Heavy-Light Tetraquarks with Lattice QCD

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Experimental and theoretical status of and perspectives for XYZ states
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Motivation (1)

Experimental background

- Experimentally observed states $Z_b(10610)^+$ and $Z_b(10650)^+$
- Mass suggests a bottomonium state $\bar{b}b$ but would be electrically neutral
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Theoretical study

- We study similar but less challenging systems
- Quark content: $\bar{Q}\bar{Q}'qq'$, here: $\bar{b}\bar{b}ud$, $\bar{b}\bar{b}us$, $\bar{b}\bar{c}ud$
- In the limit $m_Q \to \infty$ stable tetraquark was shown

[J. Carlson, L. Heller and J. A. Tjon, Phys. Rev. D **37**, 744 (1988)]
[A. V. Manohar and M. B. Wise, Nucl. Phys. B **399**, 17 (1993)]
[E. J. Eichten and C. Quigg, Phys. Rev. Lett. **119**, no. 20, 202002 (2017)]
[M. Karliner and J. L. Rosner, Phys. Rev. Lett. **119**, no.20, 202001 (2017)]

Born-Oppenheimer study of doubly-heavy tetraquarks:

- i.e. static heavy quarks ($\bar{b}$-quarks)
- Prediction of a bound tetraquark in $\bar{b}\bar{b}ud$ sector with $I(J^P) = 0(1^+)$ and $M_{\bar{b}\bar{b}ud} - (M_B + M_{B^*}) \approx -90\,\mathrm{MeV}$

  [Z. S. Brown and K. Orginos, Phys. Rev. D **86**, 114506 (2012)]
  [P. Bicudo *et al.* [ETMC], Phys. Rev. D **87**, no. 11, 114511 (2013)]
  [P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, M. Wagner, Phys. Rev. D **92**, no. 1, 014507 (2015)]
  [P. Bicudo, J. Scheunert and M. Wagner, Phys. Rev. D **95**, no. 3, 034502 (2017)]

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- Evidence for a $\bar{b}\bar{b}ud$ resonance in the $I(J^P) = 0(1^-)$ channel with $M_{\bar{b}\bar{b}ud} - (M_B + M_B) \approx +20\,\text{MeV},\ \Gamma \approx 100\,\text{MeV}$

  [P. Bicudo, M. Cardoso, A. Peters, M.P. and M. Wagner, Phys. Rev. D **96**, no. 5, 054510 (2017)]

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Searching for doubly-heavy tetraquark bound states in full lattice QCD using **Non-Relativistic QCD**:

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- For $\bar{b}\bar{c}ud$, the predictions are not as clear
  $\rightarrow$ Might be weakly bound or no binding

[A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, Phys. Rev. Lett. **118**, no. 14, 142001 (2017)]
[P. Junnarkar, N. Mathur and M. Padmanath, Phys. Rev. D **99**, no. 3, 034507 (2019)]
[A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, Phys. Rev. D **99**, no. 5, 054505 (2019)]
[L. Leskovec, S. Meinel, M.P. and M. Wagner, Phys. Rev. D **100**, no.1, 014503 (2019)]
[R. J. Hudspith, B. Colquhoun, A. Francis, R. Lewis and K. Maltman, Phys. Rev. D **102**, 114506 (2020)]
[M.P., L. Leskovec, S. Meinel and M. Wagner, arXiv:2009.10538 [hep-lat]].

