Lattice QCD in the doubly heavy sector

Sinéad M. Ryan, *Trinity College Dublin*

Experimental and theoretical status and perspectives for XYZ states, GSI, April 2021

Outline

- Heavy quarks in lattice QCD
  - General remarks on lattice formulations and huge progress as we have seen at this meeting.
- Taking a step sideways: calculation of the excited and exotic spectrum with relativistic quarks: charm and bottom.
- (Will not mention lattice in medium calculations - although significant progress there and a fruitful exchange of ideas from T=0 spectroscopy.)
- Summary and outlook

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Many details and topics omitted for time constraints - APOLOGIES!

Doubly-heavy sector - a fertile hunting ground!

- The XYZ states are an emergent property of the strong dynamics in QCD.
- In many cases, their manifest exotic nature is outside the scope of potential models.
- This meeting: have seen a range of lattice results with theoretical tools honed to explain these emergent phenomena. While XYZs were first seen and are plentiful in the charm sector - much of the focus here on *b* sector.
- The rich structure of separated scales in quarkonia makes it a versatile probe of QCD *and* allows for a complementary theoretical approaches.

Heavy quarks & lattice QCD

- An old story. Much progress driven by CKM phenomenology in the early days.
- On a lattice heavy-quark discretisation effects $\mathcal{O}(am_b)$ can be large - even for current lattice spacings. When $ma \ll 1$ control and quantify cutoff effects with Symanzik improvement. When $am \geq 1$ targeted approaches are required.

heavy-quark discretisation	EFT tools	range of m_Q
extrapolating (light) quarks	Symanzik	m_c (m_b)
static quarks (+insertions)	HQET	m_b (m_c)
non-relativistic quarks	NRQCD	m_b (m_c)
Fermilab	Synthesis: extending Symanzik w HQET & NRQCD	m_c, m_b

Adapted from A. Kronfeld's "Heavy quarks and Lattice QCD" [NPB Prof Suppl 129 (2004)]

- An interplay of methods (with pros and cons of course!)
- Useful tests and confirmation by comparing results from different methods - learned a lot about methods and physics doing this in EW scenarios.
- To understand heavy-quark mass-dependent physics (from charm to bottom) restricts the methods available.

Heavy hadrons from lattice QCD @ this meeting

Miani's *new wave* of XYZ spectroscopy

- Effective field theories for regular quarkonia and exotics - N. Brambilla (lattice input)
- Nonrelativistic effective field theories for exotic heavy hadrons - J. Tarrús (lattice input)
- Heavy fourquarks and dibaryon states - N. Mathur
- Heavy fourquark states from lattice QCD - S. Prelovsek
- Importance of meson-meson and of diquark-antidiquark creation operators for a b-bar b-bar u d tetraquar - M. Wagner
- Bottomonium resonances in the Born-Oppenjeimer approximation using static potentials from lattice QCD - L. Mueller
- Hybrid static potentials at small quark-antiquark separations - C. Riehl
- Heavy-light tetraquarks with lattice QCD - M. Pflaumer

Motivation - for another approach

- The renaissance in heavy quark spectroscopy continues to flourish with many experimental results still expected.
- New and unexpected results in charmonium (and open-charm) have prompted extensive lattice studies.
- Open question whether a similarly rich spectrum is predicted/discovered in bottomonium.
  - robust relativistic charm simulations motivate an exploratory study of relativistic quarkonia.
  - towards a rigorous treatment of heavy and heavy-light resonances?
  - complemented by NRQCD, HQET insights.

Heavy quarks & anisotropic lattices:

- Lattice heavy quarks, in particular b quarks bring potentially large discretisation errors which must be removed or controlled.
- Solutions include EFTs or extremely fine lattices such that $am_Q < 1$.
- Anisotropic lattices, $a_t \ll a_s$, already proven very useful for charm - explore their effectiveness at bottom quark masses.

