Hybrid static potentials at small quark-antiquark separations

Carolin Riehl
in collaboration with Marc Wagner

Experimental and theoretical status of and perspectives for XYZ states
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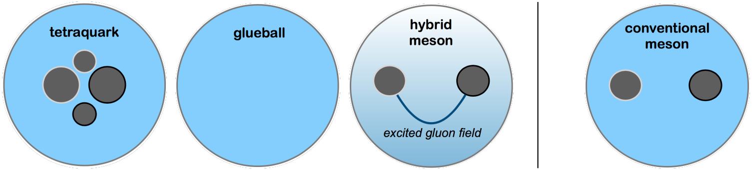
Non-quark model mesons



- active field of research, both theoretically and experimentally [1] [2] [3] [4] [5] [6]

Heavy hybrid meson

Heavy quark and antiquark surrounded by an excited gluon field → *hybrid static potential*

[1] E. Braaten, C. Langmack and D. H. Smith, Phys. Rev. Lett. 112 (2014), 222001 [arXiv:1401.7351 [hep-ph]]

[2] C. A. Meyer and E. S. Swanson, Prog. Part. Nucl. Phys. 82 (2015), 21-58 [arXiv:1502.07276 [hep-ph]]

[3] E. S. Swanson, AIP Conf. Proc. 1735 (2016) no.1, 020013 [arXiv:1512.04853 [hep-ph]]

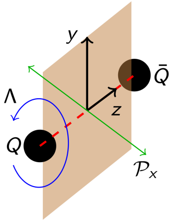
[4] S. L. Olsen, T. Skwarnicki and D. Zieminska, Rev. Mod. Phys. 90 (2018) no.1, 015003 [arXiv:1708.04012 [hep-ph]]

[5] N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C. P. Shen, C. E. Thomas, A. Vairo and C. Z. Yuan, Phys. Rept. 873 (2020), 1-154 [arXiv:1907.07583 [hep-ex]]

[6] ...

Hybrid static potentials

= gluonic static energy between quark and antiquark in a distance r



Quantum numbers Λ_η^ϵ $\quad e.g.\ \Sigma_g^+, \Pi_u, \Sigma_u^-$

- $\Lambda = \Sigma, \Pi, ..$ orbital angular momentum along quark separation axis L_z
- $\eta = u, g$ combination of parity and charge conjugation $P \circ C$
- $\epsilon = +, -$ spatial inversion P_x

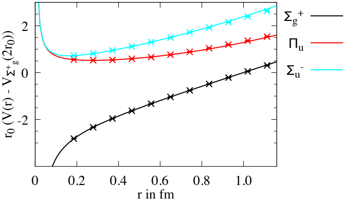
- excited gluon field contributes to the quantum numbers of the meson

$\Rightarrow$ exotic quantum numbers J^{PC} possible

$$P = (-1)^{L+1+\Lambda}$$
$$C = \eta\epsilon(-1)^{L+S+\Lambda}$$

Λ_η^ϵ	L	J^{PC} $S=0$	J^{PC} $S=1$
Σ_u^-	0	0^{++}	1^{+-}
	1	1^{--}	$\{0, \mathbf{1}, 2\}^{-+}$
	2	2^{++}	$\{1, \mathbf{2}, 3\}^{+-}$
Π_u^-	1	1^{++}	$\{\mathbf{0}, 1, \mathbf{2}\}^{+-}$
	2	2^{--}	$\{\mathbf{1}, 2, \mathbf{3}\}^{-+}$
Π_u^+	1	1^{--}	$\{0, \mathbf{1}, 2\}^{-+}$
	2	2^{++}	$\{1, \mathbf{2}, 3\}^{+-}$

Hybrid static potentials



- Computation of spectra of $\bar{b}b$ and $\bar{c}c$ hybrid mesons in the Born-Oppenheimer approximation [7] [8] [9]

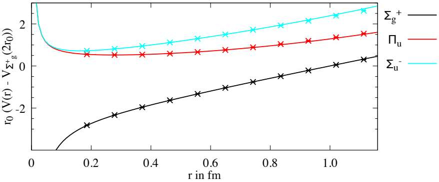
[7] K. J. Juge, J. Kuti and C. J. Morningstar, Nucl. Phys. Proc. Suppl. 63, 326 (1998) [hep-lat/9709131]

