

what can pQCD and lattice say about the thermal photon rate? ¹

mikko laine

(aec, itp, u. bern)

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disclaimer

in this talk consider QCD at $T \gtrsim T_c$, omitting pre-equilibrium effects that certainly play a role in heavy ion collision experiments

motivation / big picture

basic relations (assuming thermal equilibrium)²

dilepton rate:

$$\frac{d\Gamma_{\mu^-\mu^+}(\omega, k)}{d\omega d^3\mathbf{k}} \approx \frac{\alpha_{\text{em}}^2 n_{\text{B}}(\omega)}{3\pi^3 M^2} \sum_{i=1}^{N_f} Q_i^2 \rho_V(\omega, k) + \mathcal{O}(\alpha_{\text{em}}^3)$$

photon rate:

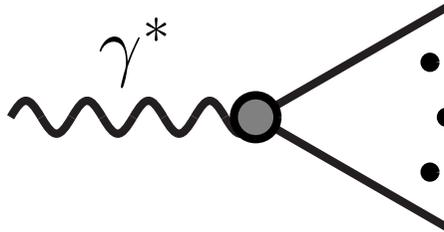
$$\frac{d\Gamma_{\gamma}(k)}{d^3\mathbf{k}} = \frac{\alpha_{\text{em}} n_{\text{B}}(k)}{2\pi^2 k} \sum_{i=1}^{N_f} Q_i^2 \rho_V(k, k) + \mathcal{O}(\alpha_{\text{em}}^2)$$

bose distribution $n_{\text{B}} \Rightarrow$ energy; spectral function $\rho_V \Rightarrow$ dynamics

² L.D. McLerran and T. Toimela, *Photon and Dilepton Emission from the Quark-Gluon Plasma: Some General Considerations*, PRD31(85)545; H.A. Weldon, *Reformulation of Finite Temperature Dilepton Production*, PRD42(90)2384; C. Gale and J.I. Kapusta, *Vector dominance model at finite temperature*, NPB357(91)65

in vacuum, only virtual photons can carry spectral weight³

ρ_V can be interpreted as decay width / interaction rate



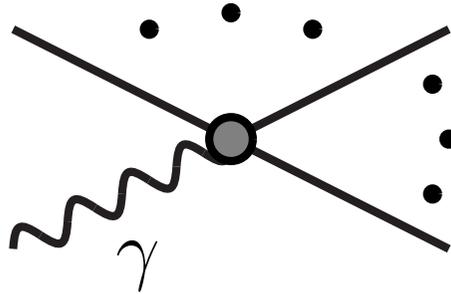
$$M^2 = \omega^2 - k^2 > 0$$

quantum-mechanical language is useful, e.g. $\gamma^* \rightarrow \pi^+ \pi^-$

³ in pQCD this has been studied up to NNNLO: P.A. Baikov, K.G. Chetyrkin and J.H. Kühn, *Order α_s^4 QCD Corrections to Z and τ Decays*, 0801.1821

in a thermal medium, spectral weight extends to $M^2 \leq 0$

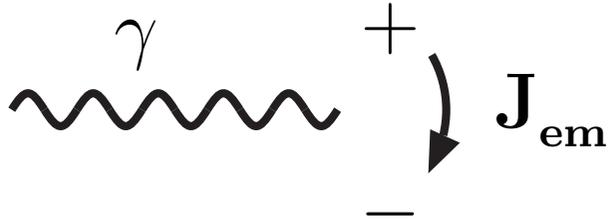
this is because one can now scatter “off the medium”