Heavy quarks and anisotropic lattices

- Not a new story. Hashimoto, Onogi, Kronfeld et al investigated the practicalities in early 2000s. [PRD64 114503(2001); PRD64 074501 (2001); PRD66 014509 (2002)].
- Investigate discretisation effects via dispersion relations

$$(a_t E)^2 = (a_t M)^2 + \left(\frac{1}{\xi_M}\right)^2 (a_s p)^2$$

- Strategy: Mass-dependent parameter tuning, determine dispersion relations in heavy and heavy-light mesons.
- Explore $\mathcal{O}((a_s p)^2)$ discretisation effects in heavy sector [NPB (proc suppl) 47 (1996); NPB (proc suppl) 53 (1997)] and $M_1 \stackrel{?}{=} M_2$.
- Noting improvements on earlier studies include stout smearing of spatial links and mass-dependent anisotropy tuning.

Lattice details

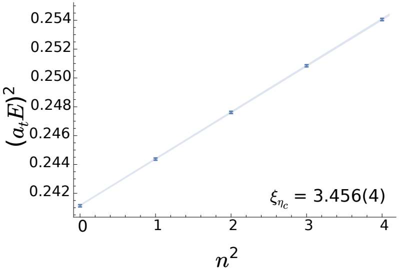
- Dynamical, anisotropic lattices with stout-smeared spatial links.
- $\xi = a_s/a_t = 3.5$; $a_s \sim 0.1227$ fm; $a_t^{-1}(m_\Omega)$, $L \sim 2-4$ fm; $m_\pi L \sim 4-6$
- Charmonium on $24^3 \times 128$ and $32^3 \times 256$ volume; Bottomonium on $20^3 \times 128$ and $24^3 \times 128$.
- $m_\pi \sim 400, 240$ MeV
- Distillation for quark propagation. [Phys.Rev.D 80 (2009) 054506]

Related work:

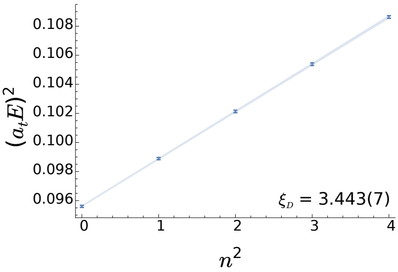
- Charmonium: JHEP1207 (2012) 126, JHEP 1612 (2016) 089.
- Open charm: JHEP 05 (2013) 021.
- Light quark mass dependence: JHEP 12 (2016) 089.
- Bottomonium JHEP02 (2021) 214.

The lattice dispersion relation - a bellwether for discretisation effects

- Energies extracted in the fine temporal direction. The dispersion relations elucidate spatial discretisation effects.
- Fermion action: tune m_q, ξ - here for the η_c.
- **Everything else a prediction.**