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- In our study: We apply a more extended operator basis
  → Enables a better treatment of threshold states.
  → *More on next slides*

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Interpolating Operators (1)

Investigated systems and quantum numbers

- $\bar{b}\bar{b}ud$ with $I(J^P) = 0(1^+)$
  $\rightarrow$ Most promising as it is closest to $\bar{Q}\bar{Q}qq$ with $m_Q \rightarrow \infty$
- $\bar{b}\bar{b}us$ with $I(J^P) = \frac{1}{2}(1^+)$
  $\rightarrow$ Slightly less promising as d replaced by s
- $\bar{b}\bar{c}ud$ with $I(J^P) = 0(1^+)$ and $I(J^P) = 0(0^+)$
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For all systems, we use two types of interpolating operators

- Local operators; basically used in all previous studies
- Nonlocal operators; unique compared to all other studies on heavy-light tetraquarks

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  - Four quarks at the same space-time position
  - Jointly projected to zero momentum
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- Expectation:
  - Local operators: good overlap to ground state (stable four-quark)
  - Nonlocal operators: sizable overlap to first excited state (2 meson state)

$\Rightarrow$ Isolate ground state from higher excitations, especially first excited state

Interpolating Operators for $\bar{b}\bar{b}ud$ and $\bar{b}\bar{b}us$

Interpolating Operators for $\bar{b}\bar{b}ud$

	$I(J^P) = 0(1^+)$
relevant thresholds	B^*B, B^*B^* ($\approx +45$ Mev)
local operators	B^*B, B^*B^*, diquark-antidiquark
nonlocal operators	B^*B, B^*B^*

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Interpolating Operators for $\bar{b}\bar{b}us$

	$I(J^P) = \frac{1}{2}(1^+)$
relevant thresholds	B^*B_s, BB_s^* ($\approx$ equal), $B^*B_s^*$ ($\approx +45$ Mev)
local operators	B^*B_s, BB_s^*, $B^*B_s^*$, diquark-antidiquark
nonlocal operators	B^*B_s, BB_s^*, $B^*B_s^*$

Interpolating Operators for $\bar{b}\bar{c}ud$

	$I(J^P) = 0(1^+)$
relevant thresholds	B^*D, $BD^*(\approx +95$ Mev$)$, $B^*D^*(\approx +140$ Mev$)$
local operators	B^*D, BD^*, diquark-antidiquark
nonlocal operators	BD^*, B^*D

	$I(J^P) = 0(0^+)$
relevant thresholds	BD, $B^*D^*(\approx +185$ Mev$)$
local operators	BD, diquark-antidiquark
nonlocal operators	BD

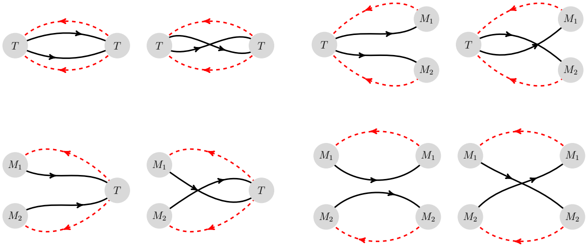
Energy Spectrum for the $\bar{Q}\bar{Q}'qq'$ system

- Due to point-to-all propagators, only non-symmetric correlation matrix available (no scattering operator at source)
- Apply multi-exponential matrix fitting: employable also for non-symmetric matrices

$$C_{jk}(t) \approx \sum_{n=0}^{N-1} Z_j^n Z_k^n \mathrm{e}^{-E_n t},$$

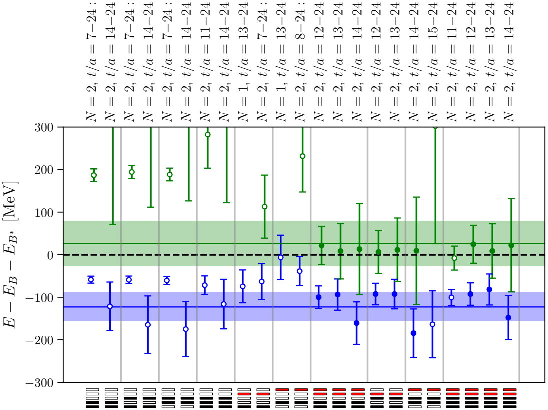
E_n : n-th energy eigenvalue

$Z_j^n = \langle \Omega | \mathcal{O}_j | n \rangle$: overlap factor



Schematic representation of Wick contractions for different correlation matrix elements