⇒ many possible reactions, providing energy / virtuality

⇒ decoherence

interpretation is simplest in terms of fluctuating plasmas



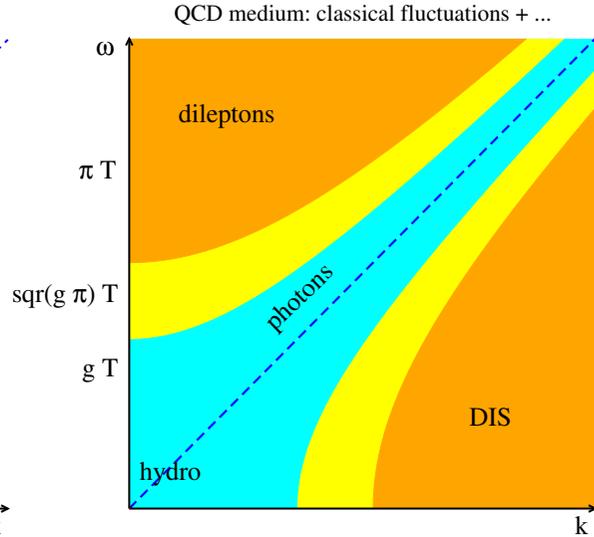
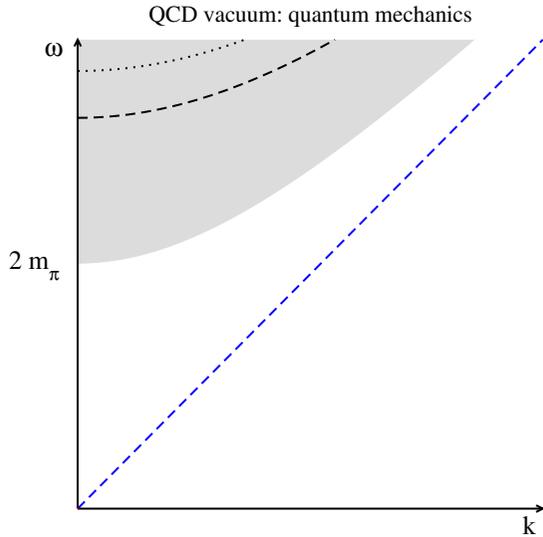
concretely, for ultra-low $\omega, k \ll T$, the general theory of statistical fluctuations⁴ leads to a “hydrodynamic” prediction:

$$\frac{\rho_V(\omega, k)}{\omega} = \left(\frac{\omega^2 - k^2}{\omega^2 + D^2 k^4} + 2 \right) \chi D$$

this holds for QED, QCD, $\mathcal{N} = 4$ SYM, ...

⁴ e.g. E.M. Lifshitz and L.P. Pitaevskii, *Statistical Physics, Part 2*, §88-89

summary for ρ_V



brief recap of pQCD predictions

leading-order result

definition of spectral function ($\eta \equiv (- + + +)$):

$$\begin{aligned}\rho_V(\omega, k) &= \text{Im } \Pi_R(\omega, k) \\ &\equiv \int_{\mathcal{X}} e^{i(\omega t - \mathbf{k} \cdot \mathbf{x})} \left\langle \frac{1}{2} [V^\mu(t, \mathbf{x}), V_\mu(0)] \right\rangle\end{aligned}$$

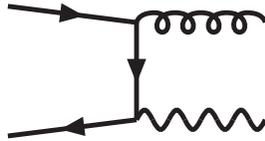
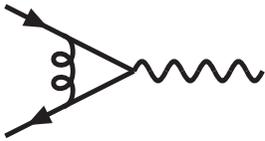
expression vanishes for $M^2 \rightarrow 0$ but is otherwise non-zero:

$$\rho_V(\omega, k) = \frac{N_c T M^2}{2\pi k} \ln \left\{ \frac{\cosh(\frac{\omega+k}{4T})}{\cosh(\frac{\omega-k}{4T})} - \frac{\omega \theta(k - \omega)}{2T} \right\}$$

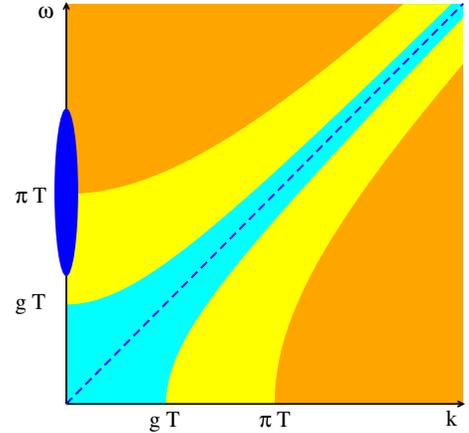


higher orders [selection]:

NLO result at vanishing momentum ($k = 0$)⁵



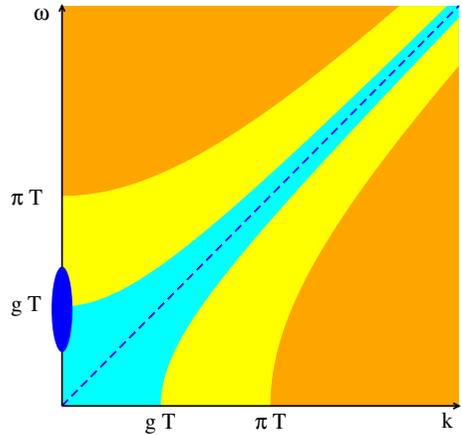
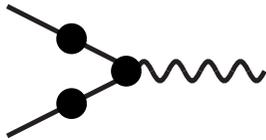
etc



⇒ only a small correction

⁵ R. Baier, B. Pire and D. Schiff, *Dilepton production at finite temperature: Perturbative treatment at order α_S* , PRD38(88)2814; Y. Gabellini, T. Grandou and D. Poizat, *Electron-positron annihilation in thermal QCD*, AP202(90)436; T. Altherr and P. Aurenche, *Finite temperature QCD corrections to lepton-pair formation in a quark-gluon plasma*, ZPC45(89)99

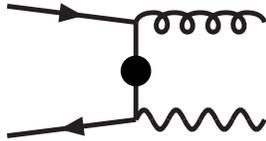
hard thermal loop (HTL) resummation in soft regime⁶



⇒ a large enhancement

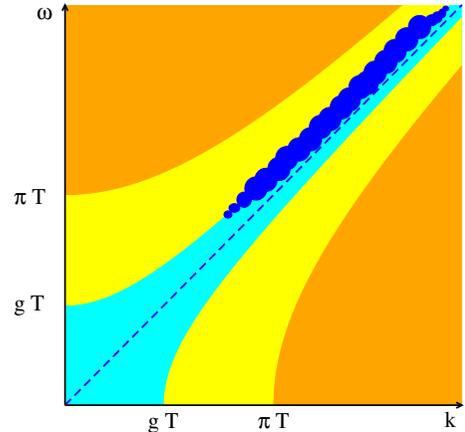
⁶ E. Braaten, R.D. Pisarski and T.-C. Yuan, *Production of soft dileptons in the quark-gluon plasma*, PRL64(90)2242

HTL resummation for hard momenta ($k \sim \pi T$)⁷



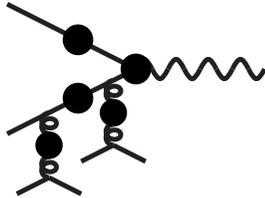
if $M \rightarrow 0$ there is a singularity from soft t -channel exchange, which is regulated by Landau damping

$$\rho_V = \dots + \frac{\alpha_s N_c C_F T^2}{4} \ln \left(\frac{T^2}{[M^2 \rightarrow \alpha_s T^2]} \right)$$

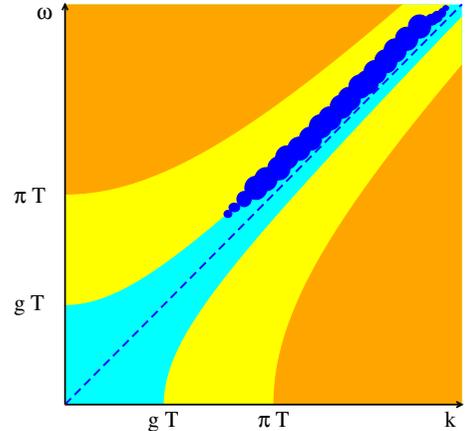


⁷ J.I. Kapusta, P. Lichard and D. Seibert, *High-energy photons from quark-gluon plasma versus hot hadronic gas*, PRD44(91)2774 [Erratum-ibid. D47(93)4171]; R. Baier, H. Nakkagawa, A. Niégawa and K. Redlich, *Production rate of hard thermal photons and screening of quark mass singularity*, ZPC53(92)433; T. Altherr and P.V. Ruuskanen, *Low mass dileptons at high momenta in ultrarelativistic heavy ion collisions*, NPB380(92)377

LPM resummation for hard momenta ($k \sim \pi T$)^{8,9}



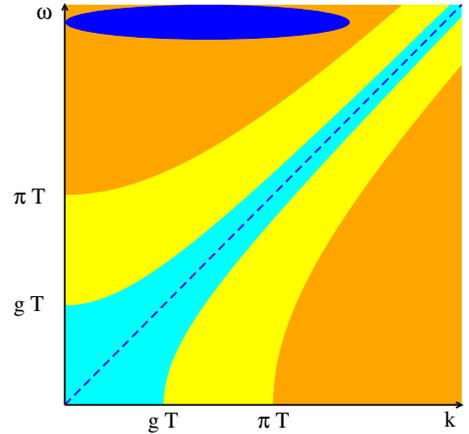
removing the divergence is not enough: there are terms of similar magnitude from multiple scatterings with collinear enhancement