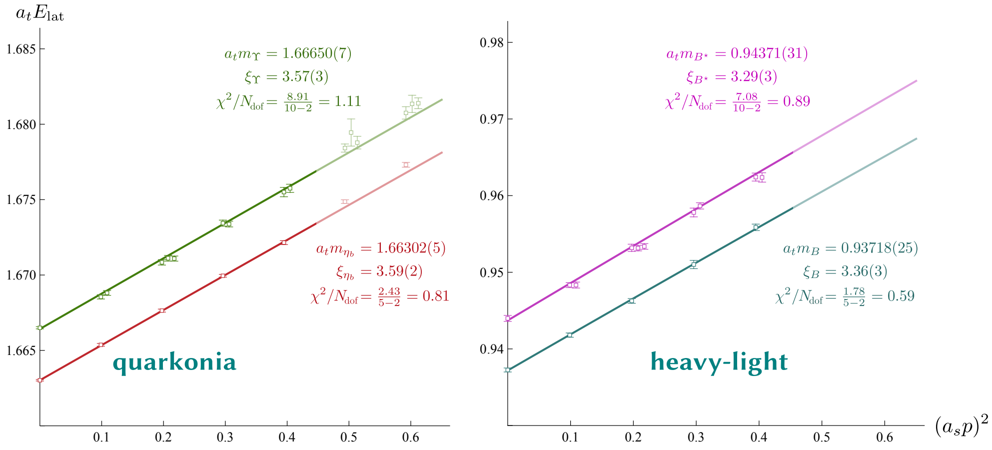
Charmonium



D meson

The lattice dispersion relation - now for b quarks

- Tune $M^{latt}_{\eta_b} = M^{expt}_{\eta_b}$ *and* ξ (from slope of dispersion) for η_b (red line).



- As in charm case, no deviation from relativistic dispersion relations for Υ, B, $B^\star$; agreement in measured ξ (dispersion slope).

The HadSpec recipe for (meson) spectroscopy

- Construct a basis of local and non-local operators $\bar{\Psi}(x)\Gamma D_i D_j \ldots \Psi(x)$ from *distilled* fields [PRD80 (2009) 054506].
- Build a correlation matrix of two-point functions

$$C_{ij} = \langle 0|\mathcal{O}_i \mathcal{O}_j^\dagger|0\rangle = \sum_n \frac{Z_i^n Z_j^{n\dagger}}{2E_n} e^{-E_n t}$$

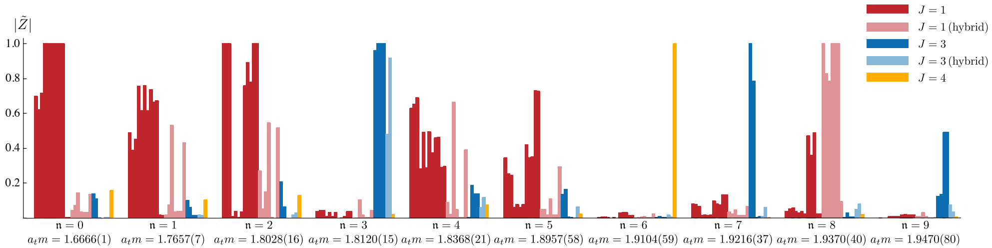
- Ground state mass can be determined from fits to $e^{-E_n t}$
- Better approach yielding ground & excited states: solve generalised eigenvalue problem $C_{ij}(t)v_j^{(n)} = \lambda^{(n)}(t)C_{ij}(t_0)v_j^{(n)}$

- Eigenvalues: $\lambda^{(n)}(t) \sim e^{-E_n t}\left[1 + O(e^{-\Delta E t})\right]$ - principal correlators
- Eigenvectors: related to overlap of operators on states of interest $Z_i^{(n)} = \sqrt{2E_n} e^{E_n t_0/2} v_j^{(n)\dagger} C_{ji}(t_0)$

- operators of definite J^{PC} are subduced into relevant lattice irrep
- a subduced irrep carries a "memory" of continuum spin J from which it was subdduced - it **overlaps** predominantly with states of this J.

J	**0**	**1**	**2**	**3**	**4**
A_1	✓				✓
A_2				✓	
E			✓		✓
T_1		✓		✓	✓
T_2			✓	✓	✓

- Use overlaps, $Z = \langle 0|\Phi|k\rangle$, to identify continuum spins
- For $J \geq 2$, look for agreement between irreps
- Example (υ): T_1^{--} irrep, with **Spin 1**, **Spin 3** and **Spin 4**.