[8] E. Braaten, C. Langmack and D. H. Smith, Phys. Rev. D 90, 014044 (2014) [arXiv:1402.0438 [hep-ph]]

[9] S. Capitani, O. Philipsen, C. Reisinger, C. Riehl and M. Wagner, Phys. Rev. D 99, no. 3, 034502 (2019) [arXiv:1811.11046 [hep-lat]]

Heavy hybrid meson masses from hybrid static potentials

- Compute hybrid static potentials in lattice gauge theory
- Parametrization of discrete lattice data $\rightarrow V_{\Lambda_\eta^\epsilon}(r)$



[10]

[10] S. Capitani, O. Philipsen, C. Reisinger, C. Riehl and M. Wagner, Phys. Rev. D 99, no. 3, 034502 (2019) [arXiv:1811.11046 [hep-lat]]

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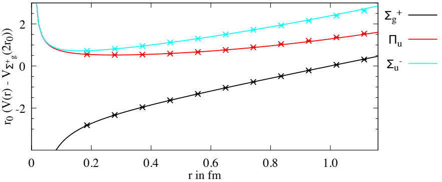

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- Solution of Schrödinger equation for heavy quarks in hybrid static potential

$$\left(\frac{-1}{2\mu}\frac{d^2}{dr^2} + \frac{\langle \mathbf{L}_{q\bar{q}}^2 \rangle}{2\mu r^2} + V_m(r) - E_n^{(m)}\right) r\psi_n^{(m)}(\vec{r}) = 0$$

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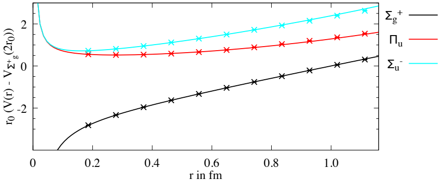

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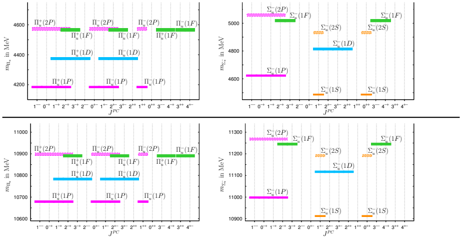
$\Rightarrow$ Heavy hybrid meson spectrum for $b\bar{b}$ hybrid mesons and $c\bar{c}$ hybrid mesons



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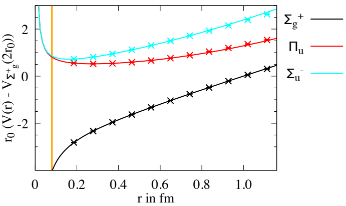
Hybrid static potentials



- Computation of spectra of $\bar{b}b$ and $\bar{c}c$ hybrid mesons in the Born-Oppenheimer approximation [11] [12] [13]
- Matching coefficients for potential Non-Relativistic QCD [14]

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[14] M. Berwein, N. Brambilla, J. Tarrus Castella and A. Vairo, Phys. Rev. D 92, 114019 (2015) [arXiv:1510.04299 [hep-ph]]

[15] K. J. Juge, J. Kuti, and C. Morningstar, Phys. Rev. Lett., 90, 161601 (2003), arXiv:hep-lat/0207004 [hep-lat].

[16] G. S. Bali and A. Pineda, Phys. Rev., D69, 094001 (2004), arXiv:hep-ph/0310130 [hep-ph]

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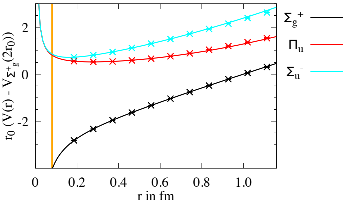

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$\rightarrow$ so far based on lattice data at $r \geq 0.16\,\text{fm}$ [10] [15] [16]