⁸ P.B. Arnold, G.D. Moore and L.G. Yaffe, *Photon emission from ultrarelativistic plasmas*, hep-ph/0109064; hep-ph/0111107; P. Aurenche, F. Gelis, G.D. Moore and H. Zaraket, *Landau-Pomeranchuk-Migdal resummation for dilepton production*, hep-ph/0211036; M.E. Carrington, A. Gyntner and P. Aurenche, *Energetic di-leptons from the Quark Gluon Plasma*, 0711.3943

⁹ NLO: J. Ghiglieri *et al*, *Next-to-leading order thermal photon production in a weakly coupled quark-gluon plasma*, 1302.5970; J. Ghiglieri and G.D. Moore, *Low Mass Thermal Dilepton Production at NLO in a Weakly Coupled Quark-Gluon Plasma*, 1410.4203

for $M \gg \pi T$ the vacuum term dominates^{10,11}



$$\rho_V = \frac{N_C M^2}{4\pi} \left(1 + \frac{3\alpha_s C_F}{4\pi}\right) + \frac{4\alpha_s N_C C_F}{9} \left(1 + \frac{4k^2}{3M^2}\right) \frac{\pi^2 T^4}{M^2} + \mathcal{O}\left(\frac{\alpha_s T^6}{M^4}\right)$$

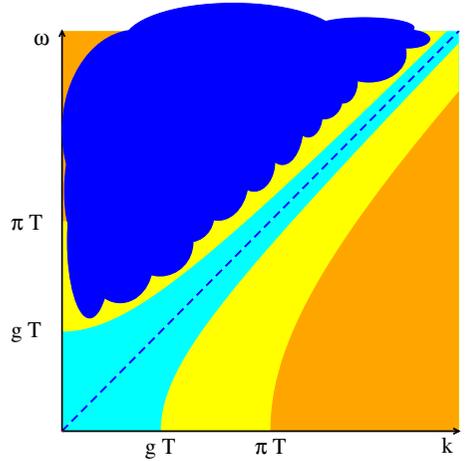
¹⁰ S. Caron-Huot, *Asymptotics of thermal spectral functions*, 0903.3958

¹¹ Y. Burnier and ML, *Towards flavour diffusion coefficient and electrical conductivity without UV contamination*, 1201.1994

away from light-cone no resummation is needed¹²



compute the NLO imaginary-time correlator and take the cut:

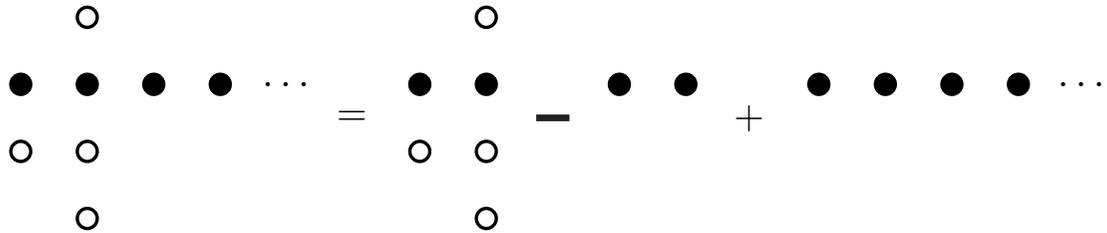


$$\Pi_E(K) \equiv \int_0^{1/T} d\tau \int_{\mathbf{x}} e^{iK \cdot X} \langle (\bar{\psi} \gamma^\mu \psi)(\tau, \mathbf{x}) (\bar{\psi} \gamma_\mu \psi)(0, \mathbf{0}) \rangle_T$$

$$\Pi_R(\mathcal{K}) = \Pi_E|_{k_n \rightarrow -i[\omega + i0^+]} \quad , \quad \rho_V = \text{Im } \Pi_R$$

¹² ML, *Thermal 2-loop master spectral function at finite momentum*, 1304.0202; *NLO thermal dilepton rate at non-zero momentum*, 1310.0164