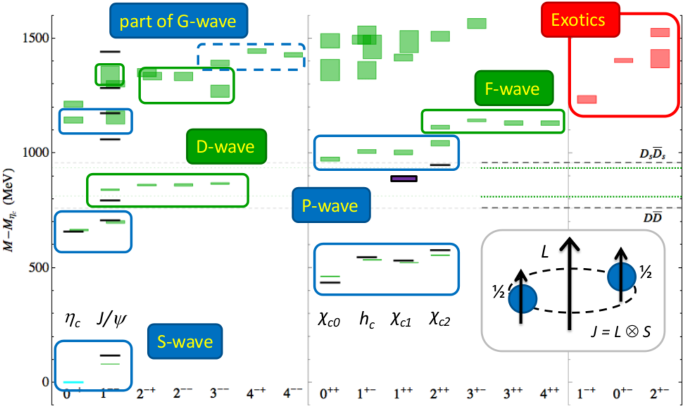
The spectrum of $c\bar{c}$ mesons

Single hadron states: charmonium

Precision calculation of high spin ($J \geq 2$) and exotic states

Caveat Emptor

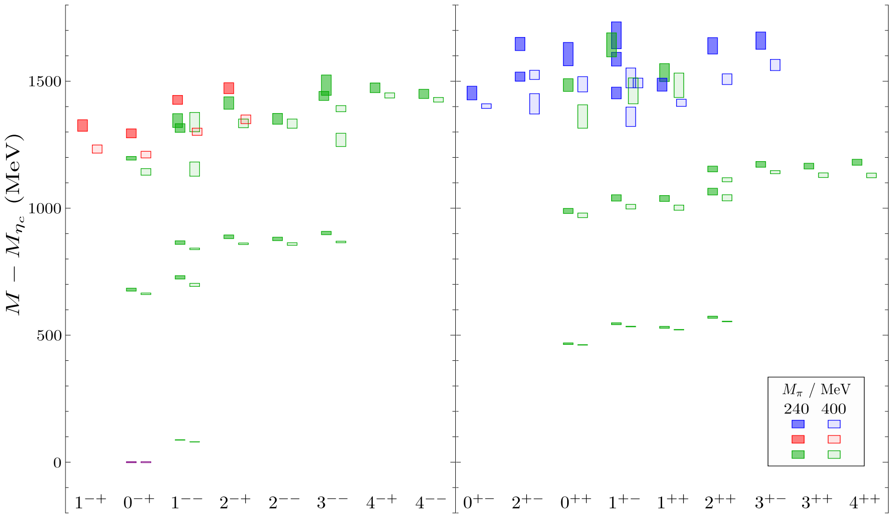
- Only single-hadron operators
- Physics of multi-hadron states appears to need relevant operators
- No continuum extrapolation
- $m_\pi = 391 MeV$
- No disconnected contributions to η_c



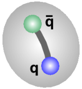
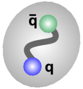
Hadron Spectrum Collaboration, JHEP1207 (2012) 126

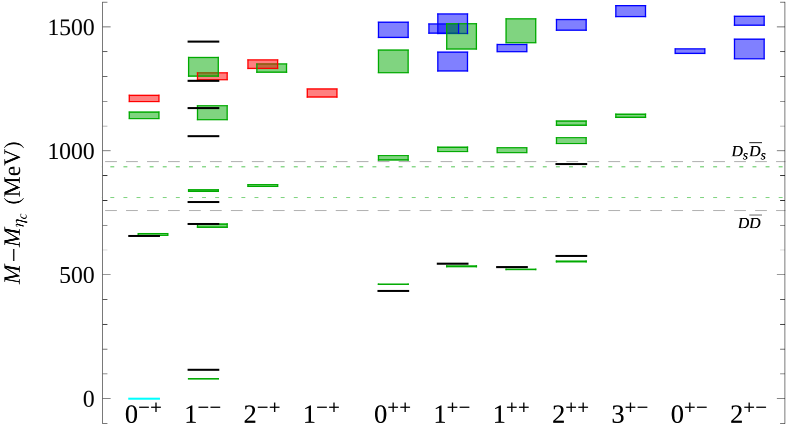
Repeated With $m_\pi = 240$ MeV with no change to pattern of states. JHEP 1612 (2016) 089