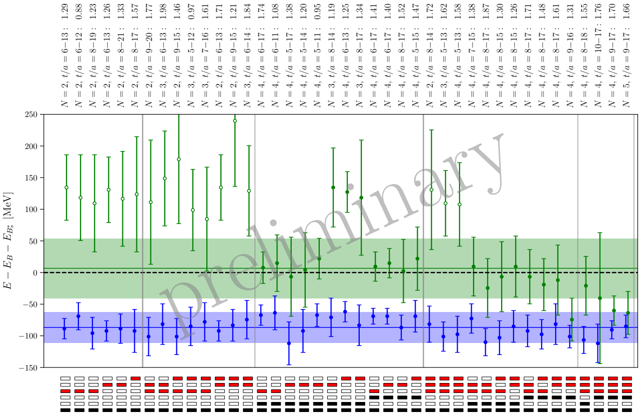
Fit Results for $\bar{b}\bar{b}ud$



Results for the lowest two $\bar{b}\bar{b}ud$ energy levels relative to the BB^* threshold. Black box: local operator included. Red box: scattering operator included.

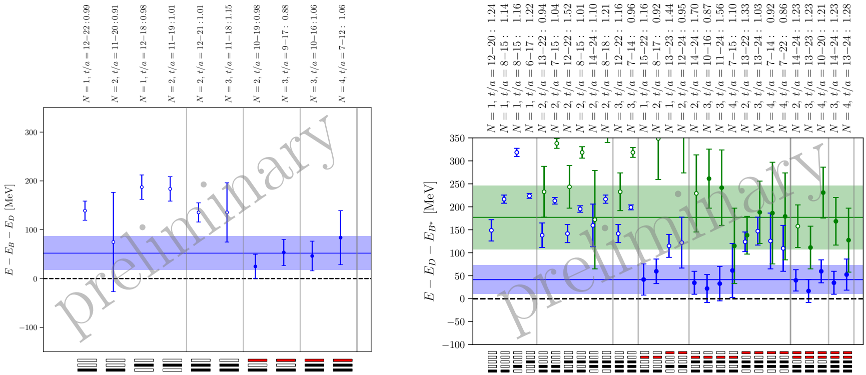
- Found evidence for bound state with $E_{\text{binding}} = -128$ MeV
- First excited state corresponds to threshold

Preliminary Results for $\bar{b}\bar{b}us$



- Found evidence for bound state with $E_{\text{binding}} \approx -80$ MeV
- First excited state corresponds to threshold

Preliminary Results for $\bar{b}\bar{c}ud$



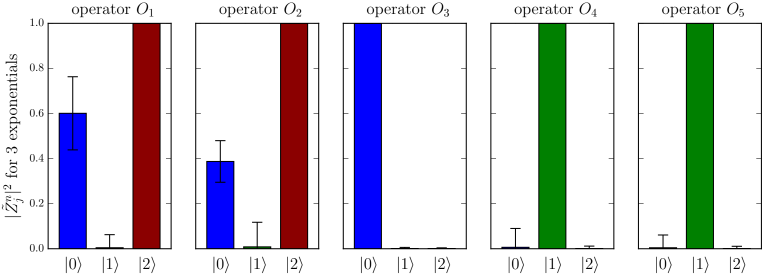
left: $\bar{b}\bar{c}ud$, $J = 0$. **right:** $\bar{b}\bar{c}ud$, $J = 1$.

- No evidence for bound states in $\bar{b}\bar{c}ud$ systems
- Lowest energy level corresponds to threshold

Overlap Factors for $\bar{b}\bar{b}ud$

For fixed j: Z_j^n indicates relative importance of energy eigenstates $|n\rangle$

$$\mathcal{O}_j^\dagger|\Omega\rangle = \sum_{n=0}^{\infty} |n\rangle\langle n|\mathcal{O}_j^\dagger|\Omega\rangle = \sum_{n=0}^{\infty} Z_j^n |n\rangle.$$



The normalized overlap factors $|\tilde{Z}_j^n|^2 = \frac{|Z_j^n|^2}{\max_m(|Z_j^m|^2)}$ as determined on ensemble C005.