$\Rightarrow$ New $SU(3)$ lattice results at r as small as $0.08\,\text{fm}$

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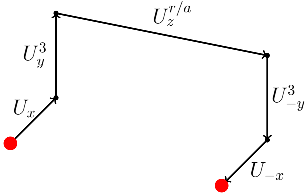
Simulations at small lattice spacings

β	6.00	6.284	6.451	6.594
a [17]	0.093 fm	0.060 fm	0.048 fm	0.040 fm

- $SU(3)$ gauge field configurations generated with a Monte Carlo heatbath algorithm and standard Wilson plaquette action
- isotropic lattices with volume $T \times L^3 \approx (2.4\,\text{fm}) \times (1.2\,\text{fm})^3$

- optimized hybrid static potential creation operators [18]
- *APE*-smearing of spatial links
  - N_{APE} optimized for each lattice spacing
- Multilevel algorithm [19]



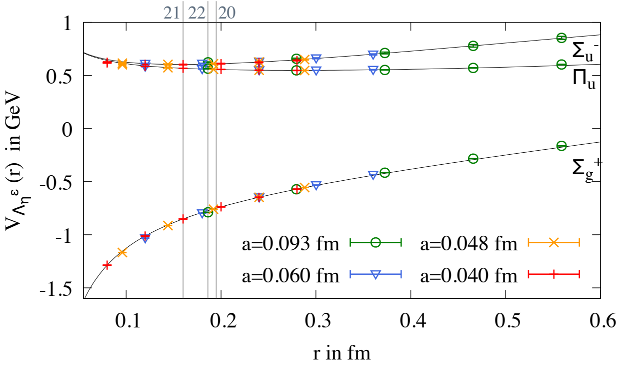
[17] S. Necco and R. Sommer, Nucl. Phys. B 622 (2002) 328–346, arXiv:hep-lat/0108008.

[18] S. Capitani, O. Philipsen, C. Reisinger, C. Riehl and M. Wagner, Phys. Rev. D 99, no. 3, 034502 (2019) [arXiv:1811.11046 [hep-lat]]

[19] M. Lüscher and P. Weisz, JHEP 09 (2001), 010 [arXiv:hep-lat/0108014 [hep-lat]

Lattice results for hybrid static potentials

- new $SU(3)$ lattice data at separations as small as $r = 0.08\,\text{fm}$
- previous lattice data at separations $r \geq 0.16\,\text{fm}$ [20] [21] [22]



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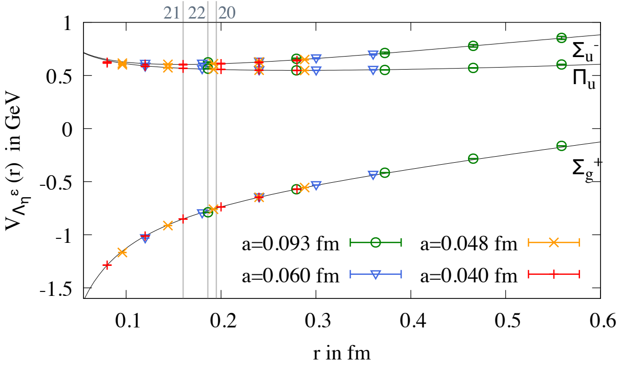

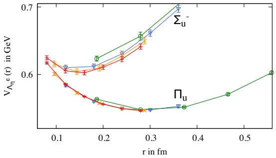

$$V_{\Lambda_\eta^\epsilon}(r) = \frac{a}{r} + b + \mathcal{O}\left(r^2\right)$$ [23]

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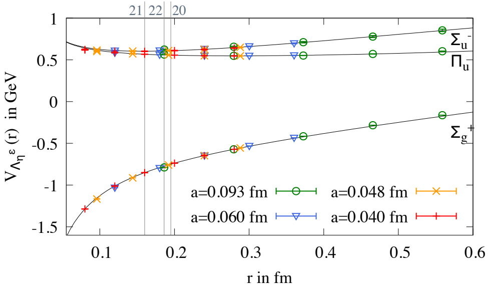

- Difficulties when $a \to 0$
  - Small lattice volume
  - Topology freezing and increase of autocorrelations