Charmonium with $m_\pi \sim 240$ MeV



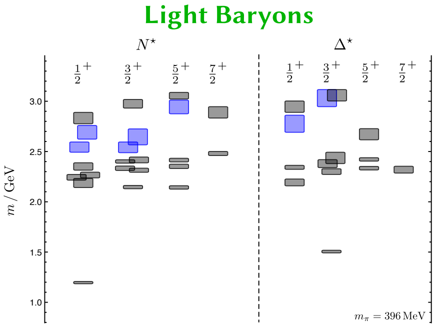
No change to the pattern, structure of states. Also for $D_{(s)}$. [JHEP 1612 (2016) 089]

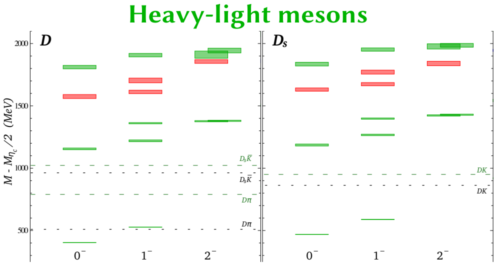
Hybrids:   large overlap with operators $\mathcal{O} \sim F_{\mu\nu}$



Lightest hybrid supermultiplet and excited hybrid supermultiplet same pattern and scale in meson and baryon, heavy and light[*HadSpec*:1106.5515] sectors. Favours bag model and P-wave quasiparticle gluon ($q\bar{q}$ in S-wave coupled to 1^{+-} chromomag gluonic excitation).

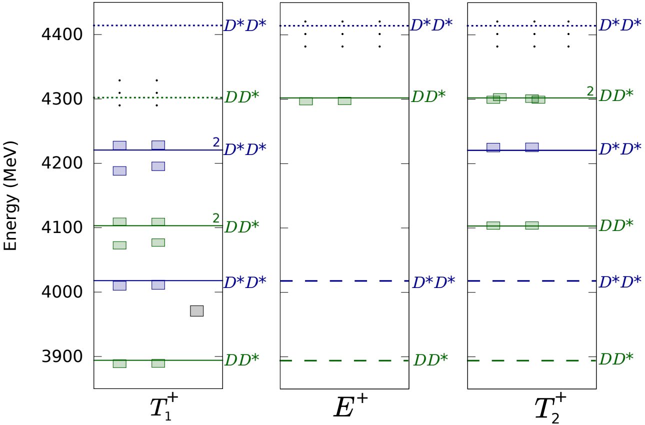
Pattern repeated in mesons and baryons (heavy and light)





Hadron Spectrum Collaboration

Charm tetraquarks: hidden charm & doubly charm



- Extensive operator basis: meson, meson-meson and tetraquarks in I=0 (shown), I=1.
- No clear signs of bound states or narrow resonances at $m_\pi = 391$ MeV in I=0,1.
- A mass-dependence study could be very fruitful: heavier heavy quarks and/or lighter light quarks. "Straightforward" when all quarks in same framework.

[Cheung et al (HadSpec) JHEP 11 (2017) 033]

A first relativistic lattice spectrum of $b\bar{b}$ mesons

- Extensive operator basis in all lattice irreps
- Variational analysis and spin identification as in charmonium

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Caveat Emptor

- Spectra determined from single-hadron operators. Physics of multi-hadron states appears to need relevant operators

- Finite lattice spacing

- Relatively heavy (∼400MeV) pions

Volume dependence & rotation breaking effects in the spectrum?