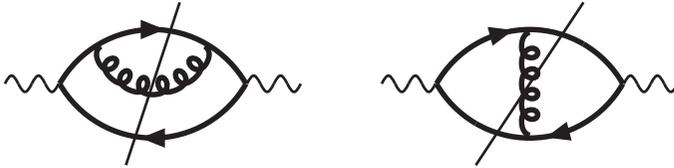
NLO and LPM results can be interpolated into each other¹³



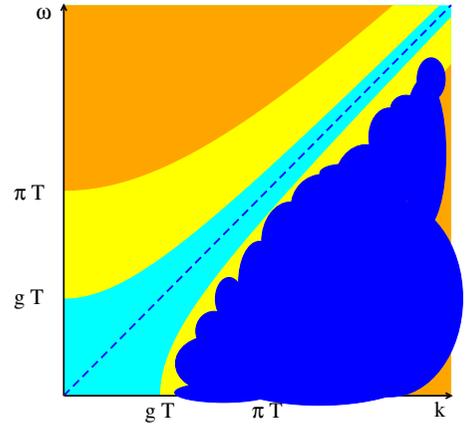
$$\rho_V|_{\text{resummed NLO}} \equiv \rho_V|_{\text{NLO}} - \rho_V|_{\text{LPM}}^{\text{expanded}} + \rho_V|_{\text{LPM}}^{\text{full}}$$

¹³ I. Ghisoiu and ML, *Interpolation of hard and soft dilepton rates*, 1407.7955

NLO below light cone, all polarizations¹⁴



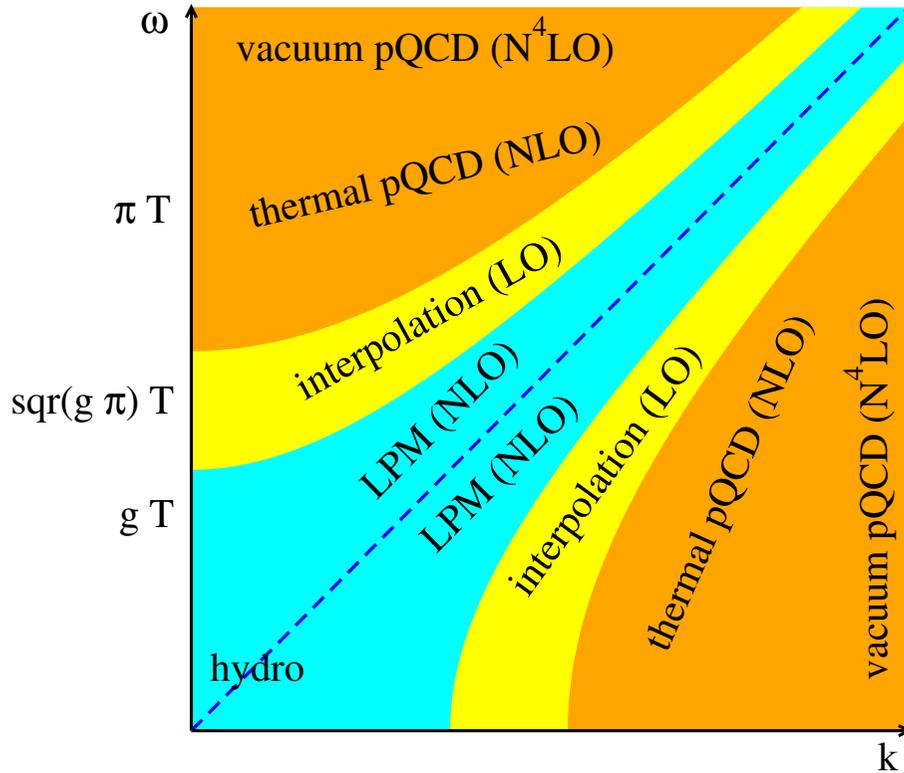
compute the NLO imaginary-time correlator and take the cut



¹⁴ G. Jackson, *Two-loop thermal spectral functions with general kinematics*, 1910.07552

... so a lot is known after 30+ years!

