) and is given by $f^2 = \frac{Q_S^4}{\Lambda_{QCD}^2}$

Thus, just as in the BHNP, the CGC entropy satisfies an area law:

$$S_{CGC} = \text{Area} * f^2 \text{ , where Area} = 4\pi\, R_S^2$$

Final remarks

The CGC/Black Hole correspondence we have outlined also extends to the evolution of the CGC/Glasma and Black Hole evaporation via Hawking radiation – the analogy is however not valid after a "quantum breaking" time beyond which the CGC forms a QGP

The study of "soft factors" in the CGC and the relation to "cusp anomalous dimensions" is under investigation in a worldline QFT formalism of radiation from sem—classical worldlines

Feal,Tarasov,RV, in progress

The classical double copy of CGC and saturated graviton states is also under investigation and can provide more quantitative evidence of the correspondence outlined

Raj, RV, in progress