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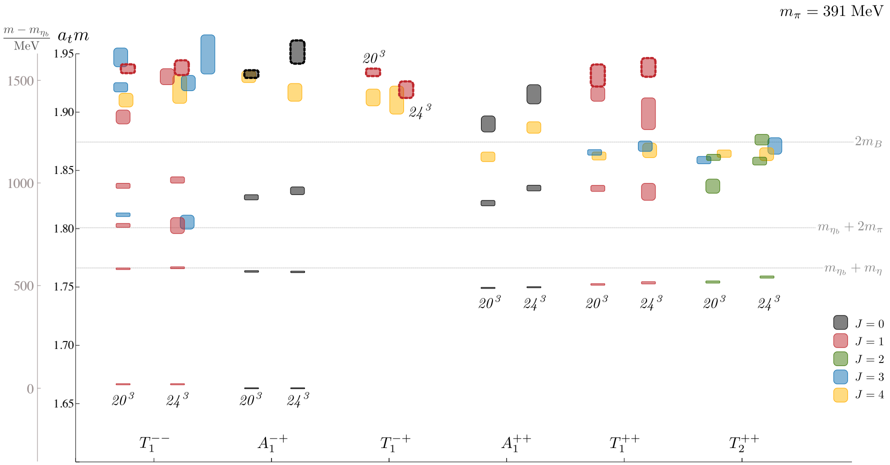
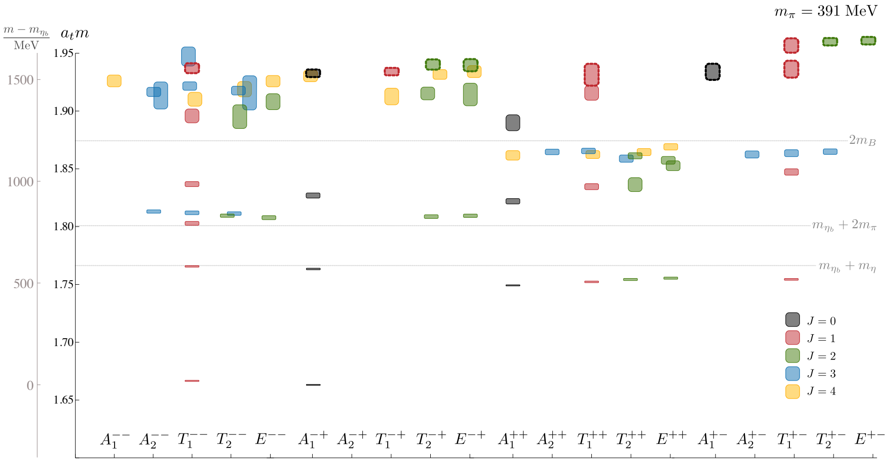


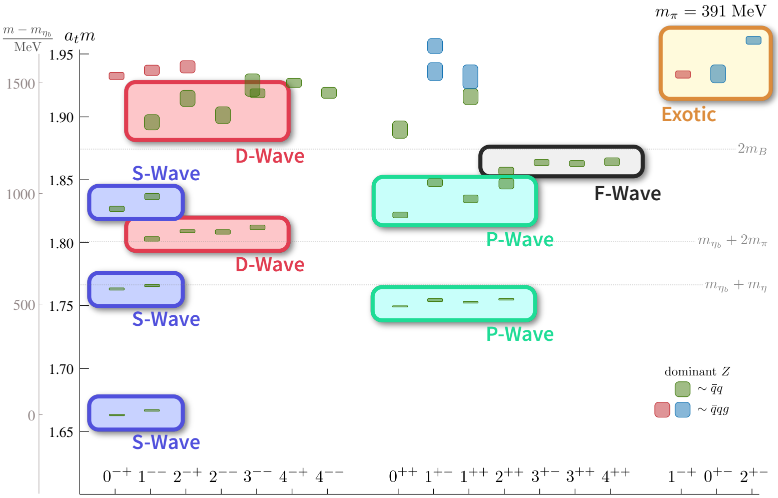
- Only mild volume dependence.