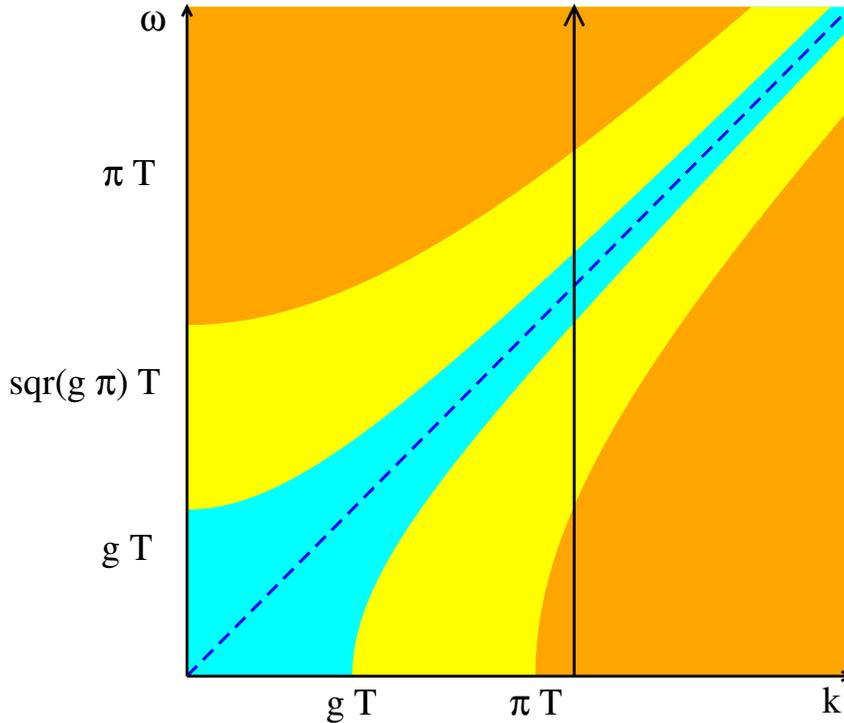
summary: different techniques needed in different regimes



what can lattice do?

basic picture

$$G_V(\tau, k) = \int_0^\infty \frac{d\omega}{\pi} \rho_V(\omega, k) \frac{\cosh[\omega(\frac{\beta}{2} - \tau)]}{\sinh[\frac{\omega\beta}{2}]}, \quad \beta \equiv \frac{1}{T}$$



major challenges

(i) UV problem

$$G_V(\tau, k) \equiv \int_{\mathbf{x}} e^{-i\mathbf{k}\cdot\mathbf{x}} \langle V^i(\tau, \mathbf{x}) V^i(0) - V^0(\tau, \mathbf{x}) V^0(0) \rangle_c$$

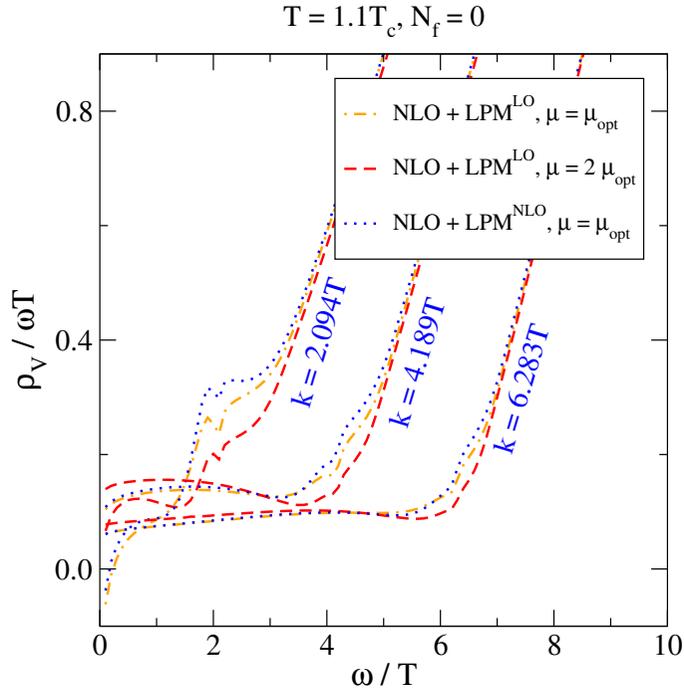
vacuum part $\sim \omega^2$ of ρ_V dominates the measured $G_V \sim \tau^{-3}$

(ii) reconstruction problem

inversion $G_V \longrightarrow \rho_V$ requires “exponentially high” precision;¹⁵
without this (or further assumptions), can only do $\rho_V \longrightarrow G_V$