[20] K. J. Juge, J. Kuti and C. J. Morningstar, Nucl. Phys. Proc. Suppl. 63, 326 (1998) [hep-lat/9709131]

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Finite volume effects

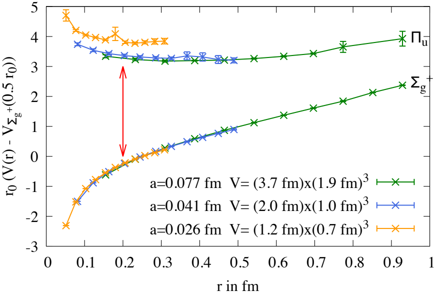


Figure: Static potential data for Σ_g^+ and Π_u at small a and small V.

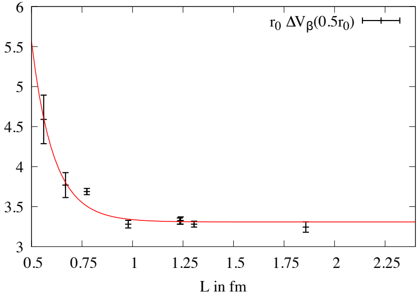


Figure: Potential difference $(V_{\Pi_u} - V_{\Sigma_g^+})$ as a function of L.

$\rightarrow$ $(V_{\Pi_u} - V_{\Sigma_g^+})$ grows with decreasing L^3

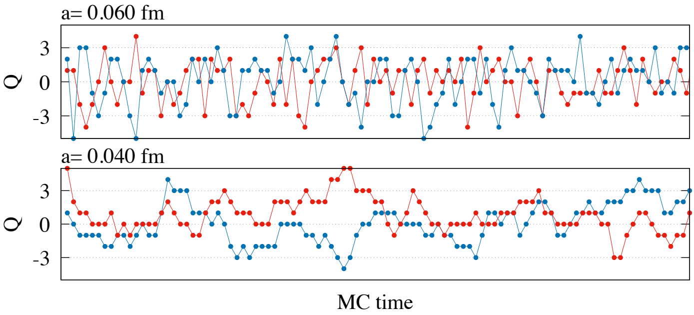
Topological freezing

Topological freezing

Rare tunneling between topological sectors when approaching continuum (lattice spacing $a \to 0$)

Topological freezing

- expected when $a \to 0$
- Topological charge computed with simple clover-leaf discretization



- topological charge distribution is sampled correctly by the algorithm at all lattice spacings

$\rightarrow$ no topological freezing

Glueball decay of hybrid static potentials

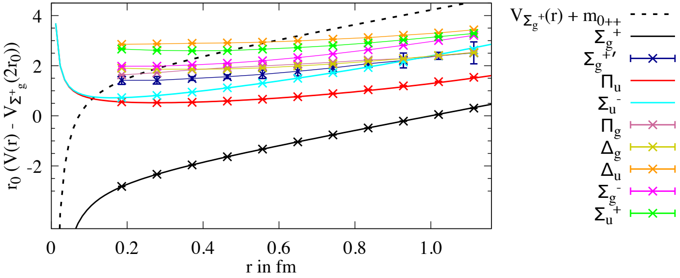


Figure: Threshold energy for a decay into the lightest glueball 0^{++} and hybrid static potentials.

Λ_η^ϵ	Π_u	Π_g	Δ_g	Δ_u	$\Sigma_g^{+\prime}$	Σ_u^+	Σ_u^-	Σ_g^-
r_{crit}/r_0	0.2	0.5	0.5	1.1	0.4	0.9	0.1	0.25

23

[23] S. Capitani, O. Philipsen, C. Reisinger, C. Riehl and M. Wagner, Phys. Rev. D 99, no. 3, 034502 (2019) [arXiv:1811.11046 [hep-lat]]