Lattice spectrum of bottomium $(M - M_{\eta_b})$

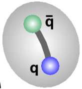
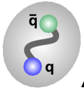


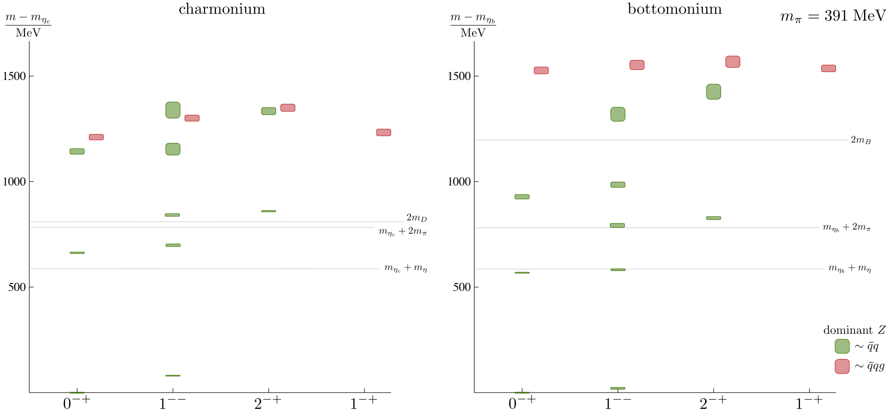
- Broadly similar to pattern of states in charmonium
- No significant rotation breaking.
- Now use overlaps to make continuum spin identifications

Bottomonium: the J^{PC} spectrum



- Used overlaps to make continuum spin identifications
- Includes exotic channels: 0^{+-}, 1^{-+}, 2^{+-} and hybrid states.
- Improvements possible with methods established

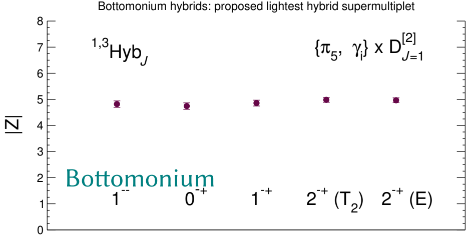
Hybrids in charmonium   and bottomonium

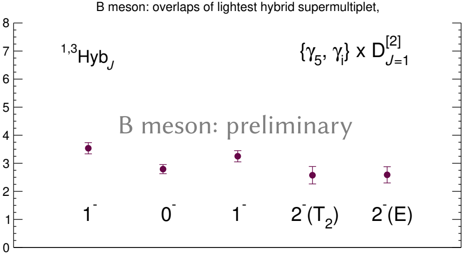


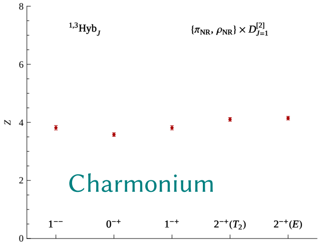
- Spin exotic and non-exotic hybrids determined
- Similar result in charmonium and in agreement with Brambilla et al PRD101 (2020).

Hybrids

Expect a large overlap with operators $\mathcal{O} \sim F_{\mu\nu}$. Used to identify hybrid multiplets.







- Similar pattern & structure of hybrids seen in light, open-charm and charm (and baryons).
- Energy scale $\sim$ 1.5GeV - as previously.

Summary & outlook

- Unexpected & puzzling experimental results are motivating lattice calculations in the doubly heavy sector.
- Full control/removal of discretisation effects and approximations in lattice formulations for heavy quark quantities remains a challenge.
- For charm and bottom, following the by-now well-established HadSpec Collaboration recipe a large basis of operators can be constructed, enabled by distillation.
  - An exploratory study of relativistic quarkonia following a mass-dependent anisotropy tuning is encouraging.
  - Extensive spectra of excited and exotic states are determined, including a first look at charm tetraquarks.
  - Evidence of a hybrid supermultiplet at a similar energy scale for all quarks masses.
- The B spectrum including the B_c underway.
- The studies can be extended to larger volumes and lighter pion masses (all available).
- Paves the way for spectroscopy and decays of b-quark hadrons and studies of heavy quark mass dependence.