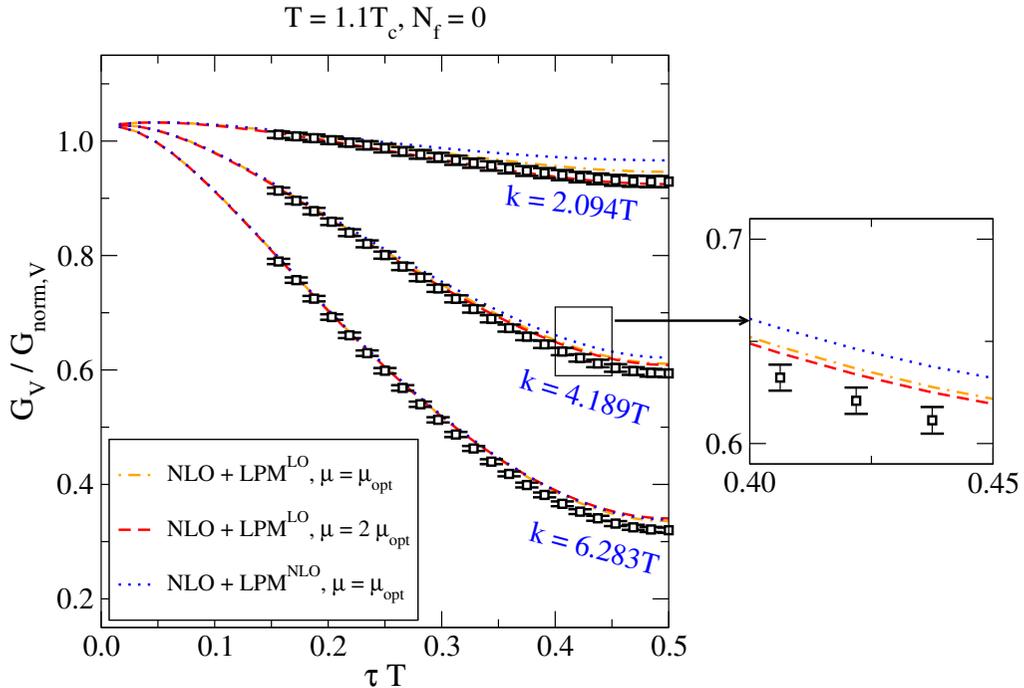
¹⁵ G. Cuniberti, E. De Micheli and G.A. Viano, *Reconstructing the thermal Green functions at real times from those at imaginary times*, cond-mat/0109175; F. Ferrari, *The Analytic Renormalization Group*, 1602.07355.

examples from pQCD¹⁶ ($\mu_{\text{opt}} \equiv \sqrt{(\pi T)^2 + |M^2|}$)



¹⁶ G. Jackson and ML *Testing thermal photon and dilepton rates*, 1910.09567

the corresponding imaginary-time correlators¹⁷



¹⁷ lattice data from J. Ghiglieri, O. Kaczmarek, ML, F. Meyer, *Lattice constraints on the thermal photon rate*, 1604.07544.

how to increase the resolution?¹⁸

longitudinal spectral function vanishes at light-cone

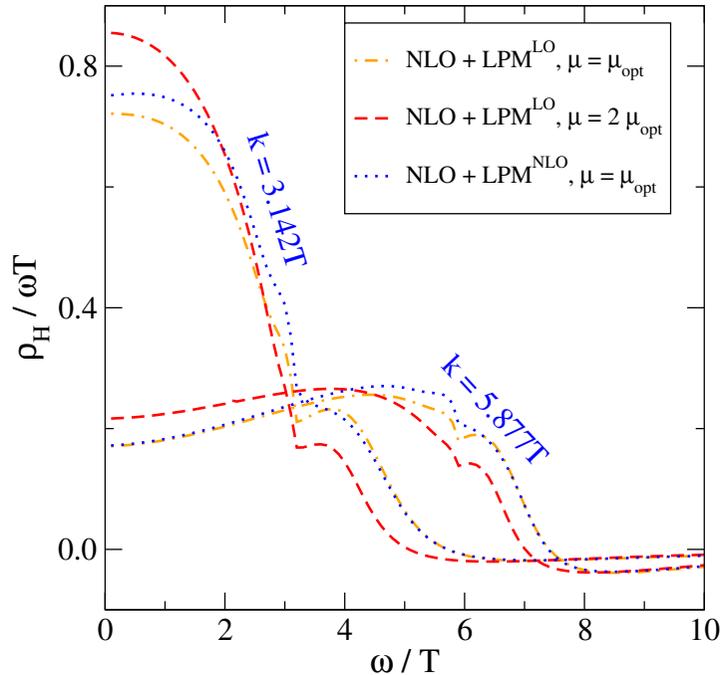
$$\begin{aligned} \stackrel{\mathbf{k}=k \mathbf{e}_z}{\Rightarrow} G_H(\tau, k) &\equiv \int_{\mathbf{x}} e^{-ikz} \langle \sum_{i=1}^2 V^i(\tau, \mathbf{x}) V^i(0) \\ &- 2[V^z(\tau, \mathbf{x}) V^z(0) - V^0(\tau, \mathbf{x}) V^0(0)] \rangle_c \end{aligned}$$