Scattering Analysis and Chiral Extrapolation for $\bar{b}\bar{b}ud$

Scattering Analysis

- Relate *finite volume* energy spectrum E_n to *infinite volume* scattering amplitude
- Use Lüscher's formula to determine phase shift and *infinite volume* binding energy
- Confirmation that ground state is stable tetraquark.

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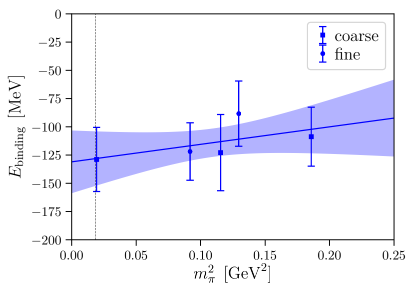
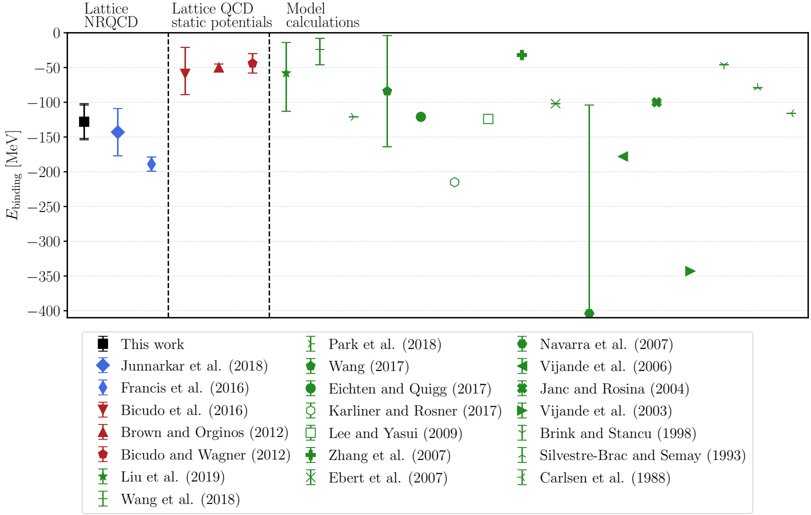
Chiral Extrapolation



Fit of the pion-mass dependence of E_{binding}. The vertical dashed line indicates the physical pion mass.

$$E_{\text{binding}}(m_{\pi,\text{phys}}) = (-128 \pm 24 \pm 10)\,\text{MeV}$$
$$m_{\text{tetraquark}}(m_{\pi,\text{phys}}) = (10476 \pm 24 \pm 10)\,\text{MeV}$$

Comparison of Different Results for $\bar{b}\bar{b}ud$



Comparison of $\bar{b}\bar{b}ud$ tetraquark binding energies with $I(J^P) = 0(1^+)$ (black: this work; blue: lattice NRQCD; red: lattice QCD computations of static $\bar{b}\bar{b}$ potentials and solving the Schrödinger equation; green: effective field theories and potential models).

Summary

- Study bound states in doubly heavy tetraquarks
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- Evidence for bound state in $\bar{b}\bar{b}us$, $I(J^P) = \frac{1}{2}(1^+)$ sector with $E_{\text{binding}} \approx -80\,\text{MeV}$
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Outlook

- Finalize evaluation of $\bar{b}\bar{b}us$ outcomes
- More detailed analysis of threshold states in $\bar{b}\bar{c}ud \rightarrow$ candidates for resonances?

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Thank You for Your Attention!