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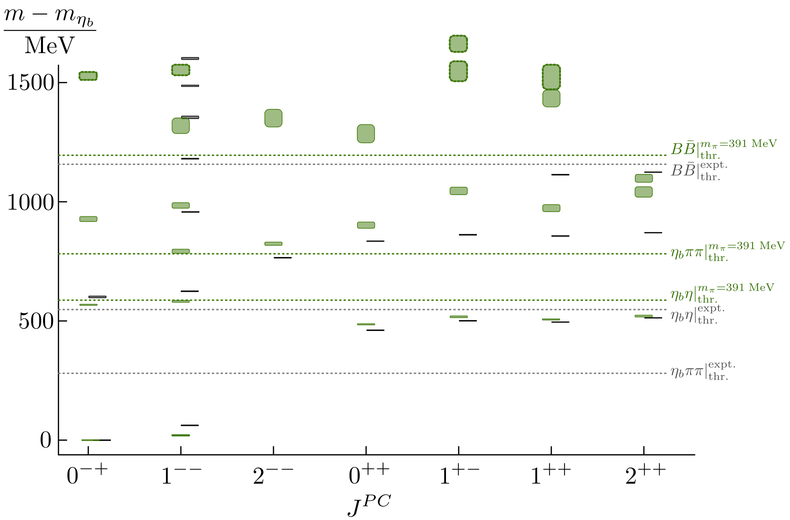
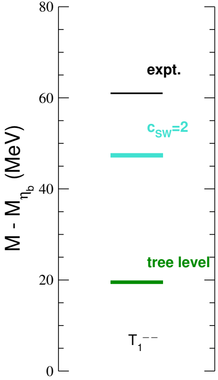
Thanks for listening!

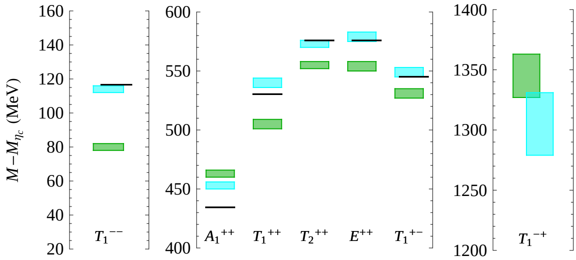
Looking to experiment

- Emphasise the pattern of states in the spectrum is reliable and broadly compatible with known experimental states.



Backup: (brief) investigation of the effect of c_{SW}

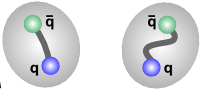


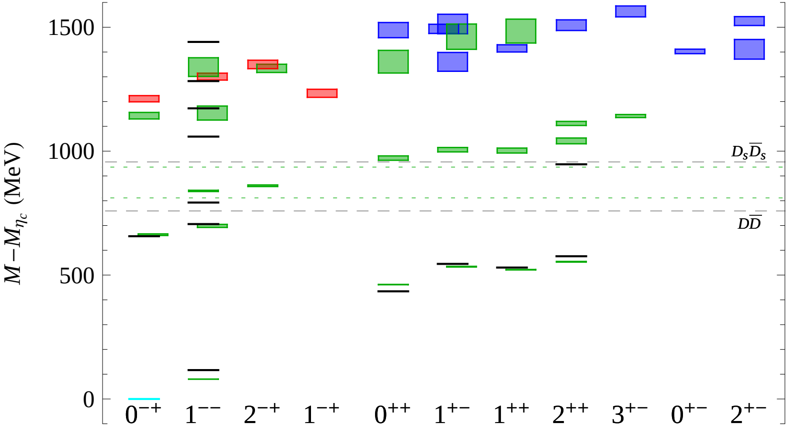


Note 1: different statistics for the tree-level and $c_{SW} = 2$ results in bottomonium.

Note 2: Same "improved" value of $c_{SW} = 2$ used for charm and bottom.

Backup: (exotic) hybrid meson in charmonium





- Spin exotic and non-exotic hybrids determined.
- Same pattern & energy scale in mesons and baryons, light and heavy.

[Hadron Spectrum Collaboration, 2012]