\Rightarrow cannot be used for dileptons but a nice idea for photons!

¹⁸ B.B. Brandt, A. Francis, T. Harris, H.B. Meyer and A. Steinberg, *An estimate for the thermal photon rate from lattice QCD*, 1710.07050

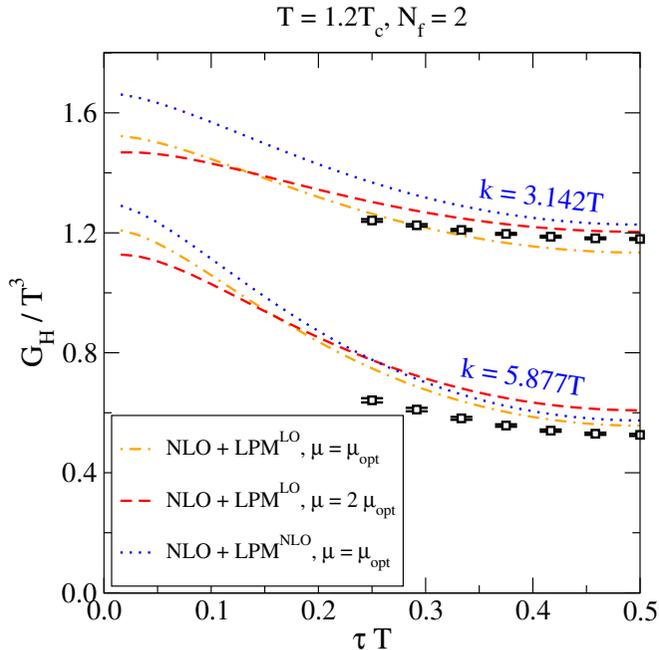
now spectral weight is in IR¹⁹ ($\mu_{\text{opt}} \equiv \sqrt{(2\pi T)^2 + |M^2|}$)

$$T = 1.2T_c, N_f = 2$$



¹⁹ G. Jackson and ML *Testing thermal photon and dilepton rates*, 1910.09567

comparison with imaginary-time correlators²⁰



⇒ the difference could be meaningful (“ $> 5\sigma$ ”)

²⁰ lattice data from M. Cè, T. Harris, H.B. Meyer, A. Steinberg and A. Toniato, *Rate of photon production in the quark-gluon plasma from lattice QCD*, 2001.03368

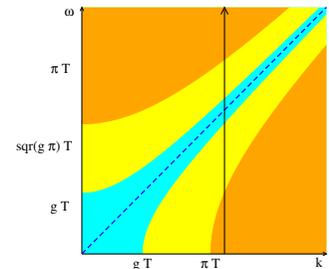
summary and outlook

advanced pQCD results have been tabulated on a broad grid²¹

comparison with lattice data suggests that they could “overshoot” the true thermal yield slightly

it would be nice to embed advanced pQCD in hydrodynamical simulations as well, and confront this with HIC data

on the theoretical side, further understanding of the spacelike (DIS) regime may lead to progress^{22,23}



²¹ <http://www.laine.itp.unibe.ch/dilepton-lpm/>

²² S. Caron-Huot, *O(g) plasma effects in jet quenching*, 0811.1603

²³ M. Cè *et al*, *Deep inelastic scattering on the quark-gluon plasma*, 2012.07522