Possible decay of hybrid static potentials at small separations

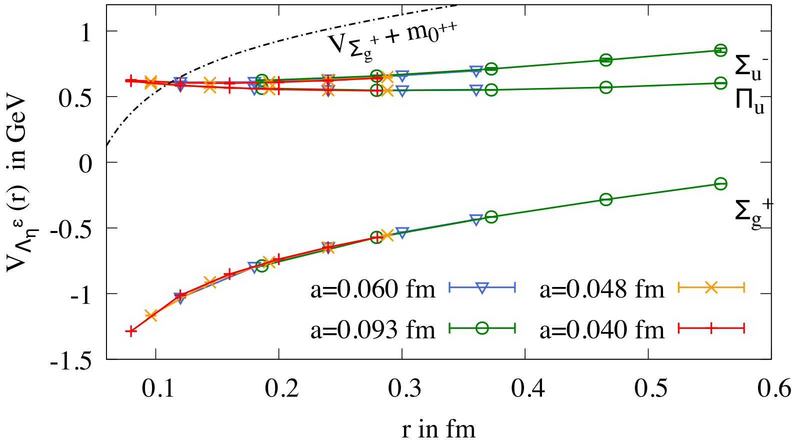


Figure: New $SU(3)$ lattice data and threshold energy for a decay into the lightest glueball 0^{++}.

Decay $\Lambda_\eta^\epsilon \to \Sigma_g^+ +$ glueball

Glueball operator with quantum numbers $L_{z\,\eta}^{\ \epsilon}$

$$\mathcal{O}_{L_{z\,\eta}^{\ \epsilon}} = \frac{1}{2}(1+\epsilon\mathcal{P}_x)\int \mathrm{d}^3r\ e^{iL_z\varphi} f(r,z)\ \mathcal{O}_{\text{glueball}}(r,\varphi,z)$$

L_z	f	$\eta = (-1)^{L_z+f}$	ϵ	$L_{z\,\eta}^{\ \epsilon}$
0	1	-1	$+1$	Σ_u^+
0	2	$+1$	$+1$	Σ_g^+
1	1	$+1$	$+1$	Π_g^+
1	1	$+1$	-1	Π_g^-
1	2	-1	$+1$	Π_u^+
1	2	-1	-1	Π_u^-
⋮	⋮	⋮	⋮	⋮

Table: Possible quantum numbers $L_{z\,\eta}^{\ \epsilon}$ with glueball 0^{++}.

Decay $\Lambda_\eta^\epsilon \rightarrow \Sigma_g^+ +$ glueball

Glueball operator with quantum numbers ${L_z}_\eta^\epsilon$

$$\mathcal{O}_{{L_z}_\eta^\epsilon} = \frac{1}{2}(1 + \epsilon \mathcal{P}_x) \int \mathrm{d}^3 r \; e^{i L_z \varphi} f(r, z) \; \mathcal{O}_{\text{glueball}}(r, \varphi, z)$$

L_z	f	$\eta = (-1)^{L_z + f}$	ϵ	${L_z}_\eta^\epsilon$
0	1	-1	$+1$	Σ_u^+
0	2	$+1$	$+1$	Σ_g^+
1	1	$+1$	$+1$	Π_g^+
1	1	$+1$	-1	Π_g^-
1	2	-1	$+1$	Π_u^+
1	2	-1	-1	Π_u^-
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Table: Possible quantum numbers ${L_z}_\eta^\epsilon$ with glueball 0^{++}.

Not possible with 0^{++}-glueball

$\Rightarrow L_z = 0$ and $\epsilon = -$

$\rightarrow \Sigma_g^-, \Sigma_u^-$

reason:

- $\mathcal{O}_{0^{++}} \xrightarrow{\mathcal{P}_x} \mathcal{O}_{0^{++}}$

Summary & Outlook

Summary

- $SU(3)$ Lattice results for ordinary and hybrid static potentials Σ_g^+, Π_u and Σ_u^- at four small lattice spacings $a = 0.093\,\text{fm} \ldots 0.040\,\text{fm}$
- Excluded systematic errors from topological freezing and finite volume effects
- Glueball decay at short separations
  - Decay of Σ_g^- and Σ_u^- into $0^{++} + \Sigma_g^+$ not allowed

Outlook

- Higher hybrid static potentials at small separations
- Computation of heavy hybrid meson spectrum based on new lattice results at small r

Thank you!