Lattice Setup

- Use gauge link configuration generated by RBC and UKQCD collaboration
  [Y. Aoki *et al.* [RBC and UKQCD Collaborations], Phys. Rev. D **83**, 074508 (2011)]
  [T. Blum *et al.* [RBC and UKQCD Collaborations],Phys. Rev. D **93**, no. 7, 074505 (2016)]
- $2 + 1$ flavours domain-wall fermions and Iwasaki gauge action
- Five different ensembles which differ in

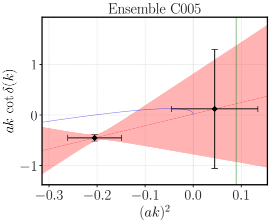
$$\begin{array}{lll} \text{lattice spacing} & a & \approx 0.083\,\text{fm} \ldots 0.114\,\text{fm}, \\ \text{lattice size} & L & \approx 2.65\,\text{fm} \ldots 5.48\,\text{fm}, \\ \text{pion mass} & m_\pi & \approx 139\,\text{MeV} \ldots 431\,\text{MeV} \end{array}$$

  $\Rightarrow$ explore dependence on L, m_π
- Smeared point-to-all propagators for the up and down quarks

Scattering Analysis

- Relate *finite volume* energy spectrum E_n to *infinite volume* scattering amplitude for 2 energy levels in T_1^+ irrep
- Use Lüscher's formula and scattering momenta k_n^2 to determine phase shift
- Apply effective-range-expansion (ERE)

$$k \cot \delta_0(k) = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 + \mathcal{O}(k^4).$$



Plot of the effective-range-expansion for C005. Blue curve: $ak \cot(\delta(k)) + |ak|$. Vertical green line: Inelastic $B^* B^*$ threshold

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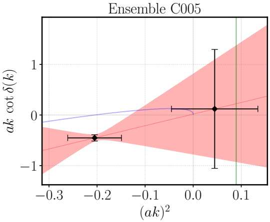

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- Search bound state pole of scattering amplitude below threshold at

$$\cot \delta_0(k_{\mathrm{BS}}) = i, \quad \text{so:} \quad -|k_{\mathrm{BS}}| = \frac{1}{a_0} - \frac{1}{2} r_0 |k_{\mathrm{BS}}|^2$$

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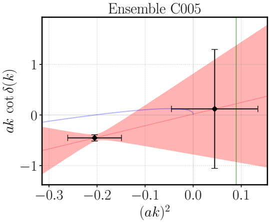

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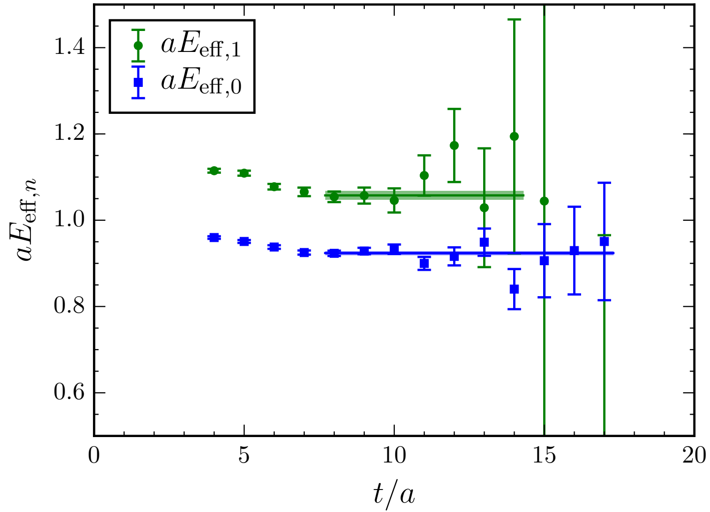
- Results essentially identical to the finite-volume energy levels
- Confirmation that ground state is stable tetraquark.

GEP-results



Effective masses $aE_{\text{eff},n}$ for $n = 0, 1$ as a function of t/a for a 3×3 correlation matrix.